SPICE SIMULATION OF MEMRISTOR AND MEMRISTOR BASED CIRCUITS

A Dissertation Submitted
In partial fulfillment of the academic requirements for
the award of

MASTER OF TECHNOLOGY
IN
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DECLARATION

I, **ASHOKREDDY KOTAMREDDY** hereby declare that the work embodied in this dissertation entitled **“SPICE SIMULATION OF MEMRISTOR AND MEMRISTOR BASED CIRCUITS”** submitted to the University Of Hyderabad, Hyderabad, for partial fulfillment of the degree of **M.Tech in Integrated Circuit Technology** has been carried out by me under the supervision of **Dr. M. GHANASHYAM KRISHNA**, School of Physics, University Of Hyderabad. To the best of my knowledge, this work has not been submitted for any other degree in any University.

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CERTIFICATE

This is to certify that the project work “**SPICE SIMULATION OF MEMRISTOR AND MEMRISTOR BASED CIRCUITS**” carried out by **ASHOKREDDY KOTAMREDDY** bearing the Reg.No.09PIMT03 under my guidance in partial fulfillment of the requirements for the award of **Master of Technology in Integrated Circuit Technology** in University of Hyderabad. The matter embodied in this dissertation has not been submitted in any other University for the award of any other degree.

Dr. M. GHANASHYAM KRISHNA,
Project Supervisor
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Dean
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To

My family and Teachers

ACKNOWLEDGEMENT

I would like to express my sincere gratitude to my project supervisor **Dr. M. GHANASHYAM KRISHNA** for his constant support, encouragement to carry out this work. He guided me with his valuable suggestions and took keen personal interest throughout the progress of my course of research during the project. I would also like to thank him for suggesting such an interesting topic for my research.

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ASHOKREDDY KOTAMREDDY

Abstract

In the past, electronic circuit theory has revolved around three fundamental components: the resistor, the capacitor, and the inductor. These elements are defined in terms of the relation between any two of the four fundamental circuit variables, namely current $\boldsymbol{i}$, voltage $\boldsymbol{v}$, charge $\boldsymbol{q}$ and flux $\boldsymbol{\varphi}$. The charge q is the time integral of the current i. According to Faraday's law, the flux φ is the time integral of the voltage v. A resistor is defined by the relationship between voltage v and current i, the capacitor is defined by the relationship between charge q and voltage v and the inductor is defined by the relationship between flux φ and current i. Out of the six possible combinations of the four fundamental circuit variables, five are defined. There is a missing link between charge q and flux φ.

Leon Chua [1] reasoned from symmetry arguments that there should be a fourth fundamental element, which he called a memristor (short for memory-resistor) which gives the functional relationship between charge $\boldsymbol{q}$ and flux $\boldsymbol{\varphi}$.

Researchers at HP Labs published an article in Nature "The missing memristor found" [2] identifying a link between the 2-terminal resistance switching behavior found in nano scale systems and Leon Chua's memristor.

The current project report focuses on the HP memristor model. The properties of memristor are studied and simulations are verified by giving various input signals using PSPICE, the parameters that decide the performance of the memristor and circuits involving memristors are discussed. Finally it provides a conclusion and a brief insight regarding possible future work.

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About the CD:

The CD-ROM provided at the end of this dissertation includes the complete thesis in PDF, spice code used in this work and the software “PSPICE 9.1 student version” used in this project.

Chapter 1

Introduction

For nearly 150 years, the known fundamental passive circuit elements were limited to the capacitor (discovered in 1745), the resistor (1827) and the inductor (1831). These elements are defined in terms of the relation between two of the four fundamental circuit variables namely current $\boldsymbol{i}$, voltage $\boldsymbol{v}$, charge $\boldsymbol{q}$ and flux $\boldsymbol{\varphi}$. The charge q is the time integral of the current i. According to Faraday's law, the flux φ is the time integral of the voltage v. A resistor is defined by the relationship between voltage v and current i $(dv=Rdi)$, the capacitor is defined by the relationship between charge q and voltage v $(dq=Cdv)$ and the inductor is defined by the relationship between flux φ and current i $(d\varphi=Ldi)$. Out of the six possible combinations of the four fundamental circuit variables, five are defined. There is a missing link between charge q flux φ.

Leon Chua reasoned from symmetry arguments [1] that there should be a fourth fundamental element, which he called a memristor (short for memory-resistor) which gives functional relationship between charge $\boldsymbol{q}$ and flux $\boldsymbol{\varphi}$ .He proved that memristor behavior could not be duplicated by any circuit built using only the other three elements, so, the memristor is truly fundamental.

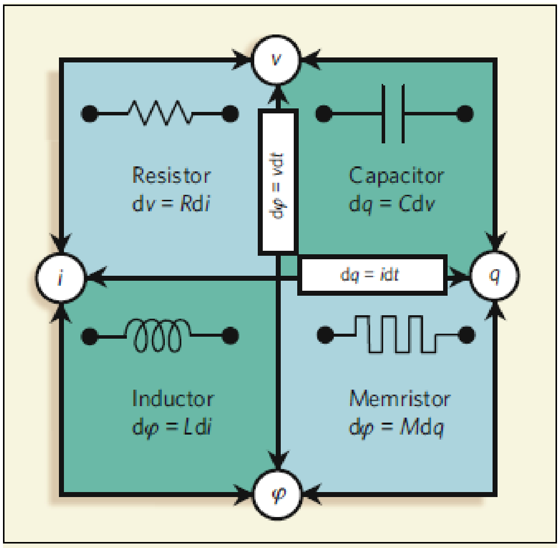


Fig.1.1 Four basic circuit elements [3]

HP Labs announced a model for the physical realization of the memristor [2] and they teamed up with Hynix to produce a commercial product dubbed "ReRam".

1.1 Motivation

Moore's law describes a long-term trend in the history of computing hardware. The number of transistors that can be placed inexpensively on an integrated circuit doubles approximately every two years. This trend has continued for more than half a century. Whether this scaling paradigm lasts for five more years or 15, it will eventually come to an end because transistor cannot be smaller than an atom. Today the smallest components of transistors on integrated circuits are roughly 45 nm wide, or about 220 silicon atoms.

Combined with transistors in a hybrid chip, memristors could radically improve the performance of digital circuits without shrinking transistors. Using transistors more efficiently could in turn give us another decade, at least, of Moore’s Law performance improvement, without requiring the costly and increasingly difficult doublings of transistor density on chips. In the end, memristors might even become the cornerstone of new analog circuits that compute using architecture much like that of the brain.

The ability to indefinitely store resistance values means that a memristor can be used as a nonvolatile memory. If our laptops were built using a memory based on memristors, when the battery is popped back in, our screen would return to life with everything exactly as we left it. No need of lengthy reboot and half dozen auto-recovered files.

A hybrid circuit containing many connected memristors and transistors could help us research actual brain function and disorders. Such a circuit might even lead to machines that can recognize patterns the way humans can, in those critical ways computers can’t—for example, picking a particular face out of a crowd even if it has changed significantly since our last memory of it. These forecasts are based on speculation of the behavior of a memristor. In the current project the extent of applicability of the memristor is investigated.

1.2 Structure of the Report

Chapter 2 presents a brief review of literature on the memristor. Chapter 3 presents the theory and properties of the memristor, the HP model for the memristor, the parameters that decide the performance of the memristor and circuits involving memristors. Chapter 4 gives the results and the simulations of the current work. Finally, Chapter 5 provides a conclusion and a brief insight regarding possible future work.

Chapter 2

Literature survey of Memristor

Memristor, the contraction of “memory resistor”, is a passive device that provides a functional relation between charge q and flux φ. It is defined as a two-terminal circuit element in which the flux φ between the two terminals is a function of the amount of electric charge q that has passed through the device. Memristor is not an energy- storage element. Interestingly, the concept of a resistor with memory existed even before Leon Chua‘s publication on the memristor in 1971.

B.Widrow reported [4] that there was a new circuit element called a “memistor” (a resistor with memory). He stated that this has been devised such that it will have general use in adaptive circuits. With such an element it is possible to get an electronically variable gain control along with the memory required for storage of the system’s experiences or training. Experiences are stored in their most compact form, and in a form that is directly usable from the standpoint of system functioning. The element consists of a resistive graphite substrate immersed in a plating bath. The resistance is reversibly controlled by electroplating. The memistor element has been applied to the realization of adaptive neurons. Memistor circuits for the “Adaline” neuron, which incorporate its simple adaption procedure, have been developed. It has been possible to train these neurons so that this training will remain effective for weeks. Steps have been taken toward the miniaturization of the memistor element. The memistor promises to be a cheap, reliable, mass-producible, adaptive element.

The memistor, three-terminal device for which the conductance between two of the terminals was controlled by the time-integral of the current into the third terminal. Thus, the resistance of the memistor was controlled by charge. Memistors formed the basic components of the neural-network architecture called ADALINE (ADAptive LInear NEuron).

F. Argall reported [5] that thin films of anodized titanium dioxide can be made to switch between three distinct conductivity states. Some electrical properties which characterize these states are given in the temperature range 4·2°K to 500°K. It is concluded that the reversible switching is not due to a phase change of the dielectric. .

L.O.Chua postulated [1] a new two-terminal circuit element-called the memristor- characterized by a relationship between the charge $q(t) \cong \int_{-\infty}^{t} i(\tau) d\tau$ and the flux linkage $\varphi(t) = \int_{-\infty}^{t} v(\tau) d\tau$ is introduced as the fourth basic circuit element. An electromagnetic field interpretation of this relationship in terms of a quasi-static expansion of Maxwell’s equations is presented. Many circuit-theoretic properties of memristors are derived. It is shown that this element exhibits some peculiar behavior different from that exhibited by resistors, inductors, or capacitors. These properties lead to a number of unique applications which cannot be realized with RLC networks alone. Although a physical memristor device without internal power supply has not yet been discovered, operational laboratory models have been built with the help of active circuits. Experimental results were presented to demonstrate the properties and potential applications of memristors.

L.O. Chua and Sung Mo Kang reported [6] the broad generalization of memristors to an interesting class of nonlinear dynamical systems called memristive system. These systems were considered unconventional in the sense that while they behave like resistive devices, they can be endowed with a rather exotic variety of dynamic characteristics. While possessing memory and exhibiting Small-signal inductive or capacitive effects, they are incapable of energy discharge and they introduce no phase shift between the input and output waveforms. This zero crossing property gives rise to a Lissajous figure which always passes through the origin. Memristive systems are hysteretic in the sense that their Lissajous figures vary with the excitation frequency. At very low frequencies, memristive systems are indistinguishable from nonlinear resistors while at extremely high frequencies, they reduce to linear resistors. These anomalous properties have misled and prevented the identification of many memristive devices and systems-including the thermistor, the Hodgkin-Huxley membrane circuit model, and the discharge tubes. Generic properties of memristive systems were derived and a canonic dynamical system model was presented along with an explicit algorithm for identifying the model parameters and functions.

S.Thakoor, et al., [7] reported on a tungsten-oxide-based, nonvolatile, electrically reprogrammable, variable resistance device as an analog synaptic memory connection for electronic neural networks .A voltage controlled, reversible injection of H(+) ions in electrochromic thin films of WO_3 is utilized to modulate its resistance. A hygroscopic thin film of Cr_2O_3 is the source of H(+) ions. The resistance of the device can be tailored and stabilized over a wide dynamic range (about 4 orders of magnitude), and the programming speed is modulated by the control voltage. The suitability of such a device in terms of its response speed, reversibility, stability, and cyclability for its use in electronic neural networks is discussed.

F. A. Buot and A. K. Rajagopal [8] argued, based on the intrinsic time-dependent behavior of double-barrier structures, that a modification of a conventional quantum-well diode with special spacer-layer structure in the source and/or the drain region will lead to two stable current-voltage and charge state behaviors all the way down to zero bias. This viewpoint explains the salient features of a recent experimental observation on quantum-well diodes with n^{-}-n^{+}-n^{-} spacer layers. A simple theory of self-consistent charge buildup and bistability was presented to show that a limited supply or highly altered distribution of electrons from the emitter at high bias leads to fractional recharging of the quantum well and fractional current values, during the decreasing voltage sweep portion of a " closed-loop" voltage sweep. This is in contrast with previous theories based on numerical simulations which allow for more than two current states, by virtue of the use of time-independent analysis and/or the use of " open-multibranch" voltage sweep which do not correspond to the " closed-loop" voltage sweep in the actual experiments mentioned above. This two charge state phenomenon then is the basis for a feasible binary-information storage device at zero bias without dissipation. This article demonstrated the existence of memristor-'bow-tie' current-voltage characteristics in AlAs/GaAs/AlAs quantum-well diodes with special spacer-layer doping design. The analysis showed no direct connection to Chua‘s memristor.

A. Beck, et al., [9] reported that thin oxide films with perovskite or related structures and with transition metal doping show a reproducible switching in the leakage current with a memory effect. Positive or negative voltage pulses can switch the resistance of the oxide films between a low- and a high-impedance state in times shorter than 100 ns. The ratio between these two states is typically about 20 but can exceed six orders of magnitude. Once a low-impedance state has been achieved it persists without a power connection for months, demonstrating the feasibility of nonvolatile memory elements. Even multiple levels can be addressed to store two bits in such a simple capacitor-like structure.

Liu, et al., [10] reported that novel electric pulse-induced resistive change (EPIR) effect has been found in thin film colossal magnetoresistive (CMR) materials, and has shown promise for the development of resistive, nonvolatile memory. The EPIR effect is induced by the application of low voltage (<4V) and short duration (< 20ns) electrical pulses across a thin film sample of a CMR material at room temperature and under no applied magnetic field. The pulse can directly either increase or decrease the resistance of the thin film sample depending on pulse polarity. The sample resistance change has been shown to be over two orders of magnitude, and is nonvolatile after pulsing. The sample resistance can also be changed through multiple levels – as many as 50 have been shown. The authors claim that such a device can provide a way for the development of a new kind of nonvolatile multiple-valued memory with high density, fast write/read speed, low power-consumption and potential high radiation-hardness.

Apart from the devices mentioned above, it is interesting to note that between 1994 and 2008 there were many other devices developed with behavior similar to that of the memristor, but only the HP scientists [2] were successful in finding a link between their work and the memristor postulated by Chua.

R.S.Williams, et al., [2] stated that anyone who ever took an electronics laboratory class will be familiar with the fundamental passive circuit elements: the resistor, the capacitor and the inductor. However, Leon Chua reasoned from symmetry arguments that there should be a fourth fundamental element, which he called a memristor (short for memory resistor). Although he showed that such an element has many interesting and valuable circuit properties, until now no one has presented either a useful physical model or an example of a memristor. They showed, using a simple analytical example, that memristance arises naturally in nanoscale systems in which solid-state electronic and ionic transport are coupled under an external bias voltage. These results serve as the foundation for understanding a wide range of hysteretic current–voltage behavior observed in many nanoscale electronic devices that involve the motion of charged atomic or molecular species, in particular certain titanium dioxide cross-point switches.

Victor Erokhin and M. P. Fontana [11] reported that, in the last few decades, the age old dream of building an artificial brain, i.e. using biological cognitive systems as a benchmark and inspiration to fabricate complex material assemblies which can learn, make decisions, analyse information in a highly parallel way: in other words, highly efficient bio-inspired information processors, has acquired the concreteness of real life research. Many programs and projects in materials science, nanotechnologies, ICT, biosciences use the relevant biological systems and processes as the basic paradigm for the research. However the enormous complexity of even the simplest brains still puts a barrier to the realization of such ambitions. Hence most of the current research (apart from theoretical modelling and simulations) deals with the fabrication and characterization of specific components which are expected to mimic in their functional behaviour neurons, synapses, or use biological molecules to build innovative sensing or electronic components. They showed that a totally different “memristor” with even better characteristics (learning, memory) was reported in the literature since 2005 and discuss its properties and functioning principles, in the framework of using it as a main building block in complex functional networks for bio-inspired information processing.

Since the announcement of the break through by Stanley Williams group, numerous papers with the aim to analyze the elementary attributes of the memristor and memristor applications in various areas of circuit design have appeared.

J. Joshua Yang, et al., [12] reported that Nanoscale metal/oxide/metal switches have the potential to transform the market for nonvolatile memory and could lead to novel forms of computing. However, progress has been delayed by difficulties in understanding and controlling the coupled electronic and ionic phenomena that dominate the behaviour of nanoscale oxide devices. An analytic theory of the 'memristor' (memory-resistor) was first developed from fundamental symmetry arguments in 1971, and they recently showed that memristor behaviour can naturally explain such coupled electron–ion dynamics. Here the authors provide experimental evidence to support this general model of memristive electrical switching in oxide systems. They have built micro- and nanoscale TiO_2 junction devices with platinum electrodes that exhibit fast bipolar nonvolatile switching. They demonstrated that switching involves changes to the electronic barrier at the Pt/TiO_2 interface due to the drift of positively charged oxygen vacancies under an applied electric field. Vacancy drift towards the interface creates conducting channels that shunt, or short-circuit, the electronic barrier to switch ON. The drift of vacancies away from the interface annilihilates such channels, recovering the electronic barrier to switch OFF. Using this model they have built TiO_2 crosspoints with engineered oxygen vacancy profiles that predictively control the switching polarity and conductance.

Yu V. Pershin, et al., [13] reported that, behavioural intelligence of the plasmodia of the true slime mold has been demonstrated.It was shown that a large amoeba-like cell Physarum polycephalum subject to a pattern of periodic environmental changes learns and changes its behavior in anticipation of the next stimulus to come. Currently, it is not known what specific mechanisms are responsible for such behaviour. In this work, they showed that such behaviour can be mapped into the response of a simple electronic circuit consisting of an LC contour and a memory-resistor (a memristor) to a train of voltage pulses that mimic environment changes. They identified a possible microscopic origin of the memristive behaviour in the Physarum polycephalum, which together with the naturally occurring biological oscillators, forms the basis of the amoeba's learning. These microscopic memristive features are likely to occur in other unicellular as well as multicellular organisms, albeit in different forms. Therefore the above memristive circuit model, which has learning properties, is useful to better understand the origins of primitive intelligence.

Sung Hyun Jo et al., [14] demonstrated large-scale (1 kb) high-density crossbar arrays using a Si-based memristive system. A two-terminal hysteretic resistive switch (memristive device) is formed at each crosspoint of the array and can be addressed with high yield and ON/OFF ratio. The crossbar array can be implemented as either a resistive random-access-memory (RRAM) or a write-once type memory depending on the device configuration. The demonstration of large-scale crossbar arrays with excellent reproducibility and reliability also facilitates further studies on hybrid nano/CMOS systems.

N.G.Hackett, et al., [15] demonstrated a rewriteable low-power operation nonvolatile physically flexible memristor device. The active component of the device is inexpensively fabricated at room temperature by spinning a TiO2 sol gel on a commercially available polymer sheet. The device exhibits memory behavior consistent with a memristor, demonstrates an on/off ratio greater than 10 000 : 1, is nonvolatile for over 1.2×10^6 s, requires less than 10 V, and is still operational after being physically flexed more than 4000 times.

A. Delgado [16] investigated the experimental characteristic of a memristor modeled with a function used to predict oscillations in closed loop systems with linear plants.

Researchers at North Carolina State University have demonstrated [17] new "soft" electronic components, built from liquid metals and hydrogels. The scientists hope that such components—quasi-liquid diodes and memristors—will work better than traditional electronics to interface with wet squishy things, such as the human brain.

Based on the literature review of the memristor [18-30], the objectives of the present project are

(1) To study and simulate the properties of memristor by giving various input signals using PSPICE,

(2) To establish the parameter(s) that determines the performance of the memristor and

(3) Simulate the performance of simple electronic circuits by replacing components with memristors and comparing the performance with and without memristors.

Chapter 3

The Missing Element: Memristor

There are four fundamental circuit variables in circuit theory. They are current ***i***, voltage ***v***, charge ***q*** and flux ***φ***. Out of the six possible combinations of the four fundamental circuit variables, five are well defined. There is a missing link between charge q and flux φ.

Leon Chua [1] reasoned from symmetry arguments that there should be a fourth fundamental element, which he called a memristor (short for memory-resistor) which gives functional relationship between charge ***q*** and flux ***φ*** .He proved that memristor behavior could not be duplicated by any circuit built using only the other three elements, so, the memristor is truly fundamental.

3.1 What is memristor?

Memristor, the contraction of "memory resistor", is a passive device that provides a functional relation between charge q and flux φ. "It is defined as a two-terminal circuit element in which the flux φ between the two terminals is a function of the amount of electric charge q that has passed through the device". Memristor is not an energy- storage element.

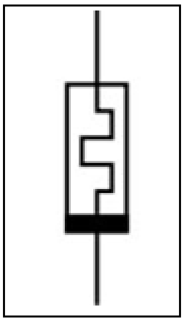


Fig. 3.1 The symbol for a memristor [18]

A memristor is said to be charge-controlled if the relation between flux and charge is expressed as a function of electric charge q and it is said to be flux-controlled if the relation between flux and charge is expressed as a function of the flux linkage φ.

For a charge-controlled memristor,

$$\varphi = f(q) \qquad \text{- (1)}$$

On differentiating equation (1)

$$\frac{d\varphi}{dt} = \frac{df(q)}{dq}\frac{dq}{dt} \qquad \text{- (2)}$$

We know that v (t) = $\frac{d\varphi}{dt}$ & $i(t) = \frac{dq}{dt}$

$$\Rightarrow \quad v(t) = M(q)i(t) \qquad \text{- (3)}$$

$$\text{Where} \quad M(q) = \frac{df(q)}{dt} \qquad \text{- (4)}$$

M(q) is called as memristance, and it has the units of resistance. Memristance defines a linear relationship between current and voltage, as long as the charge does not vary. Thus if M is constant, a memristor behaves as a resistor.

For a flux – controlled memristor,

$$q = f(\varphi) \qquad \text{- (5)}$$

On differentiating (5)

$$\frac{dq}{dt} = \frac{df(\varphi)}{d\varphi}\frac{d\varphi}{dt} \qquad \text{- (6)}$$

We know that v (t) = $\frac{d\varphi}{dt}$ & $i(t) = \frac{dq}{dt}$

$$\Rightarrow \quad i(t) = w(\varphi)v(t) \qquad \text{- (7)}$$

$$\text{where} \quad w(\varphi) = \frac{df(\varphi)}{dt} \qquad \textbf{- (8)}$$

w(φ) is called as memductance and it has the units of conductance.

3.2 What is Memristance?

Memristance is the “resistance-like” property of the memristor [19]. If charge flows in one direction through a circuit, the resistance of the memristor will increase, and if charge flows in the opposite direction in the circuit, the resistance will decrease. If the flow of charge is stopped by turning off the applied voltage, the memristor will 'remember' the last resistance that it had, and when the flow of charge starts again the resistance of the circuit will be what it was when it was last active. This is called memristance.

3.3 Analogy of a Memristor [19]

A common analogy for a resistor is a pipe that carries water. The water itself is analogous to electrical charge, the pressure at the input of the pipe is similar to voltage, and the rate of flow of the water through the pipe is like electrical current. Just as with an electrical resistor, the flow of water through the pipe is faster if the pipe is shorter and/or it has a larger diameter. An analogy for a memristor is an interesting kind of pipe that expands or shrinks when water flows through it. If water flows through the pipe in one direction, the diameter of the pipe increases, thus enabling the water to flow faster. If water flows through the pipe in the opposite direction, the diameter of the pipe decreases, thus slowing down the flow of water. If the water pressure is turned off, the pipe will retain its most recent diameter until the water is turned back on. Thus, the pipe does not store water like a bucket (or a capacitor) – it remembers how much water flowed through it.

3.4 Properties of memristor

3.4.1 $\varphi - q$ Curve of a Memristor

The $\varphi - q$ curve of a memristor is a monotonically increasing [20]. The memristance M (q) is the slope of the $\varphi - q$ curve. According to the memristor passivity criterion[1] “memristor characterized by a differentiable charge- controlled $\varphi - q$ curve is passive if, and only if, its incremental memristance M(q) is nonnegative; i.e., $M(q) \geq 0$.

The instantaneous power dissipated by a memristor is given by

$$p(t) = v(t)i(t) = M(q(t))[i(t)]^2 \qquad - (9)$$

Hence, if the incremental memristance $M(q) \geq 0$, then $p(t) \geq 0$ and the memristor is obviously passive.

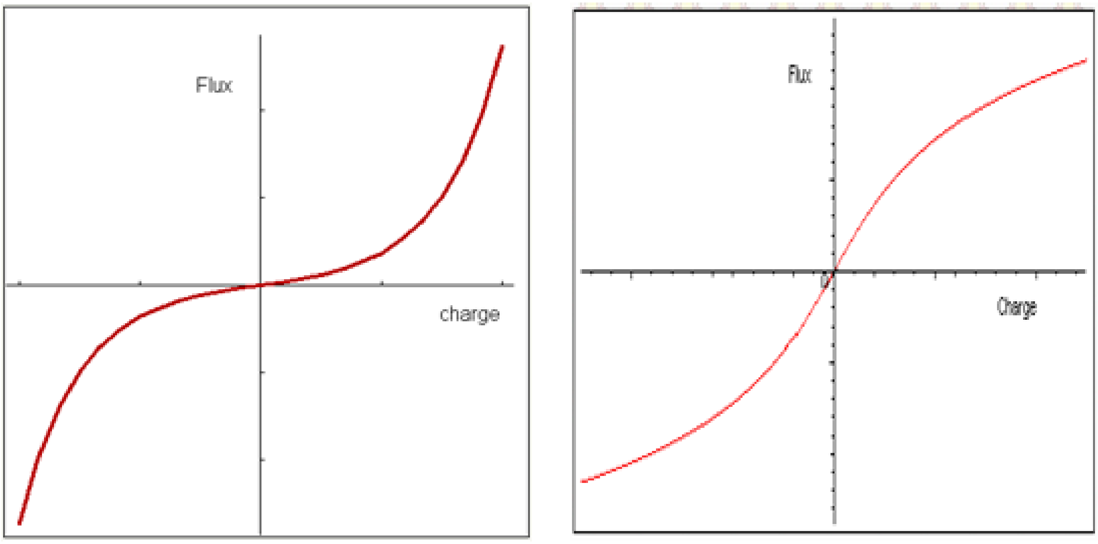


Fig. 3.2 Examples of typical φ –*q* curves of memristor

3.4.2 Current–Voltage Curve of a Memristor

The most important feature of a memristor is its pinched hysteresis loop *v-i* characteristic. For a memristor excited by a periodic signal, when the voltage v(t) is zero, the current i(t) is also zero and vice versa. Thus, both voltage v(t) and current i(t) have identical zero-crossing.

Another important feature of memristor is that the “pinched hysteresis loop” shrinks with the increase in the excitation frequency.

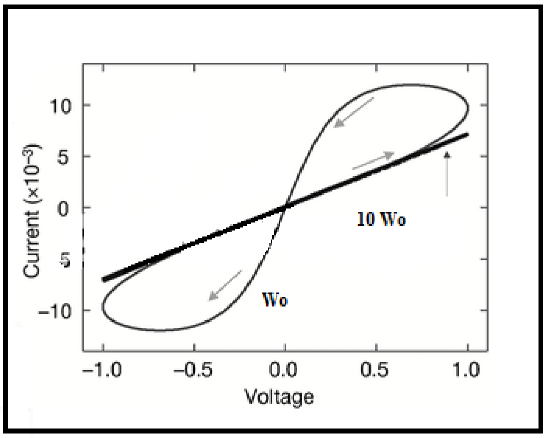


Fig.3.3 shows V– I characteristics of memristor at low and high frequencies.

Why does the "pinched hysteresis loop" shrinks with the increase in the excitation frequency?

In the low frequency case, the width of the low resistance layer, w, is driven fully within the bounds of 0 to D. Since charge carrying oxygen vacancies can drift through the full length of the device before a change in the polarity of the ac signal changes their direction and pinched hysteresis loop[2], this results in the nonlinear relationship between the memristance current and voltage. In the high frequency case the low mobility oxygen vacancies cannot drift over any appreciable distance before the polarity of the ac signal changes their direction. Thus, at high frequencies w is essentially fixed and the memristor behaves like a fixed resistor dependent on the static value of *w*. This leads to an approximate linear relationship between current and voltage

These characteristics are shown in PSPICE simulations in chapter 4.

3.5 Memristive Systems

Leon Chua and Sung Mo Kang published a paper [6] entitled "**Memristive devices and systems"** generalizing the theory of memristors and memristive systems. The memristive systems are described by

$$V=M\,(w,i)I \qquad \text{-(10)}$$

and

$$dw/dt=f(w,i) \qquad \text{-(11)}$$

Where w is a set of state variables, M and f can be functions of time and v and i are the voltage and current respectively.

The fundamental memristive system theorem [20] states that “Every two-terminal device which exhibits a pinched hysteresis loop in the current versus voltage plane when driven by a dc and/or sinusoidal signal of any frequency is a memristive system”.

3.6 Model of the memristor from HP labs [21]

In 2008, thirty-seven years after Leon Chua‘s proposal [1], the memristor in device form was developed by Stanley Williams and his group in the Information and Quantum Systems (IQS) Lab at HP. The HP memristor can be defined in terms of memristive systems. It exploits a very thin-film TiO_2 sandwiched between two platinum (Pt) contacts and one side of the TiO_2 is doped with oxygen vacancies, which are positively charged ions. Therefore, there is a TiO_2 junction where one side is doped and the other is undoped. Such a doping process results in two different resistances: one is a high resistance (undoped) and the other is a low resistance (doped). The device established by HP is shown in Fig.3.4

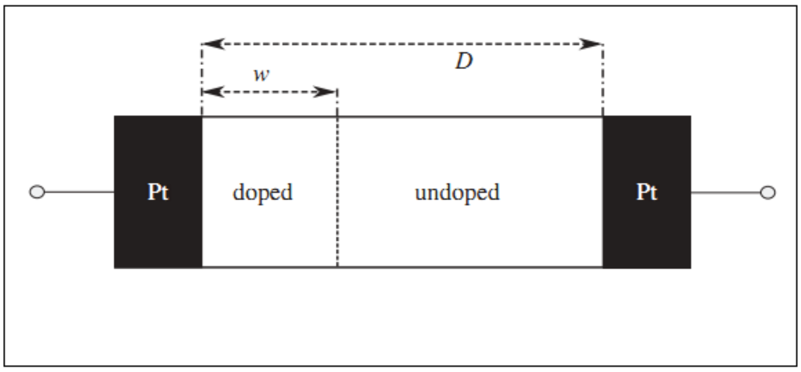


Fig.3.4 Schematic of HP memristor [21]

In fig 3.4, D is the device length and w is the length of the doped region.

Hp memristor [22] consists of titanium dioxide (TiO_2) in two layers: The undoped TiO_2 layer has a perfect 2:1 oxygen-to-titanium ratio, making it an insulator. The doped TiO_2 is missing 0.5 percent of its oxygen (TiO_{2-x}), so x is about 0.05. The vacancies make the TiO_{2-x} material metallic and conductive

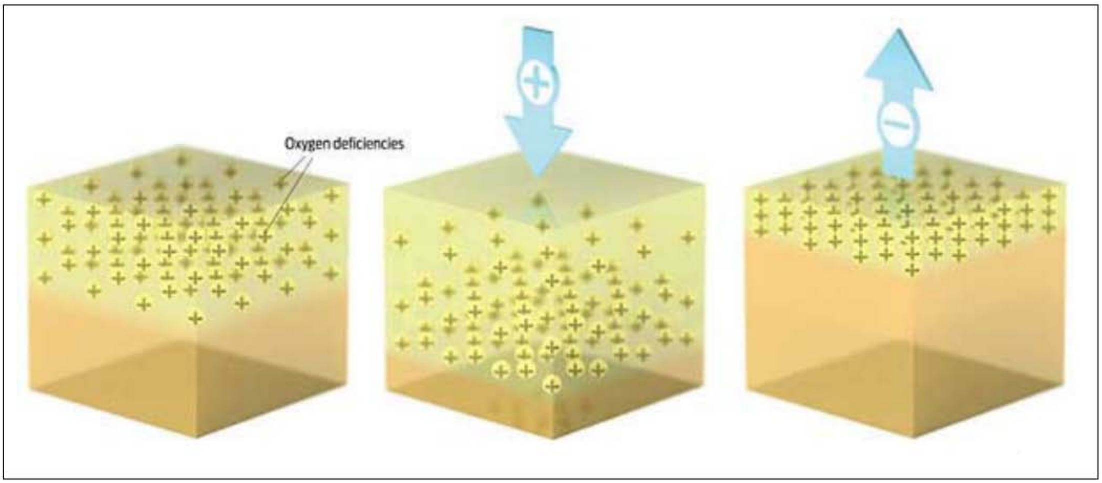


Fig.3.5 Behavior of HP memristor when positive and negative voltages are applied [22]

The oxygen deficiencies in the TiO_{2-x} manifest as “bubbles” of oxygen vacancies scattered throughout the upper layer. When positive voltage is applied to the memristor repels the (positive) oxygen vacancies in the metallic upper TiO_{2-x} layer, sending them into the insulating TiO_2 layer below. This causes the boundary between the two materials to move down, increasing the percentage of conducting TiO_{2-x} and thus the conductivity of the memristor increases. This increases the conductivity of the whole device.

When a negative voltage is applied, the positively charged oxygen vacancies are attracted, pulling them out of TiO_2 layer. This increases the amount of insulating TiO_2, thus increasing the resistivity of the whole device.

What makes the memristor special is that when the voltage is turned off, the oxygen vacancies do not move. They stay where they are, which means that the boundary between the two titanium dioxide layers is frozen. That is how the memristor “remembers” how much voltage was last applied, making it a memory-resistor or memristor.

The simple mathematical model of the HP memristor [2] is given by

$$M(q) = R_{OFF}(1 - \frac{R_{ON}}{\beta} q(t)) \qquad - (12)$$

where $\beta = \frac{D^2}{\mu_D}$ has the dimensions of magnetic flux. μ_D is the average drift velocity and has the units cm^2/sV; D is the thickness of titanium-dioxide film; R_{OFF} and R_{ON} are “on-state” and “off- state” resistances; and q(t) is the total charge passing through the memristor device.

3.7 Linear Drift Model [21]

Let us assume a uniform electric field across the device. Therefore, there is a linear relationship between drift-diffusion velocity and the net electric field. The state equation can be written as

$$\frac{1}{D}\frac{dw(t)}{dt} = \frac{R_{ON}}{\beta} i(t) \qquad \text{- (13)}$$

Integrating equation (13) gives

$$\frac{w(t)}{D} = \frac{w(t_0)}{D} + \frac{R_{ON}}{\beta} q(t) \qquad \text{- (14)}$$

where $w(t_0)$ is the initial length of w. Hence, the speed of drift under a uniform electric field across the device is given by

$$v_D = \frac{dw(t)}{dt} \qquad \text{- (15)}$$

In a uniform field, we have $D = v_D \times t$. In this case, $Q_D = i \times t$ defines the amount of required charge to move the boundary from $w(t0)$, where $w \rightarrow 0$, to distance $w(tD)$, where $w \rightarrow D$. Therefore, $Q_D = \frac{\beta}{R_{ON}}$ .Thus,

$$\frac{w(t)}{D} = \frac{w(t_0)}{D} + \frac{q(t)}{Q_D} \qquad \text{- (16)}$$

If $x(t) = \frac{w(t)}{D}$, then equation 16 can be written as

$$x(t) = x(t_0) + \frac{q(t)}{Q_D} \qquad \text{- (17)}$$

where $\frac{q(t)}{Q_D}$ describes the amount of charge that is passed through the channel over the required charge for a conductive channel.

From [2], we have

$$v(t) = (R_{ON}\frac{w(t)}{D} + R_{OFF}(1 - \frac{w(t)}{D}))i(t) \qquad \text{- (18)}$$

By inserting $x(t) = \frac{w(t)}{D}$, equation (18) can be rewritten as

$$v(t) = (R_{ON}x(t) + R_{OFF}(1 - x(t)))i(t) \qquad \text{- (19)}$$

Now assume that $q(t0)=0$, then $w(t)=w(t0) \neq 0$, and

$$M_0 = R_{ON}(x(t_0) + r(1 - x(t_0))) \qquad - (20)$$

where $r = R_{OFF}/R_{ON}$ and M_0 is the memristance value at t_0. Consequently, the following equation gives the memristance at time t:

$$M(q) = M_0 - \Delta R(\frac{q(t)}{Q_D}) \qquad - (21)$$

where $\Delta R = R_{OFF} - R_{on}$. When $R_{OFF} >> R_{ON}$, $M_0 \approx R_{OFF}$.

Substituting equation (21) into $v(t)=M(q)i(t)$, when $i(t)=\mathrm{d}q(t)/\mathrm{d}t$, we have

$$v(t) = (M_0 - \Delta R(\frac{q(t)}{Q_D}))\frac{dq(t)}{dt} \qquad - (22)$$

Recalling that $M(q) = \frac{d\varphi(q)}{dq}$, the solution is

$$q(t) = \frac{Q_D M_0}{\Lambda R}(1 \pm \sqrt{1 - \frac{2\Delta R}{Q_D M_0^2}\varphi(t)}) \qquad - (23)$$

Using $\Delta R \approx M0 \approx R_{OFF}$, equation (23) becomes

$$q(t) = Q_D(1 - \sqrt{1 - \frac{2}{Q_D R_{OFF}}\varphi(t)}) \qquad - (24)$$

if $QD = D2/\mu_D R_{ON}$, then the internal state of the memristor is

$$x(t) = 1 - (\sqrt{1 - \frac{2\mu_D}{rD^2}\phi(t)}) \qquad - (25)$$

The current–voltage relationship in this case is

$$i(t) = \frac{v(t)}{R_{OFF}(\sqrt{1 - (2\mu_D / rD^2)}\varphi(t))} \qquad - (26)$$

Equation 26 shows the inverse-square relation between memristance and TiO_2 thickness, D. Thus, for smaller values of D, the memristance shows improved characteristics. Nowadays, memristance becomes more important for understanding as the dimensions of electronic devices are shrinking to nanometer scale.

3.8 Performance Parameter

From [2] memristance M(q) is given as

$$M(q) = R_{OFF}(1 - \frac{\mu_V R_{ON}}{D^2} q(t))$$

If fabrication point of view is considered, the fundamental parameter has to be evaluated. Let us consider ($r=R_{OFF}/R_{ON}$),($s=w_o/D$) and mobility u_v

R_{ON}	-ON resistance of memristor
R_{OFF}	- OFF resistance of memristor
D	- Width of the thin film
W_0	- initial doped width
μ_v	- Migration coefficient in $m^2s^{-1}v^{-1}$

Then the performance parameter

$$\gamma = \frac{kr}{s\mu_v}$$

Where K is magnitude adjustment coefficient

If the memristor is commercialized then we have to say the range of γ (i.e $\gamma = \frac{kr}{\mu_v}$ to $\gamma = \frac{kr}{s\mu_v}$) to own the particular memristor.

3.9 Circuits involving memristor

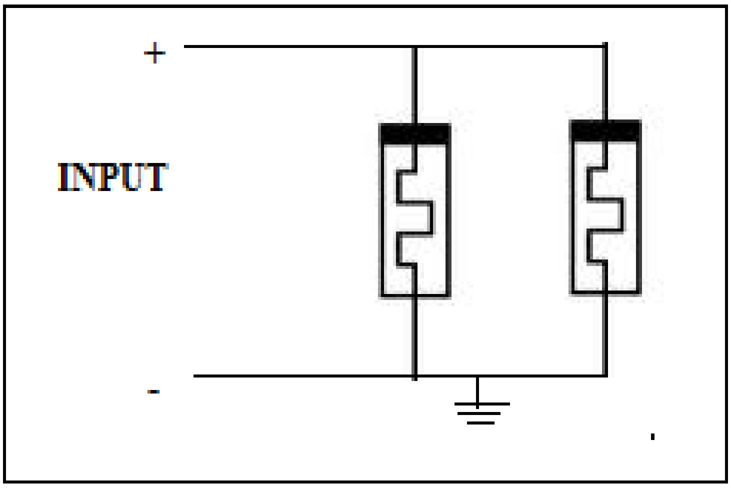


Fig .3.6 Parallel combination of memristors

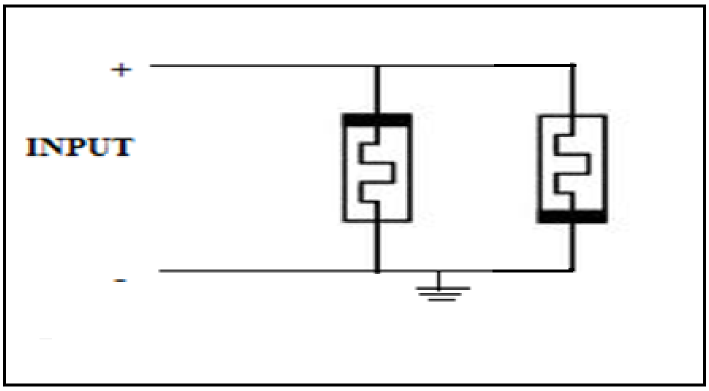


Fig .3.7 Memristors connected in parallel with one memristor connected in opposite polarity

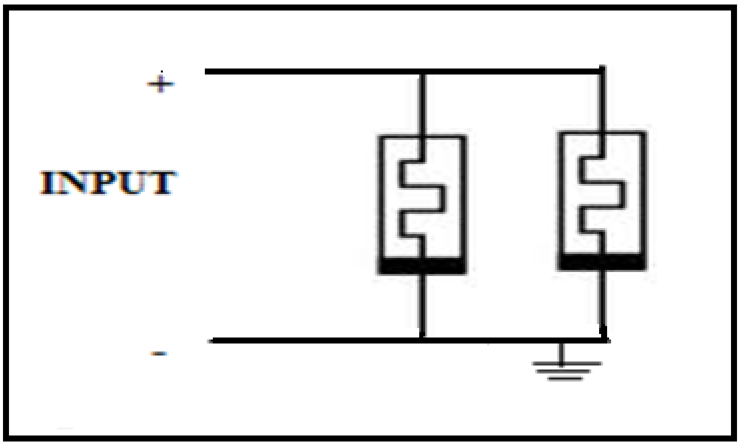


Fig .3.8 Memristors connected parallel in opposite polarity

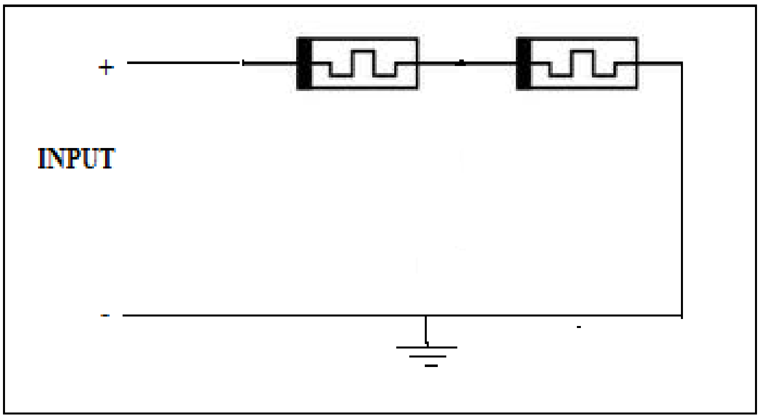


Fig .3.9 Series combination of memristors with same polarity

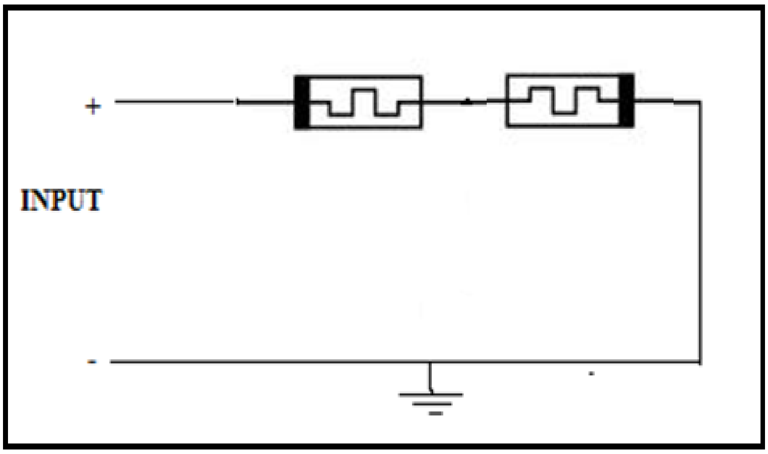


Fig .3.10 Series combination of memristors with opposite polarity

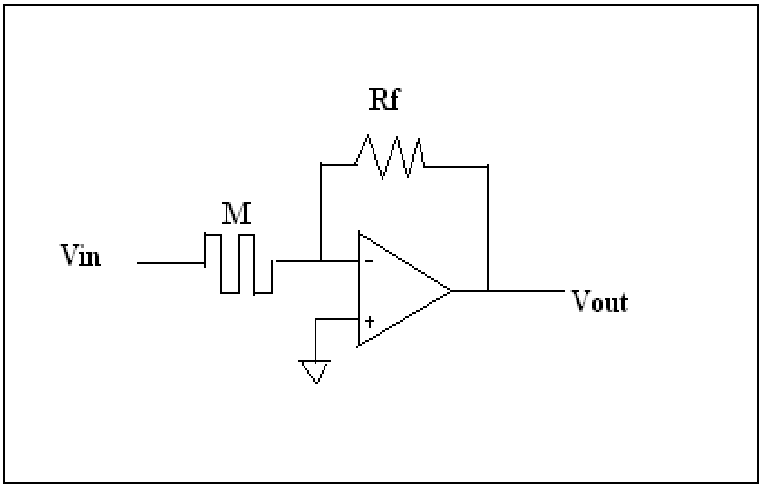


Fig .3.11 integrator circuit with memristor

This circuit acts as an integrator when square wave input is given to it. From this we can conclude that memristor can behave like a capacitor under some conditions.

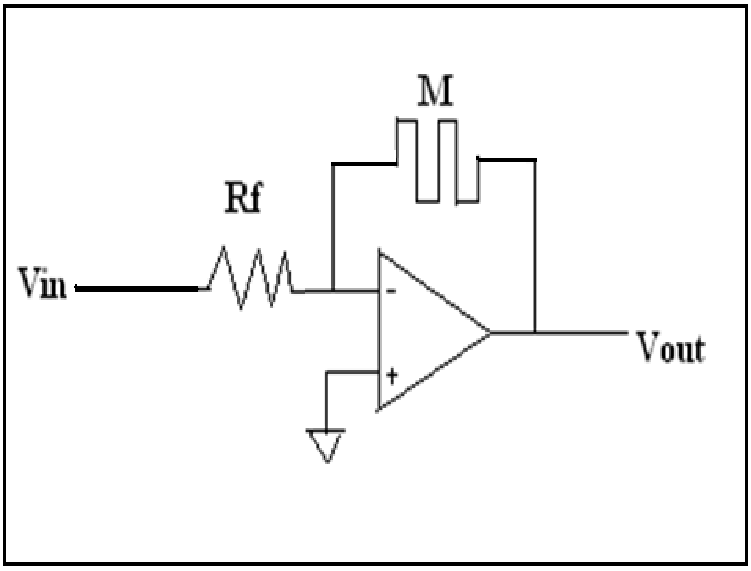


Fig .3.12 memristor circuit giving spikes output

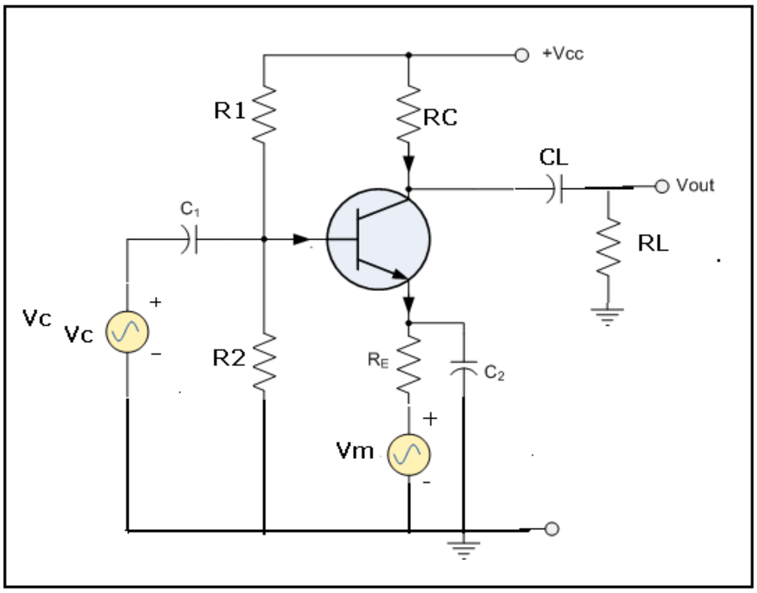


Fig .3.13 Transistor Amplitude modulator circuit

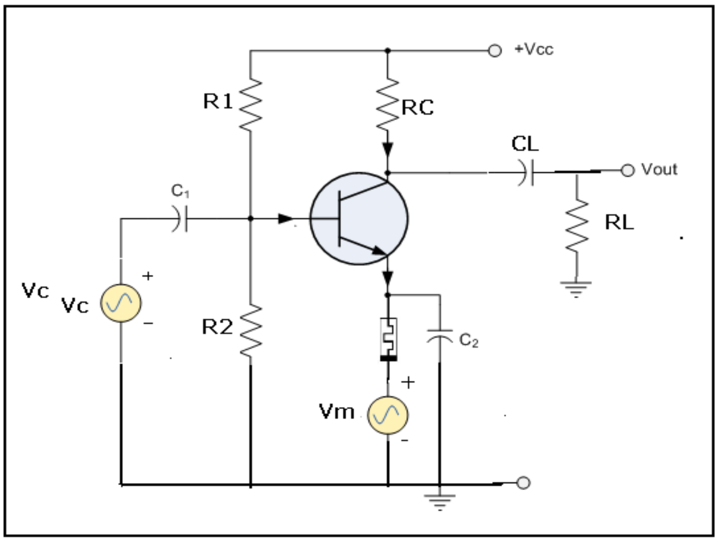


Fig .3.14 Transistor Amplitude modulator circuit with memristor

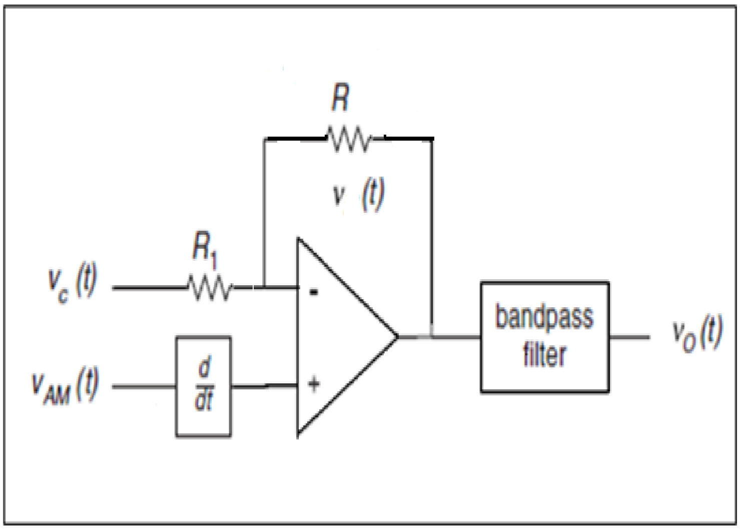


Fig.3.15 Amplitude modulator with resistor

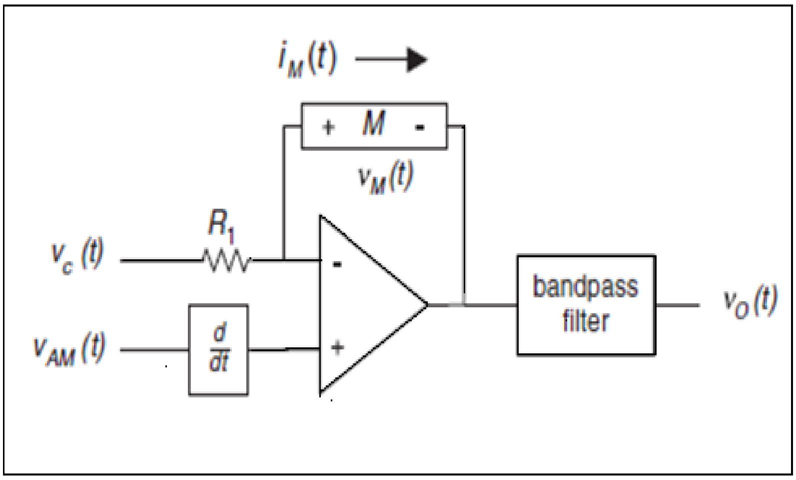


Fig.3.16 Amplitude modulator using memristor

In fig 3.14, we have replaced the resistor R in fig 3.13 with the memristor. This circuit acts as an amplitude modulator. The simulation results are shown in chapter 4.

Chapter 4

Results and simulations

4.1 SPICE Model [23]

The SPICE model of memristor provided in [23] is used for the simulations. The model of memristor is as shown in the fig 4.1

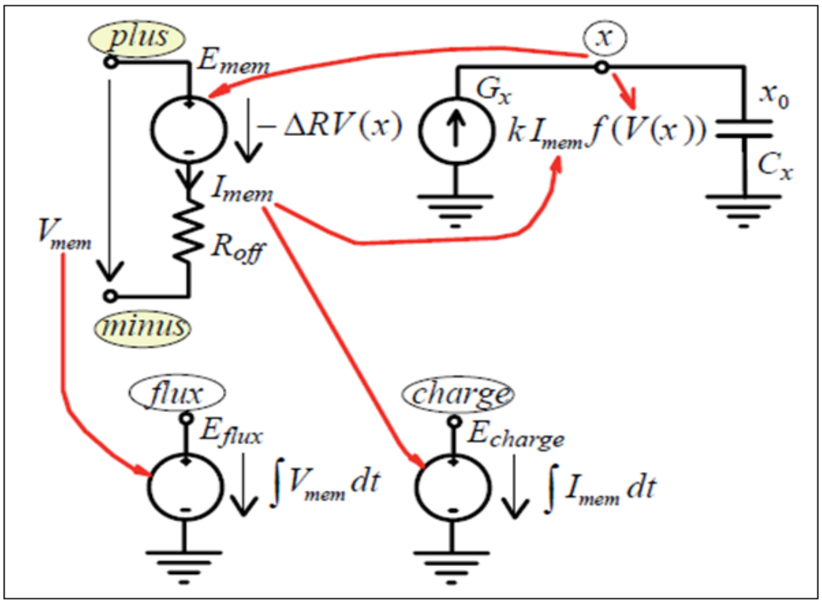


Fig. 4 .1 SPICE model of memristor.

In the above circuit, V_{mem} is the input voltage and I_{mem} is the current through the memristor. The flux is calculated by integrating the voltage V_{mem} and the charge is calculated by integrating the current I_{mem} .

From [2] The relation between the memristor voltage and current is given by

$$R_{MEM}(x) = R_{ON}\,x + R_{OFF}(1-x) \qquad \text{- (1)}$$

⇨ $$R_{MEM}(x) = R_{OFF} - x\Delta R\,,\ \Delta R = R_{OFF} - R_{ON} \qquad \text{- (2)}$$

Where R_{OFF} and R_{ON} are the limit values of the memristor resistance for w=0 and w=D.

In Fig. 4.1, equation (1) corresponds to the R_{OFF} resistor in series with the E-type voltage source Emem whose terminal voltage is controlled according to the formula “-xΔR”.

The speed of the movement of the boundary between the doped and undoped regions depends on the resistance of doped area, on the passing current, and on other factors

From [2] according to the state equation

$$\frac{dx}{dt} = k.I_{MEM}\, f(x)\ , \qquad - (3)$$

$$k = \frac{\mu_V R_{ON}}{D^2}$$

where μ_v is dopant mobility in $m^2 s^{-1} v^{-1}$.

D is thickness of the doped and undoped Tio_2 layer in nm.

We need the normalized width x of the doped layer to model the memristance R_{MEM} (x). On integrating equation (3), we will get the normalized width x of the doped layer.

We know that voltage across the capacitor is

$$v = \frac{1}{c}\int idt \qquad - (4)$$

So, Fix current source Gx whose magnitude is equal to the right hand side of the equation(3) and pass the current through the capacitor Cx of unit farad. Thus the normalized width x of the doped layer is obtained from the voltage V(x) across the capacitor Cx. This is how we can model a memristor.

For the simulations, the following parameters are used:

Ron = 1KΩ	
Roff = 100KΩ;	Ron, Roff - Resistance in ON / OFF States
Rinit = 80K ;	Rinit - Resistance at time T=0
D = 10nm ;	Width of the thin film
uv = 10^{-14} ;	Migration coefficient in $m^2 s^{-1} v^{-1}$
p = 1 ;	Parameter of the WINDOW-function

4.2 Sinusoidal input to the memristor.

Sinusoidal input of voltage (1.2v) and frequency (1HZ) is given to the memristor.

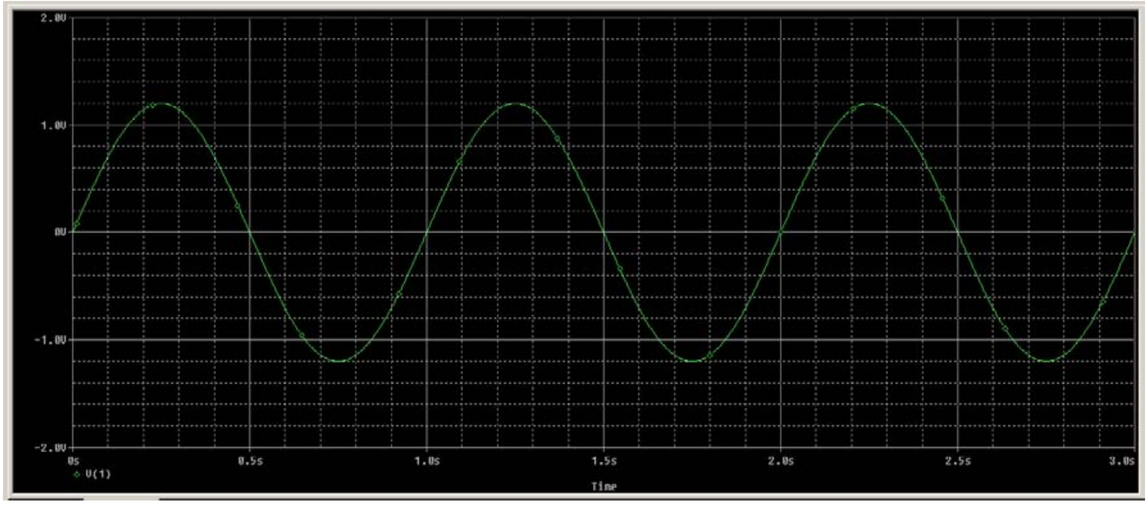


Fig. 4.2 Sinusoidal input voltage applied to the memristor.

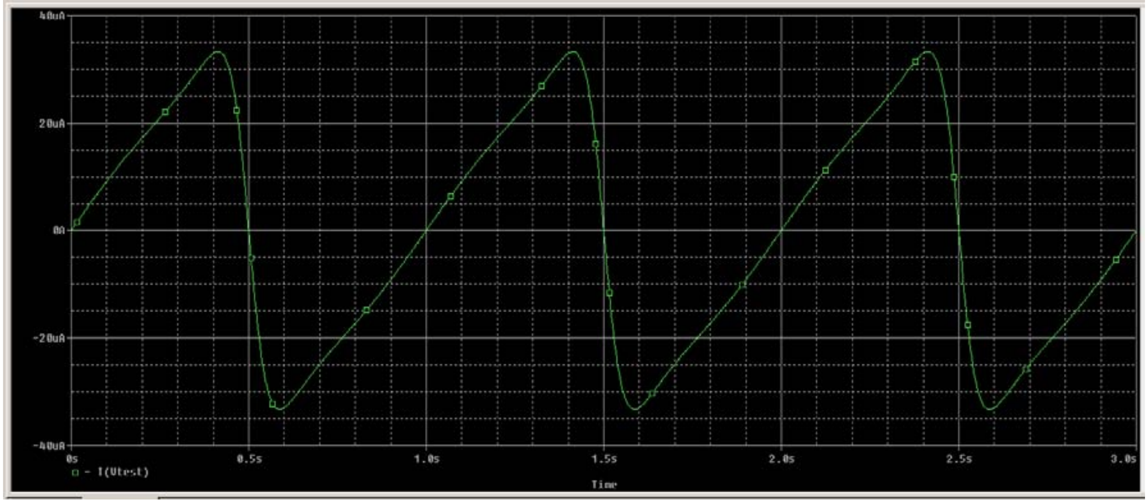


Fig. 4.3 Waveform of the current Imem through the memristor.

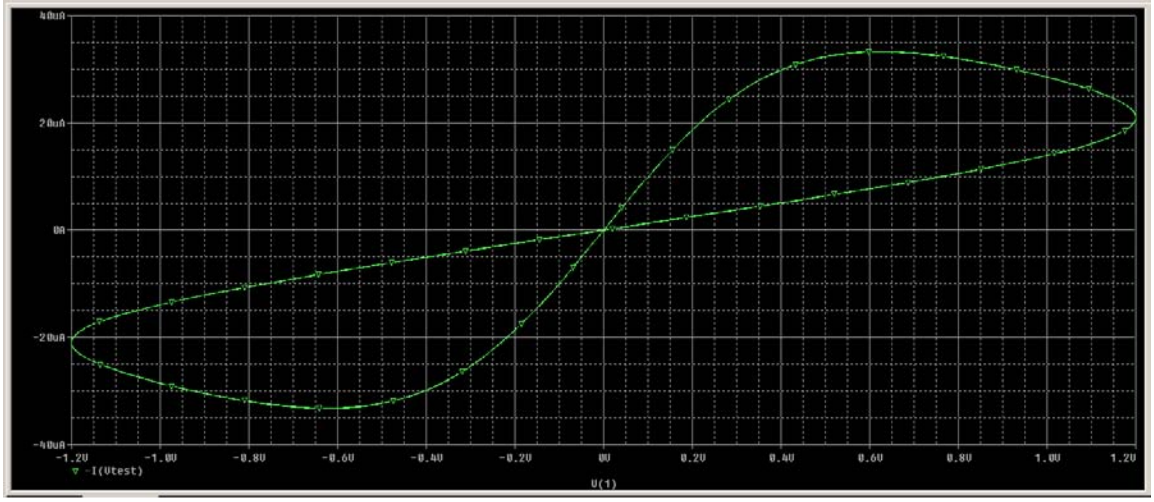


Fig. 4.4 Current-versus-voltage curve for input frequency of 1 Hz.

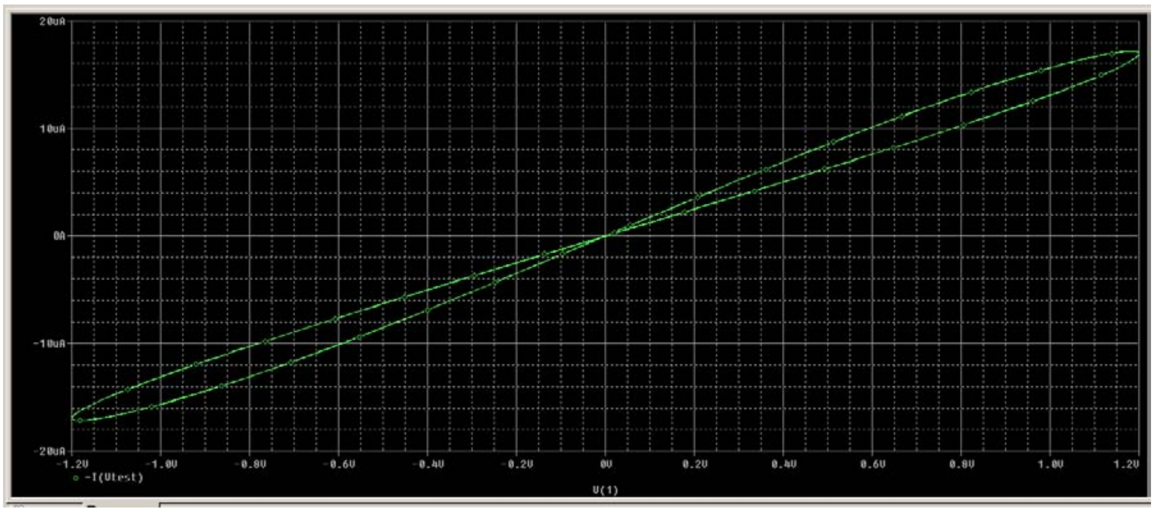


Fig. 4.5. Current-versus-voltage curve for input frequency of 2 Hz.

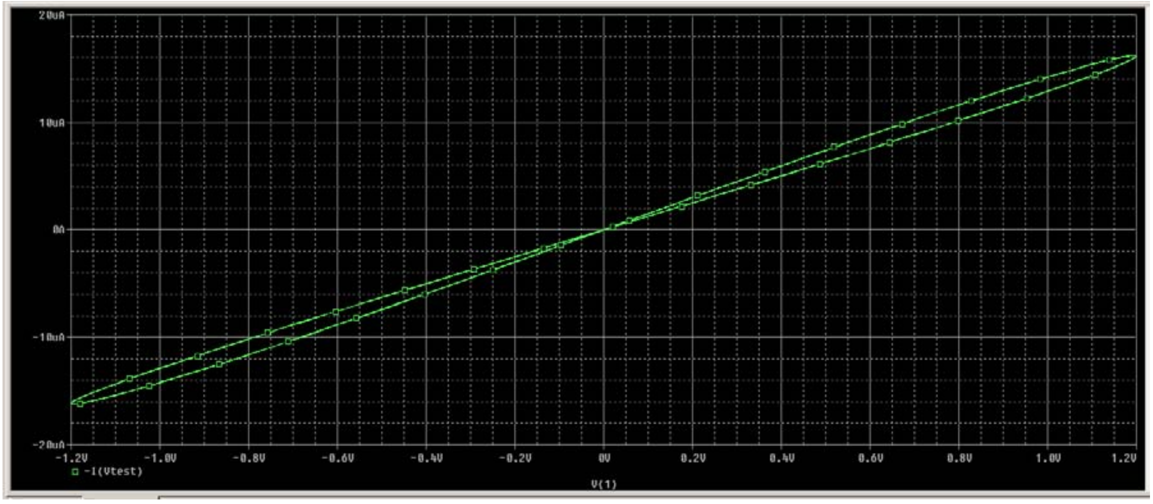
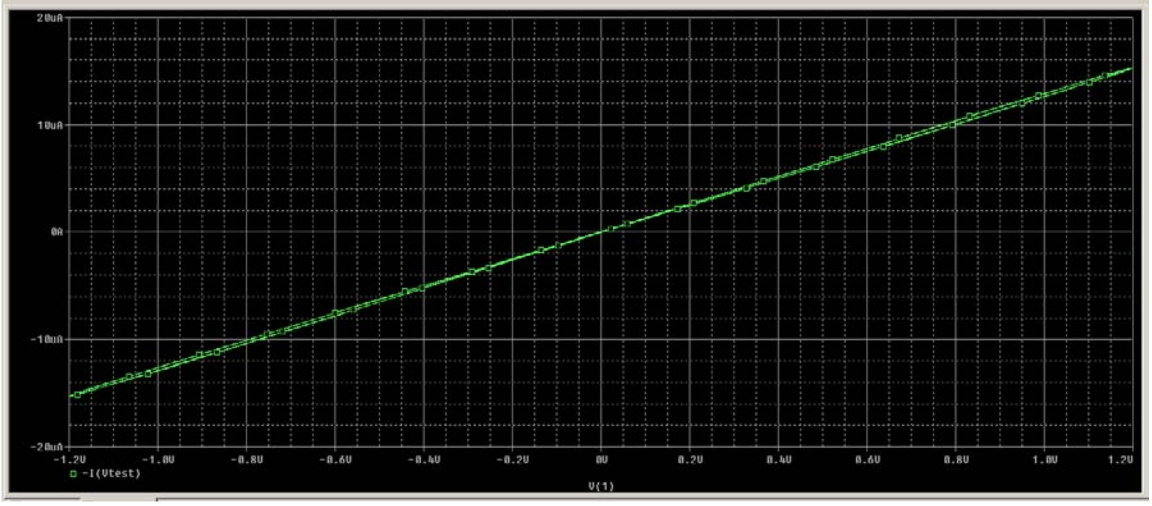


Fig. 4.6 Current-versus-voltage curve for input frequency of 3 Hz.



. 4.7 Current-versus-voltage curve for input frequency of 10 Hz.

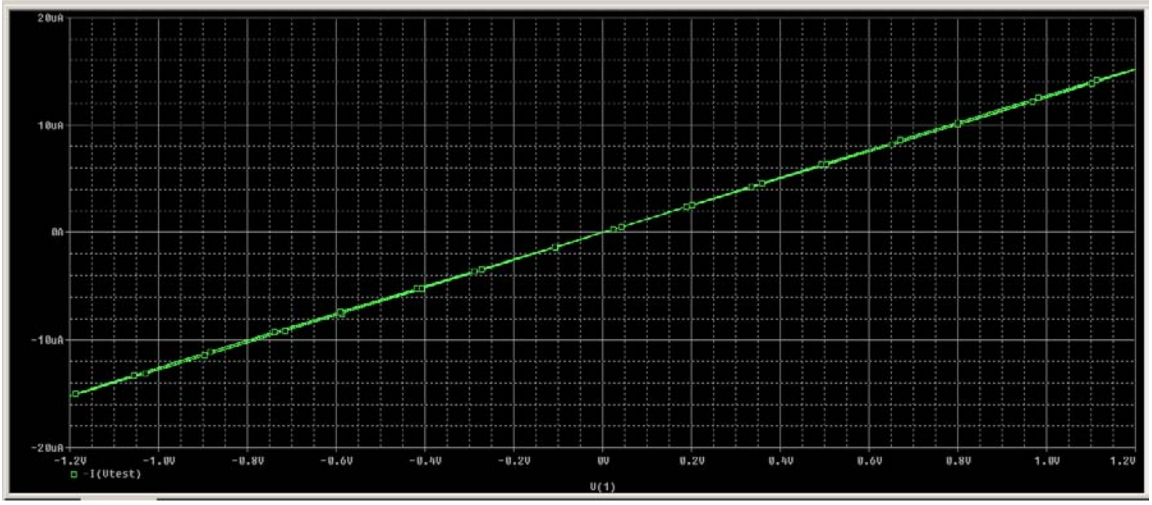


Fig. 4.8 Current-versus-voltage curve for input frequency of 15 Hz.

From the figures we can conclude that the element exhibits
1. Pinched Hysteresis loop
i.e., At origin both v(t) and i(t) are zero

⇨ No phase shift

⇨ No memory i.e., dissipative

From fig 4,5and 6 it is clear that both v(t) and i(t) are zero at origin.
2. Pinched hysteresis loop shrinks with the frequency increase.
From fig 4,5and 6 it is clear that Pinched hysteresis loop shrinks with increase in frequency.

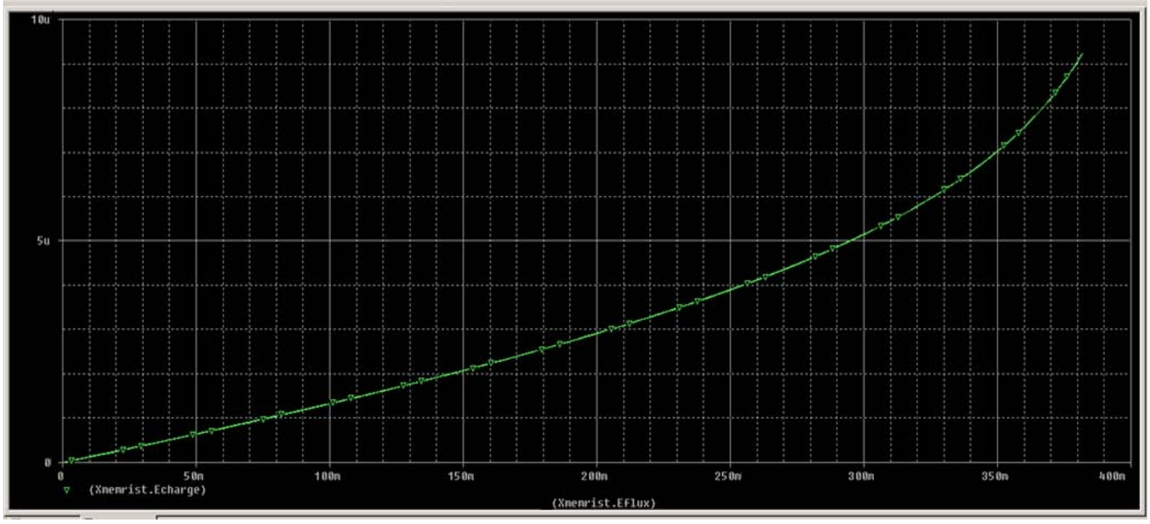


Fig. 4.9. Charge-versus-flux curve for memristor.

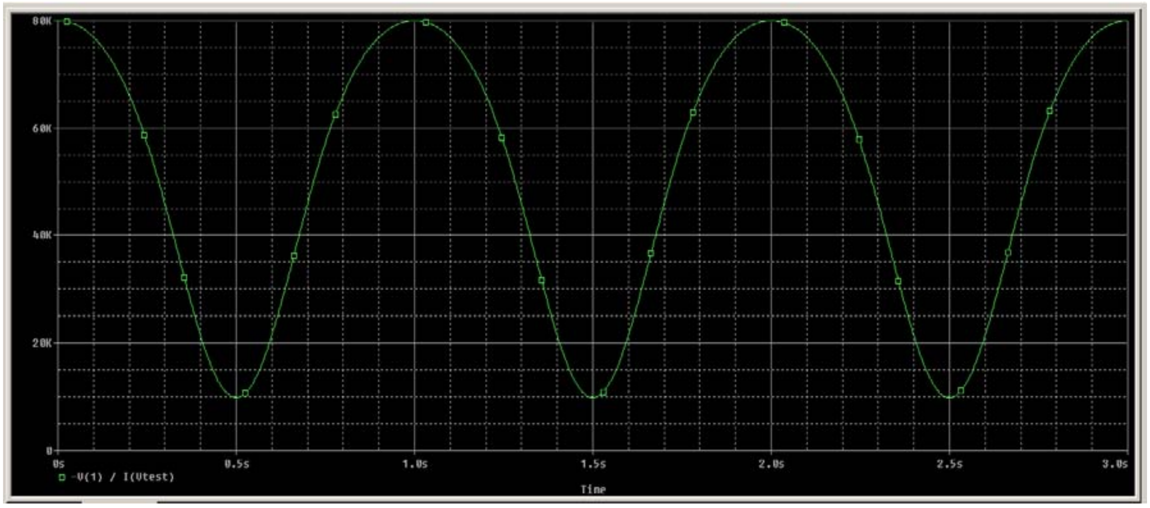


Fig. 4.10 Variation of memristance with time

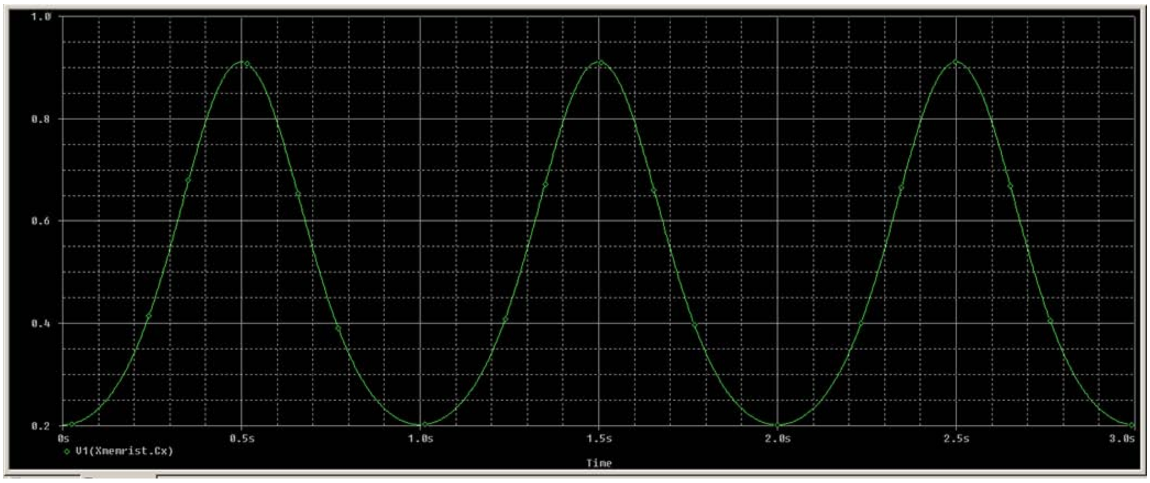


Fig. 4.11 Voltage across the capacitor Cx which gives the internal state of the device

This fig. shows that the normalized width of the doped layer varies with time

4.3 Triangular wave input to the memristor

Triangular wave input of peak to peak voltage (2v) and frequency (1HZ) is given to the memristor.

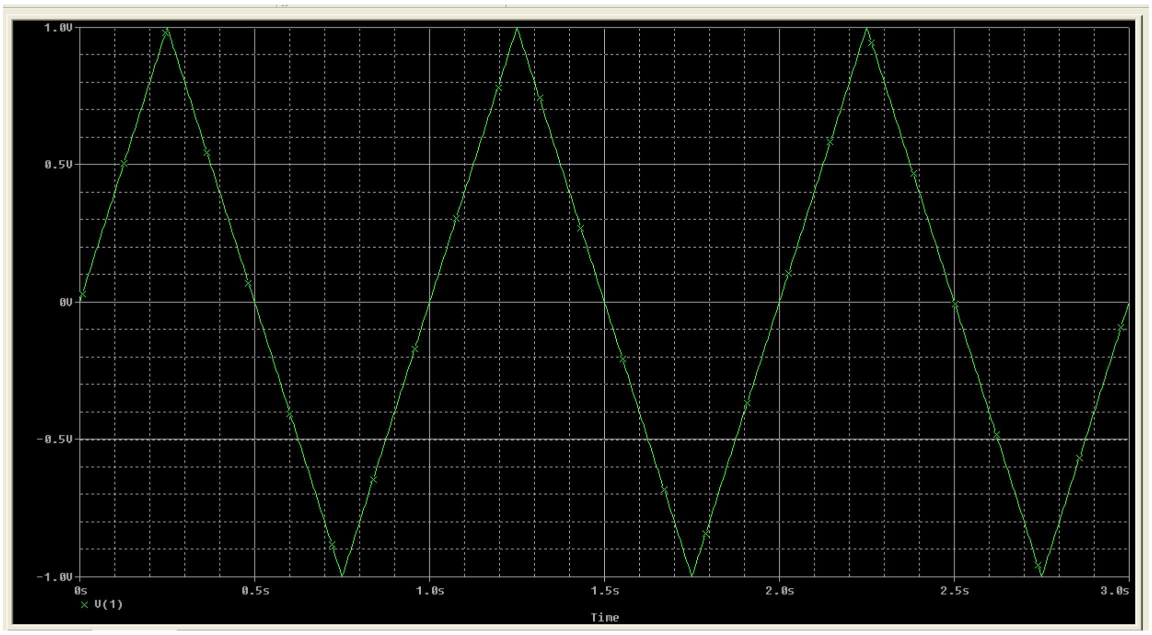


Fig. 4.12. Triangular wave input applied to the memristor.

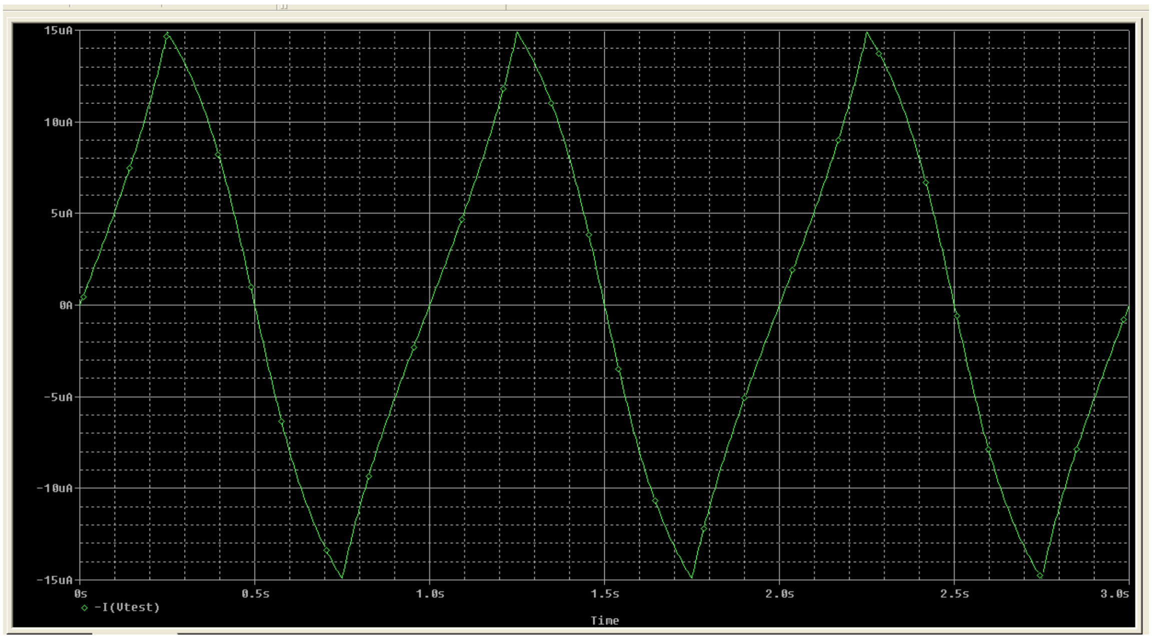


Fig. 4.13 Waveform of the current Imem through the memristor.

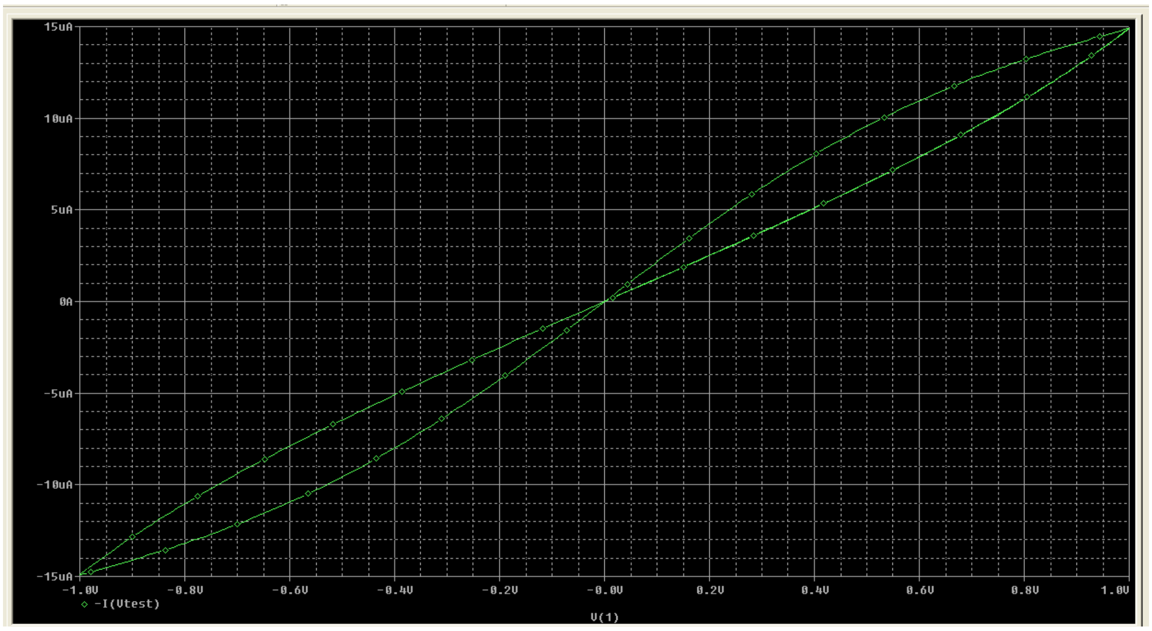


Fig. 4.14 Current-versus-voltage curve for input frequency of 1 Hz.

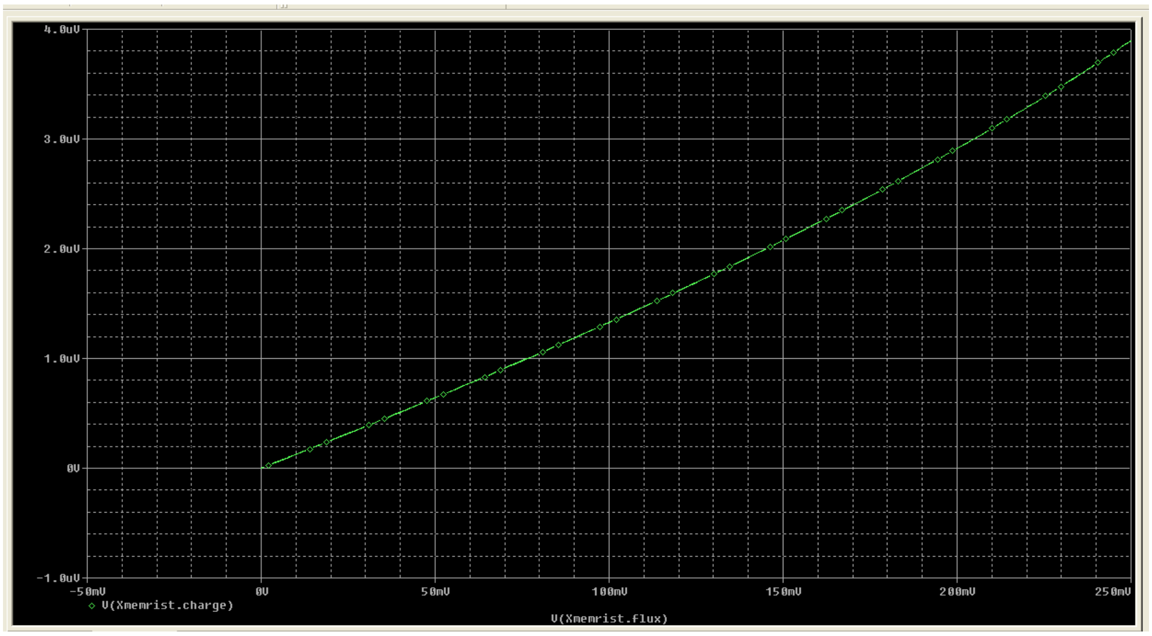


Fig. 4.15 Charge-versus-flux curve for memristor.

4.4 Square wave input to the memristor

Square wave input of peak to peak voltage (2v) and frequency (1HZ) is given to the memristor.

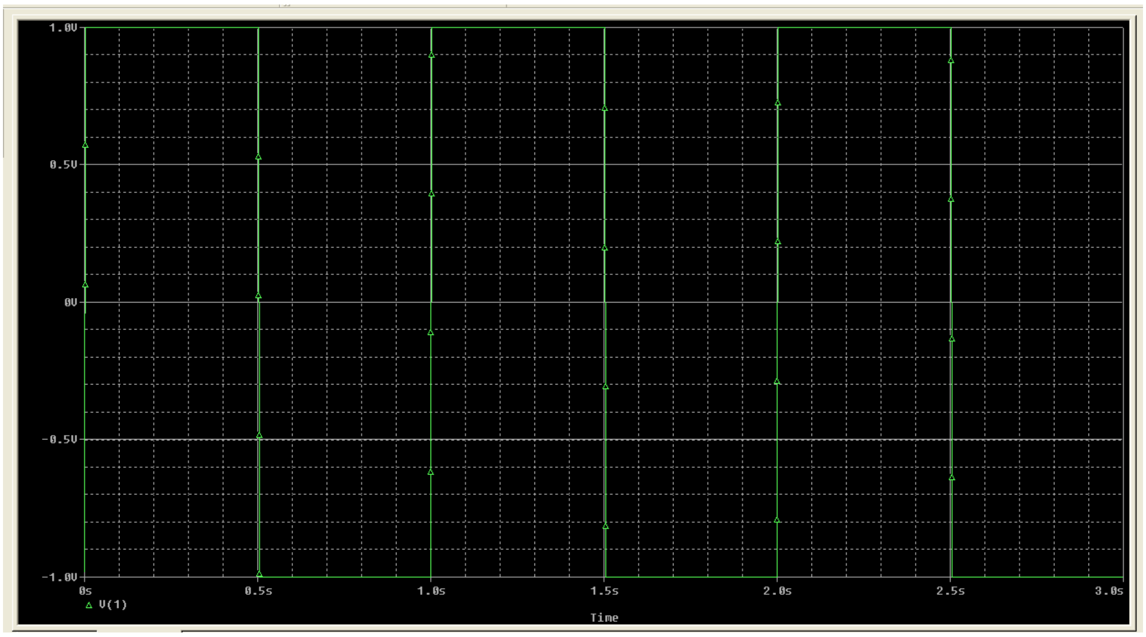


Fig. 4.16 square wave input applied to the memristor.

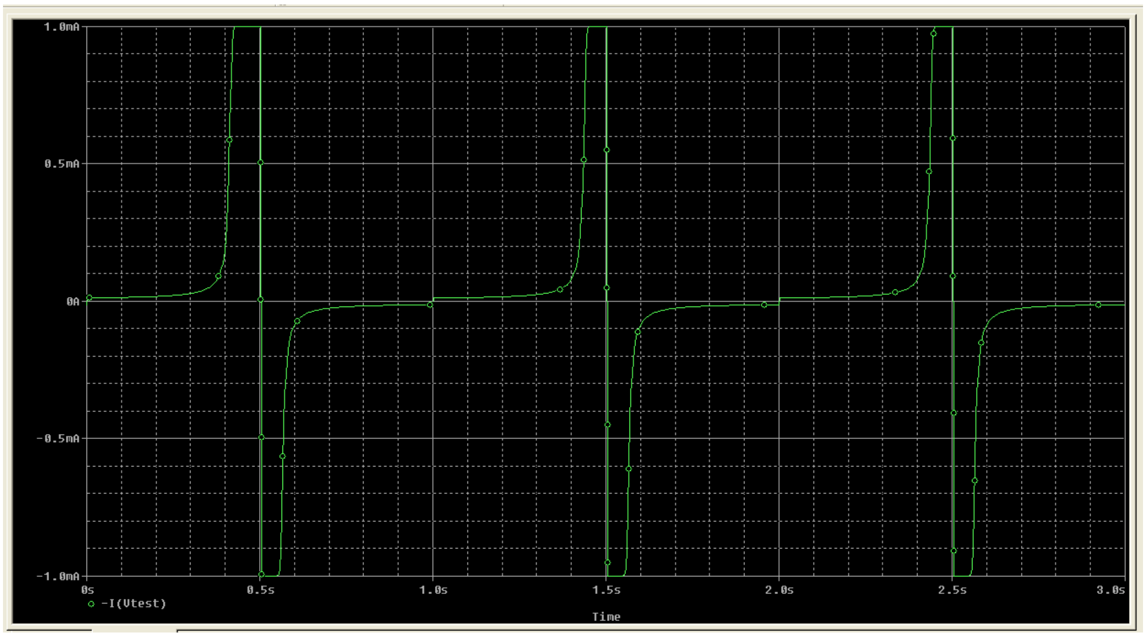


Fig. 4.17 Waveform of the current Imem through the memristor.

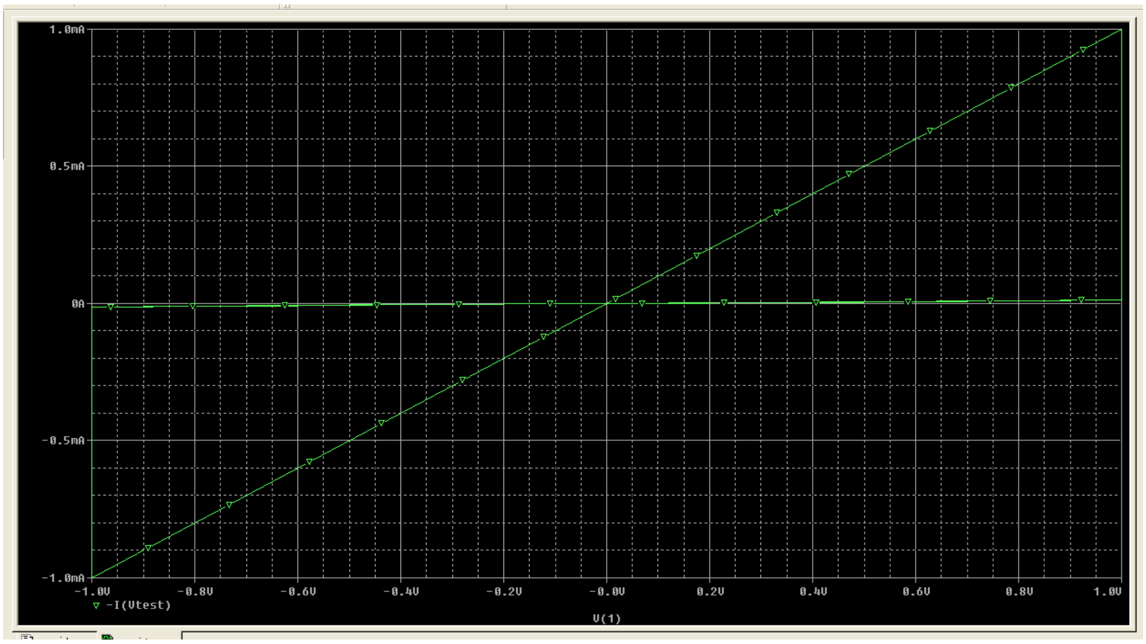


Fig. 4.18 Current-versus-voltage curve for input frequency of 1 Hz.

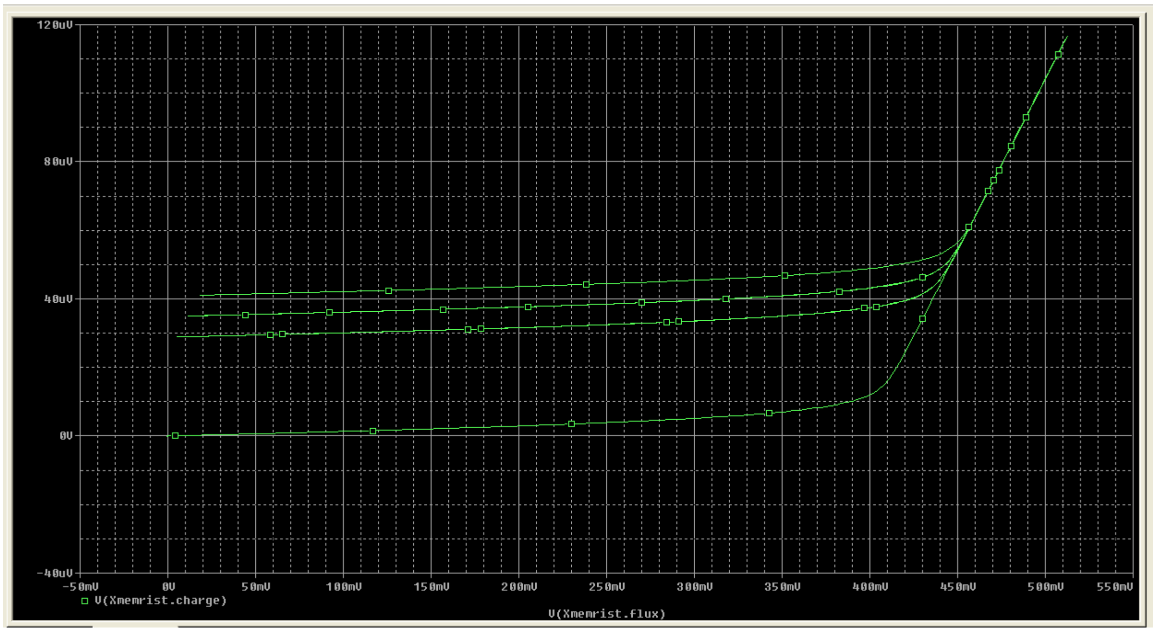


Fig.4.19 Charge-versus-flux curve for memristor.

Square wave input of peak to peak voltage (2v) and frequency (2HZ) is given to the memristor.

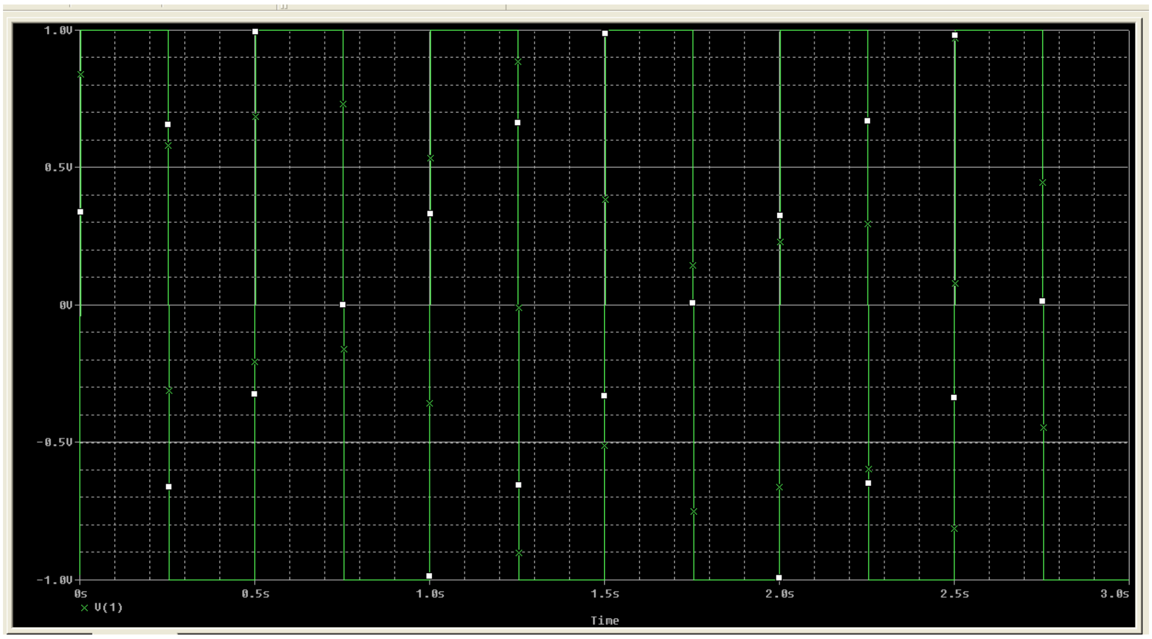


Fig. 4.20 square wave input applied to the memristor.

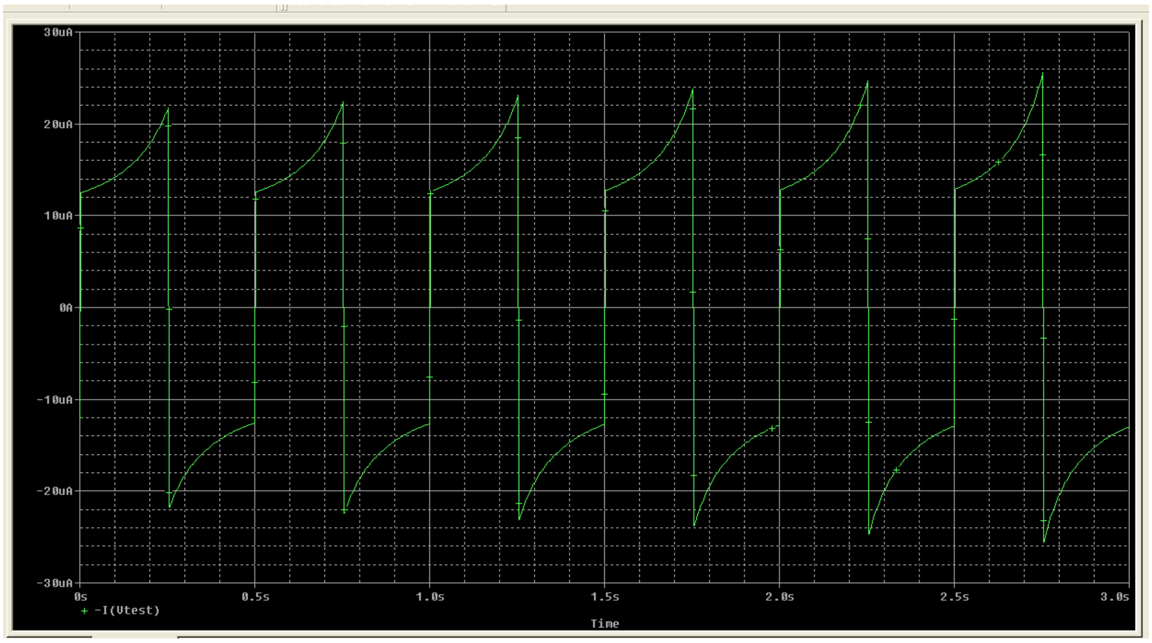


Fig. 4.21 Waveform of the current Imem through the memristor.

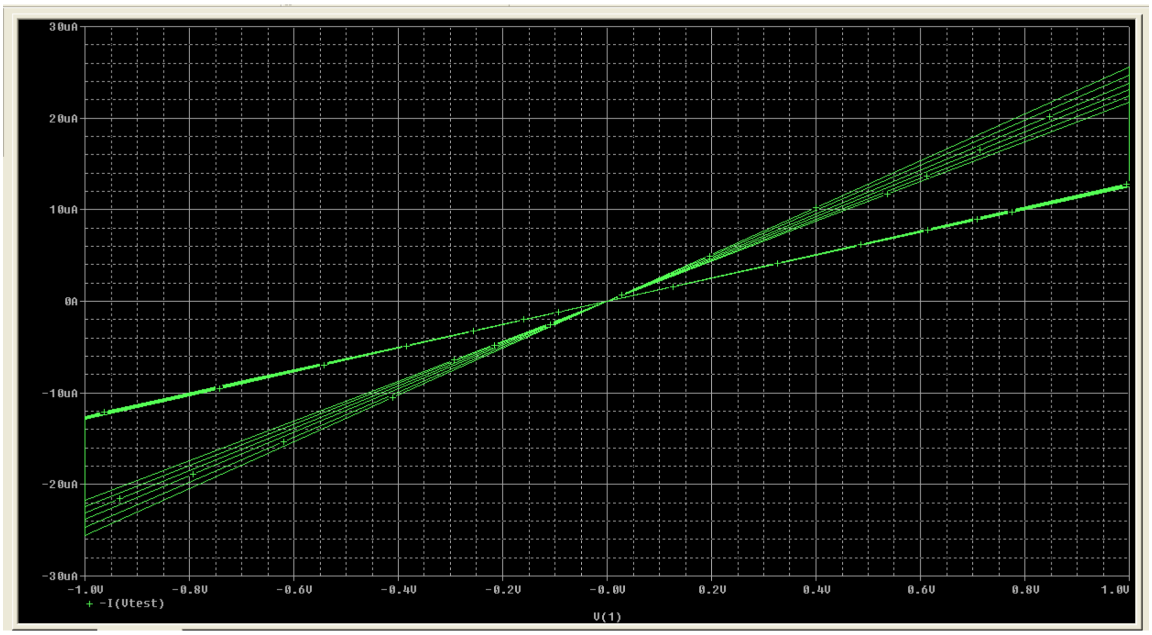


Fig. 4.22 Current-versus-voltage curve for input frequency of 2Hz.

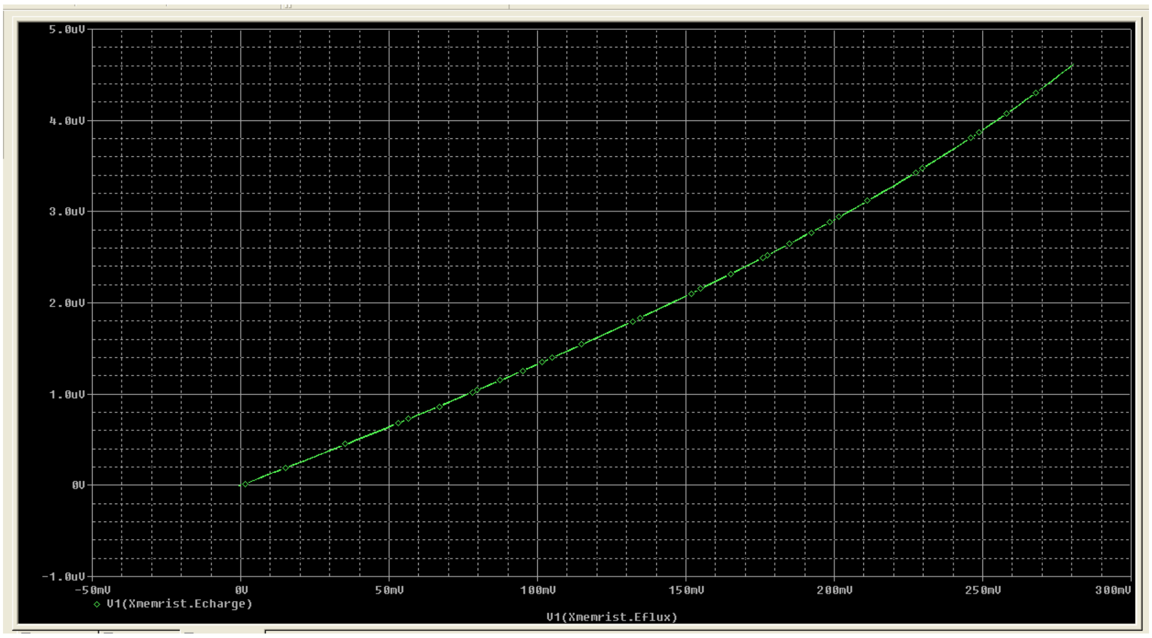


Fig. 4.23 Charge-versus-flux curve for memristor.

4.5 Parallel combination of memristors

When Sinusoidal input of voltage (1.2v) and frequency (1HZ) is given to the single memristor. The current I_{mem} through the memristor is shown below

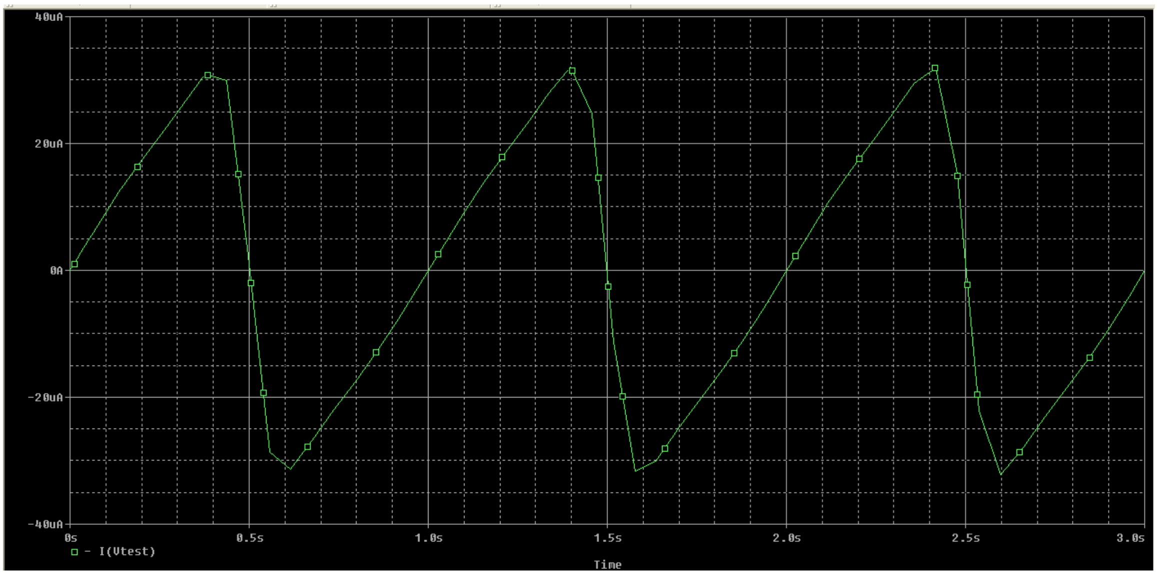


Fig. 4.24 Waveform of the current Imem through the memristor

When two memristors are connected in parallel. The resultant current Imem1 is twice the current through the single memristor.

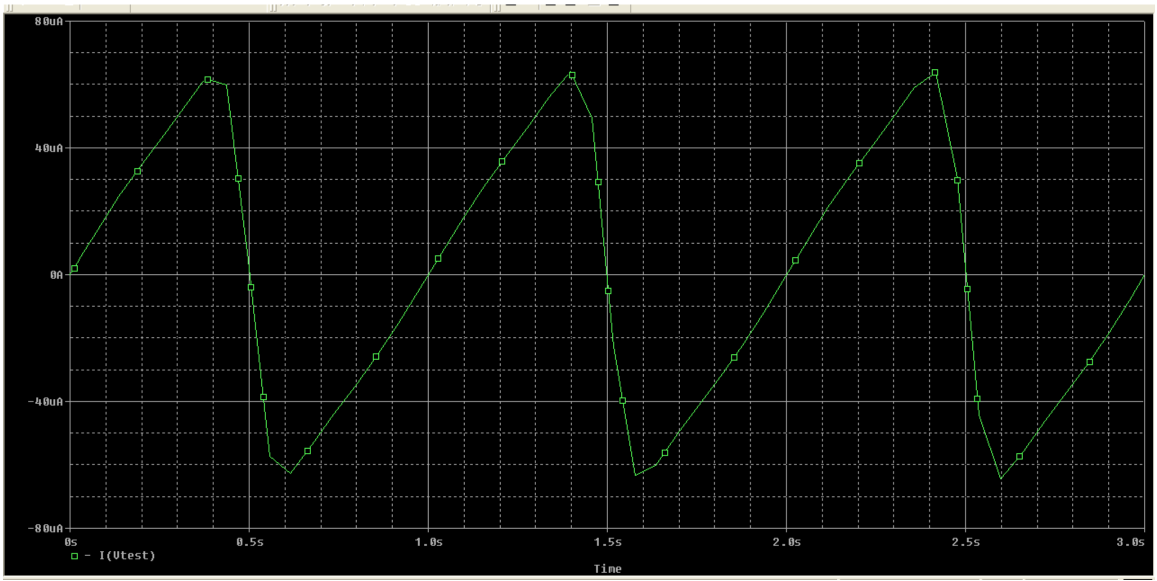


Fig. 4.25 Waveform of the resultant current Imem1 through the circuit.

One memristor is connected in opposite polarity to the other. There is change in shape of the signal and the current (Imem2)is decreased when compared to I_{mem1}.

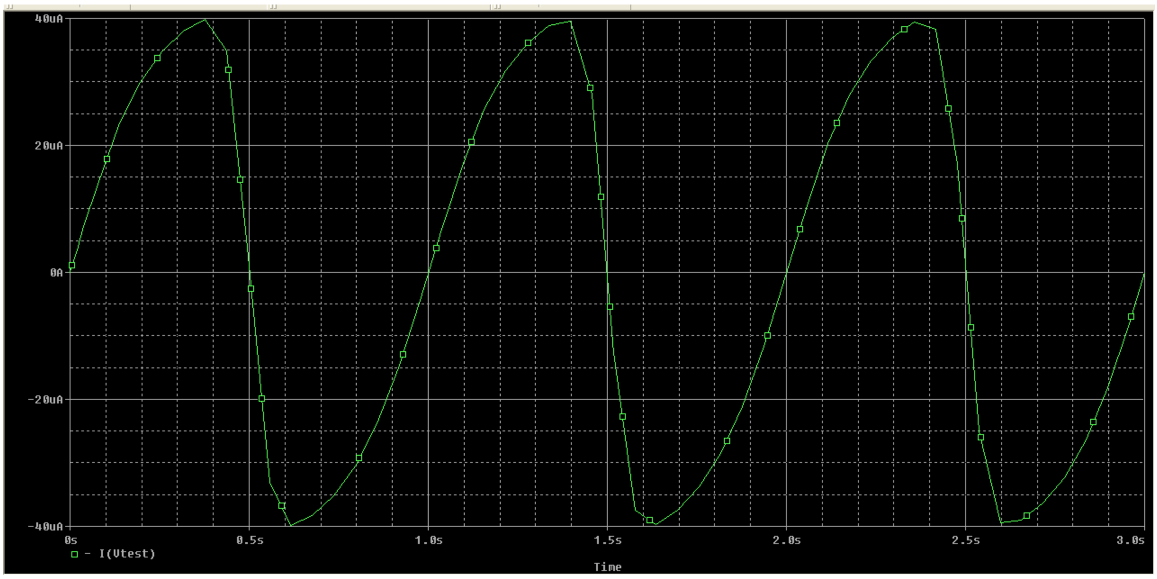


Fig. 4.26 Waveform of the resultant current Imem2 through the circuit.

When both the memristors are connected opposite polarity in parallel. The current (Imem3) changes the shape and decreases when compared to Imem1 and Imem1.

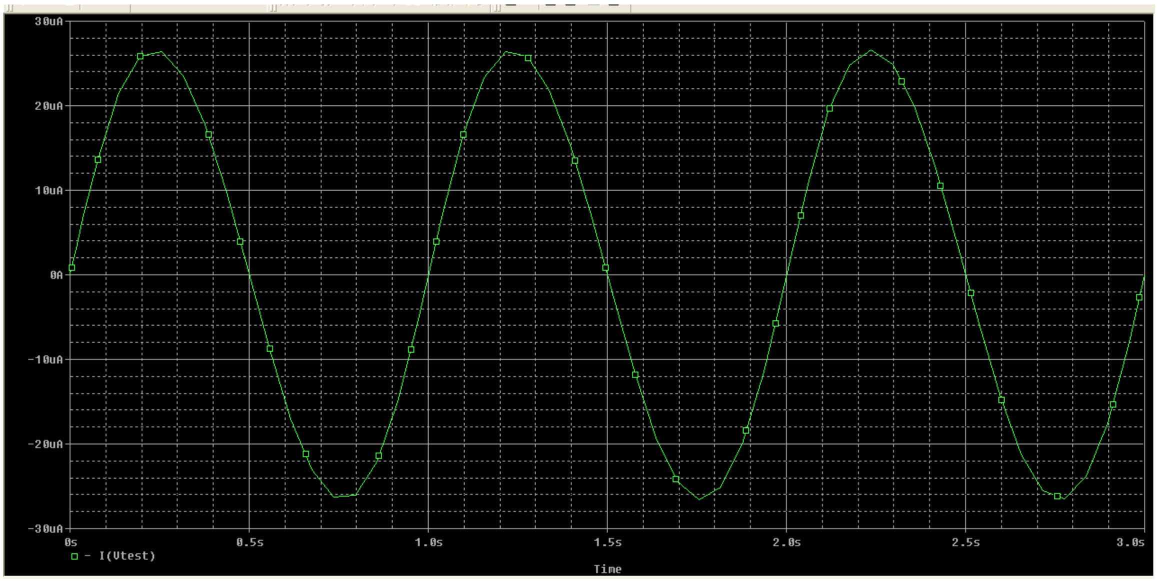


Fig. 4.27 Waveform of the resultant current Imem3 through the circuit.

4.6 series combination of memristors

When two memristors with the same polarity are connected in series i.e., doped region of second memristor is connected to the undoped region of first memristor. In this case, the memristive effect is retained because doped regions in both memristors simultaneously shrink or expand.

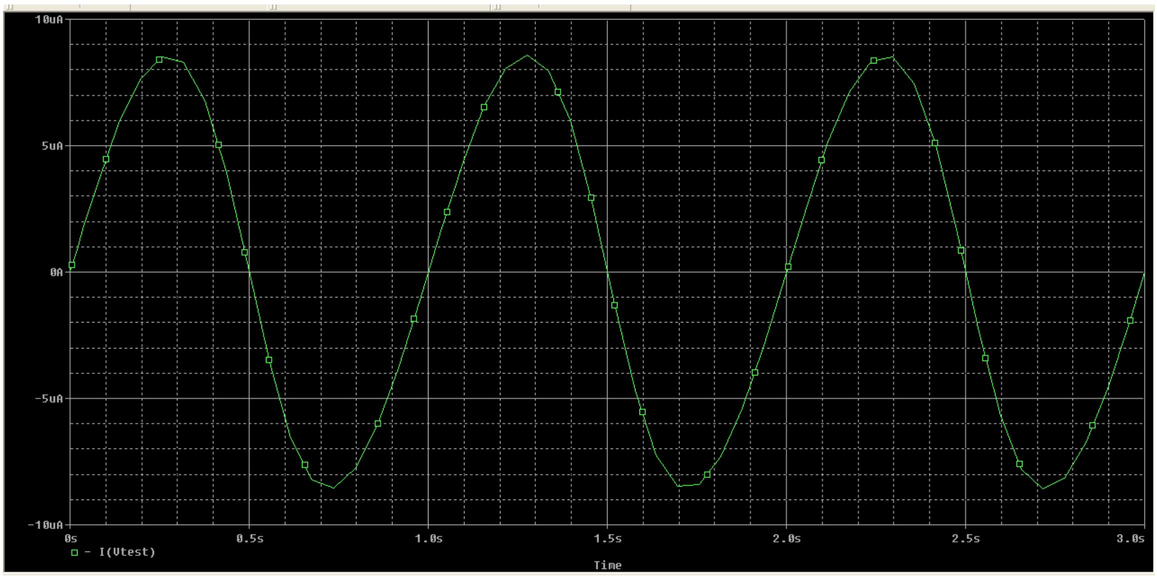


Fig. 4.28 Waveform of the resultant current Imem4 through the circuit.

When two memristors with opposite polarity in series i.e., undoped region of second memristor is connected to the undoped region of first memristor. The net memristive effect is suppressed.

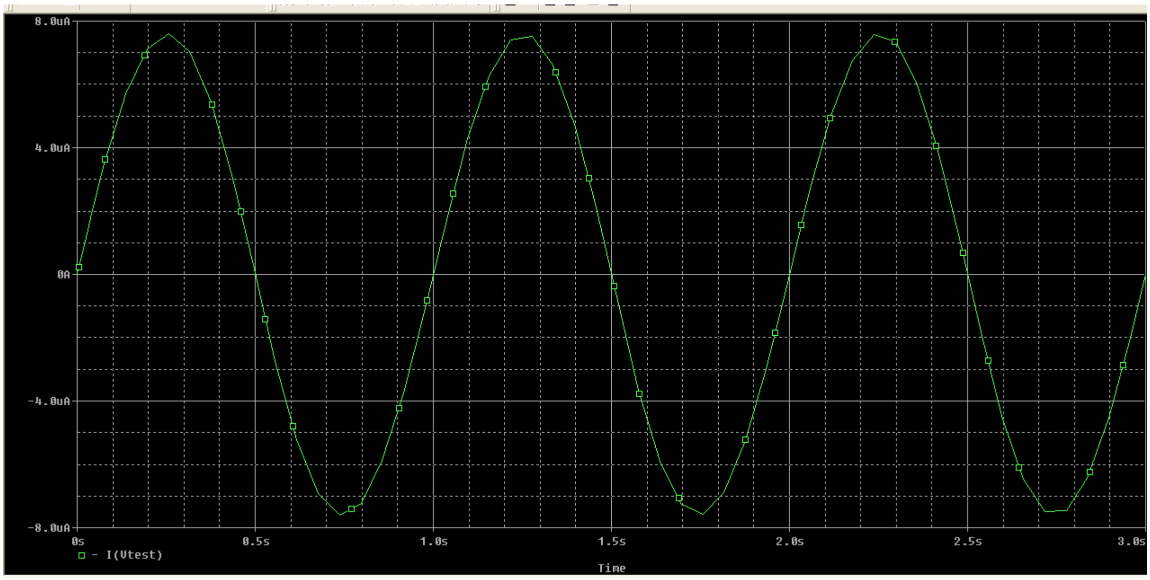


Fig. 4.29 Waveform of the resultant current Imem5 through the circuit.

4.7 Memristor integrator circuit

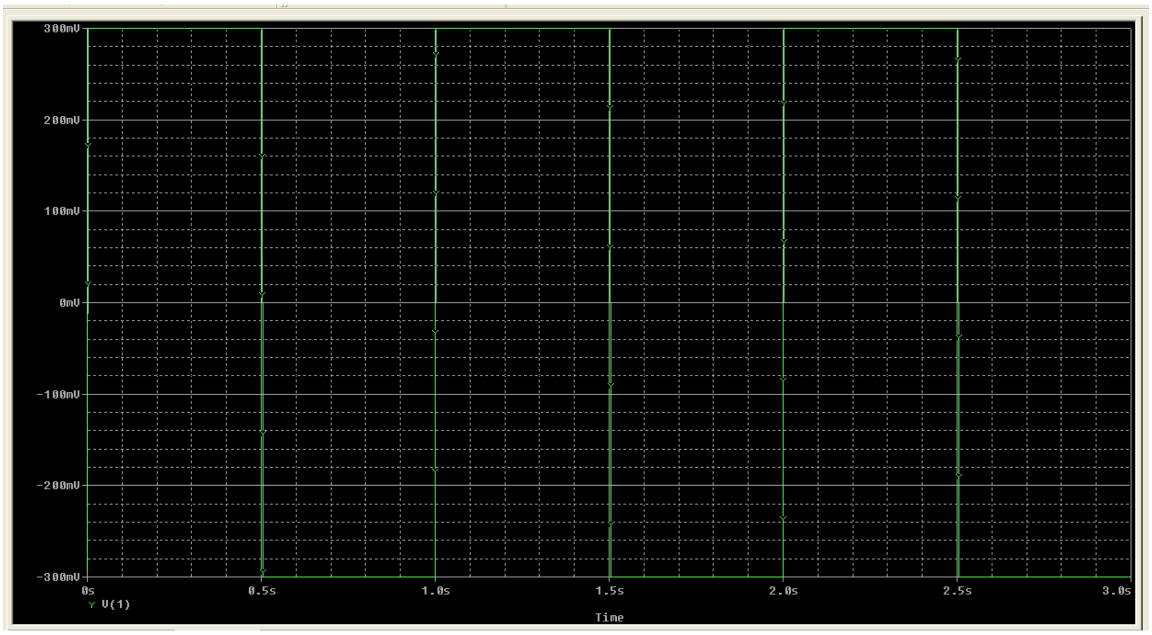


Fig. 4.30 square wave input applied to the integrator circuit

Output waveform

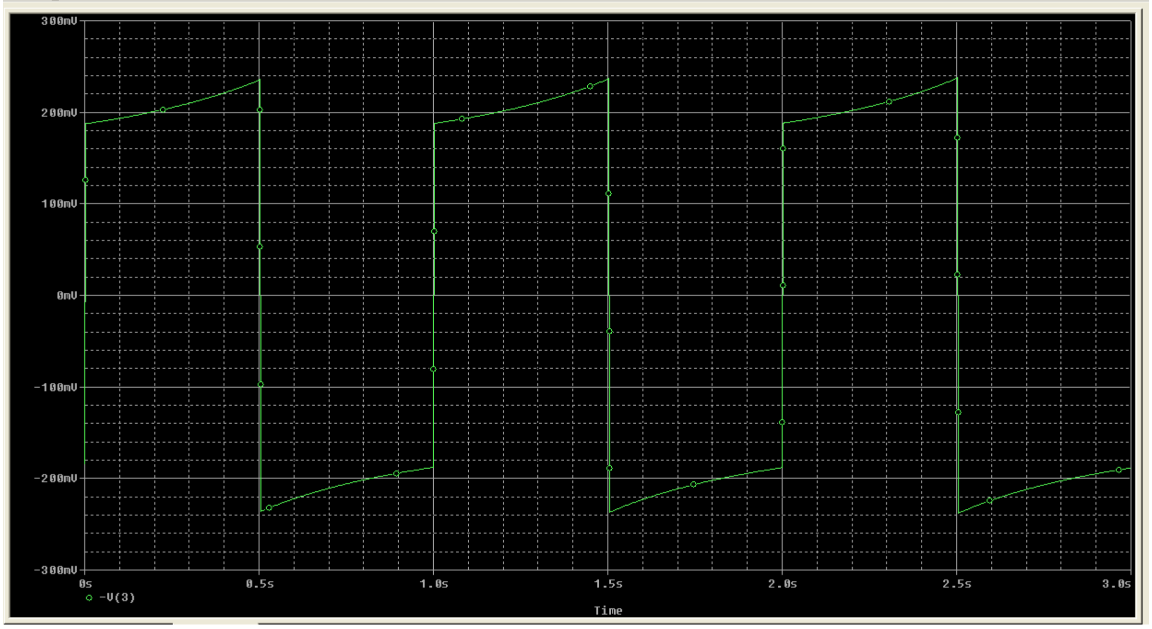


Fig. 4.31 Output waveform obtained to the integrator circuit.

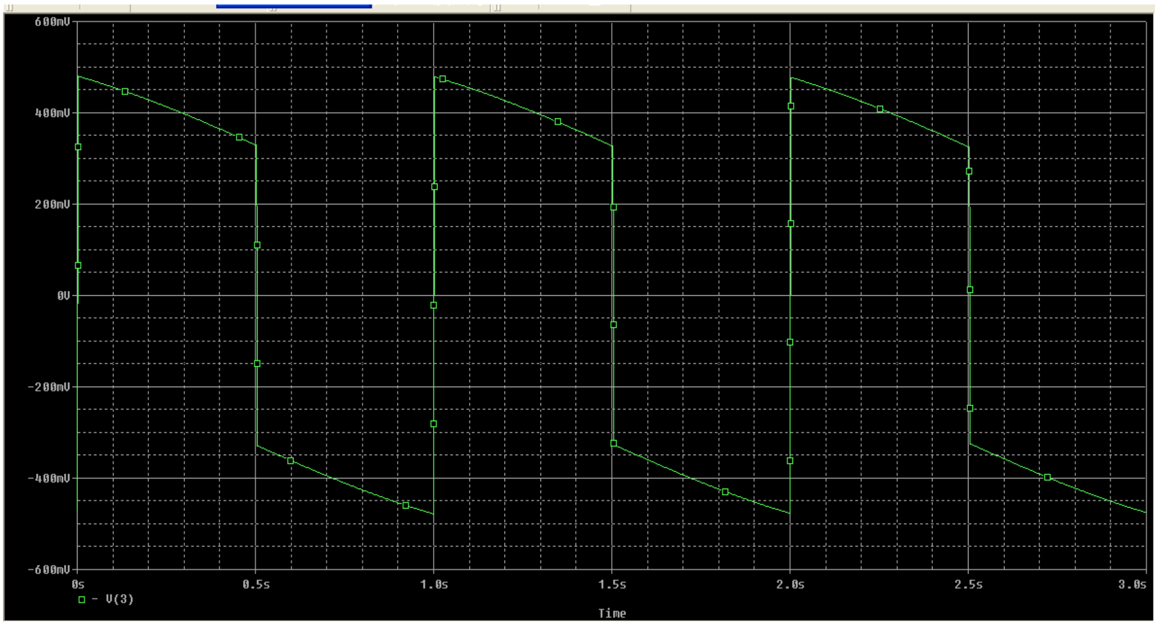


Fig 4.32.Spikes obtained at the output of memristor Opamp circuit

4.8 Transistor Amplitude Modulator:

For the simulations, the following parameters are used:

R_1=56KΩ
R_2=15KΩ
R_L=1KΩ
R_C=10KΩ
R_E=4.7KΩ
C_1=100nF, C_2=100nF and C_L=470pF

Message signal of frequency (2KHZ) and peak to peak voltage (4v)

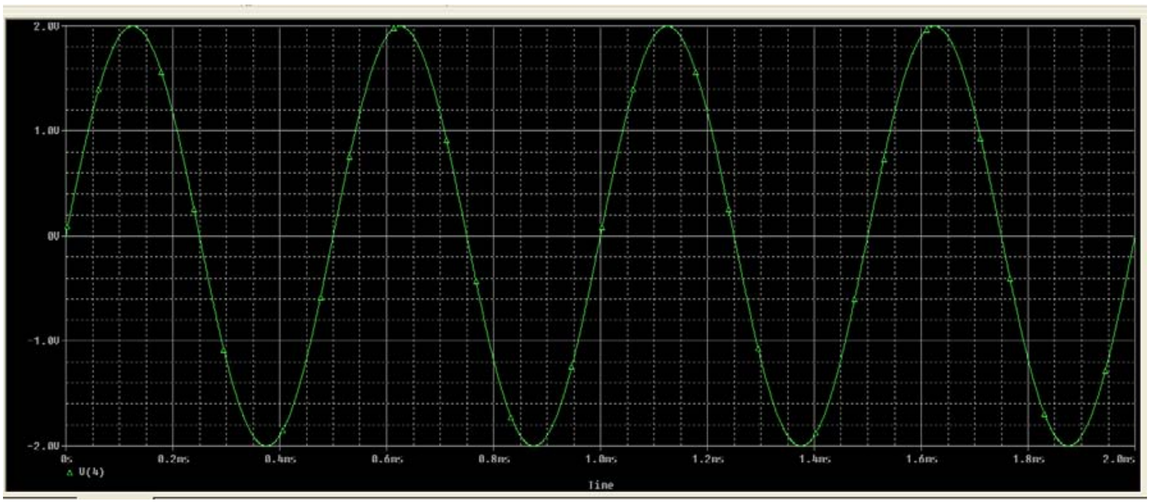


Fig 4.33 Modulating signal of frequency 2KHz

Carrier signal of frequency (200KHZ) and peak to peak voltage (20mv)

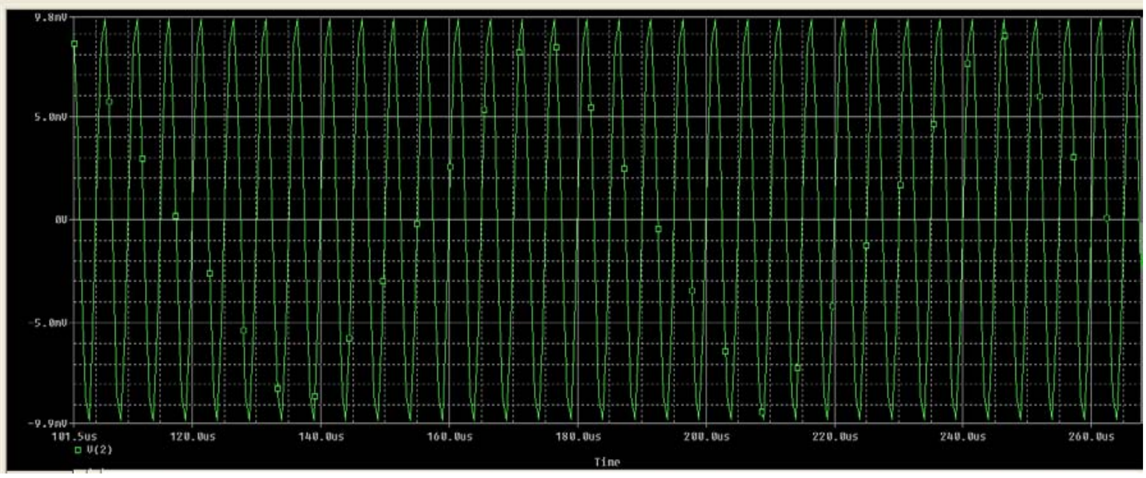


Fig 4.34 carrier signal of frequency 200KHz

Modulated signal

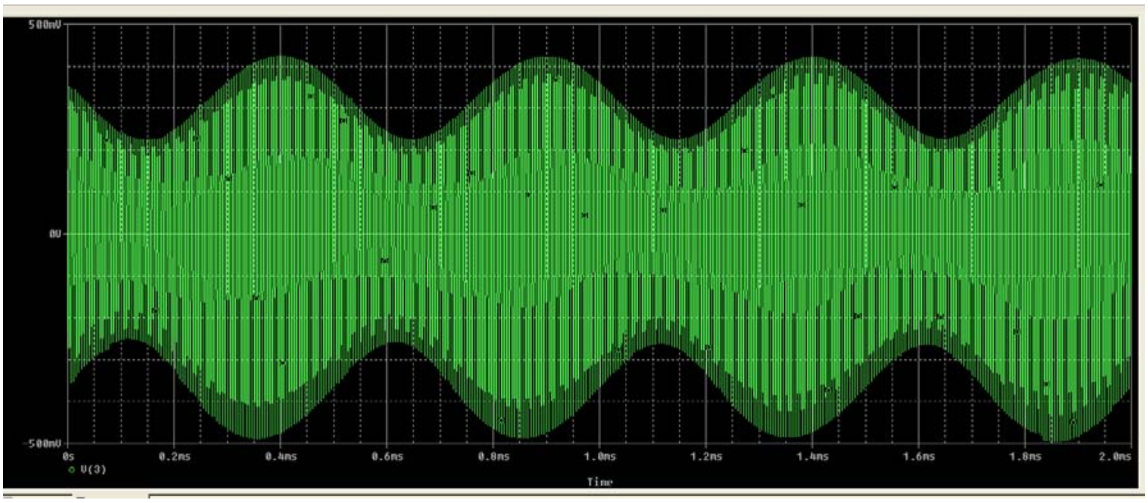


Fig 4.35 modulated signal

Spectrum of the modulated signal:

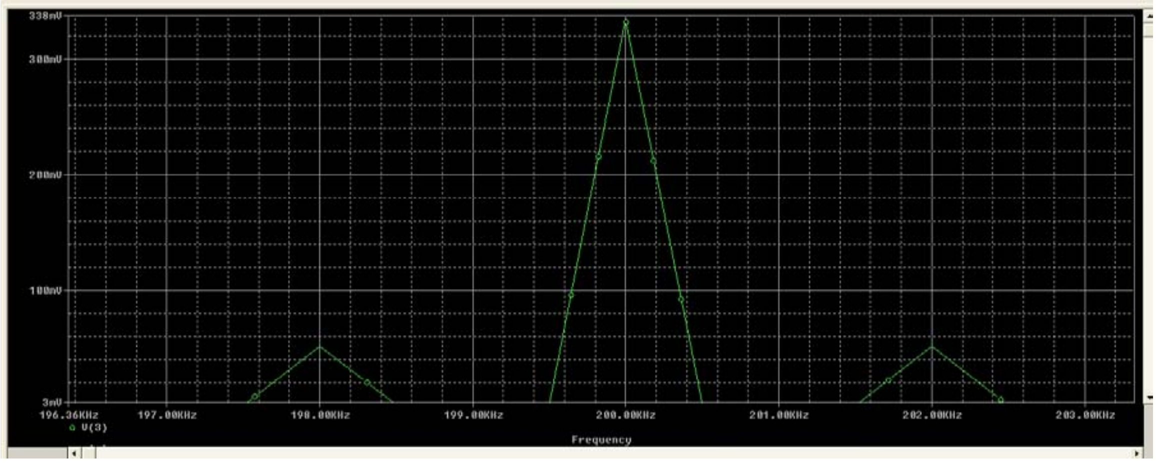


Fig 4.36 spectrum of the modulated signal

Transistor Amplitude Modulator with memristor:

In this analysis, Resistor R_E in Fig 3.13 is replaced with memristor

Message signal of frequency (2KHZ) and peak to peak voltage (4v)

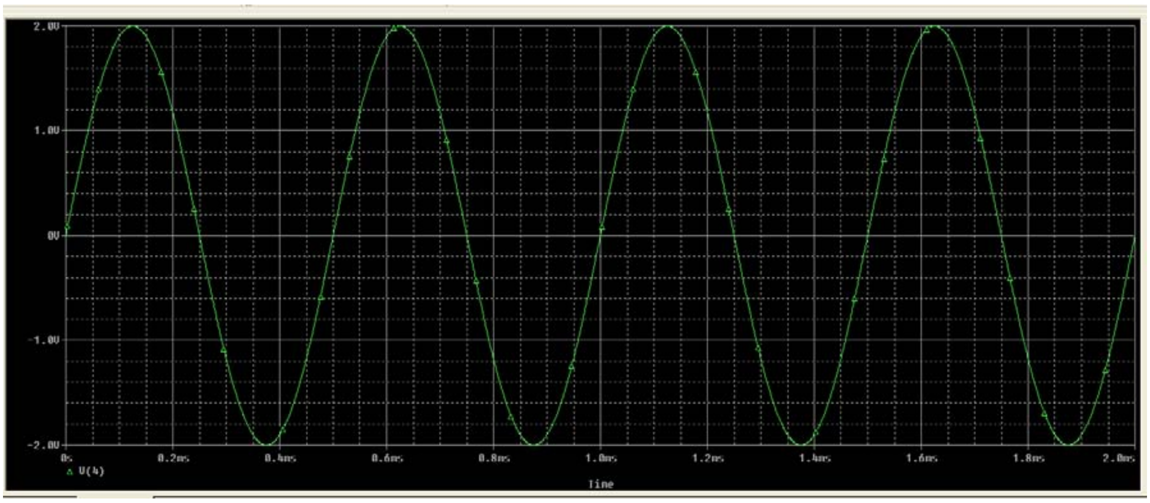


Fig 4.37 Modulating signal of frequency 2KHz

Carrier signal of frequency (200KHZ) and peak to peak voltage (20mv)

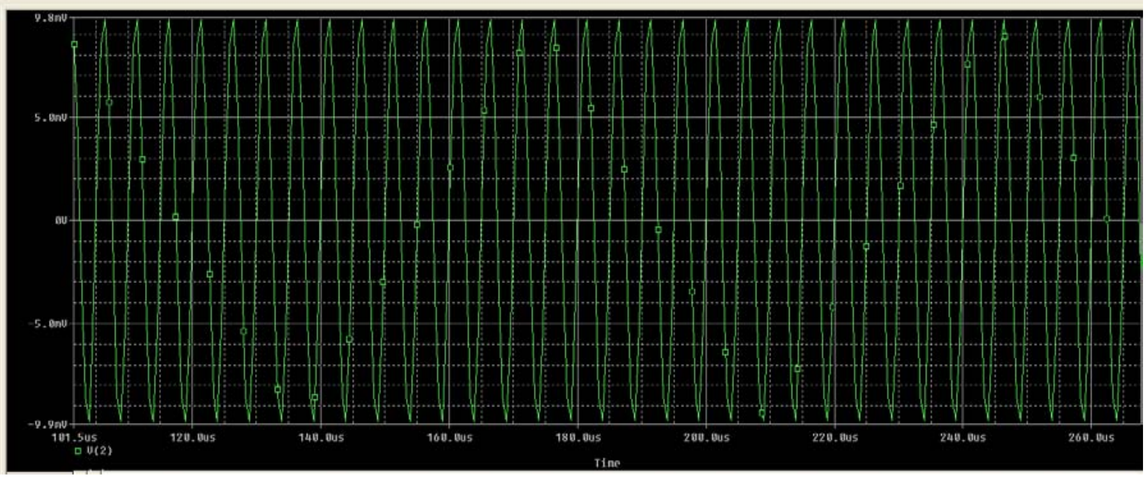


Fig 4.38 carrier signal of frequency 200KHz

Modulated signal:

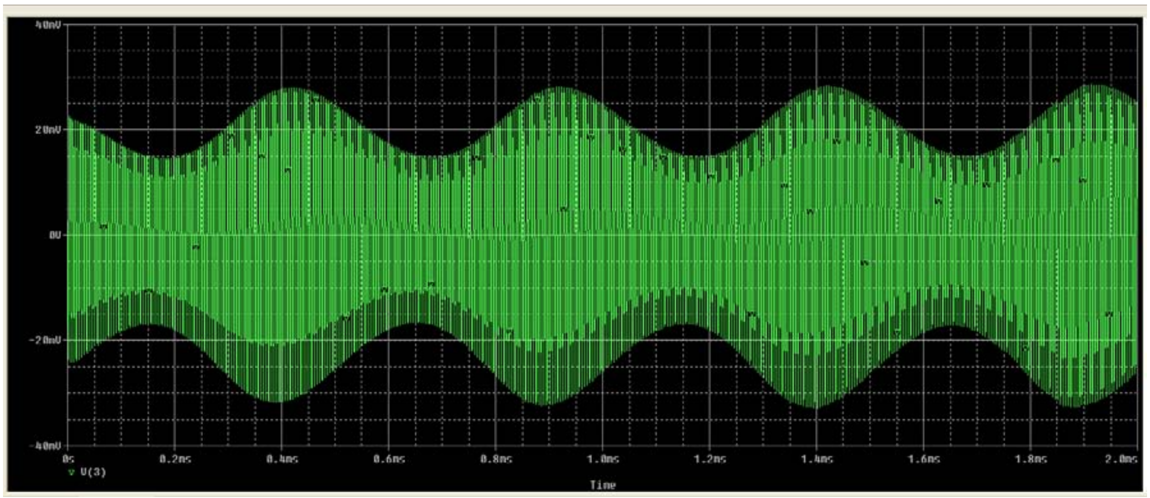


Fig 4.39 Modulated signal

Spectrum of the modulated signal:

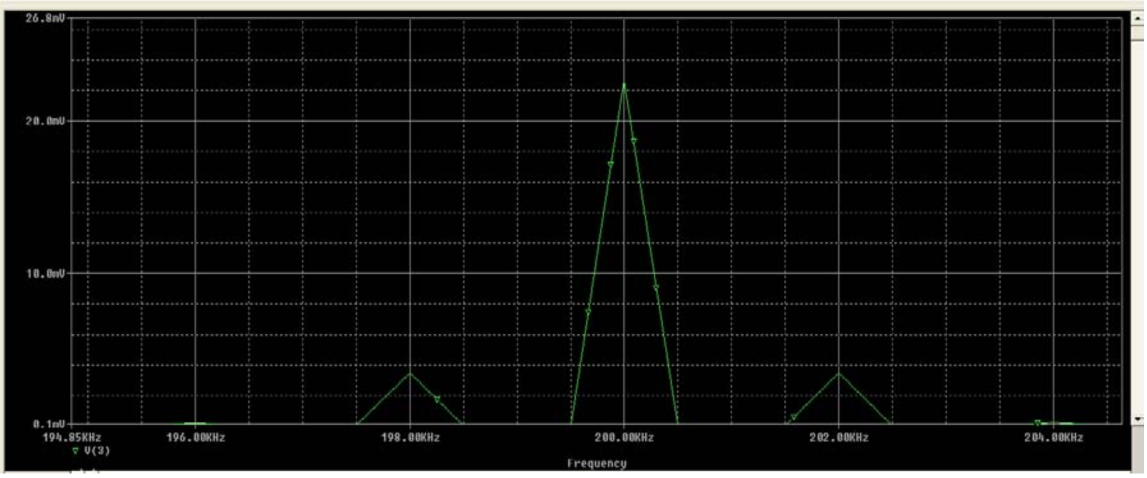


Fig 4.40 spectrum of the modulated signal

The memristor acts as a resistor because high frequency is applied to it. The amplitude change in modulated signal because of change in the resistor R_E only.

4.9 Amplitude Modulator with memristor

Amplitude modulator circuit with resistor:

Modulating signal: Sinusoidal input of voltage (0.01 v) and frequency (1Hz)

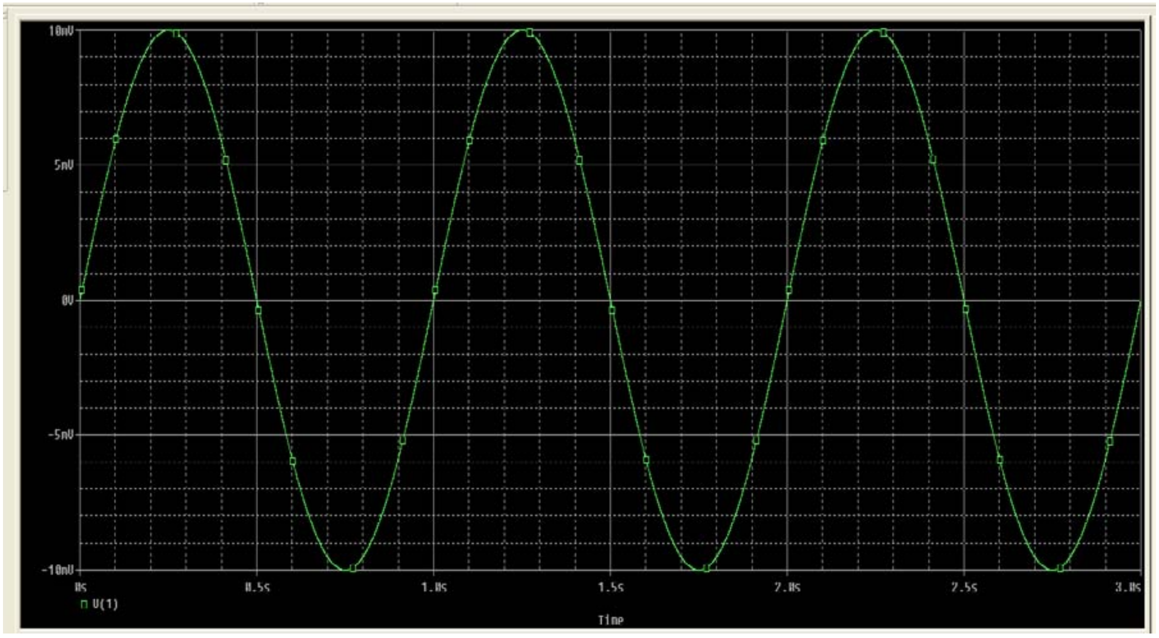


Fig 4.41 modulating signal of frequency 1Hz

Carrier signal:

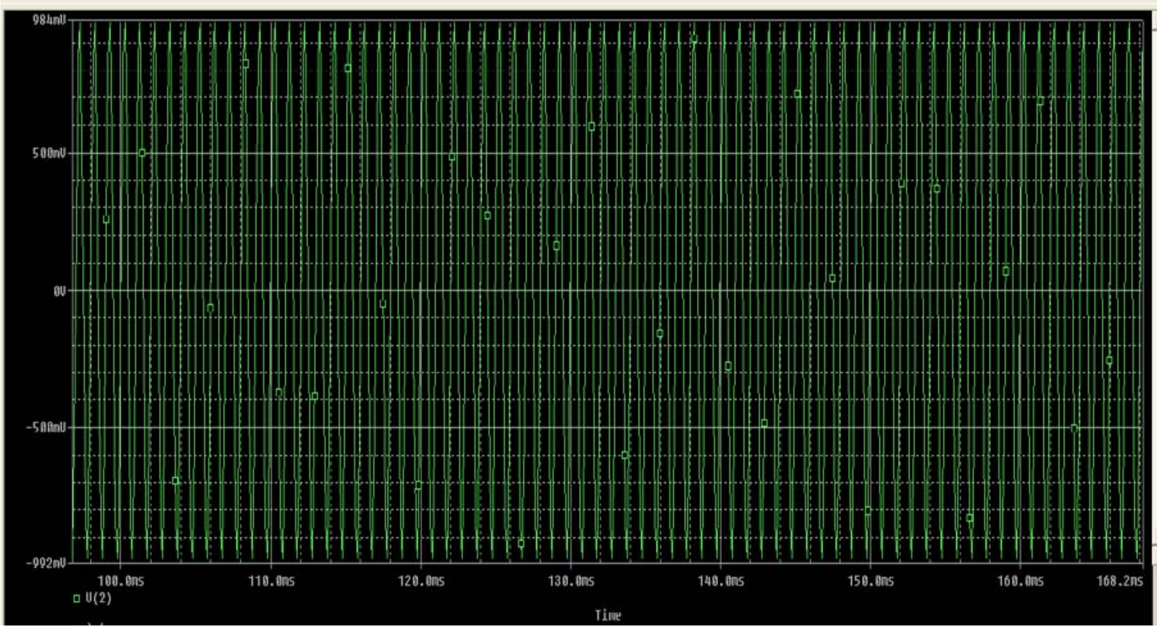


Fig 4.42 carrier signal of frequency 1KHz

Modulated signal

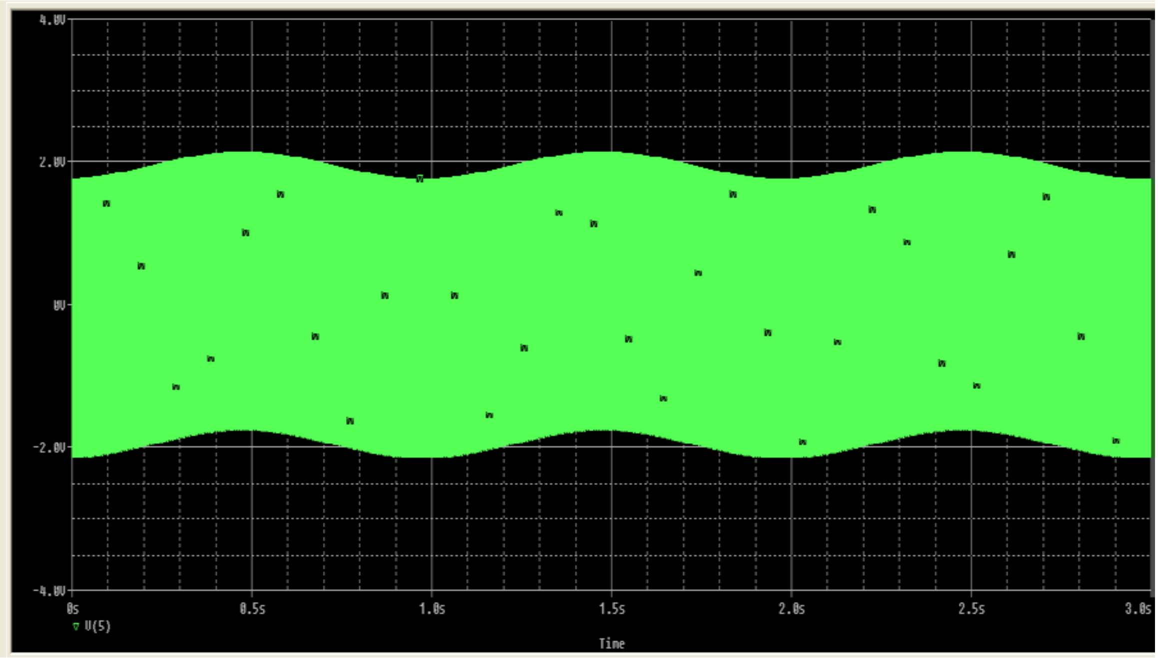


Fig 4.43 modulated signal

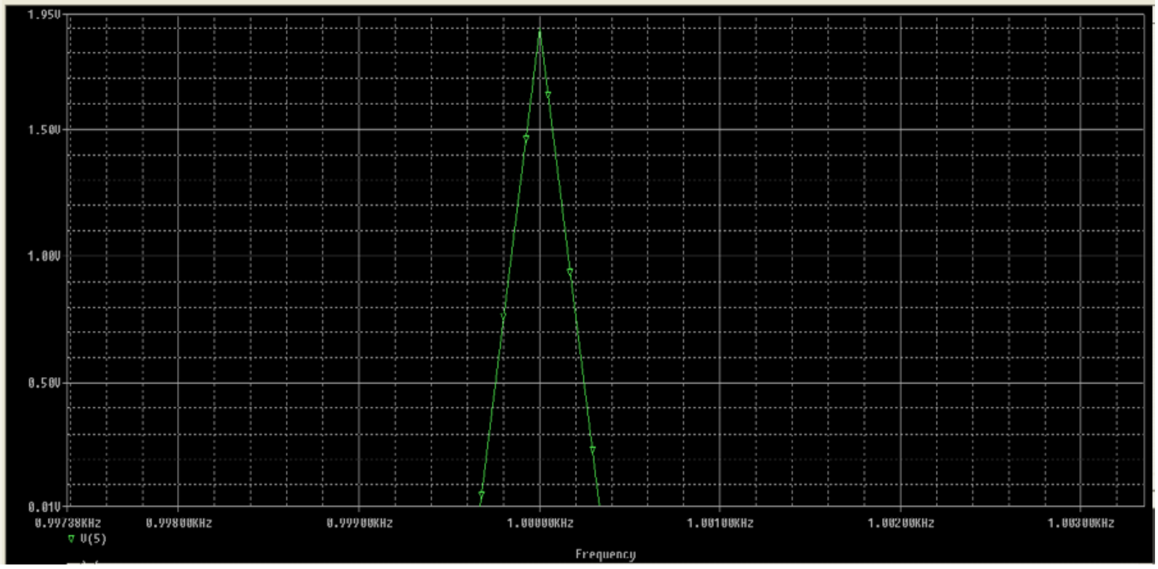


Fig 4.44 spectrum of the modulated signal

Amplitude modulator circuit with memristor:

Modulating signal: Sinusoidal input of voltage (0.01 v) and frequency (1Hz)

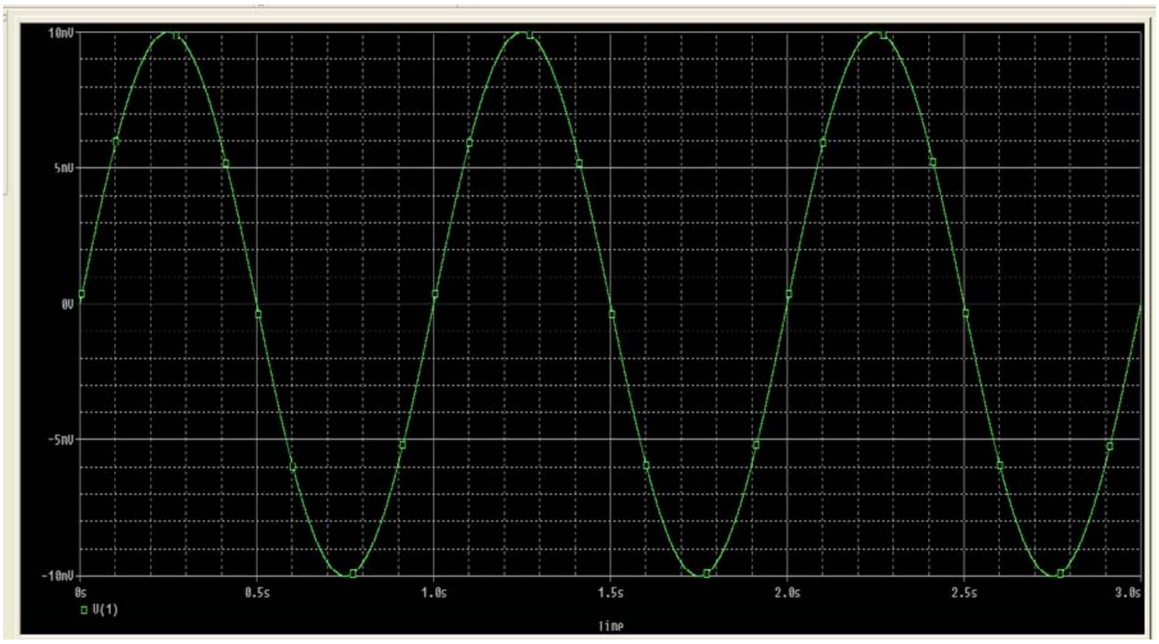


Fig 4.45 modulating signal of frequency 1Hz

Carrier signal:

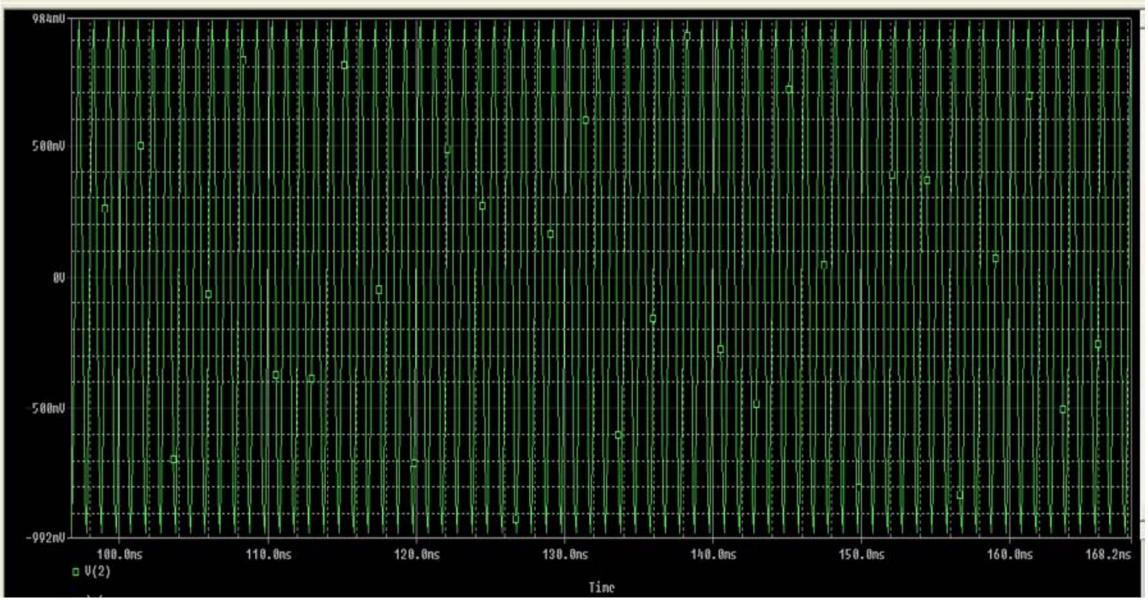


Fig 4.46 carrier signal of frequency 1KHz

Modulated signal

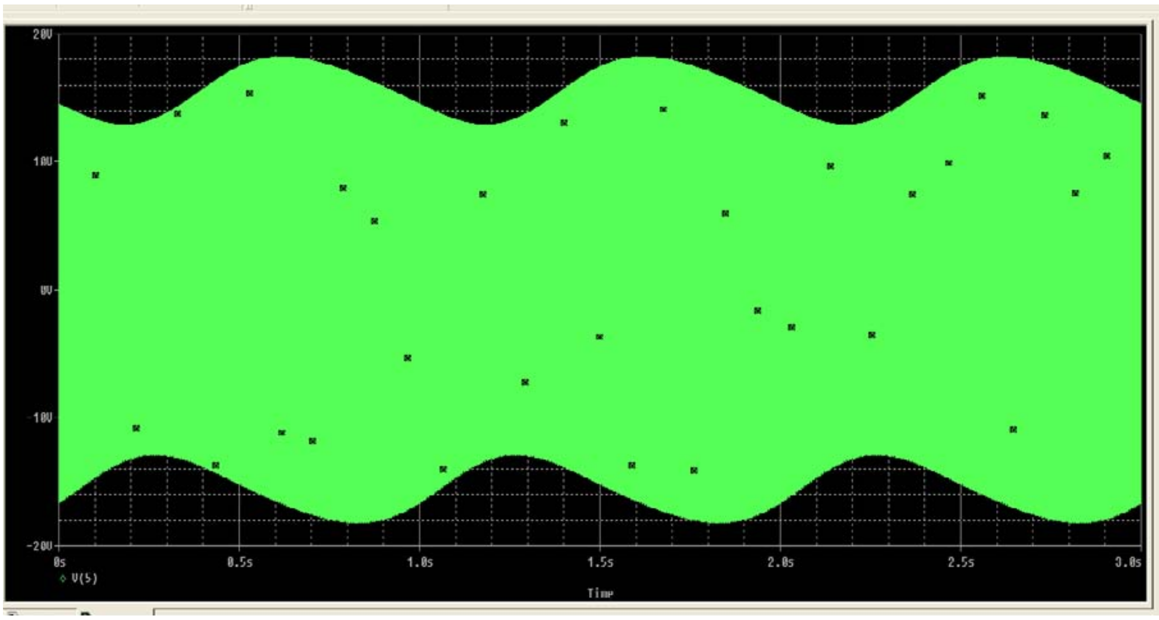


Fig 4.47 modulated signal

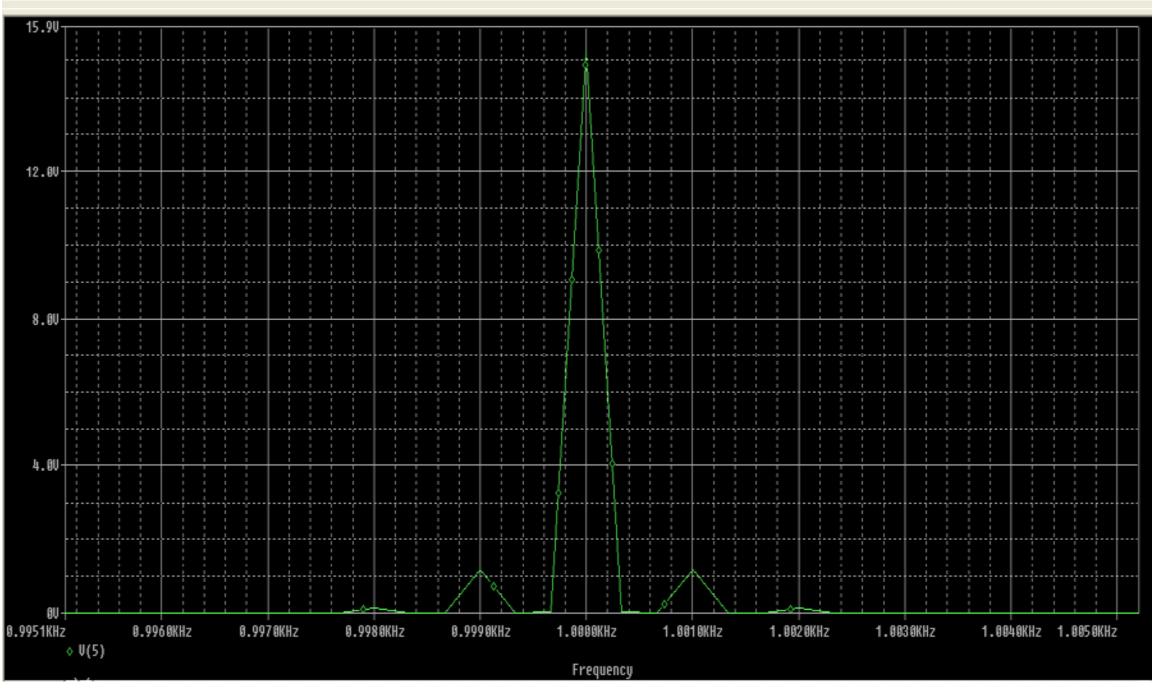


Fig 4.48 spectrum of the modulated signal

Modulating signal: Sinusoidal input of voltage (0.01 v) and frequency (10 Hz)

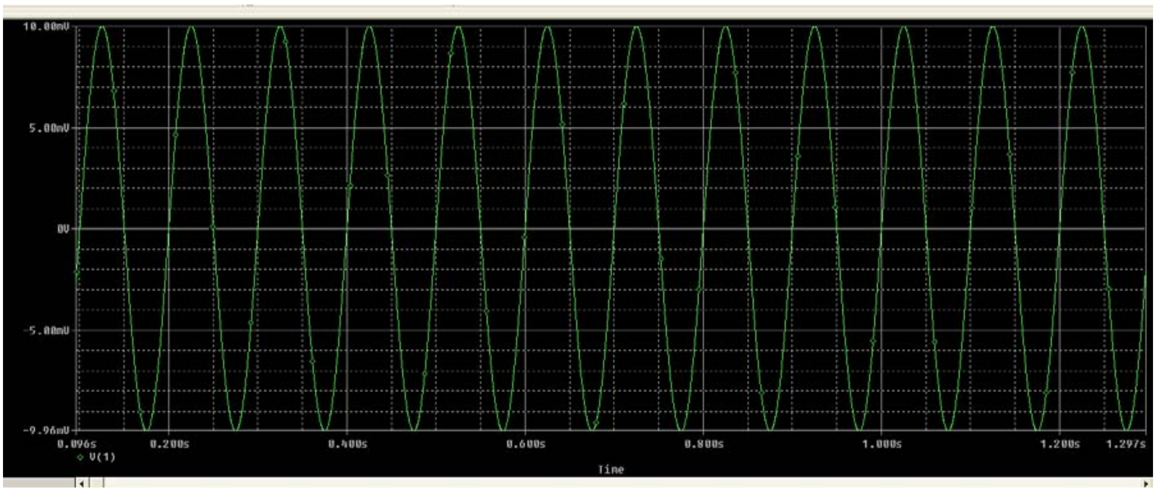


Fig 4.49 modulating signal of frequency 10 Hz

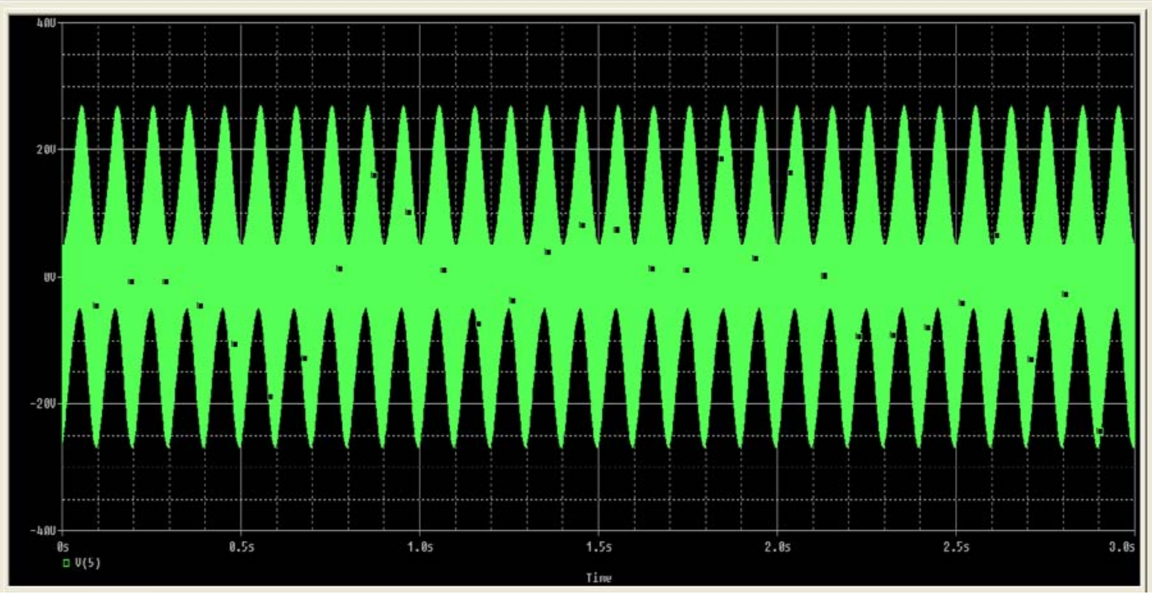


Fig 4.50 Modulated signal for 10Hz sinusoidal message signal

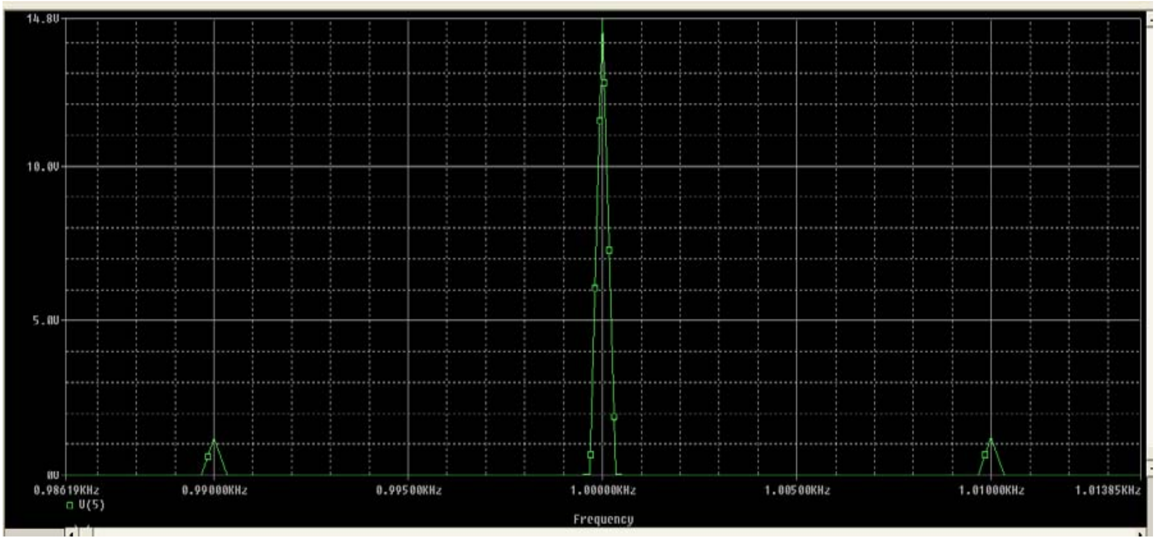


Fig 4.51 spectrum of the modulated signal

If we observe the spectrum of modulated signal of amplitude modulator with memristor and resistor, two side bands are absent in the spectrum of modulated signal of amplitude modulator resistor. Thus we can conclude that the amplitude modulator can be designed with the memristor.

Chapter 5

Conclusions and Scope for future work

5.1 Conclusions

In conclusion, the fundamentals of the memristor have been discussed. A brief survey of literature shows that the memristor is emerging as an important element in electronic circuits, although many problems need to be resolved. The PSPICE simulation of the memristor as a stand-alone element and its application in some simple circuits has been illustrated. The work shows that under the assumptions made in the current work, use of the memristor results in major changes in the low frequency regime. At high frequencies there is no difference between the memristor and a linear resistor. A performance parameter γ, that will enable the designers and fabricators to produce memristors of pre-determined performance is proposed.

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5.2 Scope for Future work

The current project has thrown up several issues for future work, such as increasing the limit of the frequency regime of operation of the memristor. Understanding on the exact nature of the memory effect is still open to debate. The dependence of performance of the memristor on physical parameters such as the width of the undoped layer, mobility of the oxygen ions and thickness of the layers is not completely understood. Recently, researchers have defined two new memdevices-memcapacitor and meminductor, thus generalizing the concept of memory devices to capacitors and inductors. These devices also show pinched hysteresis loops in two constitutive variables: charge versus voltage for the memcapacitor and current versus flux for meminductor.

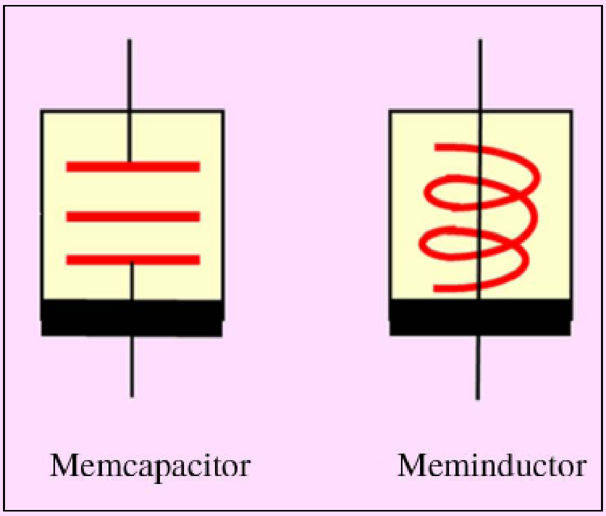

Fig. 5.1 Symbol of memcapacitor and meminductor [24]

Memristors are not lossless devices. As non-volatile memories, memristors do not consume power when idle but they do dissipate energy when they are being read or written. Hence, there is a need to invent lossless non-volatile device.

It is clear that there is much scope for work in the future both from the fundamental aspects and the applications of the memristor.

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Part2 http://www.youtube.com/watch?v=o9u9o_ToQwM

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