Some aspects of rare B decays and CP violation

by

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A thesis submitted

for the award of the degree of

Doctor of Philosophy in Physics

School of Physics

University of Hyderabad

Hyderabad 500 046

INDIA

January 2009

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Declaration

I hereby declare that the thesis entitled **Some aspects of rare B decays and CP violation** is based on the research work done by me at the School of Physics, University of Hyderabad, under the supervision of Dr. Rukmani Mohanta. No part of this thesis has been previously submitted for a degree or diploma or any other qualification at this University or any other.

(Barilang Mawlong)

Certificate

This is to certify that the thesis entitled **Some aspects of rare B decays and CP violation** is based on the research work done by Ms. Barilang Mawlong under my supervision at the School of Physics, University of Hyderabad, in fulfillment of the requirements for the award of the degree of **Doctor of Philosophy in Physics**. No part of this thesis has been previously submitted for a degree or diploma or any other qualification at this University or any other.

(Dr. Rukmani Mohanta)
Research Supervisor

(Dean, School of Physics)

Acknowledgements

At the onset, I would like to express my gratitude to my research supervisor Dr. Rukmani Mohanta for her guidance and support. I would like to thank her especially for the knowledge I gained in particle physics and also for all that I have learned from her.

I would like to thank the Dean of the School, Prof. Vipin Srivastava for the facilities provided by the School. In the same respect, I would like to thank the former Dean of the School, Prof. V. S. S. Sastry, as well.

For having imparted some of the basics of particle physics and also edifying me by posing a number of related and relevant questions, I thank Prof. Bindu Bambah. I am also thankful to Prof. M. Sivakumar and Dr. P. K. Suresh for my understanding of the other areas of high energy physics.

I would like to convey my sincere appreciation and gratitude to Prof. V. Srinivasan and Prof. S. Dutta Gupta, who with their constant support and encouragement, when I was an M.Sc. student, motivated me to aim higher even through difficult times. I would also like to acknowledge all my M.Sc. teachers for their invaluable lectures, some of which have left a longlasting impression on my mind. In this regard, I cannot fail to mention Prof. A. K. Kapoor, Prof. S. Chaturvedi and Prof. V. Seshubhai.

I would also like to acknowledge Dr. A. K. Giri, Punjabi University, Patiala, for the collaboration.

My thanks also goes to the non-teaching staff of the School who have been very helpful since my days as an M.Sc. student. I would like to acknowledge Mr. T. Abraham especially, whose patience and help, which has aided in the smooth functioning of the academic activities, can only be highly appreciated.

I acknowledge the Council of Scientific and Industrial Research (CSIR), Government of India for the financial support during my research period.

I would like to acknowledge Ajith, my high energy physics senior, for the useful discussions we had in theory room. I am extremely grateful to Sathish for his friendship and kindness and I am forever indebted to him for the continuous help and generosity. One of the most understanding persons I have ever come across is my good friend and advisor Rizwan. I thank him for the myriad discussions, some of which are very thought-provoking. His support, encouragement, practicality and maturity are highly commendable.

I cannot but appreciate my friends in the School of Physics during the three research years Preeti, Gnanavel, Joji, Lakshmi, Rajeeb and Saravanan for their friendship, the amiable atmosphere and the wonderful memories that I will forever treasure. For the memorable times in the classroom and outside it, I would like to thank all my M.Sc. classmates especially Shinto and Srinivas Rao who are also my fellow research scholars.

I would like to express my gratitude to my once-upon-a-time local guardian Cherry and her sister Samanda for the hospitality and cordiality bestowed upon me. I also would like to thank all the Khasi people who are or were in Hyderabad for their help and support in one way or the other. Two of whom I cannot discount are Dr. Gracious Temsen and Dr. Eros Kharshiing. I extend my gratitude to Palavi Borah who has been a good friend in the hostel and outside it and also for having been there for me and for actually caring. Thanks also goes to Lanu for her friendship and for the fun and memorable times we have had together.

I am forever thankful to Iada Martin for the love, succour, care and for actually believing in me. I express my deep gratitude and appreciation for the understanding, patience, loyalty and all that you have done for me. I also would like to acknowledge my best friend Beshiela for the support when I needed the most, for the good times and the bad times as well, for the beautiful friendship and for being my confidante. I ask for forgiveness and pardon if in case I have forgotten to mention anyone who has been there for me over the years.

Last but not at all the least, I would like to thank my family. I know that words will never be enough to thank my family members, especially my mother, for all that they have done for me. I am thankful to have had a wonderful human being in my father and I will forever cherish all the things he had done. The tremendous sacrifices that my mother has made over the years are very much appreciated and I am immensely grateful to her for her love and support. I would not be who and where I am today if not for her. I am also grateful and thankful to my loving aunt and uncles for their care and support. Finally, I would like to express my love and thanks to my sisters, brother-in-law, nieces and nephew who have added a new dimension to the word happiness in my life.

Abstract

In this thesis, we present our study of CP violation in the B meson system and also a study of some of the rare B decay modes, where we look for the possible manifestation of new physics.

We consider the possibility of extracting the CKM angle γ with B_c decays. The modes $B_c^{\pm} \to (D^0)D_s^{\pm} \to (K^{*+}K^-)D_s^{\pm}$ and $B_c^{\pm} \to (\bar{D}^0)D_s^{\pm} \to (K^{*+}K^-)D_s^{\pm}$ are found to be well suited for the extraction of γ. Since a large number of B_c mesons are expected to be produced at the LHC, it would be very interesting to explore the determination of γ with these modes.

The decay channels $\overset{(-)}{B}_d \to \overset{(-)}{D}{}^0 \overset{(-)}{K}{}^{*0}$, $\overset{(-)}{D}{}^{*0} \overset{(-)}{K}{}^{*0}$ are then investigated for extracting weak CKM phase information γ and $2\beta+\gamma$. These channels are described by color suppressed tree diagrams only and are free from penguin contributions. The methods presented here may be well suited to determine the CKM angle γ and the combination $2\beta+\gamma$.

We then consider the hadronic decay modes $B^{\pm(0)} \to f_0(980)K^{\pm(0)}$, involving a scalar and a pseudoscalar meson in the final state. We compute the branching ratio and the CP asymmetry parameter both in the SM and in the R-parity violating (RPV) supersymmetric model. These decay modes are dominated by the loop induced $b \to s\bar{q}q$ ($q = s,\ u,\ d$) penguins along with a small $b \to u$ tree level transition (for $B^+ \to f_0K^+$) and annihilation diagrams. Therefore, the standard model expectation of direct CP violation is negligibly small and the mixing-induced CP violation parameter in the mode $B^0 \to f_0K_S$ is expected to give the same value of $\sin(2\beta)$, as extracted from $B^0 \to J/\psi K_S$ but with opposite sign. Using the generalized factorization approach, we find the direct CP violation in the decay mode $B^+ \to f_0 K^+$ to be of the order of a few percent. We then study the effect of the R-parity violating supersymmetric model and show that the direct CP violating asymmetry in $B^+ \to f_0(980)K^+$ could be as large as $\sim 80\%$ and the mixing-induced CP asymmetry in $B^0 \to f_0 K_S$ (i.e., $-S_{f_0 K_S}$) could deviate significantly from that of $\sin(2\beta)_{J/\psi K_S}$.

The results of the study of the decay modes $B \to f_0 K(\pi)$, with f_0 being $f_0(1370, 1500)$, in the SM are also presented.

Finally, we investigate the effect of an extra fourth quark generation and FCNC mediated Z and Z' bosons on the rare decay mode $B^- \to \phi\pi^-$. In the standard model, this mode receives only $b \to d$ penguin contributions and therefore highly suppressed with branching ratio $\sim 5 \times 10^{-9}$. This in turn makes this mode a very sensitive probe for new physics. We find that due to the above mentioned new physics contributions, there is a significant enhancement in its branching ratio. Furthermore, the direct CP violation parameter which is identically zero in the SM is found to be quite significant.

List of Publications

1. "***Probing new physics in*** $B \to f_0(980)K$ ***decays*** ", A. K. Giri, **B. Mawlong** and R. Mohanta, Phys. Rev. D **74**, 114001 (2006).

2. "***Charmless nonleptonic*** B ***decays into scalar and pseudoscalar mesons***", **B. Mawlong**, Pramana - Journal of Physics **69**, 521 (2007).

3. "***Determining the CKM angle*** γ ***with*** B_c ***decays***", A. K. Giri, **B. Mawlong** and R. Mohanta, Phys. Rev. D **75**, 097304 (2007); **76**, 099902 (E) (2007).

4. "***Weak phase information from the color suppressed*** $\overset{(-)}{B}{}_d^0 \to \overset{(-)}{D}{}^{*0} \overset{(-)}{K}{}^{*0}$ ***modes***", A. K. Giri, **B. Mawlong** and R. Mohanta, Phys. Rev. D **76**, 093008 (2007).

5. "***Signature of new physics in*** $B \to \phi\pi$ ***decay***", **B. Mawlong**, R. Mohanta and A. K. Giri , Phys. Lett. B **668**, 116 (2008).

Dedicated to my parents

(L) Mr. L. B. Dunai and Mrs. B. Mawlong

Contents

Chapter 1

Introduction

1.1 The Standard Model of Particle Physics

Out of the many pictures that came up over the years, the standard model (SM) gradually emerged as the coherent picture to describe particles and their interactions. It has provided a framework for the explanation of almost all particle phenomena. In this model, one encounters two types of particles, (a) fermions which have half - integer spin and (b) bosons which have integer spin. The elementary particles which compose all matter are called quarks and leptons and they are fermions. Quarks bind together to form hadrons which are further divided into two categories, (a) baryons which are composed of three quarks and therefore have half - integer spin and (b) mesons which are composed of a quark and an antiquark and therefore have integer spin. There are six different flavors of quarks known so far. They are the up (u), down (d), strange (s), charm (c), bottom (b) and top (t) quarks. Each of the six quarks come in three different colors. Color is a quantum number introduced to explain the co-existence of quarks in hadrons. The three colors are conventionally taken to be red, green and blue. Hadrons are color-neutral. Leptons are of two types in nature, (a) electrically charged leptons and (b) neutral leptons. There are three charged leptons, namely, the electron (e^-), muon (μ^-) and tau (τ^-). The neutral leptons are called neutrinos and they are the electron neutrino (ν_e), muon neutrino (ν_μ) and tau neutrino (ν_τ).

In nature, there are four fundamental forces and they are the strong, electromagnetic, weak and gravitational forces. Gravity is the weakest force and acts only between massive objects. It therefore shows no significant effect in the subatomic world. The forces between the elementary particles are mediated by gauge vector bosons namely, the gluons (mediating strong interactions), the $W^\pm$ (mediating charged current weak interactions), the Z^0 (mediating neutral current weak interactions) and the photon (mediating electromagnetic interactions). The weak interaction experienced by all quarks and leptons is a point-like interaction according to Fermi's theory. The strength of the interaction is given by the Fermi coupling constant $G_F \sim 1.166 \times 10^{-5}$ GeV^{-2}. The electromagnetic interactions are described by a theory called Quantum Electrodynamics (QED). According to this theory, the charged particles interact via photon (γ) exchange. The strength of this interaction is given by the fine structure constant $\alpha = e^2/4\pi\epsilon_0\hbar c \sim 1/137$. The theory of strong interactions of quarks and gluons is called Quantum Chromodynamics (QCD). The gluons G_a are eight massless vector particles which also carry color and thus can interact with each other. The strength of the interaction is given by the QCD coupling constant $\alpha_s = g_s^2/4\pi$ which decreases with energy.

The electromagnetic and weak forces are unified. The electroweak (EW) interactions are described by the Glashow-Weinberg-Salam (GSW) model [1]. Together with the strong interaction they form the SM. The electroweak and strong forces are believed to be unified at some grand unification energy scale ($\sim 10^{15}$) GeV. The standard model of particle physics is described by a gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$, where C denotes color, L denotes left-handed and Y is hypercharge. The group $SU(3)$ is the gauge group of QCD which has eight generators corresponding to the eight massless gluons. The gauge symmetry group $SU(2) \times U(1)$ of electroweak interactions requires four massless gauge vector bosons. But since the weak force is a short range force, the gauge bosons must be massive. In order to generate the particle masses, the symmetry must be broken spontaneously. Through this spontaneous symmetry breaking (SSB), the massless gauge bosons acquire mass through the interaction with a complex scalar field called the Higgs field (the Higgs Mechanism). Three out of the four gauge bosons acquire mass on allowing the scalar field to have a nonzero vacuum expectation value. These three massive gauge bosons are identified with the $W^{\pm}$ and the Z^0 particles and the remaining massless gauge boson is identified with the photon γ. The gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$ is broken to $SU(3) \times U(1)_{QED}$. One neutral scalar field remains which is identified as the Higgs field. The associated particle is called the Higgs boson which is yet to be produced and observed experimentally. There is a lot of anticipation in this direction as results are awaited from the Large Hadron Collider (LHC) at CERN. Interactions with the Higgs boson also gives mass to the quarks and leptons in the SM. The masses are proportional to the strength with which the Higgs couples to the particles.

Initially, it was thought that the elementary particles (leptons and quarks) take part in weak interactions through charged $V - A$ (vector - axial-vector) currents constructed from the pairs of left-handed fermion states $\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}$ $\begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}$ $\begin{pmatrix} u \\ d \end{pmatrix}$ $\begin{pmatrix} c \\ s \end{pmatrix}$. However, with the observation of the decay $K^+ \rightarrow \mu^+ \nu_\mu$, where a u quark couples to a $\bar{s}$ quark, the above scheme is contradicted where only transitions like $u \leftrightarrow d$ and $c \leftrightarrow s$ are allowed. To reconcile this, the quark currents were assumed to couple to rotated quark states like $\begin{pmatrix} u \\ d' \end{pmatrix}$ $\begin{pmatrix} c \\ s' \end{pmatrix}$, where

$$\begin{aligned} d' &= d \cos\theta_C + s \sin\theta_C \\ s' &= -d \sin\theta_C + s \cos\theta_C \ . \end{aligned}$$

The weak eigenstates d' and s' differ from the mass eigenstates d and s.

As a result of the non-conservation of strangeness in weak interactions, d and s can mix. The doublet containing u and d' was first introduced by Cabibbo in 1963 to account for the weak decays of strange particles. The quark mixing angle θ_C is also known as the Cabibbo angle. To explain the suppression of strangeness changing neutral currents $s \to d$ in the decay $K_L \to \mu^+\mu^-$, Glashow, Illiopoulos and Maiani (GIM) proposed the existence of the c quark long before it was discovered. With the addition of the doublet $\begin{pmatrix} c \\ s' \end{pmatrix}$ to $\begin{pmatrix} u \\ d' \end{pmatrix}$, the strangeness changing term in the neutral current actually cancelled to zero. The GIM mechanism [2] allows for the suppression of flavor changing neutral currents (FCNCs) processes which first proceed at the one-loop level.

Generalizing the Cabibbo-GIM idea to more than four quark flavors, the weak interactions now operate on N doublets of left-handed quarks as

$$\begin{pmatrix} u_i \\ d'_i \end{pmatrix} ,$$

where $i = 1, 2, ..., N$ and d'_i are mixtures of the mass eigenstates d_i given as

$$d'_i = \sum_{j=1}^{N} U_{ij} d_j \; , \tag{1.1}$$

where U is a unitary $N \times N$ matrix. With $N = 3$, the mixing matrix U contains a phase factor $e^{i\delta}$ and the phase δ gives rise to complex elements in U allowing for CP (combined operation of charge-conjugation and parity transformation) violating amplitudes in the SM. Below we give a brief description of the origin of CP violation in the quark sector as a result of having three generations in the SM.

1.2 CP Violation in the Standard Model

CP symmetry implies that the physical processes in nature occur in precisely the same manner if all the particles were converted to their antiparticles using the CP transformation. It was discovered that the symmetries C (charge conjugation or particle-antiparticle interchange) and P (parity or space inversion) are violated in weak interactions but seemingly the product CP is not i.e., it is a good symmetry. It was thus thought for a long time that CP symmetry was exact in nature. However, in 1964, Christenson *et al.* [3] observed CP violation in neutral K meson system. They discovered that the long-lived neutral kaon K_L, which normally decays into three pions with CP eigenvalue -1, could also, with probability 10^{-3}, decay into two pions with CP = +1. Thus, the picture changed with the discovery of CP violation and it has led to a lot of experimental and theoretical research work to understand its mechanism and origin, more specifically in the context of the SM. CP violation is one of the necessary conditions to explain the baryon asymmetry in the universe i.e., the imbalance between matter and antimatter in the universe. It is believed that during the early universe, the condition was suitable for the production of more matter than antimatter with CP violation providing the mechanism for different decay rates to produce matter and antimatter.

1.2.1 The CKM Matrix

In the SM, quarks are grouped into three generations. The left-handed (L) quarks (i.e., quarks which have chirality -1) are put into $SU(2)_L$ doublets as

$$Q_L = \begin{pmatrix} u_L^I \\ d_L^I \end{pmatrix} \tag{1.2}$$

while the corresponding right-handed (R) quarks (with chirality +1) transform as singlets under $SU(2)_L$ denoted by u_R^I and d_R^I. The up-type quarks have charge $+\frac{2}{3}e$ and the down-type quarks have charge $-\frac{1}{3}e$.

The charged and neutral current interactions of the quarks with the $SU(2)_L$ bosons are given by

$$L_W = \frac{g}{\sqrt{2}}(W_\mu^+ \bar{u}_L^I \gamma_\mu d_L^I + W_\mu^- \bar{d}_L^I \gamma_\mu u_L^I) \tag{1.3}$$

and

$$L_Z = \frac{g}{2\cos\theta_W} Z_\mu(\bar{u}_L^I \gamma_\mu u_L^I - \bar{d}_L^I \gamma_\mu d_L^I - 2\sin^2\theta_W J_{em}^\mu) , \tag{1.4}$$

where

$$J_{em}^\mu = \frac{2}{3}(\bar{u}_L^I \gamma_\mu u_L^I + \bar{u}_R^I \gamma_\mu u_R^I) - \frac{1}{3}(\bar{d}_L^I \gamma_\mu d_L^I + \bar{d}_R^I \gamma_\mu d_R^I) \tag{1.5}$$

is the electromagnetic current.

The $SU(2)_L \times U(1)$ Yukawa couplings involving the left-handed doublets of quarks, right-handed singlets and the Higgs doublet are given in

$$\begin{aligned} L_Y &= \Big(- G\bar{Q}_L \phi d_R^I - F\bar{Q}_L \tilde{\phi} u_R^I \Big) + h.c. \\ &= -\Bigg[(\bar{u}_L^I \bar{d}_L^I) G \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} d_R^I + (\bar{u}_L^I \bar{d}_L^I) F \begin{pmatrix} \phi^{0\dagger} \\ -\phi^- \end{pmatrix} u_R^I \Bigg] + h.c. , \end{aligned} \tag{1.6}$$

where G and F are 3×3 complex matrices and ϕ is the Higgs field. When ϕ acquires a vacuum expectation value (VEV), it triggers SSB of the original gauge group as $SU(3)_C \times SU(2)_L \times U(1)_Y \to SU(3)_C \times U(1)_{QED}$. On substituting ϕ^0 by its VEV $v/\sqrt{2}$, the mass terms are obtained as

$$L_{mass} = -\bar{d}_L^I M^{d^I} d_R^I - \bar{u}_L^I M^{u^I} u_R^I + h.c. , \tag{1.7}$$

where $M^{d^I} = Gv/\sqrt{2}$ and $M^{u^I} = Fv/\sqrt{2}$.

On transforming from the weak interaction eigenstate basis to the physical mass eigenstate basis (mass basis corresponds to diagonal mass matrices), one has the following diagonalization

$$\begin{aligned} U_L^{u^I \dagger} M^{u^I} U_R^{u^I} &= M^u = \mathrm{diag}(m_u, m_c, m_t) , \\ U_L^{d^I \dagger} M^{d^I} U_R^{d^I} &= M^d = \mathrm{diag}(m_d, m_s, m_b) , \end{aligned} \tag{1.8}$$

where the matrices U are unitary, M^u and M^d are diagonal with diagonal elements $m_q (q = u, c, t, d, s, b)$ being real.

The charged current interaction in Eq.(1.3) in terms of the quark mass eigenstates is given by

$$L_W^q = \frac{g}{\sqrt{2}}(W_\mu^+ \bar{u}_L \gamma^\mu V d_L + W_\mu^- \bar{d}_L \gamma^\mu V^\dagger u_L) , \tag{1.9}$$

where

$$V = U_L^{u^{I\dagger}} U_L^{d^I} \tag{1.10}$$

is the Cabibbo-Kobayashi-Maskawa (CKM)[4, 5] matrix given as

$$V \equiv V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}.$$

The neutral current Lagrangian in the mass eigenstate basis remains unchanged i.e., there are no FCNCs at the tree level in the SM.

Thus, when we have three generations of quarks, we have three left-handed doublets $\begin{pmatrix} u \\ d' \end{pmatrix}_L$ $\begin{pmatrix} c \\ s' \end{pmatrix}_L$ $\begin{pmatrix} t \\ b' \end{pmatrix}_L$, where t and b' are the up and down quark states, respectively, in the third generation. The weak quark states d', s' and b' result from the mixing of the mass eigenstates d, s and b and they are related through the unitary CKM matrix (the unitarity of this matrix guarantees the absence of FCNCs at the tree level) in the following way

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix} \equiv V_{CKM} \begin{pmatrix} d \\ s \\ b \end{pmatrix}.$$

The unitarity of the CKM matrix and the freedom to arbitrarily choose the global phases of the quark fields reduce the nine complex elements of the CKM matrix to three real parameters and one phase, which is responsible for CP violation in the meson decays in the SM. Thus, the mixing matrix which is responsible for the weak charged current interactions of quarks contains three real parameters (Cabibbo-like mixing angles) and a phase factor $e^{i\delta}$. Due to this nonzero weak phase, the matrix is complex and this introduces the important possibility of CP violating amplitudes in the SM.

There are many ways to express the elements of V_{CKM} in terms of three rotation angles and one phase. Thus, many different parametizations for the

CKM matrix have been proposed in literature. The standard parametrization used by the particle data group (PDG) is [6]

$$V_{CKM} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} ,$$

where $c_{ij} \equiv \cos\theta_{ij}$ and $s_{ij} \equiv \sin\theta_{ij}$ $(i, j = 1, 2, 3)$ (θ_{ij} are the rotation angles) and the complex phase δ is responsible for CP violation in the SM.

One of the most important and popular parametrizations is the Wolfenstein parametrization [7] which has the following change of variables

$$s_{12} \equiv \lambda \, , \quad s_{23} \equiv A\lambda^2 \, , \quad s_{13}e^{-i\delta} \equiv A\lambda^3(\rho - i\eta) \, , \tag{1.11}$$

where λ, A and ρ are known as the Wolfenstein parameters. The CKM matrix now becomes

$$V_{CKM} = \begin{pmatrix} 1 - \frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1 - \bar{\rho} - i\bar{\eta}) & -A\lambda^2 & 1 \end{pmatrix} ,$$

where $\eta \neq 0$ is responsible for CP violation in the SM and $\bar{\rho}$ and $\bar{\eta}$ are given by

$$\bar{\rho} = \rho(1 - \frac{\lambda^2}{2}) \, , \quad \bar{\eta} = \eta(1 - \frac{\lambda^2}{2}) \, . \tag{1.12}$$

In the SM, any CP violation observable involves the product J which is independent of the parametrization and defined as

$$\mathrm{Im}(V_{ij}V_{kl}V_{il}^*V_{kj}^*) = J \sum_{m,n=1}^{3} \epsilon_{ikm}\epsilon_{jln} \, , \tag{1.13}$$

where the V's are the elements of the CKM matrix and $i, j, k, l = 1, 2, 3$. In terms of the standard parametrization

$$J = c_{12}c_{23}c_{13}^2 s_{12}s_{23}s_{13} \sin\delta \, . \tag{1.14}$$

Thus, in order to have an observable CP violation effect in the SM, the mixing angles θ_{ij} should not be zero or $\frac{\pi}{2}$ and the phase δ should not be zero or π.

The Yukawa Lagrangian in Eq.(1.6) is, in general, CP violating. More precisely, CP is violated if and only if [8]

$$\mathrm{Im}(\det[GG^\dagger, FF^\dagger]) \neq 0. \tag{1.15}$$

The relation of CP violation to the complex Yukawa couplings can be explained as follows. The hermiticity of the Lagrangian implies that L_Y has its terms in pairs of the form

$$Y_{ij}\overline{\psi_{Li}}\phi\psi_{Rj} + Y_{ij}^*\overline{\psi_{Rj}}\phi^\dagger\psi_{Li}\ , \tag{1.16}$$

where Y denotes G or F. A CP transformation exchanges the operators

$$\overline{\psi_{Li}}\phi\psi_{Rj} \leftrightarrow \overline{\psi_{Rj}}\phi^\dagger\psi_{Li}, \tag{1.17}$$

but leaves their coefficients, Y_{ij} and Y_{ij}^*, unchanged. This means that CP is a symmetry of L_Y if $Y_{ij} = Y_{ij}^*$.

In the mass basis, the condition (1.15) translates to a necessary and sufficient condition for CP violation in the quark sector of the SM as

$$\Delta m_{tc}^2 \Delta m_{tu}^2 \Delta m_{cu}^2 \Delta m_{bs}^2 \Delta m_{bd}^2 \Delta m_{sd}^2 J \neq 0\ , \tag{1.18}$$

where $\Delta m_{ij}^2 \equiv m_i^2 - m_j^2$.

Thus, in order that CP be violated in the SM, the following requirements must be met

(a) the quarks of the same given charge should not be degenerate in mass.

(b) None of the three mixing angles should be zero or $\frac{\pi}{2}$.

(c) The phase δ of the CKM matrix should neither be zero nor π.

1.2.2 The Unitarity Triangle

The CKM matrix is unitary as it is given by the product of two unitary matrices $U_L^{u^I}$ and $U_L^{d^I}$. Unitarity of the matrix indicates various relations between its elements and the following sets of equations are obtained :

$$\begin{aligned}|V_{ud}|^2 + |V_{cd}|^2 + |V_{td}|^2 &= 1 \, , \\ |V_{us}|^2 + |V_{cs}|^2 + |V_{ts}|^2 &= 1 \, , \\ |V_{ub}|^2 + |V_{cb}|^2 + |V_{tb}|^2 &= 1 \, . \end{aligned} \tag{1.19}$$

$$\begin{aligned}|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 &= 1 \, , \\ |V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2 &= 1 \, , \\ |V_{td}|^2 + |V_{ts}|^2 + |V_{tb}|^2 &= 1 \, . \end{aligned} \tag{1.20}$$

$$\begin{aligned}V_{ud}V_{us}^* + V_{cd}V_{cs}^* + V_{td}V_{ts}^* &= 0 \, , \\ V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* &= 0 \, , \\ V_{us}V_{ub}^* + V_{cs}V_{cb}^* + V_{ts}V_{tb}^* &= 0 \, . \end{aligned} \tag{1.21}$$

$$\begin{aligned}V_{ud}V_{cd}^* + V_{us}V_{cs}^* + V_{ub}V_{cb}^* &= 0 \, , \\ V_{ud}V_{td}^* + V_{us}V_{ts}^* + V_{ub}V_{tb}^* &= 0 \, , \\ V_{cd}V_{td}^* + V_{cs}V_{ts}^* + V_{cb}V_{tb}^* &= 0 \, . \end{aligned} \tag{1.22}$$

The relations (1.21) and (1.22) can be represented as six unitarity triangles in the complex $(\bar{\rho}, \bar{\eta})$ plane. Considering the most relevant relation describing B decays i.e.,

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0 \, , \tag{1.23}$$

a rescaled unitarity triangle in the complex $(\bar{\rho}, \bar{\eta})$ plane is obtained which is shown in Figure 1.1. The areas of all unitarity triangles are equal and are related to the measure of CP violation J

$$J = 2 \cdot A \, , \tag{1.24}$$

where A denotes the area of the triangle.

The rescaled triangle contains sides of length 1, R_t and R_b defined by

$$\begin{aligned} R_t &\equiv \frac{|V_{td}V_{tb}^*|}{|V_{cd}V_{cb}^*|} , \\ R_t &\equiv \frac{|V_{ud}V_{ub}^*|}{|V_{cd}V_{cb}^*|} , \end{aligned} \tag{1.25}$$

where $R_b = \sqrt{\bar{\rho}^2 + \bar{\eta}^2}$ and $R_t = \sqrt{(1-\bar{\rho})^2 + \bar{\eta}^2}$ and the side of length 1 is real.

The angles of the triangle are labelled as α, β and γ. β and γ are directly related to the complex phases of the CKM elements V_{td} and V_{ub}, respectively, as

$$V_{td} = |V_{td}|e^{-i\beta} , \quad V_{ub} = |V_{ub}|e^{-i\gamma} . \tag{1.26}$$

The angle α can be obtained through the relation

$$\alpha + \beta + \gamma = 180^\circ . \tag{1.27}$$

It can be seen that a nonzero value of β or γ implies that η is nonzero and thus CP violation cannot be ruled out. It is, therefore, imperative that the three angles of the triangle be measured independently to get decisive information on the origin of CP violation.

1.2.3 CP violation in the K system

In the SM with six fundamental quark flavors, weak transitions can occur between different flavors and a CP violating phase angle can be involved. CP violation is elegantly described by the CKM mechanism in the SM. A nonzero weak phase in the complex CKM matrix gives rise to CP violation. More specifically, it occurs in weak interactions when quarks undergo weak transformations and turn into quarks of different electric charges.

We now briefly discuss the manifestation of CP violation in neutral K system. The two neutral kaons K^0 and $\bar{K}^0$ can decay to pions via the weak

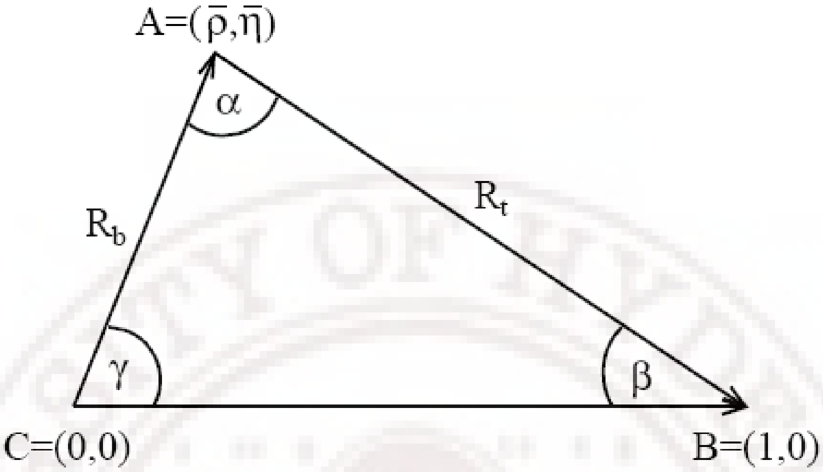

Figure 1.1: The rescaled Unitarity Triangle

.

interaction $|\Delta S| = 1$. Thus, mixing can occur via (virtual) intermediate pion states. These transitions are $|\Delta S| = 2$ transitions and are thus second-order weak transitions. Thus, if at time $t = 0$, we have a pure K^0 state, then at any later time t, we can have a superposition of both K^0 and $\bar{K}^0$. Therefore, we can form the linear combination (CP eigenstates)

$$K_{L,S} = \frac{K^0 \pm \bar{K}^0}{\sqrt{2}} , \tag{1.28}$$

where K_S and K_L are the particles associated with the short-lived and long-lived 2π (CP even) and 3π (CP odd) states, respectively. A diagram called the box diagram as shown in Figure 1.2 depicts the $K^0 - \bar{K}^0$ mixing.

With the discovery of the decay $K_L \to 2\pi$ in 1964, K_L and K_S are not CP eigenstates anymore but the new CP eigenstates are K_1 and K_2 defined as

$$\begin{aligned} K_1 &= \frac{1}{\sqrt{2}}(K^0 - \bar{K}^0) , \qquad CP|K_1\rangle = K_1 , \\ K_2 &= \frac{1}{\sqrt{2}}(K^0 + \bar{K}^0) , \qquad CP|K_2\rangle = -K_2 \end{aligned} \tag{1.29}$$

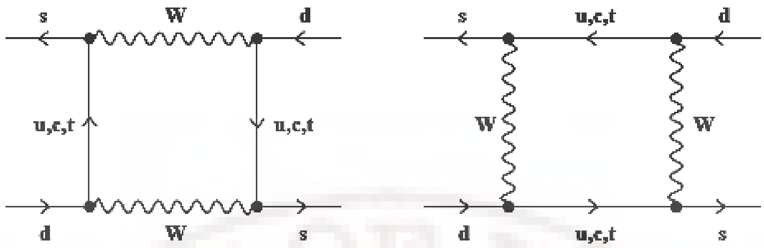

Figure 1.2: Box diagrams depicting $K^0 - \bar{K}^0$ mixing.

and the mass eigenstates K_S and K_L are given by

$$K_S = \frac{K_1 + \bar{\varepsilon} K_2}{\sqrt{(1+ \mid \bar{\varepsilon} \mid^2)}} \;, \qquad K_L = \frac{K_2 + \bar{\varepsilon} K_1}{\sqrt{(1+ \mid \bar{\varepsilon} \mid^2)}} \;, \tag{1.30}$$

where $\bar{\varepsilon}$ parametizes the deviation from the CP conserving limit. This violation is called indirect CP violation as it arises from the fact that the weakly decaying eigenstates of definite lifetimes, K_S and K_L, are each an admixture of the wrong CP to a degree $\bar{\varepsilon}$. The measure for this type of CP violation is defined as

$$\varepsilon \equiv \frac{A(K_L \to (\pi\pi)_{I=0})}{A(K_S \to (\pi\pi)_{I=0})} \;, \tag{1.31}$$

where $\varepsilon = \bar{\varepsilon} + i\xi$ and $\xi = \mathrm{Im}A_0/\mathrm{Re}A_0$.

CP violation can also be direct and is realized via a direct transition of a CP odd to a CP even state: $K_2 \to \pi\pi$. A measure of such a violation is given by the complex parameter

$$\varepsilon' = \frac{1}{\sqrt{2}} \mathrm{Im}\left(\frac{A_2}{A_0}\right) \exp(i\phi_{\varepsilon'}) \;, \tag{1.32}$$

where A_I is the amplitude for K^0 to decay into a two pion final state with isospin I, with the strong phase $\phi_{\varepsilon'}$ factored out. Experimentally, the two parameters ε and ε' can be determined by measuring the ratios

$$\eta_{00} = \frac{A(K_L \to \pi^0\pi^0)}{A(K_S \to \pi^0\pi^0)} \;, \qquad \eta_{+-} = \frac{A(K_L \to \pi^+\pi^-)}{A(K_S \to \pi^+\pi^-)} \tag{1.33}$$

or

$$\eta_{00} = \varepsilon - 2\varepsilon' \;, \qquad \eta_{+-} = \varepsilon + \varepsilon' \;. \tag{1.34}$$

The three types of CP violation observed in $K \to \pi\pi$ decays are [9]

$$\mathrm{Re}(\varepsilon') = (2.5 \pm 0.4) \times 10^{-6} \,, \tag{1.35}$$

$$\mathrm{Re}(\varepsilon) = (1.657 \pm 0.021) \times 10^{-3} \,, \tag{1.36}$$

$$\mathrm{Im}(\varepsilon) = (1.572 \pm 0.022) \times 10^{-3} \,. \tag{1.37}$$

As can be seen from the expressions (1.35 - 1.37), the CP violation measures ε and ε' in the K systems show small effects and since the kaons are light, there are not too many decay modes available, therefore it is difficult to relate these CP violation effects to CKM parameters. However, it came to be realized that CP violation may not be restricted to neutral kaon systems but may also be present in the neutral mesons containing charm and bottom quarks. Investigations have shown that CP violation in charmed D systems may not be observable or is small in the SM. It is expected that in B mesons, the effects will be larger and so it will be easy to relate them to SM parameters.

1.3 B Physics

After the discovery of CP violation in K system in 1964, there was a lot of enthusiasm to look for CP violation in other systems as well. It was found that CP violation in K systems is rather small. However, theoretical predictions have suggested that the B meson system may be an ideal place for detecting sufficiently large CP violating effects. In fact, large CP violation in B systems has been observed in the two dedicated B factories, namely, BABAR at SLAC and BELLE at KEK. When we consider CP violation in the B meson system, we obtain an interesting and relevant unitarity triangle whose angles are denoted by α, β and γ. The nonzero values of these angles indicate CP violation. It is therefore imperative that these angles be determined experimentally.

With the advent of B factories dedicated to the precise measurements of SM parameters, certain hints of discrepancies in the measured and the theoretical predictions have been noticed. In addition to these, the SM falls short of being the complete theory because it (i) does not include the fourth force gravity, (ii) cannot explain the observed neutrino oscillations, (iii) does not explain why there are three generations of particles and many other problems. As they cannot be explained by the SM alone, it is believed that there exists some kind of new physics (NP) beyond that of the SM.

One of the most promising and interesting fields of particle physics today is B physics which basically deals with b quarks and, in particular, their decay modes or their transformations to other quarks. The B mesons are strongly interacting particles and are made up of a b quark (antiquark) and another light (u, d, s) antiquark (quark) and have a spin-parity combination $J^P = 0^-$ and therefore are pseudoscalars. The B meson can also contain the charm (c) quark and it is denoted as B_c. The resonance $\Upsilon = b\bar{b}$ produced at the e^+e^- colliders of the B factories provides a clean source of b quarks. It decays into $B^+ - B^-$ pairs and into $B_d^0 - \bar{B}_d^0$ pairs with branching ratios close to 50% each. Experiments involving neutral B mesons involve the determination of the flavor of the neutral meson at the time of its production and/or at the time of its decay i.e., determining if it is a B^0 meson or its antiparticle $\bar{B}^0$. This is achieved through a method known as tagging. The production of the neutral B mesons is dominated either by the strong interaction $p\bar{p}$ collisions or by the electromagnetic interaction $e^+e^- \to \Upsilon(4S) \to B_d^0\bar{B}_d^0$. Due to flavor conservation in both strong and electromagnetic interactions, the quark b is always produced in association with its antiquark $\bar{b}$. One of the ways of tagging a neutral meson is by identifying the flavor of the charged meson it was produced together with, through the decay of the latter. We thus have a single tagged neutral meson. The flavor of a neutral meson at the time of its decay can be determined from flavor-specific final states i.e., final states that can be reached either from B^0 but not from $\bar{B}^0$ or the other way round. One can also tag the neutral meson via its semileptonic decay and then look for the decay of the other neutral meson into a CP eigenstate. But when we have the two neutral mesons produced via the strong or the electromagnetic processes as mentioned, both of them oscillate or mix as in the neutral kaon case. So detecting a flavor-specific final state in one side of a detector informs us about the flavor of that meson at its decay time but not about its flavor at production time. This is because the tagged neutral meson might have oscillated in between the time of its creation and the time of its decay. Also identifying the flavor of one of them at decay time does not identify the initial flavor of the other one, which we wish to tag in order to follow its tagged, time-dependent decay rate. However, one can have a situation in which the semileptonic decay of one meson really identifies the flavor of the other meson. This is possible when the two neutral mesons are produced in an antisymmetric wave as at the $\Upsilon(4S)$. As the two identical bosons cannot be in an overall antisymmetric wave, tagging is possible and thus the determination of the flavor of one neutral meson through its semileptonic decay at time t_2 ensures that the other neutral meson has the opposite flavor at the same time and evolves thereafter as a tagged meson with time $t = t_1 - t_2$.

The study of B meson decays plays a very prominent role as it provides valuable information on the SM, which includes interesting insights into the various realms of particle physics, in particular flavor physics and CP violation. Numerous studies have been carried out in this direction before. The system of B particle decays provides us with avenues of measuring the SM parameters which includes measurement of various CP violating parameters and determination of the CKM angles and to look for possible hints of new physics beyond the SM. In fact, the main objective of the two B factories is to test the SM and to look for possible signals of new physics. They have started a new era in the exploration of CP violation. Now one of the ways of searching for new physics is by studying rare B decay modes i.e., decay modes that are suppressed in the SM. Thus, with B decays, not only can the KM (Kobayashi - Maskawa) mechanism, which allows CP violation in the SM of electroweak interactions, be tested and the CKM unitarity angles be determined but models beyond the SM can also be explored to explain the observed discrepancies. Therefore, there is a strong motivation to carry out a theoretical study in the area of B physics. The study would supplement the existing studies and also be of valuable contribution to particle phenomenology especially in the LHC era as a lot of data is expected to be collected. In this thesis, we intend to study some of the rare B decay modes and CP violation in the b-sector.

1.3.1 Theoretical Framework for Studying Nonleptonic Two-Body B decays

A framework is developed for the theoretical formulation of weak nonleptonic two-body B decays. The requirement is a low energy effective Hamiltonian which drives the nonleptonic decays of B mesons. In this work, we consider only nonleptonic B decays. In order to compute decay amplitudes, one needs to know the structure of the effective Hamiltonian which governs the transitions. It is constructed using a technique known as the Operator Product Expansion (OPE) which gives it as a sum of local operators multiplied by effective coupling constants. Its generic structure is given as

$$\mathcal{H}_{eff} = \frac{G_F}{\sqrt{2}} V_{CKM} \sum_i C_i(\mu) O_i \ , \tag{1.38}$$

where G_F is the Fermi constant, V_{CKM} are the CKM elements in the SM, O_i are the relevant local operators or effective vertices in the effective theory and C_i are the effective coupling constants associated with the effective vertices and are known as the Wilson coefficients.

In the full theory, the local operator is represented by Feynman diagrams with full W, Z^0 and top-quark (t) exchanges and these represent situations at short distance scales $\mathcal{O}(M_W, M_Z, m_t)$. However, the actual situation when a hadron with mass $\mathcal{O}(m_b)$ decays is more correctly described by effective point-like vertices and these are given by the local operators O_i. Thus, starting at a high energy scale $\mathcal{O}(M_W)$, the heavy degrees of freedom are consecutively integrated out from appearing explicitly in the theory. In the effective theory, the heavy propagators W, Z^0 and t are therefore removed from the theory and the operators do not involve the heavy degrees of freedom. However, their effects are merely hidden in the effective gauge coupling constants and in the Wilson coefficients.

Having known the structure of the effective Hamiltonian, one needs to evaluate the decay amplitudes. The amplitude for a B meson to decay to a final state f is given by

$$A(B \to f) = \langle f|\mathcal{H}_{eff}|B\rangle = \frac{G_F}{\sqrt{2}} V_{CKM} \sum_i C_i(\mu)\langle f|O_i(\mu)|B\rangle \ , \qquad (1.39)$$

where $\langle f|O_i(\mu)|B\rangle$ are the hadronic transition matrix elements of O_i between B and f and they also depend on the renormalization scale μ as do the Wilson coefficients C_i. The renormalization scale μ separates the physics contributions to the decay into two distinct parts: the short distance contributions (high energy QCD corrections) coming from scales higher than μ which are contained in the Wilson coefficients C_i and the long distance contributions (low energy non-perturbative confinement effects) coming from scales lower than μ contained in the hadronic matrix element $\langle f|O_i|B\rangle$. This scale is usually chosen to be of the order of the mass of the decaying hadron.

It can be seen that the Wilson coefficients C_i depend on the renormalization scale μ. These coefficents can be computed by matching the standard model and the effective theory at a scale $\mu \sim M_W$. The evolution of these coefficients down to lower scales can be computed perturbatively (as a result of the asymptotic freedom of QCD) with the help of renormalization group (RG) techniques as long as μ is not too small. This is also known as the running of Wilson coefficients from high to low energies. The detailed discussions of the calculations can be found in [10]. During the running of these coefficients, one encounters large logarithms ($\ln M_W/\mu$) due to two vastly different scales ($M_W \gg \mu$). In the perturbative expansion, there are terms proportional to the QCD coupling constant α_s and also terms proportional to $\alpha_s \ln(M_W^2/\mu^2)$. Even when α_s is small, $\alpha_s \ln(M_W^2/\mu^2)$ can be large when μ is low. At leading order, terms of the type $\alpha_s^n \ln^n(M_W^2/\mu^2)$ have to be summed to all orders in α_s (leading logarithmic approximation or LLA) and at next-to-leading order (next-to-leading order approximation or NNLA), terms of the type $\alpha_s^n \ln^{n-1}(M_W^2/\mu^2)$ have to be summed to all orders in α_s. This is efficiently carried out using the renormalization group and results in a renormalization group improved perturbative expansion for the C_i's which are more reliable.

Now the full decay amplitude does not depend on the renormalization scale μ. Therefore, the μ-dependence of the Wilson coefficients must cancel the μ-dependence of the matrix elements of the operators. Since the local operators represent effective vertices, they undergo renormalization and as such the matrix elements $\langle f|O_i|B\rangle$ are also dependent on the renormalization scheme used. The C_i's depend on the same scheme also. As the physical amplitudes are independent of renormalization schemes, the scheme dependence of the C_i's have to cancel that of the matrix elements. The cancellations may involve several terms in the expansion and may in principle be quite complicated.

We now give a brief explanation of the quark diagram description which is useful in working out decay amplitudes. There are two such diagrams depicting the decay and they are the tree and penguin diagrams and therefore we have two interfering amplitudes, (a) the tree amplitude and (b) the penguin amplitude, in the decay. The spectator approximation is considered where it is assumed that the B meson decays through the decay of its b quark (or b antiquark) without the interference of the other quark in the B meson (hence, the spectator quark). The simplest diagram in this approximation is the tree diagram which involves only one intermediate $W^{\pm}$ boson for the decay of the b quark (or b antiquark). This is shown in Figure 1.3 where q_1 and q_2 represent up-type quarks u, c. The other diagram which is a one-loop diagram is called the penguin diagram and we have two types of penguin diagrams and they are the QCD and EW penguin diagrams. The QCD penguin diagram involving one $W^{\pm}$ and one gluon is shown in Figure 1.4. As can be seen, the pair $q_1\bar{q}_1$ is created from the gluon G. The intermediate particles running in the loop are the up-type quarks u, c, t. Besides the gluonic penguins, we can have penguins in which the gluon is substituted by a Z boson or by a photon γ. These are the EW penguins (Figure 1.5) and they involve the same CKM factors as gluonic penguins, the pair $q_1\bar{q}_1$ is created from the Z boson or the photon γ and the particles in the loop are the up-type quarks as in the gluon penguins. The basic difference is that EW penguins couple differently to up-type quark-antiquark pairs and to down-type quark-antiquark pairs whereas the gluonic penguins couple to all quark-antiquark pairs equally. Besides the tree and penguin diagrams, we can also have diagrams that involve the spectator quark but they are usually neglected as they are suppressed by f_B/m_B compared to the tree and penguin diagrams. Here f_B and m_B are, respectively, the decay constant and mass of the B meson.

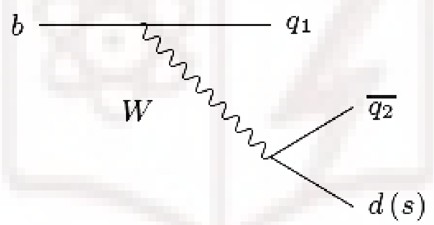

Figure 1.3: Tree diagram with $q_1, q_2 \in \{u, c\}$

Due to the presence of electroweak and strong interactions in the weak decays, the local operators can be classified according to the color structure, the Dirac structure and the type of quarks relevant for a given decay. We give below the operators that are needed for weak B decays.

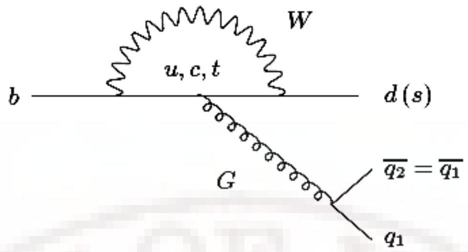

Figure 1.4: QCD Penguin diagram with $q_1 = q_2 \in \{u, d, c, s\}$

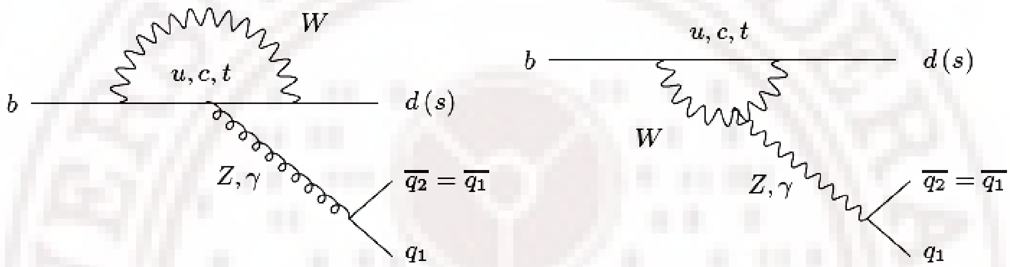

Figure 1.5: Electroweak Penguin diagram with $q_1 = q_2 \in \{u, d, c, s\}$

Current-current operators ($q = d, s$ **and** $q_1, q_2 \in u, c$)

$$\begin{aligned} O_1 &\equiv (\bar{q}_1 b)_{V-A}(\bar{q} q_2)_{V-A} \ , \\ O_2 &\equiv (\bar{q}_{1\alpha} b_\beta)_{V-A}(\bar{q}_\beta q_{2\alpha})_{V-A} \ , \end{aligned} \tag{1.40}$$

QCD penguin operators ($q = d, s$)

$$\begin{aligned} O_3 &\equiv (\bar{q} b)_{V-A} \sum_{q'} (\bar{q}' q')_{V-A} \ , \\ O_4 &\equiv (\bar{q}_\alpha b_\beta)_{V-A} \sum_{q'} (\bar{q}'_\beta q'_\alpha)_{V-A} \ , \\ O_5 &\equiv (\bar{q} b)_{V-A} \sum_{q'} (\bar{q}' q')_{V+A} \ , \\ O_6 &\equiv (\bar{q}_\alpha b_\beta)_{V-A} \sum_{q'} (\bar{q}'_\beta q'_\alpha)_{V+A} \ , \end{aligned} \tag{1.41}$$

Electroweak penguin operators ($q = d, s$)

$$
\begin{aligned}
O_7 &\equiv \frac{3}{2}(\bar{q}b)_{V-A}\sum_{q'} e_{q'}(\bar{q}'q')_{V+A} \ , \\
O_8 &\equiv \frac{3}{2}(\bar{q}_\alpha b_\beta)_{V-A}\sum_{q'} e_{q'}(\bar{q}'_\beta q'_\alpha)_{V+A} \ , \\
O_9 &\equiv \frac{3}{2}(\bar{q}b)_{V-A}\sum_{q'} e_{q'}(\bar{q}'q')_{V-A} \ , \\
O_{10} &\equiv \frac{3}{2}(\bar{q}_\alpha b_\beta)_{V-A}\sum_{q'} e_{q'}(\bar{q}'_\beta q'_\alpha)_{V-A} \ ,
\end{aligned}
\tag{1.42}
$$

where α and β are color indices, V and A are the Dirac structures which refer to the vector and axial-vector currents, respectively and $V \pm A$ refers to $\gamma_\mu(1 \pm \gamma_5)$. In the sums, the quark q' runs over the quark fields that are active at the scale $\mu = \mathcal{O}(m_b)$ i.e., $q' \in u, d, c, s$ and $e_{q'}$ are the corresponding electric charges of the quarks. The tree, QCD and EW penguin processes in the full theory originate from the above current-current, QCD and EW penguin operators and have been depicted in Figures 1.3, 1.4 and 1.5, respectively.

What remains is the computation of decay amplitudes. In order to do so, the matrix elements $\langle f|O_i|B\rangle$ have to be evaluated. As they involve long distance contributions, one has to resort to non-perturbative methods to compute them. These methods have some limitations and therefore a major part of the theoretical uncertainties in the decay amplitude come from the hadronic matrix elements. The standard procedure in computing the B decay amplitudes is the factorization hypothesis. This consists of the decay amplitudes being factorized into products of two current matrix elements by inserting the vacuum. This approximation amounts to evaluating the matrix elements of the four-quark operators, given in (1.40 - 1.42), between the decaying B meson and the final hadronic states $f_1 f_2$ as the product of two matrix elements of the type $\langle f_1|\bar{q}b|B\rangle$ which mediates the $B \to f_1$ transition and $\langle f_2|\bar{q}'q'|0\rangle$ which describes vacuum $\to f_2$ transition. The resulting matrix elements are parametrized in terms of form factors and decay constants. The form factors are usually calculated using a model and therefore the results are model-dependent. Form factors for $B \to P, S, V$ transitions are defined by

$$\begin{aligned}
\langle 0|A_\mu|P(q)\rangle &= if_P q_\mu \,, \\
\langle 0|q\bar{q}|S\rangle &= m_S \bar{f}_S^q \,, \\
\langle 0|V_\mu|V(p,\varepsilon)\rangle &= f_V m_V \varepsilon_\mu \,, \\
\langle P'(p')|V_\mu|P(p)\rangle &= \left(p_\mu + p'_\mu - \frac{m_P^2 - m_{P'}^2}{q^2} q_\mu\right) F_1(q^2) \\
&+ F_0(q^2) \frac{m_P^2 - m_{P'}^2}{q^2} q_\mu \,, \\
\langle S(p')|A_\mu|P(p)\rangle &= i\left(p_\mu + p'_\mu - \frac{m_P^2 - m_S^2}{q^2} q_\mu\right) F_1(q^2) \\
&+ F_0(q^2) \frac{m_P^2 - m_S^2}{q^2} q_\mu \,, \\
\langle V(p',\varepsilon)|V_\mu|P(p)\rangle &= \frac{2}{m_P + m_V} \epsilon_{\mu\nu\alpha\beta} \varepsilon^{*\nu} p^\alpha p'^\beta V(q^2) \,, \\
\langle V(p',\varepsilon)|A_\mu|P(p)\rangle &= i\Big[(m_P + m_V)\varepsilon^*_\mu A_1(q^2) \\
&- \frac{\varepsilon^* \cdot p}{m_P + m_V} (p + p')_\mu A_2(q^2) \\
&- 2m_V \frac{\varepsilon^* \cdot p}{q^2} q_\mu [A_3(q^2) - A_0(q^2)]\Big] \,, \qquad (1.43)
\end{aligned}$$

where $P^{(\prime)}$, S and V denote pseudoscalar, scalar and vector mesons, respectively, V_μ and A_μ denote the vector and axial-vector currents, respectively, ε is the polarization vector of V and $q = p - p'$. The decay constants are given by f_P, $\bar{f}_s^q$, f_V and the form factors by $F_1(q^2)$, $F_0(q^2)$, $V(q^2)$, $A_1(q^2)$, $A_2(q^2)$, $A_3(q^2)$. Further, $F_1(0) = F_0(0)$, $A_3(0) = A_0(0)$ and

$$A_3(q^2) = \frac{m_P + m_V}{2m_V} A_1(q^2) - \frac{m_P - m_V}{2m_V} A_2(q^2) \,. \qquad (1.44)$$

1.3.2 CP Violation in B meson system

CP violation has been extensively described in [11]. Its effects are expected to be larger in B meson systems and in fact large CP violation has been observed in the B factories. A lot of study in the area of CP violation has been carried out by many facilities collecting data with B events. CP violation effects can manifest themselves in the B decay modes or in particle-antiparticle mixing i.e., $B^0 - \bar{B}^0$ mixing where B^0 is either a B_d^0 or a B_s^0.

$B^0 - \bar{B}^0$ Mixing

Particle-antiparticle mixing is related to CP violation and is the reason for the mass differences between the mass eigenstates of neutral mesons. It is a second-order weak transition and in the SM first appears at the one-loop level and is induced via a box diagram as in the kaon system with s replaced by b in the case of $B_d - \bar{B}_d^0$ mixing. In the case of the B_s system, the s is replaced by the b and the d by the s. The flavor eigenstates in the $B_{d,s}^0 - \bar{B}_{d,s}^0$ mixing are given by

$$B_d^0 = (\bar{b}d) \ , \quad \bar{B}_d^0 = (b\bar{d}) \ , \quad B_s^0 = (\bar{b}s) \ , \quad \bar{B}_s^0 = (b\bar{s}) \ . \tag{1.45}$$

When we have flavor mixing, the time evolution of the $B^0 - \bar{B}^0$ system is described by

$$i\frac{d\psi(t)}{dt} = \hat{H}\psi(t), \quad \psi(t) = \begin{pmatrix} \mid B^0(t)\rangle \\ \mid \bar{B}^0(t)\rangle \end{pmatrix} \ , \tag{1.46}$$

where

$$\hat{H} = \hat{M} - i\frac{\hat{\Gamma}}{2} = \begin{pmatrix} M_{11} - i\frac{\Gamma_{11}}{2} & M_{12} - i\frac{\Gamma_{12}}{2} \\ M_{21} - i\frac{\Gamma_{21}}{2} & M_{22} - i\frac{\Gamma_{22}}{2} \end{pmatrix} \tag{1.47}$$

with $\hat{M}$ and $\hat{\Gamma}$ being the mass matrix and decay width matrix, respectively and they are hermitian. Due to hermiticity of the M and Γ matrices, $M_{21} = M_{12}^*$, $\Gamma_{21} = \Gamma_{12}^*$ and due to CPT invariance, $M_{11} = M_{22} \equiv M$, $\Gamma_{11} = \Gamma_{22} \equiv \Gamma$. The Hamiltonian thus becomes

$$\hat{H} = \begin{pmatrix} M - i\frac{\Gamma}{2} & M_{12} - \frac{\Gamma_{12}}{2} \\ M_{12}^* - i\frac{\Gamma_{21}^*}{2} & M - i\frac{\Gamma}{2} \end{pmatrix} \ . \tag{1.48}$$

With the diagonalization of the Hamiltonian matrix, one obtains the two physically observed mass eigenstates given by

$$B_H = pB^0 + q\bar{B}^0 \ , \quad B_L = pB^0 - q\bar{B}^0 \ , \tag{1.49}$$

where

$$p = \frac{1+\bar{\varepsilon}_B}{\sqrt{2(1+\mid \bar{\varepsilon}_B \mid^2)}}\ , \quad q = \frac{1-\bar{\varepsilon}_B}{\sqrt{2(1+\mid \bar{\varepsilon}_B \mid^2)}} \tag{1.50}$$

and $\bar{\varepsilon}_B$ corresponds to $\bar{\varepsilon}$ in the kaon system and H and L indicates heavy and light, respectively. In the $B^0 - \bar{B}^0$ system, the lifetime difference $\Delta\Gamma = \Gamma_H - \Gamma_L$ is much smaller as compared to the mass difference $\Delta M = M_H - M_L$ i.e., $\Delta\Gamma \ll \Delta M$. Therefore, the mass eigenstates B_H and B_L are usually distinguished by their masses and not by their lifetimes.

The strength of the $B^0_{d,s} - \bar{B}^0_{d,s}$ mixing is described by the mass difference

$$\Delta M_{d,s} = M_H^{d,s} - M_L^{d,s} \tag{1.51}$$

and one obtains [12] in terms of the off-diagonal elements in the B^0 meson mass matrix and decay width matrix, respectively

$$\Delta M_q = 2|M_{12}^{(q)}|\ , \quad \Delta\Gamma_q = 2\frac{\mathrm{Re}(M_{12}\Gamma_{12}^*)}{|M_{12}|} \ll \Delta M_q\ , \quad q = d, s. \tag{1.52}$$

The quantity $\bar{\varepsilon}$ is given in terms of p and q as

$$\frac{1-\bar{\varepsilon}}{1+\bar{\varepsilon}} = \frac{q}{p} = \sqrt{\frac{M_{12}^* - \frac{i}{2}\Gamma_{12}^*}{M_{12} - \frac{i}{2}\Gamma_{12}}} \tag{1.53}$$

and after calculating the box diagrams, the off-diagonal element M_{12} is given as [13]

$$(M_{12})_q = \frac{G_F^2}{12\pi^2} F_{B_q}^2 \hat{B}_{B_q} m_{B_q} M_W^2 (V_{tq}^* V_{tb})^2 S_0(x_t)\eta_B, \tag{1.54}$$

where F_{B_q} is the B_q meson decay constant, $\hat{B}_{B_q}$ is known as the renormalization group invariant parameter or the bag parameter, $S_0(x_t)(x_t = m_t^2/M_W^2)$ is the Inami-Lim function [14] which parametrizes the electroweak contributions without any gluon (QCD) corrections and η_B parametrizes short distance QCD corrections. Thus, we have

$$(M_{12}^*)_d \propto (V_{td}V_{tb}^*)^2\ , \quad (M_{12}^*)_s \propto (V_{ts}V_{tb}^*)^2 \tag{1.55}$$

and since the CKM elements V_{td} and V_{ts} are expressed in terms of the angle $\beta_{(s)}$ (which in turn is related to the complex phases of the CKM matrix) of the unitarity triangle as

$$V_{td} = |V_{td}|e^{-i\beta} \;, \qquad V_{ts} = |V_{ts}|e^{-i\beta_s} \;, \tag{1.56}$$

we have to an approximation

$$\left(\frac{q}{p}\right)_{d,s} = e^{i2\phi_M^{d,s}} \;, \quad \phi_M^d = -\beta \;, \quad \phi_M^s = -\beta_s \;, \tag{1.57}$$

where $\phi_M^{d,s}$ is given in terms of the weak phases of the CKM matrix.

Various Types of CP Violation

The complex phases of the CKM matrix can manifest themselves in the decay processes and in the particle-antiparticle mixing. When the decay of a hadron is considered, theoretical uncertainties are encountered as a consequence of the hadronic matrix elements. It is important that the CP violating measurements be free from such uncertainties. Three types of CP violation are considered, (1) ***CP Violation in Mixing***, (2) ***CP Violation in Decay***, and (3) ***CP Violation in the interference of mixing and decay***.

(1) ***CP Violation in Mixing*** : This type of CP violation is a result of the fact that the mass eigenstates are different from the CP eigenstates and is defined by $\mathrm{Re}(\varepsilon) \neq 0$ or $|q/p| \neq 1$. The effect can be observed in semileptonic decays of B and K where the final states contain "wrong charge" leptons and can be attained only through $B^0 - \bar{B}^0$ mixing. The asymmetry is defined as

$$a_{SL}(B) = \frac{\Gamma(\bar{B}^0(t) \to l^+\nu X) - \Gamma(B^0(t) \to l^-\bar{\nu}X)}{\Gamma(\bar{B}^0(t) \to l^+\nu X) + \Gamma(B^0(t) \to l^-\bar{\nu}X)} = \frac{1 - |q/p|^4}{1 + |q/p|^4} \;, \tag{1.58}$$

where $B^0(0) = B^0$, $\bar{B}^0(0) = \bar{B}^0(0)$ and the time evolution of these states are given by

$$\begin{aligned} |B^0(t)\rangle &= g_+(t)|B^0\rangle + \frac{q}{p}g_-(t)|\bar{B}^0\rangle \;, \\ |\bar{B}^0(t)\rangle &= \frac{p}{q}g_-(t)|B^0\rangle + g_+(t)|\bar{B}^0\rangle \;, \end{aligned} \tag{1.59}$$

where

$$g_{\pm}(t) = \frac{1}{2}(e^{-i(M_H - \frac{i}{2}\Gamma_H)t} \pm e^{-i(M_L - \frac{i}{2}\Gamma_L)t}) \ . \tag{1.60}$$

The asymmetry becomes nonzero as the phases in the transitions $B^0 \to \bar{B}^0$ and $\bar{B}^0 \to B^0$ differ from each other.

(2) ***CP Violation in Decay*** : This type of CP violation is also known as direct CP violation and is best described in charged B and K decays. It can also be measured in the neutral modes. Defining

$$\mathcal{A}_{f^+} = \langle f^+ | \mathcal{H}^{weak} | B^+ \rangle \ , \quad \bar{\mathcal{A}}_{f^-} = \langle f^- | \mathcal{H}^{weak} | B^- \rangle \ , \tag{1.61}$$

the asymmetry is given as

$$\mathcal{A}_{CP}^{dir}(B^\pm \to f^\pm) = \frac{\Gamma(B^+ \to f^+) - \Gamma(B^- \to f^-)}{\Gamma(B^+ \to f^+) + \Gamma(B^- \to f^-)} = \frac{1 - \mid \bar{A}_{f^-}/A_{f^+} \mid^2}{1 + \mid \bar{A}_{f^-}/A_{f^+} \mid^2} \ . \tag{1.62}$$

For direct CP violation, one requires at least two different interfering contributions to the decay amplitude having different weak (ϕ_i) and strong (δ_i) phases. For example, they can be two tree diagrams, two penguin diagrams or one tree and one penguin diagram. In this way, we can write the decay amplitude A_{f^+} and its CP conjugate $\bar{A}_{f^-}$ as

$$A_{f^+} = \sum_{i=1,2} A_i e^{i(\delta_i + \phi_i)} \ , \quad \bar{A}_{f^-} = \sum_{i=1,2} A_i e^{i(\delta_i - \phi_i)} \ . \tag{1.63}$$

The weak phases ϕ_i have opposite sign. But the strong phases δ_i have the same sign as CP is conserved in strong interactions. Thus, we can have $|\bar{A}_{f^-}/A_{f^+}| \neq 1$ and the direct CP asymmetry is, therefore, nonzero and is given as

$$\mathcal{A}_{CP}^{dir}(B^\pm \to f^\pm) = \frac{-2A_1A_2 \sin(\delta_1 - \delta_2)\sin(\phi_1 - \phi_2)}{A_1^2 + A_2^2 + 2A_1A_2\cos(\delta_1 - \delta_2)\cos(\phi_1 - \phi_2)} \ . \tag{1.64}$$

(3) ***CP Violation in the interference of Mixing and Decay*** : This type of CP violation occurs in neutral B decays only where the final states are common to both B^0 and $\bar{B}^0$. The effect can be observed by comparing

the time-dependent decays into final CP eigenstates. A complex quantity λ_f is introduced which describes the interference phenomena between $B^0 \to f$ and $\bar{B}^0 \to f$ and is defined as

$$\lambda_f = \frac{q}{p}\frac{A(\bar{B}^0 \to f)}{A(B^0 \to f)} = e^{i2\phi_M}\frac{A(\bar{B}^0 \to f)}{A(B^0 \to f)} \ , \tag{1.65}$$

where ϕ_M denotes the weak phase in the $B^0 - \bar{B}^0$ mixing and $A(B^0 \to f)$ and $A(\bar{B}^0 \to f)$ are the decay amplitudes. The time-dependent CP asymmetry is defined as

$$\begin{aligned}\mathcal{A}_{CP}(t,f) &= \frac{\Gamma(B^0(t) \to f) - \Gamma(\bar{B}^0(t) \to f)}{\Gamma(B^0(t) \to f) + \Gamma(\bar{B}^0(t) \to f)} \\ &= \mathcal{A}_{CP}^{dir}(f)\cos(\Delta M t) + \mathcal{A}_{CP}^{mix}(f)\sin(\Delta M t) \ ,\end{aligned} \tag{1.66}$$

where $\mathcal{A}_{CP}^{dir}$ are the decay direct CP violating contributions and $\mathcal{A}_{CP}^{mix}$ are the contributions describing CP violation in the interference of mixing and decay which is also usually called mixing-induced CP violation. In terms of λ_f, they are defined as

$$\mathcal{A}_{CP}^{dir}(f) = \frac{1- \mid \lambda_f \mid^2}{1+ \mid \lambda_f \mid^2} \equiv C_f \ , \quad \mathcal{A}_{CP}^{mix}(f) = \frac{2\mathrm{Im}\lambda_f}{1+ \mid \lambda_f \mid^2} \equiv -S_f \tag{1.67}$$

and

$$\mathcal{A}_{CP}(t,f) = C_f\cos(\Delta M t) - S_f\sin(\Delta M t) \ . \tag{1.68}$$

The amplitude $A(B^0 \to f)$ can, in general, have several different contributions like tree, QCD penguin and electroweak penguin contributions. Therefore, we can write

$$\frac{A(\bar{B}^0 \to f)}{A(B^0 \to f)} = -\eta_f\left[\frac{A_T e^{i(\delta_T - \phi_T)} + A_P e^{i(\delta_P - \phi_P)}}{A_T e^{i(\delta_T + \phi_T)} + A_P e^{i(\delta_P + \phi_P)}}\right] \ , \tag{1.69}$$

with $\eta_f = \pm 1$ being the CP-parity of the final state, $A_{T(P)}$ (T denotes tree and P denotes penguin) contains the hadronic matrix elements, $\delta_{T(P)}$ and $\phi_{T(P)}$ are the strong and weak phases, respectively. The minus sign comes from the CP phase convention $CP|B^0\rangle = -|\bar{B}^0\rangle$. The direct CP violation contribution C_f and the mixing-induced CP violation contribution S_f now become

$$C_f = -2r\sin(\phi_1 - \phi_2)\sin(\delta_1 - \delta_2)\ , \tag{1.70}$$

$$S_f = -\eta_f[\sin 2(\phi_1 - \phi_M) + 2r\cos 2(\phi_1 - \phi_M)\sin(\phi_1 - \phi_2)\cos(\delta_1 - \delta_2)]\ , \tag{1.71}$$

where $r = A_2/A_1 \ll 1$ is assumed and ϕ_i and δ_i are the weak and strong phases, respectively. It can be seen that by measuring the CP asymmetry parameters, the CKM weak phases can be measured.

If there is only one weak phase which dominates in the decay amplitude or if the different contributions to the decay amplitude have the same weak phases, then the hadronic matrix elements and the strong phases drop out and one obtains

$$\frac{A(\bar{B}^0 \to f)}{A(B^0 \to f)} = -\eta_f e^{-i2\phi_D} \tag{1.72}$$

with ϕ_D being the weak phase in the decay amplitude $A(B^0 \to f)$. Hence,

$$\lambda_f = -\eta_f \exp(i2\phi_M)\exp(-i2\phi_D)\ , \quad |\lambda_f|^2 = 1 \tag{1.73}$$

and

$$\mathcal{A}_{CP}^{dir}(f) = C_f = 0\ , \tag{1.74}$$

$$\mathcal{A}_{CP}^{mix}(f) = \mathrm{Im}\lambda_f = \eta_f\sin(2\phi_D - 2\phi_M) = -S_f\ . \tag{1.75}$$

Consequently, the asymmetry is given as

$$\mathcal{A}_{CP}(t, f) = -S_f\sin(\Delta M t)\ . \tag{1.76}$$

After having an idea of CP violation and its materialization in the B meson system, we now proceed with presenting a study of the B_c meson decay modes from which we can determine the angle γ of the unitarity triangle in chapter 2. Chapter 3 is dedicated to the extraction of the weak CKM phase $(\gamma/2\beta + \gamma)$ information from the decay modes $\overset{(-)}{B}_d \to \overset{(-)}{D}{}^0\, \overset{(-)}{K}{}^{*0}$, $\overset{(-)}{D}{}^{*0}\, \overset{(-)}{K}{}^{*0}$. Chapter 4 contains a study of the rare decay mode $B \to f_0 K(\pi)$ both in the SM and in some extensions of the SM (new physics models). In chapter 5, we present a study of another rare decay mode $B \to \phi\pi$ and show how some new physics models can provide an enhancement in the branching ratio and CP asymmetry parameters. Chapter 6 contains the summary.

Chapter 2

Determination of the CKM angle γ with B_c decays

It is now known that in the standard model (SM), CP violation arises from the nonzero weak phase in the complex CKM matrix which is responsible for the charged current weak interaction. One of the main ingredients of the SM description of CP violation is the CKM unitarity triangle (UT). When we consider the most relevant unitarity relation describing B decays, we obtain the angles of the UT termed as α (ϕ_2), β (ϕ_1) and γ (ϕ_3) [15]. Therefore, to have a more precise information of CP violation, it is important that we find new ways and methods to extract the three angles α, β and γ of this triangle. Large CP violation, as was expected, has already been established in B systems in the running B factories at SLAC and KEK. The present status is that we have measured, with the huge data sets available, the angle β (actually, sin (2β)) with a reasonable accuracy and we expect to have a precision measurement of angle β in the years to come, with the help of the golden mode $B_d^0 \to J/\psi K_S$. Unfortunately, we do not have three golden modes to determine the three angles of the UT. So we have to be contented with the best available modes like $B \to \pi\pi$ (and some related modes) for the determination of the angle α, but these modes are accompanied by a generic problem called penguin contamination. So finally, we are left with the angle $\gamma = \arg(-V_{ud}V_{ub}^*/V_{cd}V_{cb}^*)$, which was believed to be the most difficult one, among all the three angles, at the beginning. But, fortunately, in this case, nature has been very kind to provide us with many options to determine the angle γ in various avenues.

There have been many attempts in the past to devise methods to determine the CKM angle γ as cleanly as possible. One of the methods to determine γ is the Gronau-London-Wyler (GLW) method [16], which uses the interference of two amplitudes ($b \to c\bar{u}s$ and $b \to u\bar{c}s$) in $B \to DK$ modes. In this method, γ can be determined by measuring the decay rates $B^- \to D^0K^-$, $B^- \to \bar{D}^0K^-$ and $B^- \to D^0_+K^-$ (where D^0_+ is the CP-even eigenstate of neutral D meson system) and their corresponding CP conjugate modes. However, because the mode $B^- \to \bar{D}^0K^-$ is both color and CKM suppressed with respect to $B^- \to D^0K^-$, the corresponding amplitude triangles are expected to be highly squashed and it is also very difficult to measure the rate of $B^- \to \bar{D}^0K^-$. To overcome the problems of the GLW method, Atwood-Dunietz-Soni (ADS) [17] proposed an improved method where they have considered the decay chains $B^- \to K^-D^0[\to f]$ and $B^- \to K^-\bar{D}^0[\to f]$, where f is the doubly Cabibbo suppressed (Cabibbo favored) non-CP eigenstate of $D^0(\bar{D}^0)$. These methods are being explored in the B-factory experiments and will also be taken up at the collider experiments along with another method called the Aleksan-Dunietz-Kayser (ADK) method [18], which uses the time-dependent measurement of $B^0_s(\bar{B}^0_s) \to D^\mp_s K^\pm$ modes. Because of its importance and, of course, possible options available, there are many methods that exist in the literature. Some of the alternative methods to obtain γ are those using B and B_s decays [19]-[27], B_c decays [28] and also Λ_b decays [29].

Another method, the Giri-Grossman-Soffer-Zupan (GGSZ) method (otherwise also known as the Dalitz method) [19] has also been proposed (using $B \to D^0(\bar{D}^0)K \to K_S\pi\pi K$), which has many attractive features and has already been explored at both the B factories. It should be noted here that the GGSZ method uses the ingredients of GLW and ADS methods where the

$D^0(\bar{D}^0)$ decays to multi-particle final states. This method in turn helps us to constrain the angle γ directly from the experiments. But at present the error bars are quite large, which are expected to come down in the coming years. It may be worthwhile to emphasize here that one has to measure the angle with all possible clean methods available to arrive at a conclusion and thereby reducing the error in γ to a minimum.

2.1 The Method

In the continued effort, we now wish to explore yet another method with the decays $B_c^\pm \to D_s^\pm D^0 \to D_s^\pm (K^{*+}K^-)_{D^0}$ and $B_c^\pm \to D_s^\pm \bar{D}^0 \to D_s^\pm (K^{*+}K^-)_{\bar{D}^0}$. It has been shown earlier in [28] that the decay $B_c^\pm \to D^0(\bar{D}^0)D_s^\pm$ modes can be used to determine the CKM angle γ in a better way since the interfering amplitudes in B_c case are roughly of equal sizes, whereas the corresponding ones in GLW method (using B mesons) are not so. In [28], it has been shown that γ can be determined from the decay rates $B_c^\pm \to D^0 D_s^\pm$, $B_c^\pm \to \bar{D}^0 D_s^\pm$ and $B_c^\pm \to D_\pm^0 D_s^\pm$ (where $D_\pm^0$ are the CP eigenstates of neutral D meson system with CP eigenvalues ± 1, which can be identified by the CP-even and CP-odd decay products of neutral D meson). In this work, another method is proposed where we consider the $B_c^\pm \to D^0(\bar{D}^0)D_s^\pm$ decay modes, that are followed by $D^0(\bar{D}^0)$ decaying to $K^{*+}K^-$, which is a non-CP eigenstate.

The decay modes $B_c^- \to D_s^- D^0$ and $B_c^- \to D_s^- \bar{D}^0$ are described by the quark level transition $b \to c\bar{u}s$ and $b \to u\bar{c}s$, respectively and the amplitudes for these processes are given as

$$\begin{aligned}
\mathcal{A}(B_c^- \to D^0 D_s^-) &= \frac{G_F}{\sqrt{2}} V_{cb} V_{us}^* (C + A)\,, \\
\mathcal{A}(B_c^- \to \bar{D}^0 D_s^-) &= \frac{G_F}{\sqrt{2}} V_{ub} V_{cs}^* (\tilde{C} + \tilde{T})\,, \qquad (2.1)
\end{aligned}$$

where C and A denote the color suppressed tree and annihilation topologies for $b \to c$ transition and $\tilde{C}$ and $\tilde{T}$ denote the color suppressed tree and color allowed tree contributions for $b \to u$ transition. Now let us denote these amplitudes as

$$A_B = \mathcal{A}(B_c^- \to D^0 D_s^-) \ , \quad \bar{A}_B = \mathcal{A}(B_c^- \to \bar{D}^0 D_s^-) \ , \tag{2.2}$$

and their ratios as

$$\frac{\bar{A}_B}{A_B} = r_B e^{i(\delta_B - \gamma)} \ , \quad \text{with} \quad r_B = \left|\frac{\bar{A}_B}{A_B}\right| \quad \text{and} \quad \arg\left(\bar{A}_B/A_B\right) = \delta_B - \gamma \ , \tag{2.3}$$

where δ_B and $(-\gamma)$ are the relative strong and weak phases between the two amplitudes. The ratio of the corresponding CP conjugate processes are obtained by changing the sign of the weak phase γ. One can then obtain a rough estimate of r_B from dimensional analysis, i.e.,

$$r_B = \left|\frac{V_{ub}V_{cs}^*}{V_{cb}V_{us}^*}\right| \cdot \frac{a_1^{eff}}{a_2^{eff}} \approx \mathcal{O}(1) \ , \tag{2.4}$$

where a_1^{eff} and a_2^{eff} are the effective QCD coefficients describing the color allowed and color suppressed tree level transitions. For the sake of comparison, we would like to point out here that the corresponding ratio between the $B^- \to D^0(\bar{D}^0)K^-$ amplitudes are given as $|\mathcal{A}(B^- \to \bar{D}^0 K^-)/\mathcal{A}(B^- \to D^0 K^-)| = |(V_{ub}V_{cs}^*)/(V_{cb}V_{us}^*)| \cdot (a_2^{eff}/a_1^{eff}) \approx \mathcal{O}(0.1)$. The D^0 decay amplitudes are denoted as

$$A_D = \mathcal{A}(D^0 \to K^{*+}K^-) \ , \quad \bar{A}_D = \mathcal{A}(\bar{D}^0 \to K^{*+}K^-) \ , \tag{2.5}$$

and their ratios as

$$\frac{\bar{A}_D}{A_D} = r_D e^{i\delta_D} \ , \quad \text{with} \quad r_D = \left|\frac{\bar{A}_D}{A_D}\right| \ . \tag{2.6}$$

It is interesting to note that the parameters r_D and δ_D have been measured by CLEO collaboration [30], with values $r_D = 0.52 \pm 0.05 \pm 0.04$ and $\delta_D = 332^\circ \pm 8^\circ \pm 11^\circ$, rendering our study more appealing.

With these definitions the four amplitudes are given as

$$\begin{aligned}
\mathcal{A}_1(B_c^- \to D_s^-(K^{*+}K^-)_D) &= |A_B A_D|\Big[1 + r_B r_D e^{i(\delta_B+\delta_D-\gamma)}\Big] \,, \\
\mathcal{A}_2(B_c^- \to D_s^-(K^{*-}K^+)_D) &= |A_B A_D| e^{i\delta_D}\Big[r_D + r_B e^{i(\delta_B-\delta_D-\gamma)}\Big] \,, \\
\mathcal{A}_3(B_c^+ \to D_s^+(K^{*-}K^+)_D) &= |A_B A_D|\Big[1 + r_B r_D e^{i(\delta_B+\delta_D+\gamma)}\Big] \,, \\
\mathcal{A}_4(B_c^+ \to D_s^+(K^{*+}K^-)_D) &= |A_B A_D| e^{i\delta_D}\Big[r_D + r_B e^{i(\delta_B-\delta_D+\gamma)}\Big] \,. \quad (2.7)
\end{aligned}$$

From these amplitudes one can obtain the four observables $(R_1, \cdots, R_4)$, with the definition

$$R_i = \left|\mathcal{A}_i(B_c^\mp \to D_s^\mp(K^{*\pm}K^\mp)_D)/A_B A_D\right|^2 \,. \quad (2.8)$$

We can now write $R_1 = 1 + r_B^2 r_D^2 + 2 r_B r_D \cos(\delta_B + \delta_D - \gamma)$ and similarly for R_2, R_3 and R_4.

Here we assume that the amplitudes $|A_B|$ and $|A_D|$ are known (so also r_B, which is $\mathcal{O}(1)$).

Thus, one can obtain an analytical expression for γ as

$$\sin^2\gamma = \frac{[R_1 - R_3]^2 - [R_2 - R_4]^2}{4\Big[[R_2 - R_{BD}^2][R_4 - R_{BD}^2] - [R_1 - \tilde{R}_{BD}^2][R_3 - \tilde{R}_{BD}^2]\Big]} \,, \quad (2.9)$$

where $R_{BD}^2 = (r_B^2 + r_D^2)$ and $\tilde{R}_{BD}^2 = (1 + r_B^2 r_D^2)$.

We then study the sensitivity of γ in some limiting cases in the method described above.

(a) If the relative strong phase between $\bar{A}_B$ and A_B is zero then Eq.(2.9) can no longer be used to extract the angle γ as both numerator and denominator vanish in this limit. However, still γ can be extracted, in this limit, from either the observables R_1 and R_3 or R_2 and R_4. Now, considering the observables R_2 and R_4, for example, one can obtain an expression for γ as

$$\tan\gamma = \frac{\cot\delta_D (R_4 - R_2)}{R_2 + R_4 - 2(r_B^2 + r_D^2)} \,. \quad (2.10)$$

An analogous expression for γ can also be obtained from R_1 and R_3 with the replacement of $R_{2,4} \leftrightarrow R_{3,1}$ and $(r_B^2 + r_D^2) \leftrightarrow (1 + r_B^2 r_D^2)$.

(b) If $r_B = 1$ and $\delta_B = 0$, then the four observables $(R_1, \cdots, R_4)$ are no longer independent of each other and we have two degenerate sets with $(R_1 = R_4)$ and $(R_2 = R_3)$. One can then define two parameters

$$\begin{aligned} C_- &\equiv \cos(\delta_D - \gamma) = \frac{1}{2r_B r_D}(R_4 - r_B^2 - r_D^2)\ , \\ C_+ &\equiv \cos(\delta_D + \gamma) = \frac{1}{2r_B r_D}(R_2 - r_B^2 - r_D^2)\ , \end{aligned} \tag{2.11}$$

where we have deliberately retained the r_B term in the above expressions, so that one can still use this method for $r_B \neq 1$ case. Thus, one can now obtain the solution for γ, in terms of these observables, as

$$\sin^2\gamma = \frac{1}{2}\left[1 - C_+C_- \pm \sqrt{(1 - C_+^2)(1 - C_-^2)}\right]\ , \tag{2.12}$$

one solution of which will give $\sin^2\gamma$ while the other being $\sin^2\delta_D$. Since δ_D has already been measured, $\sin^2\gamma$ could be extracted from these observables, once we know the values of R_2, R_4 (otherwise R_1 and R_3) and r_B (it may be noted that the value of r_D is already known now).

Our method consists of two parts, the first one being the $B_c^\pm \to D^0(\bar{D}^0)D_s^\pm$, which will be measured at the hadron colliders, such as LHC, whereas the second part consists of the measurement of $D^0(\bar{D}^0) \to K^{*+}K^-$, which can also be measured at the same collider experiments. Moreover, since we already have experiments and there are upcoming dedicated experiments to measure the parameters in the charm-sector, like at CLEO-c and the BEPCII, which will provide us half of the parameters needed in our study, it is meaningful to combine the data from various experiments, mentioned above, to obtain γ with a better accuracy.

The possible effect of $D^0 - \bar{D}^0$ mixing for the determination of γ is not taken into account in our analysis since it has been well studied in the literature [19, 31] and found that the effect is very small, unless we are doing a precision measurement of γ. To be quantitative, the error could be around 1°, with the present data available, which for all practical purposes is ignored at the moment.

Now, with r_D already known (so also δ_D), we are left with only two unknowns (δ_B and γ). Therefore, we have two unknowns and four observables. We can consider different non-CP eigenstates (like $\rho^+\pi^-$), which will increase the observables by four and unknowns by two (r'_D and δ'_D) for each additional eigenstate. One can also take $B_c^\pm \to D^0(\bar{D}^0)D_s^{*\pm}$ mode, thereby further increasing number of observables by four and unknowns by two (say r'_B and δ'_B, in fact it could be just δ'_B). Hence we hope to have enough observables and at best half the number of unknowns (actually, it will always be less than half since new unknown parameters, namely, r'_D and δ'_D can also be inferred from the D decay data) and we can obtain the value of γ without hadronic uncertainties.

Now we estimate the branching ratios for these modes. Using the generalized factorization approximation, the amplitudes are given as

$$\begin{aligned}
\mathcal{A}(B_c^- \to D^0 D_s^-) &= \frac{G_F}{\sqrt{2}} V_{cb} V_{us}^* (a_2^{eff} X + a_1^{eff} Y) \, , \\
\mathcal{A}(B_c^- \to \bar{D}^0 D_s^-) &= \frac{G_F}{\sqrt{2}} V_{ub} V_{cs}^* (a_1^{eff} X_1 + a_2^{eff} X) \, , \qquad (2.13)
\end{aligned}$$

where $X = if_{D^0}(m_{B_c}^2 - m_{D_s}^2) F_0^{B_c D_s}(m_{D^0}^2)$, $X_1 = if_{D_s}(m_{B_c}^2 - m_{D^0}^2) F_0^{B_c D^0}(m_{D_s}^2)$ and $Y = if_{B_c}(m_{D_s}^2 - m_{D^0}^2) F_0^{D_s D^0}(m_{B_c}^2)$ are the factorized hadronic matrix elements. For numerical evaluation we use the values of the form factors at zero recoil from [32] as $F_0^{B_c D^0}(0) = 0.352$, $F_0^{B_c D_s}(0) = 0.37$, the decay constants (in MeV) as $f_{D^0} = 235$, $f_{D_s} = 294$, $f_{B_c} = 360$, the QCD coefficients $a_1^{eff} = 1.01$, $a_2^{eff} = 0.23$, particle masses, lifetime of B_c and CKM matrix elements from [33]. We thus obtain the branching ratios as

$$\mathrm{BR}(B_c^- \to D^0 D_s^-) = 7.0 \times 10^{-6} \, , \quad \mathrm{BR}(B_c^- \to \bar{D}^0 D_s^-) = 4.5 \times 10^{-5} \, . \qquad (2.14)$$

Let us now make a crude estimate of the number of reconstructed events that could be observable at LHC per year of running. At LHC, one expects about 10^{10} untriggered B_c's per year [34]. For the estimation, we use the branching ratios as $\mathrm{BR}(B_c^- \to D^0 D_s^-) = 7.0\times 10^{-6}$ and $\mathrm{BR}(D^0 \to K^{*+} K^-) = 3.7\times 10^{-3}$

[33] and assume that the D_s can be reconstructed efficiently by combining a number of hadronic decay modes. As the LHCb trigger system has a good performance for hadronic modes, we assume an overall efficiency of 30% and hence we expect to get nearly 80 events per year of running at LHC.

2.2 Discussion and Conclusion

We have outlined here that $B_c^\pm \to (D^0)D_s^\pm \to (K^{*\pm}K^\mp)D_s^\pm$ and $B_c^\pm \to (\bar{D}^0)D_s^\pm \to (K^{*\pm}K^\mp)D_s^\pm$ can be used to determine the CKM angle γ at the LHC. Since the interfering amplitudes are of equal order (which is not the case with $B \to DK$ methods) and furthermore neither tagging nor time-dependent studies are required to undertake this strategy and above all the final particles are charged ones (and of course with reduced background), this method may be very well suited for the determination of γ without hadronic uncertainties. But one has to pay the price for all the niceties of this method in the sense that the branching ratios are smaller by an order compared to the earlier modes. Nevertheless, we hope that this should not cause any hindrance for the clean determination of the angle γ using this method and even if we get lesser number of events, the predictive power will not be diluted.

In conclusion, in this study we have looked into the possibility of extracting the CKM angle γ using multibody B_c decays and in view of the fact that LHC is already underway, this method can be found to be very useful to obtain γ and to supplement the results from other methods.

Chapter 3

Extraction of $\gamma/2\beta+\gamma$ from $\overset{(-)}{B}{}^0_d \to \overset{(-)}{D}{}^{*0}\ \overset{(-)}{K}{}^{*0}$

In the context of CP violation, we have already stressed the importance of extracting the three angles α, β and γ of the unitarity triangle. Usually, these angles are extracted from CP violating rate asymmetries in B decays. The angle β (or $\sin 2\beta$) has been cleanly determined from the measurement of the time-dependent CP asymmetry in the golden decay mode $B^0_d \to J/\psi K_S$ [35]. The angle α can be measured using the CP asymmetries in $B^0_d \to \pi^+\pi^-$ [36], but due to the existence of penguin diagrams there are theoretical hadronic uncertainties which are very difficult to quantify. The last angle which is hoped to be determined cleanly is γ. There have been many attempts, suggestions and discussions to measure this angle as cleanly as possible without hadronic uncertainties. The Gronau, London and Wyler (GLW) [16] method to extract γ has been cited in the preceding chapter. However, as pointed out, we encounter difficulties for the extraction of γ with the $B \to DK$ modes.

There exist many studies in the literature [17, 19, 22, 25, 28, 37] to help overcome the difficulty and to provide improved ways to determine the angle γ. It is highly desirable to have independent measurement of the angle γ (or otherwise the angle α), at least to the precision of the angle β as of today, to understand better the CKM mechanism of CP violation under the framework of the SM. But so far we have not been able to succeed in this effort. Given the various methods and wide range of options available, the measurement of the angle γ seems to be a better option. This is currently being done and will also be taken up in the second generation experiments. There is also another parameter, namely, $2\beta+\gamma$ which is discussed in the literature [38] to be measured. Since β is well measured by now, therefore, the measurement of $2\beta+\gamma$ will be very much useful for the clean determination of γ. It should be noted here that we should measure the angle γ in all possible ways (and as cleanly as possible) to independently verify the measurements, improve the statistics and to help resolve discrete ambiguities. To this end, we intend to present here another important and simple way to extract the weak phase $\gamma/(2\beta+\gamma)$ from the decay modes $\overset{(-)}{B}_d \to \overset{(-)}{D}{}^0 \overset{(-)}{K}{}^{*0}$, $\overset{(-)}{D}{}^{*0} \overset{(-)}{K}{}^{*0}$.

In this study, we consider the color suppressed decay modes $\overset{(-)}{B}_d \to \overset{(-)}{D}{}^0\ (\overset{(-)}{D}{}^{*0}\ )\ \overset{(-)}{K}{}^{*0}$, to extract the CKM phase information. Several studies [39, 40, 41] have been carried out using these decay modes for the extraction of the angle γ. In this investigation, we present another alternative method, which is very clean and simple to extract the weak phase $\tan^2\gamma$ $(\tan^2(2\beta+\gamma))$.

3.1 γ from the decay modes $B^0 \to D^0 K^{*0}$ and $B^0 \to \bar{D}^0 K^{*0}$

First let us consider the decay channels $B^0 \to D^0 K^{*0}$ and $B^0 \to \bar{D}^0 K^{*0}$. It has been shown in Ref. [40] that the CKM angle γ can be determined by measuring the following six decay rates: $B^0 \to D^0 K^{*0}$ and $B^0 \to \bar{D}^0 K^{*0}$, $B \to D_{\rm CP} K^{*0}$ (where $D_{\rm CP} = (D^0 + \bar{D}^0)/\sqrt{2}$, is the CP-even eigenstate of the neutral D meson) and the corresponding conjugate processes. The $D^0(\bar{D}^0)$ meson is considered to decay subsequently to the flavor state $K^+\pi^-$ for which the ratio of the two amplitudes is found to be very tiny i.e., $r_D = |A(B^0 \to K^+\pi^-)/A(\bar{D}^0 \to K^+\pi^-)| = 0.06 \pm 0.003$ [33]. Here we show that if we consider the decay of the D meson to the non-CP final state i.e., $K^{*+}K^-$ for which $r_D \sim \mathcal{O}(1)$, then it is possible to extract the CKM angle γ by measuring only four decay rates. The method presented here is similar to the one presented in chapter 2 but with the final state decay particle $D_s^\pm$ being replaced by the K^{*0} meson. This method is very promising because the experimental branching ratio for the process $B^0 \to \bar{D}^0 K^{*0}$ is already known with value $\mathrm{BR}(B^0 \to \bar{D}^0 K^{*0}) = (5.3 \pm 0.8) \times 10^{-5}$ and for the $B^0 \to D^0 K^{*0}$ process, we have the upper limit as $\mathrm{BR}(B^0 \to D^0 K^{*0}) < 1.8 \times 10^{-5}$ [33]. The advantage of using the non-CP eigenstate has been discussed in [25], in connection with the charged B decays $B^\pm \to K^\pm D^0(\bar{D}^0)$, which renders the corresponding interfering amplitudes to be of the same order.

Now let us denote the amplitudes for these processes as

$$A_B = \mathcal{A}(\bar{B}^0 \to D^0 \bar{K}^{*0})\ , \quad \bar{A}_B = \mathcal{A}(\bar{B}^0 \to \bar{D}^0 \bar{K}^{*0}) \tag{3.1}$$

and their ratio as

$$\frac{\bar{A}_B}{A_B} = r_B e^{i(\delta_B - \gamma)}\ , \quad \text{with} \quad r_B = \left|\frac{\bar{A}_B}{A_B}\right| \quad \text{and} \quad \arg\left(\bar{A}_B/A_B\right) = \delta_B - \gamma\ , \tag{3.2}$$

where δ_B and $(-\gamma)$ are the relative strong and weak phases between the two amplitudes. The ratio of the corresponding CP conjugate processes is obtained by changing the sign of the weak phase γ. One can then obtain a rough estimate of r_B from dimensional analysis, i.e.,

$$r_B = \left|\frac{V_{ub}V_{cs}^*}{V_{cb}V_{us}^*}\right| \approx 0.4\ . \tag{3.3}$$

Now we consider that both D^0 and $\bar{D}^0$ will decay into the common non-CP final state $(K^{*+}K^-)$. Denoting the D^0 decay amplitudes as

$$A_D = \mathcal{A}(D^0 \to K^{*+}K^-)\ , \quad \bar{A}_D = \mathcal{A}(\bar{D}^0 \to K^{*+}K^-)\ , \tag{3.4}$$

one can write their ratio

$$\frac{\bar{A}_D}{A_D} = r_D e^{i\delta_D}\ , \quad \text{with} \quad r_D = \left|\frac{\bar{A}_D}{A_D}\right|\ , \tag{3.5}$$

where δ_D is the relative strong phase between them. As mentioned in the previous chapter, the parameters r_D and δ_D have been measured by CLEO collaboration [30].

With these definitions the four amplitudes are given as

$$\begin{aligned}
\mathcal{A}_1(\bar{B}_d^0 \to (K^{*+}K^-)_D\bar{K}^{*0}) &= |A_BA_D|\left[1+r_Br_De^{i(\delta_B+\delta_D-\gamma)}\right] , \\
\mathcal{A}_2(\bar{B}_d^0 \to (K^{*-}K^+)_D\bar{K}^{*0}) &= |A_BA_D|e^{i\delta_D}\left[r_D+r_Be^{i(\delta_B-\delta_D-\gamma)}\right] , \\
\mathcal{A}_3(B_d^0 \to (K^{*-}K^+)_DK^{*0}) &= |A_BA_D|\left[1+r_Br_De^{i(\delta_B+\delta_D+\gamma)}\right] , \\
\mathcal{A}_4(B_d^0 \to (K^{*+}K^-)_DK^{*0}) &= |A_BA_D|e^{i\delta_D}\left[r_D+r_Be^{i(\delta_B-\delta_D+\gamma)}\right] . \qquad (3.6)
\end{aligned}$$

From these amplitudes, one can obtain the four observables $(R_1, \cdots, R_4)$, with the definition

$$R_i = \left|\mathcal{A}_i(\overset{(-)}{B_d} \to (K^{*\pm}K^{\mp})_D\ \overset{(-)}{K^{*0}})/(A_BA_D)\right|^2 . \qquad (3.7)$$

We can now write $R_1 = 1 + r_B^2r_D^2 + 2r_Br_D\cos(\delta_B+\delta_D-\gamma)$ and similarly for R_2, R_3 and R_4. Thus, we get four observables and three unknowns, namely, r_B, δ_B and γ. Hence, γ can in principle be determined from these four observables.

Assuming that the amplitudes $|A_B|$ and $|A_D|$ are known (so also r_B, which is expected to be $\sim \mathcal{O}(0.4)$), we obtain an analytical expression for γ as

$$\tan^2\gamma = \frac{(R_1-R_3)^2-(R_2-R_4)^2}{[R_2+R_4-2(r_B^2+r_D^2)]^2-[R_1+R_3-2(1+r_B^2r_D^2)]^2} . \qquad (3.8)$$

Thus the measurement of the four observables $R_{1,\cdots,4}$ can be used to extract cleanly the CKM angle γ.

3.2 $2\beta+\gamma$ from $B^0 \to D^{*0}(\bar{D}^{*0})K^{*0}$

Next, we consider the decay channels $B_d^0 \to D^{*0}K^{*0}$, $\bar{D}^{*0}K^{*0}$ and $\bar{B}_d^0 \to D^{*0}\bar{K}^{*0}$, $\bar{D}^{*0}\bar{K}^{*0}$ with two vector mesons in the final state. Considering the decay of a B meson into two vector mesons V_1 and V_2, which subsequently decays into pseudoscalar mesons i.e., $V_1 \to P_1 P_1'$ and $V_2 \to P_2 P_2'$, one can write the normalized differential angular distribution as [42]

$$\frac{1}{\Gamma}\frac{d^3\Gamma}{d\cos\theta_1\, d\cos\theta_2\, d\psi} = \frac{9}{8\pi\Gamma}\Bigg\{L_1 \cos^2\theta_1 \cos^2\theta_2 + \frac{L_2}{2}\sin^2\theta_1 \sin^2\theta_2 \cos^2\psi$$
$$+\frac{L_3}{2}\sin^2\theta_1 \sin^2\theta_2 \sin^2\psi + \frac{L_4}{2\sqrt{2}}\sin 2\theta_1 \sin 2\theta_2 \cos\psi$$
$$-\frac{L_5}{2\sqrt{2}}\sin 2\theta_1 \sin 2\theta_2 \sin\psi - \frac{L_6}{2}\sin^2\theta_1 \sin^2\theta_2 \sin 2\psi\Bigg\}, \quad (3.9)$$

where θ_1 (θ_2) is the angle between the three-momentum of P_1 (P_2) in the V_1 (V_2) rest frame and the three-momentum of V_1 (V_2) in the B rest frame, and ψ is the angle between the normals to the planes defined by $P_1 P_1'$ and $P_2 P_2'$, in the B rest frame. The coefficients L_i can be expressed in terms of three independent amplitudes, A_0, $A_\parallel$ and $A_\perp$, which correspond to the different polarization states of the vector mesons as

$$\begin{aligned} L_1 &= |A_0|^2\,, & L_4 &= \mathrm{Re}[A_\parallel A_0^*]\,,\\ L_2 &= |A_\parallel|^2\,, & L_5 &= \mathrm{Im}[A_\perp A_0^*]\,,\\ L_3 &= |A_\perp|^2\,, & L_6 &= \mathrm{Im}[A_\perp A_\parallel^*]\,. \end{aligned} \quad (3.10)$$

In the above A_0, $A_\parallel$, and $A_\perp$ are complex amplitudes of the three helicity states in the transversity basis. These observables can be efficiently extracted from the angular distribution (3.9) using the appropriate weight functions as discussed in Ref. [43].

The decay mode $B \to V_1 V_2$ can also be described in the helicity basis, where the amplitude for the helicity matrix element can be parameterized as [44]

$$\begin{aligned} H_\lambda &= \langle V_1(\lambda) V_2(\lambda)|\mathcal{H}_{eff}|B^0\rangle \\ &= \varepsilon^*_{1\mu}(\lambda)\,\varepsilon^*_{2\nu}(\lambda)\left[a g^{\mu\nu} + \frac{b}{m_1 m_2}p^\mu p^\nu + \frac{ic}{m_1 m_2}\epsilon^{\mu\nu\alpha\beta}p_{1\alpha}p_\beta\right]\,, \end{aligned} \quad (3.11)$$

where p is the B meson momentum, $\lambda = 0, \pm 1$ are the helicity of both the vector mesons and m_i, p_i and ε_i ($i = 1, 2$) denote their masses, momenta and polarization vectors, respectively. Furthermore, the three invariant amplitudes a, b, and c are related to the helicity amplitudes by

$$H_{\pm 1} = a \pm c\sqrt{x^2-1}\ , \qquad H_0 = -ax - b\,(x^2-1)\ , \tag{3.12}$$

where $x = (p_1 \cdot p_2)/m_1 m_2 = (m_B^2 - m_1^2 - m_2^2)/(2m_1 m_2)$.

The corresponding decay rate using the helicity basis amplitudes can be given as

$$\Gamma = \frac{p_{cm}}{8\pi m_B^2}\bigg(|H_0|^2 + |H_{+1}|^2 + |H_{-1}|^2\bigg)\ , \tag{3.13}$$

where p_{cm} is the magnitude of the center-of-mass momentum of the outgoing vector particles.

The amplitudes in the transversity and helicity bases are related to each other through the following relations

$$A_\perp\ =\ \frac{H_{+1}-H_{-1}}{\sqrt{2}}, \qquad A_\parallel\ =\ \frac{H_{+1}+H_{-1}}{\sqrt{2}}, \qquad A_0\ =\ H_0. \tag{3.14}$$

The corresponding helicity amplitudes $\bar{H}_\lambda$ for the complex conjugate decay process $\bar{B} \to \bar{V}_1 \bar{V}_2$ have the same decomposition with $a \to \bar{a}, b \to \bar{b}$ and $c \to -\bar{c}$. The amplitudes $\bar{a}, \bar{b}$ and $\bar{c}$ can be obtained from a, b and c by changing the sign of the weak phases.

In order to study the feasibility of this method, first we would like to estimate the branching ratios of the above mentioned decay modes. Only the experimental upper limits for these modes are known so far i.e., BR($B^0 \to \bar{D}^{*0} K^*$) $< 6.9 \times 10^{-5}$ and BR($B^0 \to D^{*0} K^*$) $< 4.0 \times 10^{-5}$ [33]. We expect that these modes will be well measured in the asymmetric B factories or in the LHCb experiment.

In the SM, these decays proceed through color suppressed tree diagrams only and are free from penguin contributions. The decay $B^0 \to D^{*0} K^{*0}$ arises from the quark level transition $\bar{b} \to \bar{u} c \bar{s}$ and the process $\bar{B}^0 \to D^{*0} \bar{K}^{*0}$ arises from $b \to c \bar{u} s$. To evaluate the hadronic matrix element $\langle O_i \rangle \equiv \langle D^{*0} \overset{(-)}{K^{*0}}\ | O_i | \overset{(-)}{B_d}\ \rangle$, the factorization approximation has been used. Thus, in this approach, we obtain the factorized amplitude for the $B^0 \to D^{*0}K^{*0}$ modes as

$$
\begin{aligned}
H &= \frac{G_F}{\sqrt{2}}\lambda_u^*\ a_2\langle K^{*0}(\varepsilon_1,p_1)|(\bar{s}b)_{V-A}|B_d^0(p)\rangle\langle D^{*0}(\varepsilon_2,p_2)|(\bar{u}c)_{V-A}|0\rangle \\
&= \frac{G_F}{\sqrt{2}}\lambda_u^*\ a_2\ if_{D^{*0}}m_{D^{*0}}\Bigg[(m_{B^0}+m_{K^{*0}})A_1^{BK^*}(m_{D^{*0}}^2)(\varepsilon_1^*\cdot\varepsilon_2^*) \\
&- \frac{2A_2^{BK^*}(m_{D^{*0}}^2)}{(m_{B^0}+m_{K^{*0}})}(\varepsilon_1^*\cdot p)(\varepsilon_2^*\cdot p) \\
&- i\frac{2V^{BK^*}(m_{D^{*0}}^2)}{(m_{B^0}+m_{K^{*0}})}\epsilon_{\mu\nu\alpha\beta}\varepsilon_2^{*\mu}\varepsilon_1^{*\nu}p^\alpha p_1^\beta\Bigg]\ ,
\end{aligned}
\tag{3.15}
$$

where $f_{D^{*0}}$ is the decay constant of the vector meson D^{*0} and $\lambda_u^* = V_{ub}^*V_{cs}$. Furthermore, $A_1^{BK^*}(m_{D^{*0}}^2), A_2^{BK^*}(m_{D^{*0}}^2)$ and $V^{BK^*}(m_{D^{*0}}^2)$ are the form factors involved in the transition $B^0 \to K^{*0}$. The coefficient a_2 is given by $a_2 = C_2 + C_1/N_C$, with N_C as the number of colors. Thus, in this way, we can have the invariant amplitudes a, b and c (in the unit of $G_F/\sqrt{2}$) as

$$
\begin{aligned}
a &= ia_2\lambda_u^*\ f_{D^{*0}}m_{D^{*0}}(m_{B^0}+m_{K^{*0}})A_1^{BK^*}(m_{D^{*0}}^2)\ , \\
b &= -ia_2\lambda_u^*\ f_{D^{*0}}m_{D^{*0}}\frac{2m_{D^{*0}}m_{K^{*0}}}{(m_{B^0}+m_{K^{*0}})}A_2^{BK^*}(m_{D^{*0}}^2)\ , \\
c &= -ia_2\lambda_u^*\ f_{D^{*0}}m_{D^{*0}}\frac{2m_{D^{*0}}m_{K^{*0}}}{(m_{B^0}+m_{K^{*0}})}V^{BK^*}(m_{D^{*0}}^2)\ .
\end{aligned}
\tag{3.16}
$$

Substituting the values of the effective coefficient $a_2 = 0.23$, the Wolfenstein parameters $A = 0.801, \lambda = 0.2265, \bar{\rho} = 0.189$ and $\bar{\eta} = 0.358$ from [45], the decay constant $f_{D^{*0}} = 240$ MeV, the particle masses and lifetimes from [33] and the form factors $A_1^{BK^*}(m_{D^{*0}}^2) = 0.32, A_2^{BK^*}(m_{D^{*0}}^2) = 0.31$ and $V^{BK^*}(m_{D^{*0}}^2) = 0.52$ from [46], we obtain the branching ratio for the $B^0 \to D^{*0}K^{*0}$ as

$$
\mathrm{BR}(B^0 \to D^{*0}K^{*0}) = 3.87\times10^{-6}\ . \tag{3.17}
$$

Similarly, one can obtain the transition amplitude for the $\bar{B}^0 \to D^{*0}\bar{K}^{*0}$ process, which is analogous to (3.15) with the replacement of λ_u^* by $\lambda_c = V_{cb}V_{us}^*$ and hence the corresponding branching ratio as

$$
\mathrm{BR}(\bar{B}^0 \to D^{*0}\bar{K}^{*0}) = 2.3\times10^{-5}\ . \tag{3.18}
$$

Since the branching ratios of the above two processes are very much within the reach of the present experiments, we expect that these processes will be measured soon by the running B factories and one will have a plenty of such events in the upcoming LHCb factory.

Now, we consider the extraction of $(2\beta+\gamma)$ from the modes $\overset{(-)}{B}_d \to \overset{(-)}{D}{}^{*0}\overset{(-)}{K}{}^{*0}$. Since it is possible to obtain the different helicity contributions by performing an angular analysis [43, 47], from now onward we will concentrate on the longitudinal (i.e., A_0) component, which is the dominant one. The $K_S\pi^0$ mode of $\overset{(-)}{K}{}^{*0}$ allows the $B^0 \to D^{*0}K^{*0}$ and $\bar{B}^0 \to D^{*0}\bar{K}^{*0}$ amplitudes to interfere with each other. As discussed earlier, the decay amplitude for the mode $B_d^0 \to D^{*0}K^{*0}$ arises from $\bar{b} \to \bar{u}c\bar{s}$ and carries the weak phase $e^{i\gamma}$ while $\bar{B}_d^0 \to D^{*0}\bar{K}^{*0}$ arises from the quark transition $b \to c\bar{u}s$ and carries no weak phase. The amplitudes also carry strong phases $e^{i\delta_1}$ and $e^{i\delta_2}$. Thus, we can write the longitudinal components of the decay amplitudes as

$$\begin{aligned}
A_0(f) &= \mathrm{Amp}(B_d^0 \to f)_0 = M_1 e^{i\gamma} e^{i\delta_1}\ , \\
\bar{A}_0(f) &= \mathrm{Amp}(\bar{B}_d^0 \to f)_0 = M_2 e^{i\delta_2}\ , \\
\bar{A}_0(\bar{f}) &= \mathrm{Amp}(\bar{B}_d^0 \to \bar{f})_0 = M_1 e^{-i\gamma} e^{i\delta_1}\ , \\
A_0(\bar{f}) &= \mathrm{Amp}(B_d^0 \to \bar{f})_0 = M_2 e^{i\delta_2}.
\end{aligned} \tag{3.19}$$

Since the final state $f = D^{*0}\overset{(-)}{K}{}^{*0}$ is accessible to B^0 and $\bar{B}^0$, inserting the time evolution of the observables $A_0(t)$ as in [48], one arrives at the usual expression for the longitudinal component of the time-dependent decay widths [49] as

$$\begin{aligned}
\Gamma_0(B^0(t) \to f) &= \frac{e^{-\Gamma t}}{2}\Big\{ \left(|A_0(f)|^2 + |\bar{A}_0(f)|^2\right) \\
&+ \left(|A_0(f)|^2 - |\bar{A}_0(f)|^2\right)\cos\Delta m t \\
&- 2\mathrm{Im}\left(\frac{q}{p}A_0(f)^*\bar{A}_0(f)\right)\sin\Delta m t\Big\}, \\
\Gamma_0(\bar{B}^0(t) \to f) &= \frac{e^{-\Gamma t}}{2}\Big\{ \left(|A_0(f)|^2 + |\bar{A}_0(f)|^2\right) \\
&- \left[|A_0(f)|^2 - |\bar{A}_0(f)|^2\right)\cos\Delta m t \\
&+ 2\mathrm{Im}\left(\frac{q}{p}A_0(f)^*\bar{A}_0(f)\right)\sin\Delta m t\Big\}, \qquad (3.20)
\end{aligned}$$

where $q/p = \exp(-2i\beta)$ is the $B^0 - \bar{B}^0$ mixing parameter and Γ and Δm denote the average width and the mass difference of the heavy and light B mesons and we have neglected the small width difference $\Delta\Gamma$ between them.

Thus, the time-dependent measurement of the longitudinal component of $B^0(t) \to f$ decay rates allows one to obtain the following observables :

$$|A_0(f)|^2 + |\bar{A}_0(f)|^2, \quad |A_0(f)|^2 - |\bar{A}_0(f)|^2, \text{ and } \mathrm{Im}[\frac{q}{p}A_0(f)^*\bar{A}_0(f)], \qquad (3.21)$$

i.e., the longitudinal components of CP averaged branching ratio, the direct CP violation and the mixing-induced CP violation parameters.

Similarly, one can obtain the time-dependent decay rates for the final state $\bar{f}$ i.e., $\Gamma_0(\bar{B}^0(t) \to \bar{f})$ from $\Gamma_0(B^0(t) \to f)$ by replacing $A_0(f)$ by $\bar{A}_0(\bar{f})$ and $\bar{A}_0(f)$ by corresponding CP conjugate $A_0(\bar{f})$. $\Gamma_0(B^0(t) \to \bar{f})$ can be obtained from $\Gamma_0(\bar{B}^0(t) \to f)$ with similar substitution.

Now substituting the decay amplitudes as defined in Eq. (3.19) in (3.20), we get the decay rates as

$$\begin{aligned}
\Gamma_0(B^0(t)\to f) &= \frac{e^{-\Gamma t}}{2}\Big\{(M_1^2+M_2^2)+(M_1^2-M_2^2)\cos\Delta mt \\
&- 2M_1M_2\sin(\delta-\phi)\sin\Delta mt\Big\}\,, \\
\Gamma_0(B^0(t)\to \bar f) &= \frac{e^{-\Gamma t}}{2}\Big\{(M_1^2+M_2^2)-(M_1^2-M_2^2)\cos\Delta mt \\
&+ 2M_1M_2\sin(\delta+\phi)\sin\Delta mt\Big\}\,, \\
\Gamma_0(\bar B^0(t)\to \bar f) &= \frac{e^{-\Gamma t}}{2}\Big\{(M_1^2+M_2^2)+(M_1^2-M_2^2)\cos\Delta mt \\
&- 2M_1M_2\sin(\delta+\phi)\sin\Delta mt\Big\}\,, \\
\Gamma_0(\bar B^0(t)\to f) &= \frac{e^{-\Gamma t}}{2}\Big\{(M_1^2+M_2^2)-(M_1^2-M_2^2)\cos\Delta mt \\
&+ 2M_1M_2\sin(\delta-\phi)\sin\Delta mt\Big\}\,,
\end{aligned} \tag{3.22}$$

where $\delta=\delta_2-\delta_1$ is the strong phase difference between the longitudinal components of the two amplitudes $\bar B^0\to f$ and $B^0\to f$ and $\phi=2\beta+\gamma$. Thus, through the measurements of the time-dependent rates, it is possible to measure the amplitudes M_1 and M_2 and the CP violating quantities $S_+\equiv\sin(\delta+\phi)$ and $S_-\equiv\sin(\delta-\phi)$. In turn these quantities will determine $\tan^2\phi$ up to a four fold ambiguity via the expression

$$\tan^2\phi[\cot^2\delta] = \frac{(S_+-S_-)^2}{2-S_-^2-S_+^2\pm 2\sqrt{(1-S_+^2)(1-S_-^2)}}\,, \tag{3.23}$$

where one sign will give $\tan^2\phi$ and the other $\cot^2\delta$.

Let us now estimate the number of reconstructed events that could be observed at the B factories assuming that 3×10^8 (10^{12}) $B\bar B$'s are (will be) available at the e^+e^- asymmetric B factories (hadronic B machines like LHCb). Let us first estimate the number of $B^0\to D^0(\bar D^0)K^{*0}$ events that will be available in the upcoming LHCb experiment. Assuming the branching ratio for the process $B^0\to D^0K^{*0}$ to be $|(V_{ub}V_{cs}^*)/(V_{cb}V_{us}^*)|^2\times \mathrm{BR}(B^0\to$

$\bar{D}^0K^{*0}) \approx 8.5\times10^{-6}$, BR$(D^0 \to K^{*+}K^-) = 3.7\times10^{-3}$ [33] and 10% overall reconstruction efficiency, we expect to get nearly 9×10^3 events per year of running at LHCb. For the corresponding vector-vector modes, we use the longitudinal component of the branching ratio as BR$_0(B^0 \to D^{*0}K^{*0}) = 0.65\times$ BR$(B^0 \to D^{*0}K^{*0}) \approx 2.51\times10^{-6}$, BR$(D^{*0} \to D^0\pi^0) = 62\%$ [33], BR$(K^{*0} \to K_S\pi^0) =$ Br$(K^{*0} \to K\pi)/3$, and an overall efficiency of 10%. Thus we expect to get approximately 15 (5×10^4) reconstructed events at the e^+e^- (hadronic) machines per year of running. This crude estimate indicates that this method may be well suited for the extraction of the weak phase $\gamma(2\beta+\gamma)$ at LHCb.

3.3 Conclusion

We have carried out a study of the color suppressed decay modes $B^0 \to D^0(D^{0*})K^{0*}$ to extract the weak phase $\gamma(2\beta+\gamma)$. For the extraction of γ, we considered the decay modes $B^0 \to D^0(\bar{D}^0)K^{0*}$, with subsequent decay of $D^0(\bar{D}^0)$ into the non-CP state $K^{*+}K^-$. The use of the non-CP state allows the two interfering amplitudes to be of the same order and hence one can cleanly extract the CKM angle γ. Next, we considered the processes $B^0 \to D^{*0}(\bar{D}^{*0})K^{0*}$, where the final states are admixtures of CP-even and CP-odd states. However, it is possible to disentangle them using the angular distributions of the final decay products. Now considering the longitudinal component of the time-dependent decay rates of these modes, we have shown that $\phi \equiv (2\beta+\gamma)$ can be cleanly obtained. Since these modes are free from penguin pollution and also the branching ratios are measurable at hadron factories such as the LHCb, we feel that they could be very much suited for determining the phase $\gamma(2\beta+\gamma)$.

Chapter 4

Probing new physics with the decay mode $B \to f_0 K(\pi)$

In B physics domain, the main interest lies in critically testing the SM and looking for possible signatures of new physics. Towards this end, a variety of useful observables are being measured and are compared with the corresponding theoretical predictions. If a discrepancy is noted, new physics or physics beyond the SM is assumed as a plausible explanation, although theoretical uncertainties are present which also have to be accounted for. A study of some B decay mode is carried out with this aim in mind, with emphasis on certain observables like the CP asymmetry parameter and the branching ratio of these modes.

The measurement of rare hadronic B decays induced by FCNC transitions $b \to s, d$ which are loop suppressed in the SM can help us to understand and test QCD and EW penguin mechanisms and also to look for any NP contribution as they are sensitive to physics beyond the SM. One of such decay modes that is of interest is the $B \to f_0 K(\pi)$ decay. We have considered the scalar f_0 in the final state being $f_0(980, 1370, 1500)$. We first present the decay mode involving $f_0(980)$ in the next section and the one involving $f_0(1370, 1500)$ is presented after that.

4.1 $B \to f_0(980)K$

We consider the hadronic decay modes $B^{\pm(0)} \to f_0(980)K^{\pm(0)}$, involving a scalar and a pseudoscalar meson in the final state. These decay modes are dominated by the loop induced $b \to s\bar{q}q$ ($q = s,\ u,\ d$) penguins along with a small $b \to u$ tree level transition (for $B^+ \to f_0 K^+$) and annihilation diagrams. The two B factories Belle [50, 51, 52, 53] and Babar [54, 55, 56, 57, 58] have both reported the measurement of the branching ratios and CP violating parameters in the rare decay modes $B^{0,+} \to f_0(980)K^{0,+}$. The measured decay rates (in units of 10^{-6}) for the mode $B^+ \to f_0 K^+$ are

$$\begin{aligned} \mathrm{BR}(B^+ \to f_0(980)K^+ \to \pi^+\pi^- K^+) &= (8.78 \pm 0.82^{+0.85}_{-1.76}) \ , [52] \\ \mathrm{BR}(B^+ \to f_0(980)K^+ \to \pi^+\pi^- K^+) &= (9.47 \pm 0.97^{+0.62}_{-0.88}) \ , [57] \end{aligned} \tag{4.1}$$

with an average (in units of 10^{-6})

$$\mathrm{BR}(B^+ \to f_0(980)K^+ \to \pi^+\pi^- K^+) = (9.21 \pm 0.97) \ . \tag{4.2}$$

For the process $B^0 \to f_0 K^0$, the measured rates (in units of 10^{-6}) are

$$\begin{aligned} \mathrm{BR}(B^0 \to f_0(980)K^0 \to \pi^+\pi^- K^0) &= (7.60 \pm 1.66^{+0.76}_{-0.89}) \ , \ [53] \\ \mathrm{BR}(B^0 \to f_0(980)K^0 \to \pi^+\pi^- K^0) &= (5.5 \pm 0.7 \pm 0.7) \ . \ [58] \end{aligned} \tag{4.3}$$

The absolute branching ratios for the $B \to f_0 K$ processes depend on the branching fraction of $f_0 \to \pi^+\pi^-$ process. Using the results from [59] for $\Gamma(f_0 \to \pi\pi) = 64 \pm 8$ MeV, $\Gamma^{tot}_{f_0} = 80 \pm 10$ MeV along with the relation $\Gamma(f_0 \to \pi^+\pi^-) = \frac{2}{3}\Gamma(f_0 \to \pi\pi)$, we obtain the branching ratios for $B \to f_0 K$ processes as

$$\begin{aligned} \mathrm{BR}(B^+ \to f_0(980)K^+) &= (17.38 \pm 3.47) \times 10^{-6} \ , \\ \mathrm{BR}(B^0 \to f_0(980)K^0) &= (11.26 \pm 2.52) \times 10^{-6} \ . \end{aligned} \tag{4.4}$$

The mixing-induced parameter for the process $B^0 \to f_0 K_S$, observed by both Babar and Belle is

$$\begin{aligned} \sin(2\beta)_{f_0 K_S} &= 0.95^{+0.23}_{-0.32} \pm 0.10 \;, && [60] \\ \sin(2\beta)_{f_0 K_S} &= 0.18 \pm 0.23 \pm 0.11 \;, && [61] \end{aligned} \tag{4.5}$$

with an average

$$\sin(2\beta)_{f_0 K_S} = 0.51 \pm 0.19 \tag{4.6}$$

which has nearly one sigma deviation from that of $\sin(2\beta)_{b\to c\bar{c}s} = 0.687 \pm 0.032$ [62]. These observations not only provide us another way to test the SM and/or to look for new physics but also may help us to understand the nature of the light scalar meson $f_0(980)$. It should be noted here that the mixing-induced CP violation parameter seems to be, at present, not deviated significantly from its SM expectation. But since the error bars are quite large, the situation is still very much conducive to explore some non-standard physics.

The light scalar mesons with masses below 1 GeV are considered as a controversial issue for a long time. Even today, there exists no consensus on the nature of the $f_0(980)$ and $a_0(980)$ mesons. While the low energy hadron phenomenology has been successfully understood in terms of the constituent quark model, the scalar mesons are still puzzling and the quark composition of the light scalar mesons are not understood with certainty. The structure of the scalar meson $f_0(980)$ has been discussed for decades and appears to be still not clear. There were attempts to interpret it as $K\bar{K}$ molecular states [63], four-quark states [64] and normal $q\bar{q}$ states [65]. However, studies of $\phi \to \gamma f_0$ ($f_0 \to \gamma\gamma$) [59, 66] and $D_s^+ \to f_0 \pi^+$ decays [67] favor the $q\bar{q}$ model. Since $f_0(980)$ is produced copiously in D_s decays, this supports the picture of large $s\bar{s}$ component in its wave function, as the dominant mechanism in the D_s decay is $c \to s$ transition. The prominent $s\bar{s}$ nature of $f_0(980)$ has been supported by the radiative decay $\phi \to f_0(980)\gamma$ [68]. In this interpretation, the flavor content of f_0 is given by $f_0 = n\bar{n}\sin\theta + s\bar{s}\cos\theta$ with $n\bar{n} = (u\bar{u} +$

$d\bar{d})/\sqrt{2}$. A mixing angle of $\theta = 138^\circ \pm 6^\circ$ has been experimentally determined from $\phi \to \gamma f_0$ decays [59]. We follow this structure for our study.

Theoretically, these decay modes have been studied in the standard model using perturbative QCD [69] and QCD factorization approach [70, 71]. In our study, we study the decay modes $B^0 \to f_0(980)K^0$ and $B^+ \to f_0(980)K^+$ using the generalized factorization approach. We consider $f_0(980)$ to be composed of $f_0(980) = n\bar{n}\sin\theta + s\bar{s}\cos\theta$ with dominant $s\bar{s}$ composition. Therefore, these processes may be considered, at the leading order, as dominated by $b \to s\bar{s}s$ penguin amplitudes. Hence, the mixing-induced CP violation in the decay mode $B^0 \to f_0 K_S$ is expected to give the same value of $\sin(2\beta)$ as extracted from $B^0 \to J/\psi K_S$, with an uncertainty of 5%. Comparison of these two values, therefore, could be a sensitive probe for physics beyond the SM. Since the predicted branching ratios available from previous studies [69, 70, 71] are not in agreement with the experimental values, we would like to see the effect of the R-parity violating (RPV) supersymmetric model in these modes. Moreover, since we are interested to see whether it is possible to extract any signature of new physics from these modes or not, we resort to generalized factorization approach in analyzing these modes.

4.1.1 Standard Model Contribution

The effective Hamiltonian describing the charmless hadronic B decays is given as

$$H_{eff} = \frac{G_F}{\sqrt{2}}\left[V_{ub}V_{us}^* \sum_{i=1}^{2} C_i O_i - V_{tb}V_{ts}^* \sum_{j=3}^{10} C_j O_j\right], \tag{4.7}$$

where G_F is the Fermi coupling constant, C_i's are the Wilson coefficients, $O_{1,2}$ are the tree operators and O_{3-10} are the QCD and electroweak penguin operators.

To calculate the branching ratios of the $B \to f_0 K$ decay processes, we adopt the generalized factorization framework to evaluate the hadronic matrix elements i.e., $\langle O_i \rangle = \langle f_0 K | O_i | B \rangle$. In this approximation, these hadronic matrix elements can be parametrized in terms of the decay constants and the form factors which are defined as

$$\langle 0|A^\mu|K(k)\rangle = i f_K k^\mu \,, \qquad \langle 0|\bar{q}q|f_0\rangle = m_{f_0} \bar{f}_{f_0}^q \,, \tag{4.8}$$

$$\begin{aligned}\langle K(k)|(V-A)_\mu|B(P)\rangle &= \left[(P+k)_\mu - \left(\frac{m_B^2 - m_K^2}{q^2}\right) q_\mu\right] F_1^{BK}(q^2) \\ &+ \left(\frac{m_B^2 - m_K^2}{q^2}\right) q_\mu F_0^{BK}(q^2) \,,\end{aligned} \tag{4.9}$$

$$\begin{aligned}\langle f_0(q)|(V-A)_\mu|B(P)\rangle &= i\Bigg\{ \left[(P+q)_\mu - \left(\frac{m_B^2 - m_{f_0}^2}{k^2}\right) k_\mu\right] F_1^{Bf_0}(k^2) \\ &+ \left(\frac{m_B^2 - m_{f_0}^2}{k^2}\right) k_\mu F_0^{Bf_0}(k^2)\Bigg\} \,,\end{aligned} \tag{4.10}$$

where V and A denote the vector and axial-vector currents, f_K and $\bar{f}_{f_0}$ are the decay constants of K and f_0 mesons, $F_{0,1}(q^2)$ are the form factors and P, q, k are the momenta of B, f_0 and K mesons satisfying the relation $q = P - k$.

Now, let us first consider the process $B^+ \to f_0 K^+$. Within the SM, it receives contributions from $b \to u$ tree, $b \to s\bar{q}q$ (with $q = u, s$) penguins and annihilation diagrams. Using Eqs. (4.7)-(4.10), one can obtain the amplitude in the SM as

$$\begin{aligned}\mathcal{A}(B^+ \to f_0 K^+) &= -\frac{G_F}{\sqrt{2}}\Bigg\{\Big[V_{ub}^* V_{us} a_1 - V_{tb}^* V_{ts}(a_4 + a_{10} - r_\chi(a_6 + a_8))\Big] X \\ &- V_{tb}^* V_{ts}(2a_6 - a_8) Y - \Big[V_{ub}^* V_{us} a_1 \\ &- V_{tb}^* V_{ts}\Big(a_4 + a_{10} - \frac{2(a_6 + a_8) m_B^2}{(m_b + m_u)(m_s + m_u)}\Big)\Big] Z\Bigg\} \,,\end{aligned} \tag{4.11}$$

where

$$\begin{aligned} r_\chi &= \frac{2m_K^2}{(m_b + m_u)(m_s + m_u)} \,, \\ X &= f_K (m_B^2 - m_{f_0}^2) F_0^{Bf_0}(m_K^2) \,, \\ Y &= \bar{f}_{f_0}^s \, m_{f_0} \frac{m_B^2 - m_K^2}{m_b - m_s} F_0^{BK}(m_{f_o}^2) \,, \\ Z &= f_B (m_{f_0}^2 - m_K^2) F_0^{f_0 K}(m_B^2) \end{aligned} \tag{4.12}$$

and a_i's are the combinations of Wilson coefficients given by

$$a_{2i-1} = C_{2i-1} + \frac{1}{N_C} C_{2i} \ , \quad a_{2i} = C_{2i} + \frac{1}{N_C} C_{2i-1} \ , \qquad (i = 1, 2, 3, 4, 5) \quad (4.13)$$

with N_C as the number of colors.

The corresponding neutral process $B^0 \to f_0 K^0$ receives contributions only from $b \to s\bar{q}q$ (with $q = s, d$) penguins and annihilation diagrams. Thus, one can write the amplitude for this process as

$$\begin{aligned} \mathcal{A}(B^0 \quad \to \quad f_0 K^0) = \frac{G_F}{\sqrt{2}} V_{tb}^* V_{ts} \Big\{ \Big[a_4 - \frac{a_{10}}{2} - r_{\chi_1} (a_6 - \frac{a_8}{2}) \Big] X \\ + \quad (2a_6 - a_8) Y - \Big[a_4 - \frac{a_{10}}{2} - \frac{(2a_6 - a_8) m_B^2}{(m_b + m_d)(m_s + m_d)} \Big] Z \Big\} \end{aligned} \quad (4.14)$$

where r_{χ_1} can be obtained from r_χ by replacing the K^+ and u-quark masses by K^0 and d-quark masses. The branching ratios can be obtained from these amplitudes as

$$\mathrm{BR}(B \to f_0 K) = \frac{|\mathrm{p_c}| \tau_B}{8\pi m_B^2} |\mathcal{A}(B \to f_0 K)|^2 \ , \quad (4.15)$$

where $|\mathrm{p_c}|$ is the c.m. momentum of the final mesons and τ_B is the lifetime of the B meson. For numerical analysis, we have used the particle masses and lifetimes from [72]. The current quark masses are taken as $m_b = 4.88$ GeV, $m_s = 122$ MeV, $m_d = 7.6$ MeV and $m_u = 4.2$ MeV. The values of the effective QCD parameters (a_i's) are taken from [73], which are evaluated at the scale $\mu = m_b/2$. For the CKM matrix elements, we have used the Wolfenstein parametrization with the parameters A=0.801, $\lambda = 0.2265$, $\bar{\rho} = 0.189$ and $\bar{\eta} = 0.358$ [45]. The form factors describing the transition $B \to f_0$ are given as [70]

$$F_0^{B^- f_0} = \frac{1}{\sqrt{2}} \sin\theta F_0^{B^- f_0^{u\bar{u}}} \ , \qquad F_0^{B^0 f_0} = \frac{1}{\sqrt{2}} \sin\theta F_0^{B^0 f_0^{d\bar{d}}} \ , \quad (4.16)$$

with $F_0^{B f_0^{q\bar{q}}}(0)$ ($q\bar{q} = u\bar{u}$ or $d\bar{d}$) being of the order of 0.25 [74]. For the q^2 dependence, we assume the simple pole dominance as

$$F_0^{B f_0}(q^2) = \frac{F_0^{B f_0}(0)}{1 - q^2/m_P^2} \ , \quad (4.17)$$

with m_P being the mass of the 0^- pole state with the same quark content as the current under consideration. For the form factors describing $B \to K$ transition, we use the corresponding QCD sum rule value [75]

$$F_0^{BK}(m_{f_0}^2) = \frac{0.3302}{1 - \frac{m_{f_0}^2}{37.46}} . \tag{4.18}$$

The annihilation form factor $F_0^{f_0 K}(q^2)$ is expected to be suppressed at large momentum transfer (i.e., $q^2 = m_B^2$) due to helicity suppression. However, it may receive long distance contributions from nearby resonances via final state interactions. In Ref. [76], its value is extracted using the experimental values of BR($B \to f_0 K$), where it has been shown that in order to explain the observed data in the SM, one requires large value of annihilation form factor if the $B \to f_0$ form factor will be $F_0^{Bf_0} \le 0.2$. Since we are interested to look for new physics signature in this mode, here we use the lowest value of $|F_0^{f_0 K}|$, which is around 0.03, as seen from Figure 2 of [76]. Furthermore, since both the components of f_0 ($n\bar{n}$ and $s\bar{s}$) are involved in the annihilation topology, the corresponding amplitude should be multiplied by $(\sin\theta/\sqrt{2} + \cos\theta)$.

The decay constants used are $f_K = 0.16$ GeV, $f_B = 0.19$ GeV and $\bar{f}_0^s = \frac{m_{f_0}^{(s)}}{m_{f_0}} \tilde{f}_s \cos\theta$ with $\tilde{f}_s(\mu = 2.1\text{GeV}) = 0.39$ GeV and $m_{f_0}^{(s)} \simeq (1.02 \pm .05)$ GeV [70].

Using these values and the mixing angle $\theta = 138^\circ$, we obtain the branching ratios for the $B \to f_0(980)K$ processes as

$$\begin{aligned} \text{BR}(B^+ \to f_0(980)K^+) &= 6.56 \times 10^{-6} , \\ \text{BR}(B^0 \to f_0(980)K^0) &= 4.73 \times 10^{-6} , \end{aligned} \tag{4.19}$$

which are quite below the experimental values (4.4). The variation of the branching ratios for the strange, non-strange mixing angle θ between 0 and π are shown in Figures 4.1 and 4.2. Thus, one can see from Figure 4.1 that for $B^+ \to f_0 K^+$ process, generalized factorization approach cannot accommodate the experimental data for any value of the mixing angle θ. For the $B^0 \to f_0 K^0$ mode also, it cannot explain the data unless θ is very close to 0 or π as seen from Figure 4.2.

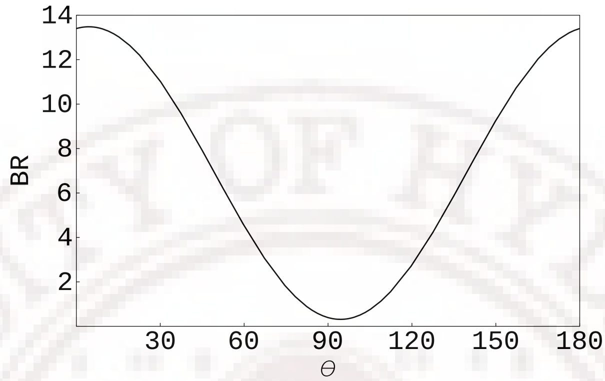

Figure 4.1: The branching ratio for the process $B^- \to f_0(980)K^-$ (in units of 10^{-6}), versus the mixing angle θ in degrees.

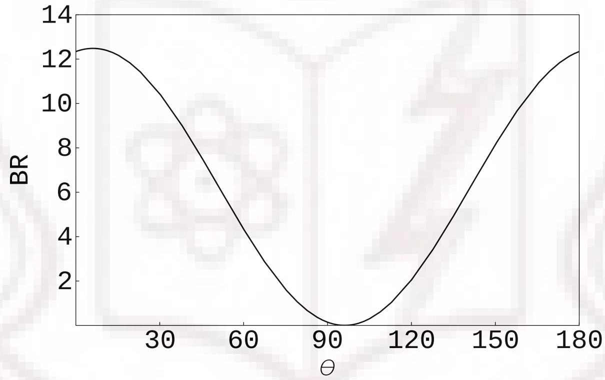

Figure 4.2: Same as Figure-4.1, for the $B^0 \to f_0 K^0$ process.

4.1.2 CP Violation Parameters

Here, we briefly present the basic and well known formula for the CP violating parameters. Let us first consider the process $B^+ \to f_0 K^+$, which has only direct CP violation. The amplitude for this process can be symbolically written as

$$\begin{aligned} \mathcal{A}(B^+ \to f_0 K^+) &= \lambda_u^* |A_u| e^{i\delta_u} + \lambda_t^* |A_t| e^{i\delta_t} , \\ \mathcal{A}(B^- \to f_0 K^-) &= \lambda_u |A_u| e^{i\delta_u} + \lambda_t |A_t| e^{i\delta_t} , \end{aligned} \tag{4.20}$$

where $\lambda_q = V_{qb} V_{qs}^*$ with $(q = u, t)$ denote the product of CKM matrix elements which contain the weak phase information. It should be noted that the weak phase of λ_u^* is $\arg(V_{ub}^* V_{us}) = \gamma$ and that of λ_t^* is $\arg(V_{tb}^* V_{ts}) = \pi$. A_u and A_t denote the contributions arising from the current operators proportional to λ_u and λ_t, respectively and the corresponding strong phases are taken as δ_u and δ_t.

For the charged $B^\pm \to f_0 K^\pm$ decays, the CP violating rate asymmetry in the partial rates is defined as follows :

$$\begin{aligned} A_{\rm CP} &= \frac{\Gamma(B^+ \to f_0 K^+) - \Gamma(B^- \to f_0 K^-)}{\Gamma(B^+ \to f_0 K^+) + \Gamma(B^- \to f_0 K^-)} \\ &= \frac{2r \sin\gamma \sin(\delta_u - \delta_t)}{1 + r^2 - 2r \cos\gamma \cos(\delta_u - \delta_t)} , \end{aligned} \tag{4.21}$$

where $r = |\lambda_u A_u / \lambda_t A_t|$. Thus, to obtain significant direct CP asymmetry, one requires the two interfering amplitudes to be of the same order and their relative strong phase should be significantly large (i.e., close to $\pi/2$). However, in the SM, the ratio of the CKM matrix elements of the two terms in Eq. (4.20) can be given (in the Wolfenstein parametrization) as $|\lambda_u/\lambda_t| \simeq \lambda^2 \sqrt{\rho^2 + \eta^2} \simeq 2\%$. Therefore, the first amplitude will be highly suppressed with respect to the second unless $A_u >> A_t$. Hence, the naive expectation is that the direct CP violation in the SM in this mode will be negligibly small. Using the generalized factorization approach, we find $r = 0.15$, $\delta_t - \delta_u \sim 7^\circ$ and the direct CP violation in the mode $B^+ \to f_0 K^+$ is $\sim (-4\%)$. This in turn makes the mode interesting to look for NP in terms of large direct CP asymmetry.

In the presence of new physics, the amplitude can be written as

$$\mathcal{A}(B^+ \to f_0 K^+) = A_{SM} + A_{NP} = A_{SM} \left[1 + r_{NP}\, e^{i\phi_{NP}} \right] , \tag{4.22}$$

where $r_{NP} = |A_{NP}/A_{SM}|$, (A_{SM} and A_{NP} correspond to the SM and NP contributions to the $B^+ \to f_0 K^+$ decay amplitude, respectively) and $\phi_{NP} = \arg(A_{NP}/A_{SM})$, which contains both strong and weak phase components. The branching ratio for the $B^+ \to f_0 K^+$ decay process can be given as

$$\mathrm{BR}(B^+ \to f_0 K^+) = \mathrm{BR}^{\mathrm{SM}} \left(1 + r_{NP}^2 + 2 r_{NP} \cos\phi_{NP}\right) \,, \tag{4.23}$$

where $\mathrm{BR}^{\mathrm{SM}}$ represents the corresponding standard model value.

Now, we present the basic formula of CP asymmetry parameters in the presence of new physics. Due to the contributions from new physics, these parameters deviate substantially from their standard model values. To find the CP asymmetry, it is necessary to represent explicitly the strong and weak phases of the SM as well as of NP amplitudes. Although it is expected that the SM amplitude $\lambda_u A_u$ is highly suppressed with respect to its $\lambda_t A_t$ counterpart, for completeness we will keep this term for the evaluation of A_{CP}. We denote the NP contribution to the decay amplitude as $A_{NP} = |A_{NP}| e^{i(\delta_n + \theta_n)}$, where δ_n and θ_n denote the strong and weak phases of the NP amplitude, respectively. Thus in the presence of NP, we can explicitly write the decay amplitude for $B^+ \to f_0 K^+$ mode as

$$\mathcal{A}(B^+ \to f_0 K^+) = \lambda_u^* |A_u| e^{i\delta_u} + \lambda_t^* |A_t| e^{i\delta_t} + |A_{NP}| e^{i(\delta_n + \theta_n)} \,. \tag{4.24}$$

The amplitude for $B^- \to f_0 K^-$ mode is obtained by changing the sign of the weak phases of the amplitude (4.24). Thus, the CP asymmetry parameter is given as

$$A_{CP} = \frac{2\Big(r \sin\gamma \sin\delta_{ut} + r_N \sin\theta_n \sin\delta_{nt} - r r_N \sin(\gamma - \theta_n) \sin\delta_{un}\Big)}{|\mathcal{A}|^2 - 2\Big(r \cos\gamma \cos\delta_{ut} + r_N \cos\theta_n \cos\delta_{nt} - r r_N \cos(\gamma - \theta_n) \cos\delta_{un}\Big)}, \tag{4.25}$$

where $|\mathcal{A}|^2 = 1 + r^2 + r_N^2$, $r_N = |A_{NP}/\lambda_t A_t|$ and $\delta_{ij} = \delta_i - \delta_j$ are the relative strong phases between different amplitudes.

Next, we consider the CP violation parameters in the neutral B meson decays, which has both direct and mixing-induced components. Let us consider the B^0 and $\bar{B}^0$ decay into a CP eigenstate f_{CP} (we consider $f_{CP} = f_0 K_S$ with CP eigenvalue +1).

The time-dependent CP asymmetry for $B \to f_0 K_S$ is [77]

$$\begin{aligned} A_{f_0 K_S}(t) &= \frac{\Gamma(B^0(t) \to f_0 K_S) - \Gamma(\bar{B}^0(t) \to f_0 K_S)}{\Gamma(B^0(t) \to f_0 K_S) + \Gamma(\bar{B}^0(t) \to f_0 K_S)} \\ &= C_{f_0 K_S} \cos(\Delta M_{B_d} t) - S_{f_0 K_S} \sin(\Delta M_{B_d} t) , \end{aligned} \tag{4.26}$$

where we identify

$$C_{f_0 K_S} = \frac{1 - |\lambda|^2}{1 + |\lambda|^2}, \qquad S_{f_0 K_S} = \frac{2\mathrm{Im}(\lambda)}{1 + |\lambda|^2} , \tag{4.27}$$

as the direct and the mixing-induced CP asymmetries. The parameter λ corresponds to

$$\lambda = \frac{q}{p} \frac{\mathcal{A}(\bar{B}^0 \to f_0 K_S)}{\mathcal{A}(B^0 \to f_0 K_S)} , \tag{4.28}$$

where q and p are the mixing parameters and are represented by the CKM elements in the standard model as

$$\frac{q}{p} = \frac{V_{tb}^* V_{td}}{V_{tb} V_{td}^*} \sim \exp(-2i\beta) . \tag{4.29}$$

Now the amplitude for $\bar{B}^0 \to f_0 K_S$ can be symbolically written as

$$\mathcal{A}(\bar{B}^0 \to f_0 K_S) = \lambda_t A_t , \tag{4.30}$$

where $\lambda_t = V_{tb} V_{ts}^*$, which is real in the SM. Thus, the mixing-induced CP asymmetry is given as $S_{f_0 K_S} = -\sin 2\beta$, same in magnitude as the one for $B \to \psi K_S$ but with opposite sign and the direct CP asymmetry turns out to be identically zero.

However, the decay amplitude also receives some contributions from the internal *up* and *charm* quarks in the loop. Therefore, the CP violating parameters may deviate from their expected values. Now including the effects of u, c, t quarks in the loop and using CKM unitarity ($\lambda_u + \lambda_c + \lambda_t = 0$), one can write the decay amplitude as

$$\mathcal{A}(B^0 \to f_0 K^0) = \lambda_u^* A_u + \lambda_c^* A_c = \lambda_c^* A_c \left[1 + r' e^{i(\delta'+\gamma)}\right] , \tag{4.31}$$

where the amplitude A_u contains contributions from u and t quarks in the loop (i.e., $A_u = P_u - P_t$, where $P_{u,c,t}$ are the penguin amplitudes corresponding to u, c, t quark exchange in the loop) and the same argument holds for A_c. The parameter r' is the ratio of the two amplitudes, i.e., $r' = |\lambda_u A_u / \lambda_c A_c|$, $\delta' = \delta_u - \delta_c = \arg(A_u/A_c)$ is the relative strong phase between them and γ is the weak phase. The explicit expressions for these amplitudes (in units of $-G_F/\sqrt{2}$) are given as

$$\begin{aligned} A_q &= \left[a_4^q - \frac{a_{10}^q}{2} - r_{\chi_1}(a_6^q - \frac{a_8^q}{2})\right] X + (2a_6^q - a_8^q) Y \\ &- \left[a_4^q - \frac{a_{10}^q}{2} - \frac{(2a_6^q - a_8^q) m_B^2}{(m_b + m_d)(m_s + m_d)}\right] Z , \end{aligned} \tag{4.32}$$

with $q = u$ and c, Y and Z are given in Eq. (4.12). Thus, one obtains the CP asymmetries as

$$\begin{aligned} S_{f_0 K_S} &= -\frac{\sin 2\beta + 2r' \cos\delta' \sin(2\beta+\gamma) + r'^2 \sin(2\beta + 2\gamma)}{1 + r'^2 + 2r' \cos\delta' \cos\gamma} , \\ C_{f_0 K_S} &= \frac{-2r' \sin\delta' \sin\gamma}{1 + r'^2 + 2r' \cos\delta' \cos\gamma} . \end{aligned} \tag{4.33}$$

In order to know the precise value of the CP violating asymmetries one should know the values of r' and δ'. Using the QCD coefficients from [70], we obtain $r' = 0.02$, $\delta' = 12^\circ$ and hence the CP asymmetries as

$$S_{f_0 K_S} = -0.672 \quad \text{and} \quad C_{f_0 K_S} = -0.007 , \tag{4.34}$$

which are in accordance with the results of top quark dominance in the penguin loop. Therefore, here onwards we consider the SM amplitude for the $B^0 \to f_0 K_S$ process to be dominated by the top quark penguin.

New physics could in principle contribute to both mixing and decay amplitudes. The new physics contribution to mixing is universal while it is non-universal and process dependent in the decay amplitudes. As the NP contribution to mixing phenomena is universal, it will still set $S_{\psi K_S} = -S_{f_0 K_S}$. Therefore, to explain the deviation between $S_{\psi K_S}$ and $(\sin 2\beta)_{f_0 K_S} = -S_{f_0 K_S}$, here we explore the NP effects only in the decay amplitudes. Thus, including the NP contributions, we can write the decay amplitude for $B \to f_0 K$ process as

$$\mathcal{A}(B^0 \to f_0 K^0) = A_{SM} + A_{NP} = \lambda_t^* A_t \left[1 - r_N \ e^{i(\delta_{nt}+\theta_n)}\right] , \tag{4.35}$$

where $r_N = |A_{NP}/\lambda_t A_t|$, δ_{nt} and θ_n are the relative strong and weak phases between the new physics contributions to the decay amplitude and that of the SM part. The negative sign before r_N in Eq. (4.35) arises because the weak phase π of λ_t^* has been factored out. Thus, one can then obtain the expressions for the CP asymmetries in the presence of NP as

$$S^{NP} = -\frac{\sin 2\beta - 2r_N \cos\delta_{nt} \sin(2\beta + \theta_n) + r_N^2 \sin(2\beta + 2\theta_n)}{1 + r_N^2 - 2r_N \cos\delta_{nt} \cos\theta_n} \tag{4.36}$$

and

$$C^{NP} = \frac{2r_N \sin\delta_{nt} \sin\theta_n}{1 + r_N^2 - 2r_N \cos\delta_{nt} \cos\theta_n} . \tag{4.37}$$

Having obtained the CP asymmetry parameters in the presence of new physics, we now proceed to evaluate the same in the R-parity violating supersymmetric model.

4.1.3 Contribution from R-parity violating (RPV) supersymmetric model

Despite its many spectacular successes, the SM still has some unresolved issues. We do not know for sure that the electroweak symmetry is broken by the vacuum expectation value (VEV) of a spin zero elementary field (the Higgs field). Also there is no proper explanation of particle masses or mixing patterns. It offers no explanation for the replication of generations as well. The SM does not incorporate gravity also. With regards to the Higgs sector, the Higgs mass is quadratically divergent. The Higgs boson itself has not yet been discovered and is only expected to be so at the LHC. As such, new physics or physics beyond the SM is expected to provide an explanation for the aforementioned issues of the SM. One of such attractive and promising extensions of the SM is supersymmetry (SUSY). It offers the simplest and natural solution to the gauge hierarchy problem. With SUSY, the particle spectrum of the SM is doubled with every fermion having a bosonic partner and vice versa. SUSY also inter-relates properties of matter fields (leptons and quarks) and force fields (gauge and/or Higgs boson). In [78], it was concluded that the main reason behind all the phenomenological interest in SUSY is that it provides a solution for the naturalness problem, if the masses of the superpartners are below 1 TeV. SUSY cannot be an exact symmetry of nature but broken and this is justified by the fact that the superpartners of the known particles with the same mass have not yet been discovered.

The minimal supersymmetric extension of the SM with minimal particle content is called the minimal supersymmetric standard model (MSSM). The MSSM contains the particles of the SM with two Higgs doublets and their superpartners. The gauge group is considered to be that of the SM i.e., $SU(3)_C \times SU(2)_L \times U(1)_Y$. In [78], a more detailed description of the particle spectrum has been given. The MSSM assumes the conservation of a discrete symmetry called R-parity defined as $R_p = (-1)^{(3B+L+2S)}$ where B is the baryon number, L is the lepton number and S is the spin of a particle. R_p is $+1$ for all particles and -1 for all superparticles. In the SM, baryon and lepton numbers are automatically conserved as a result of gauge invariance and renormalizability in the model. As a consequence of R-parity conservation, the SUSY particles are produced in pairs, they cannot decay to ordinary particles but only to SUSY particles and the lightest SUSY particle is absolutely stable. However, no compelling theoretical motivation such as gauge invariance nor SUSY requires the conservation of R_p. In fact, the most general SUSY extension of the SM contains explicit R_p violating interactions that are consistent with both gauge invariance and SUSY. The most general super-potential of the MSSM can be written as

$$\begin{aligned} W &= \mu H_1 H_2 + h_{ij} L_i H_2 E_j^c + h'_{ij} Q_i H_2 D_j^c + h''_{ij} Q_i H_1 U_j^c \\ &+ \frac{1}{2}\lambda_{ijk} L_i L_j E_k^c + \lambda'_{ijk} L_i Q_j D_k^c + \frac{1}{2}\lambda''_{ijk} U_i^c D_j^c D_k^c + \mu_{2i} L_i H_2 \ , \end{aligned} \tag{4.38}$$

where $L_i(Q_i)$ and $E_i(U_i, D_i)$ are left-handed lepton(quark) doublet and lepton(quark) singlet chiral superfields, respectively, i, j, k are generation indices, c denotes a charge conjugate field and $H_{1,2}$ are the chiral superfields representing the two Higgs doublets.

The couplings λ and λ' in Eq. (4.38) violate lepton number conservation while the λ'' violate baryon number conservation. Because of gauge invariance, the λ and λ'' couplings are antisymmetric with respect to the first two indices and the last two indices, respectively. The simultaneous presence of both the baryon number and lepton number violating interactions may induce Lepton Flavor Violating (LFV) processes and also rapid proton decay through product of couplings $\lambda' \times \lambda''$ which may contradict strict experimental bound. Therefore, in order to keep the proton lifetime within experimental limit, one needs to impose an additional symmetry beyond the SM gauge symmetry to force the unwanted baryon and lepton number violating interactions to vanish. This symmetry is the one we have already defined i.e., R-parity. However, this symmetry is ad-hoc in nature. There is no theoretical argument in support of this discrete symmetry. Hence, it is interesting to see the phenomenological consequences of the breaking of R-parity in such a way that either B or L number is violated, both not simultaneously violated, thus avoiding rapid proton decay.

Therefore, in our study, we analyze the considered decay modes in the MSSM with R-parity violation. For our purpose, we consider only the lepton number violating super-potential with only λ' couplings, which is given as

$$W_{\not{L}} = \lambda'_{ijk} L_i Q_j D^c_k \; . \tag{4.39}$$

Thus the effective Hamiltonian for charmless hadronic B decays can be given as [79]

$$\begin{aligned}
\mathcal{H}^{\lambda'}_{eff} &= d^R_{jkn}[\bar{d}_{n\alpha}\gamma^\mu_L d_{j\beta}\bar{d}_{k\beta}\gamma_{\mu R} b_\alpha] + d^L_{jkn}[\bar{d}_{n\alpha}\gamma^\mu_L b_\beta \bar{d}_{k\beta}\gamma_{\mu R} d_{j\alpha}] \\
&+ u^R_{jkn}[\bar{u}_{k\alpha}\gamma^\mu_L u_{j\beta}\bar{d}_{n\beta}\gamma_{\mu R} b_\alpha] \; ,
\end{aligned} \tag{4.40}$$

α, β are the color indices, $\gamma^\mu_{R,L} = \gamma^\mu(1 \pm \gamma_5)$ and

$$d^R_{jkn} = \sum_{i=1}^{3} \frac{\lambda'_{ijk}\lambda'^*_{in3}}{8m^2_{\tilde{\nu}_{Li}}} \; , \qquad d^L_{jkn} = \sum_{i=1}^{3} \frac{\lambda'_{i3k}\lambda'^*_{inj}}{8m^2_{\tilde{\nu}_{Li}}} \; , \qquad u^R_{jkn} = \sum_{i=1}^{3} \frac{\lambda'_{ijn}\lambda'^*_{ik3}}{8m^2_{\tilde{e}_{Li}}} \; . \tag{4.41}$$

Thus one can write the transition amplitudes as

$$\begin{aligned}
\mathcal{A}^{\lambda'}(B^+ \to f_0 K^+) &= -2(d^L_{222} + d^R_{222})Y + u^R_{112} r_\chi X - (d^R_{112} + d^L_{121})2Y_d \; , \\
\mathcal{A}^{\lambda'}(B^0 \to f_0 K^0) &= -2(d^L_{222} + d^R_{222})Y + (d^R_{121} + d^L_{112}) r_{\chi_1} X \\
&+ (d^R_{112} + d^L_{121})\left(\frac{X}{N} - 2Y_d\right) \; ,
\end{aligned} \tag{4.42}$$

where Y_d is the value of Y with $\bar{f}^s_{f_0}$ replaced by $\bar{f}^d_{f_0}$.

Following the standard practice, we assume that the RPV couplings are hierarchical, i.e., only one combination of the coupling is numerically significant. Furthermore, we also assume that both the transitions $B^{+,0} \to f_0 K^{+,0}$ receive dominant contribution from the quark level transition $b \to s\bar{s}s$, and hence we consider only d^L_{222} coupling to be nonzero. As discussed in Ref. [80], we also discard the d^R_{222} coupling in our analysis as it is related to u^R_{222} by $SU(2)$ isospin symmetry and its effect in the mode $B \to J/\psi K_S$ is found to be negligibly small. Thus, with these approximations, the transition amplitudes for both the processes can be given as

$$\mathcal{A}^{\lambda'}(B \to f_0 K) = -\frac{1}{8m^2_{\tilde{\nu}_{Li}}}\left(\lambda'_{i32}\lambda'^*_{i22}\right) 2m_{f_0}\bar{f}^s_{f_0}\frac{m^2_B - m^2_K}{m_b - m_s} F^{BK}_0(q^2) \; , \tag{4.43}$$

where the summation over $i = 1, 2, 3$ is implied. Now we consider the values of R-parity couplings as

$$\lambda'_{i32}\lambda'^{*}_{i22} = Re^{i\theta_n} \ , \qquad (4.44)$$

where $R = |\lambda'_{i32}\lambda'^{*}_{i22}|$ and θ_n is the new weak phase with range $-\pi \leq \theta_n \leq \pi$. It should be noted that since the dominant SM amplitude (i.e., the t-quark dominated penguin amplitude A_t) contains the weak phase π, we vary the weak phase θ_n between $[-\pi, \pi]$ so that the NP amplitude will interfere constructively with the SM amplitude when the relative weak phase between them is zero. To see the effect of R-parity violation in the decay modes $B \to f_0(980)K$, it is essential to know the value of the RPV couplings (R). We first present a crude estimation of R by assuming that R-parity will explain the observed discrepancy between the observed and SM predicted branching ratios for $B \to f_0 K$ modes. We further assume that the new physics amplitude will interfere constructively with the standard model amplitude (i.e., $\phi_{NP} = 0$ in Eq. (4.22)), so that one can obtain a lower bound on r_{NP} from Eq. (4.23). Now using the values of the experimental branching ratios from Eq. (4.4) and the corresponding SM values from (4.19), we obtain the lower bound as $r_{NP} \geq 0.6$. This, in turn with Eqns. (4.11), (4.14) and (4.43) gives

$$R \geq 1 \times 10^{-3} \qquad (4.45)$$

for $m_{\tilde{\nu}_{Li}} = 100$ GeV. In Ref. [81], it has been shown that the branching ratio and the polarization anomaly in $B \to \phi K^*$ modes can be resolved in the R-parity violating supersymmetric model for a very narrow interval in the parameter space as $|\lambda'_{i32}\lambda'^{*}_{i22}|/m^2_{\tilde{\nu}_{Li}} \in [1.5 \times 10^{-3}, 2.1 \times 10^{-3}]$, for the sneutrino mass scale $m_{\tilde{\nu}_{Li}} = 100$ GeV. Therefore, in this analysis we consider the lowest value for R i.e., 1.5×10^{-3} from the above allowed range, which also satisfies the constraint (4.45). Using this value, we obtain the ratios of RPV to SM amplitude as

$$
\begin{aligned}
& r_{NP} = 0.81 \, , \quad r_N = 0.87 \quad (\text{for} \;\; B^+ \to f_0 K^+) \, , \\
& r_N = 0.92 \, , \quad (\text{for} \;\; B^0 \to f_0 K^0) \, . \qquad (4.46)
\end{aligned}
$$

Therefore, the upper limits in the branching ratios (for $\phi_{NP} = 0$ in Eq.(4.23)) in the RPV model are found to be

$$
\begin{aligned}
\mathrm{BR}(B^+ \to f_0(980) K^+) &\leq 21.6 \times 10^{-6} \, , \\
\mathrm{BR}(B^0 \to f_0(980) K^0) &\leq 17.4 \times 10^{-6} \, . \qquad (4.47)
\end{aligned}
$$

Thus one can see that the observed branching ratios (4.4) can be accommodated in the RPV model.

Now assuming the strong phase difference δ_{nt} to be small (e.g., $\sim 10^\circ$), direct CP violation for $B^+ \to f_0 K^+$ process and the mixing-induced CP violating parameter from $B^0 \to f_0 K^0$ are shown in Figures 4.3 and 4.4, respectively. Thus, as seen from the figures, the observed $(\sin 2\beta)_{f_0 K_S} = -S_{f_0 K_S} = 0.51 \pm 0.19$ can be explained in the RPV model and large direct CP violation (upto 80 %) in $B^+ \to f_0 K^+$ mode could be obtainable in this model. However, there is no obvious reason why the strong phase difference δ_{nt} could be small. To see the impact of the strong phase, we vary it between the range of $-\pi$ and π and plot the correlation between direct and mixing-induced CP asymmetries for $B^0 \to f_0 K_S$, for two representative values of weak phase $\theta_n = \pi/2$ and $\pi/4$, in Figure 4.5. From the figure, it is seen that R-parity violating supersymmetric model can accommodate large CP violation in the $B^0 \to f_0 K_S$ decay mode.

4.2 $B \to f_0(1370, 1500) K(\pi)$

Here we consider the two-body decay of the B meson to the scalar mesons $f_0(1370, 1500)$ and the pseudoscalar mesons $K(\pi)$. The scalars $f_0(1370, 1500)$ also have an uncertain structure. In [82], the scalar mesons above 1 GeV are

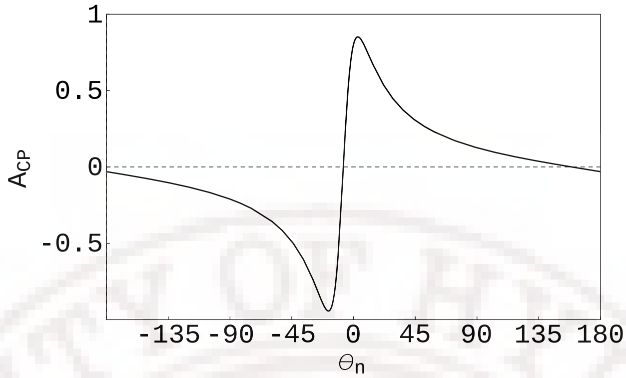

Figure 4.3: Direct CP violation for the process $B^+ \to f_0(980)K^+$ versus the new weak phase θ_n in degrees.

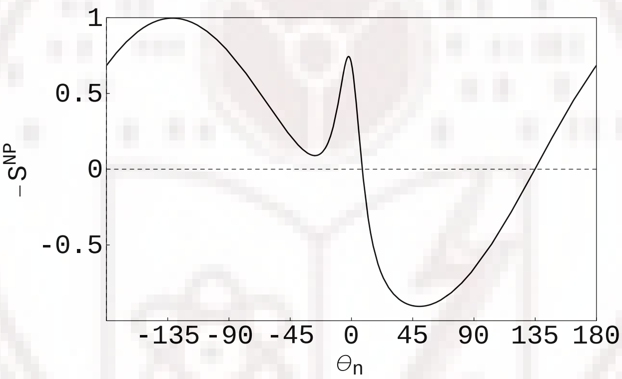

Figure 4.4: Mixing-induced CP violation for the process $B^0 \to f_0(980)K^0$ versus the new weak phase θ_n in degrees.

identified as a conventional $q\bar{q}$ nonet with some possible glue content. In our case, we assume $f_0(1500)$ to be dominated by the quarkonium content as in [83] i.e., $f_0(1500) = n\bar{n}\sin\theta + s\bar{s}\cos\theta$. The scalar $f_0(1370)$ flavor content is taken as in [84] i.e., $f_0(1370) = s\bar{s}\sin\theta + n\bar{n}\cos\theta$. Here also we follow the generalized factorization approach for the decay modes $B \to f_0(1370, 1500)K(\pi)$.

The decay mode $B^- \to f_0(1370, 1500)\ K^-$ receives contributions from $b \to u$ tree and $b \to s\bar{q}q (q = u, s)$ penguins within the SM. We have the

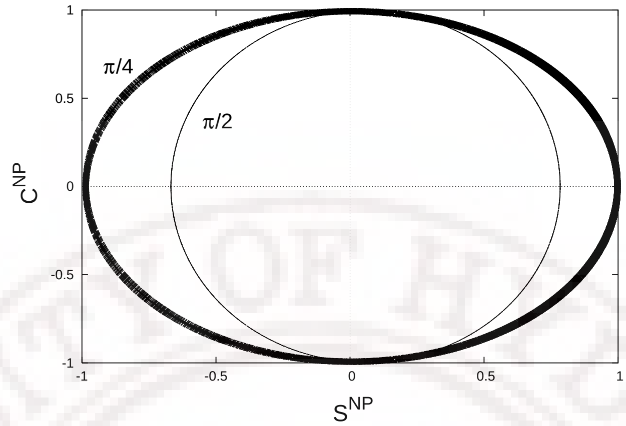

Figure 4.5: The correlation plot between S^{NP} and C^{NP} for the process $B^0 \to f_0(980)K_S$ in the RPV model for two representative values of weak phases ($\theta_n = \pi/2,\ \pi/4$), where we have used $r_N = 0.92$, and varied the strong phase δ_{nt} between $-\pi$ and π.

amplitude for this charged mode defined as

$$\begin{aligned}\mathcal{A}(B^- \to f_0 K^-) &= \frac{G_F}{\sqrt{2}}\Big[- V_{ub}V_{us}^* a_1 X_2 + V_{tb}V_{ts}^* \Big\{ (2a_6 - a_8)X_1 \\ &+ (a_4 + a_{10})X_2 - 2(a_6 + a_8)Y_1 \Big\}\Big] , \end{aligned} \tag{4.48}$$

where

$$\begin{aligned} X_1 &= \left[\frac{(m_B^2 - m_K^2)}{(m_b - m_s)}\right] F_0^{BK}(m_{f_0}^2) m_{f_0} \bar{f}_{f_0} , \\ X_2 &= (m_B^2 - m_{f_0}^2) F_0^{Bf_0}(m_{K^-}^2) f_{K^-} , \\ Y_1 &= \frac{(m_B^2 - m_{f_0}^2) m_{K^-}^2 F_0^{Bf_0}(m_{K^-}^2) f_{K^-}}{(m_b + m_u)(m_u + m_s)} , \end{aligned} \tag{4.49}$$

with the a_i's given as in Eq.(4.13). Here we have neglected the annihilation diagrams as they behave as (f_B/m_B) in comparison with the tree and penguin contributions and are also helicity suppressed by $(m_{u,d,s}/m_B)$ since the B mesons are pseudoscalars [85].

The mode $B^- \to f_0(1370)\ \pi^-$ receives contributions only from the $n\bar{n}$

component of f_0. The transition amplitude therefore has the form

$$\begin{aligned}
\mathcal{A}(B^- \to f_0\, \pi^-) &= \frac{G_F}{\sqrt{2}}\Big[- V_{ub}V_{ud}^* a_1 X_3 + V_{tb}V_{td}^* \Big\{ (a_4 + a_{10})] X_3 \\
&+ (2a_6 - a_8) X_4 + 2(a_6 + a_8) Y_2 \Big\}\Big] , \qquad (4.50)
\end{aligned}$$

where

$$\begin{aligned}
X_3 &= (m_B^2 - m_{f_0}^2) F_0^{Bf_0}(m_\pi^2) f_\pi , \\
X_4 &= \frac{(m_B^2 - m_\pi^2) F_0^{B\pi}(m_{f_0}^2) m_{f0} \bar{f}_{f_0}}{(m_b - m_d)} , \\
Y_2 &= \frac{m_\pi^2 \; X_5}{(m_b + m_u)(m_d + m_u)} , \qquad (4.51)
\end{aligned}$$

Here, the flavor wave function for $f_0(1500)$ is taken as $s\bar{s}\cos\theta + n\bar{n}\sin\theta$ where $n\bar{n} = (u\bar{u} + d\bar{d})/\sqrt{2}$. The θ dependence of the form factor and decay constant involving $f_0(1500)$ is therefore given as

$$F_0^{B^- f_0} = \frac{1}{\sqrt{2}} \sin\theta F_0^{B^- f_0^{u\bar{u}}} \qquad (4.52)$$

and

$$\bar{f}_{f_0}^n = \frac{1}{\sqrt{2}} \sin\theta \tilde{f}_{f_0}^n, \qquad \bar{f}_{f_0}^s = \cos\theta \tilde{f}_{f_0}^s \; . \qquad (4.53)$$

The expressions for the $f_0(1370)$ form factors and decay constants are similar to the above but with the replacement $\sin\theta \leftrightarrow \cos\theta$ since the flavor content for $f_0(1370)$ is taken as opposite to that of $f_0(1500)$. As the mixing angle θ for the scalars $f_0(1370, 1500)$ is not well known, the variation of the branching ratio with this angle is shown in plots for both decay processes.

For our numerical evaluation, we use the particle masses and lifetimes of B mesons from [33]. The current quark masses are taken as $m_b = 4.2$ GeV, m_s = 95 MeV, m_u = 3.0 MeV and m_d = 6.0 MeV. The Wolfenstein parameters used for the CKM matrix elements are given as λ =0.2205, A=0.815, ρ=0.175 and η=0.370 [86]. The effective coefficients a_i's used are also taken from [86].

For the form factors $F_0^{B^- f_0^{u\bar{u}}}$ involved in the transition $B \to f_0$, the ISGW2 model [87] is used. They are calculated at m_K^2 or m_π^2 depending on the decay mode involving K or π and are given as :

$$\begin{aligned}
F_0^{B^- f_0(1370)^{u\bar{u}}}(m_{K^-}^2) &= 0.112 \,, \\
F_0^{B^- f_0(1370)^{u\bar{u}}}(m_{\pi^-}^2) &= 0.112 \,, \\
F_0^{B^- f_0(1500)^{u\bar{u}}}(m_{K^-}^2) &= 0.114 \,.
\end{aligned} \tag{4.54}$$

For the form factors involved in the transition $B \to K(\pi)$, the results from the relativistic covariant light front quark model [74] are taken where the form factor is parametrized as :

$$F(q^2) = \frac{F(0)}{1 - a(q^2/m_B^2) + b(q^2/m_B^2)^2} \,, \tag{4.55}$$

where for $B \to \pi$ transition, $F_0^{B\pi}(0) = 0.25$, $a = 0.84$, $b = 0.10$ and for $B \to K$ transition, $F_0^{BK}(0) = 0.35$, $a = 0.71$, b=0.04.

The decay constants used are $f_K = 160$ MeV, $f_\pi = 130.7$ MeV. The decay constant $\tilde{f}_{f_0(1500)}$ is taken from [71] (here, $\tilde{f}_{f_0}^s = \tilde{f}_{f_0}^n$) where $\tilde{f}_{f_0(1500)}(2.1$ GeV$)$ $= 315$ MeV. The decay constant for $f_0(1370)$ is taken as $\tilde{f}_{f_0(1370)}(2.1$ GeV$)$ $= 460$ MeV.

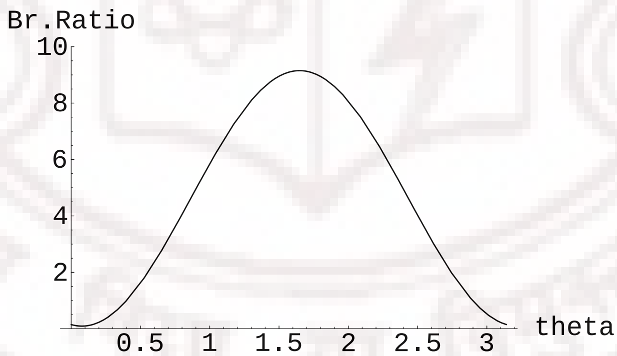

Figure 4.6: Branching Ratio (in units of 10^{-6}) for the process $B^- \to f_0(1370)K^-$ versus the mixing angle θ.

The variation of the branching ratio with the mixing angle θ for the processes involving $f_0(1370, 1500)$ is shown as plots in Figures 4.6, 4.7 and 4.8. The flavor wave function for $f_0(1370)$ is taken as opposite to that of

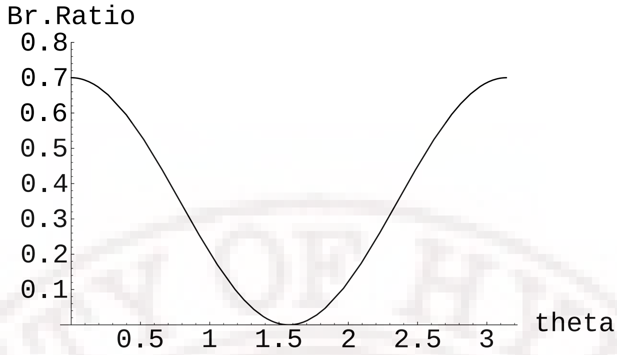

Figure 4.7: Branching Ratio (in units of 10^{-6}) for the process $B^- \to f_0(1370)\pi^-$ versus the mixing angle θ.

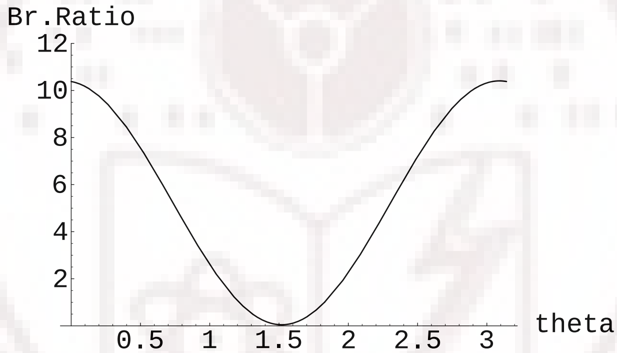

Figure 4.8: Branching Ratio (in units of 10^{-6}) for the process $B^- \to f_0(1500)K^-$ versus the mixing angle θ.

$f_0(980)$ i.e., $s\bar{s}\sin\theta + n\bar{n}\cos\theta$, therefore, the plot for $f_0(1370)K$, which has dominant $s\bar{s}$ contribution, has the form as shown in Figure 4.6 and for the mode $f_0(1370)\pi$, which has only $n\bar{n}$ contribution, the form is opposite, as shown in Figure 4.7. It is found that the branching ratio for the mode $B^- \to f_0(1370)K^-$ lies within the range $(0.15 - 9.09) \times 10^{-6}$, for $B^- \to f_0(1370)\pi^-$ within the range $(2.80 \times 10^{-13} - 0.70 \times 10^{-6})$ and that for $B^- \to f_0(1500)K^-$ within the range $(0.07 - 10.4) \times 10^{-6}$.

4.2.1 Conclusion

In this study, we have considered the rare decay modes $B \to f_0(980)K$, involving a scalar and a pseudoscalar meson in the final state. Since the structure of the f_0 meson is not well established till now, we consider it as a $q\bar{q}$ state, comprising of both $s\bar{s}$ and $(u\bar{u}+d\bar{d})/\sqrt{2}$ components with a mixing angle of 138°, which appears to be the most preferable one. Using the generalized factorization approach, we find that the branching ratios in the standard model are below the current experimental values, as was obtained in previous studies using different approaches. The average value of the observed mixing-induced CP asymmetry, i.e., $(\sin 2\beta)_{f_0 K_S} = -S_{f_0 K_S}$, also has about one sigma deviation from that of $S_{J/\psi K}$. To explain the observed discrepancy in the branching ratios and CP asymmetry parameter, we considered the R-parity violating model. Since these processes receive dominant contribution from $b \to s\bar{s}s$ loop induced penguins, we assumed that the new physics parameters will affect such transitions strongly. We found that the R-parity violating model can explain the observed discrepancy in the branching ratios and the CP violation parameter $-S_{f_0 K_S}$. It can accommodate large CP violation even for small relative strong phase between SM and RPV amplitudes. In this analysis, we have considered a representative value for the RPV coupling $|\lambda'_{i32}\lambda'^{*}_{i22}|$. But it should be noted that using the data on the branching ratios and CP asymmetries of the processes which have dominant $b \to s\bar{s}s$ quark level transitions, it would be possible to obtain the allowed parameter space for the magnitudes and phases of the RPV couplings.

We also considered the rare decay modes $B \to f_0 K(\pi)$ involving the scalars $f_0(1370, 1500)$ in the final state. As the mixing angle for the processes involving $f_0(1370, 1500)$ is not very well known, the variation of the branching ratios with the mixing angle is shown as plots only.

If in future, the $q\bar{q}$ structure for f_0 is established then we can understand their nature properly and these modes could also play an important role to look for new physics beyond the standard model or else, at least, it will certainly enrich our understanding regarding the nature of the light scalar mesons.

Chapter 5

Signature of new physics in $B \to \phi\pi$ decay

Over the years, there has been profound interest in the search for physics beyond the SM. The observed discrepancy between the measured $S_{\phi K_S}$ and $S_{\psi K_S}$ [88] already gave an indication of the possible existence of NP in the $B \to \phi K_S$ decay amplitude and this has, in one way, motivated many to carry out an intensive search for NP. Although the presence of NP in the b-sector is not yet firmly established, but there exists several smoking gun signals [89] which will be verified in the upcoming LHCb experiment or super B factories. As stated earlier, one of the ways of searching for new physics is by studying the rare decay modes arising at the one-loop level, which are induced by flavor changing neutral current (FCNC) transitions. Thus the study of the same will provide us with an excellent testing ground for NP. Therefore, it is interesting to examine as many different rare decay channels as possible to have an indication of new physics.

In this study, we explore the effect of the extra fourth generation of quarks and FCNC mediated $Z(Z')$ boson(s) on the rare decay mode $B^- \to \phi\pi^-$, which is a pure penguin induced process, mediated by the quark level transition $b \to d\bar{s}s$. The interesting feature of this process is that it is dominated by the electroweak penguin contributions as the QCD penguins are Okubo-Zweig-Iizuka (OZI) suppressed and therefore expected to be highly suppressed in the SM. It therefore serves as a suitable place to search for new physics. At present, only the upper limit of its branching ratio is known [90]

$$\mathrm{BR}(B^- \to \phi\pi^-) < 0.24 \times 10^{-6} \, . \tag{5.1}$$

This decay mode has been analyzed both in the SM [91] and in various extensions of it [92] where it has been found that in some of these new physics models, the branching ratio can be enhanced significantly from its corresponding SM value.

5.1 The standard model result

In order to discuss the effect of the fourth quark generation and FCNC mediated $Z(Z')$ boson, we would first like to present the SM result using the QCD factorization [93]. As the decay mode $B^- \to \phi\pi^-$ proceeds through the quark level transition $b \to d\bar{s}s$ and is a pure penguin induced process occurring at the one-loop level, the relevant effective Hamiltonian describing this process is given by

$$\mathcal{H}_{eff}^{SM} = \frac{G_F}{\sqrt{2}} V_{pb} V_{pd}^* \sum_{i=3}^{10} C_i(\mu) O_i \, , \tag{5.2}$$

where $p = u, c$, $C_i(\mu)$'s are the Wilson coefficients evaluated at the b-quark mass scale and O_i's are the QCD and electroweak penguin operators.

In QCD factorization [93], the decay amplitude can be represented in the form

$$\begin{aligned} A(B^-(p_B) \to \phi(\epsilon, p_1)\pi^-(p_2)) &= -i\frac{G_F}{\sqrt{2}} 2m_\phi f_\phi(\epsilon^* \cdot p_B) F_+^{B\pi}(0) \\ &\times \sum_{p=u,c} \lambda_p(\alpha_3^p - \frac{1}{2}\alpha_{3,EW}^p) \, , \end{aligned} \tag{5.3}$$

where $\lambda_p = V_{pb}V_{pd}^*$, the QCD coefficients $\alpha_{3(3,EW)}^p$ are related to the Wilson coefficients as defined in [93] and $F_+^{B\pi}$ is the form factor describing $B \to \pi$ transition. It should be noted that the QCD coefficients contributing to

$B^- \to \phi\pi^-$ are independent of $p = u, c$, (i.e., the virtual particles in the loop). Therefore, one can also represent the above amplitude using CKM unitarity $\lambda_u + \lambda_c + \lambda_t = 0$, as

$$A(B^- \to \phi\pi^-) = i\frac{G_F}{\sqrt{2}} 2m_\phi f_\phi(\epsilon^* \cdot p_B) F_+^{B\pi}(0)\lambda_t(\alpha_3 - \frac{1}{2}\alpha_{3,EW}) , \qquad (5.4)$$

where we have now omitted the superscripts on α's. The above amplitude can be simplified by replacing $2m_\phi\epsilon^* \cdot p_B \to m_B^2$. The branching ratio thus can be obtained using the formula

$$\mathrm{BR}(B^- \to \phi\pi^-) = \frac{\tau_B}{16\pi m_B}|A(B^- \to \phi\pi^-)|^2 , \qquad (5.5)$$

where τ_B is the lifetime of B^- meson. Another possible observable in this decay mode is the direct CP violation parameter, defined as

$$A_{CP} = \frac{\Gamma(B^+ \to \phi\pi^+) - \Gamma(B^- \to \phi\pi^-)}{\Gamma(B^+ \to \phi\pi^+) + \Gamma(B^- \to \phi\pi^-)} . \qquad (5.6)$$

In order to have nonzero direct CP violation, it is necessary that the corresponding decay amplitude should contain at least two interfering contributions with different strong and weak phases. Since in the SM this decay mode does not have two such different contributions in its amplitude, the direct CP violation turns out to be identically zero.

For the numerical evaluation, we use the input parameters as given in the S4 scenario of QCD factorization approach [93]. The particle masses and lifetime of the B meson are taken from [33]. The value of the form factor at zero recoil is taken as $F_+^{B\pi}(0) = 0.28$. The value of the CKM matrix elements used are [33], $|V_{ub}| = 3.96 \times 10^{-3}$, $|V_{ud}| = 0.97383$, $|V_{cb}| = 42.21 \times 10^{-3}$, $|V_{cd}| = 0.2271$ and γ the phase associated with V_{ub} as 70°. With these values as input parameters, the branching ratio obtained in the SM is

$$\mathrm{BR}^{\mathrm{SM}}(B^- \to \phi\pi^-) = 4.45 \times 10^{-9} , \qquad (5.7)$$

which is quite below the experimental upper limit as given in Eq. (5.1).

5.2 In the presence of new physics

Now in the presence of NP, the transition amplitude (5.4) receives an additional contribution and can be symbolically represented as

$$A^T(B^- \to \phi\pi^-) = A^{SM} + A^{NP} = A^{SM}(1 + r\ e^{i\delta}\ e^{-i(\beta-\phi)})\ , \tag{5.8}$$

where β is the weak phase of the SM amplitude i.e., we have used $V_{td} = |V_{td}|e^{-i\beta}$ with the value $\beta = 0.375$, ϕ is the weak phase associated with the NP amplitude and δ is the relative strong phase between these two amplitudes. It should be noted that the strong phases are generated by the final state interactions (FSI) and at the quark level they arise through absorptive parts of the perturbative penguin diagrams. Furthermore, r denotes the magnitude of the ratio of NP to SM amplitude. Thus, we obtain the CP averaged branching ratio $\langle \mathrm{BR} \rangle \equiv [\mathrm{BR}(B^- \to \phi\pi^-) + \mathrm{BR}(B^+ \to \phi\pi^+)]/2$, including the new physics contribution as

$$\langle \mathrm{BR} \rangle = \mathrm{BR}^{\mathrm{SM}}(1 + r^2 + 2r\cos(\beta - \phi)\cos\delta)\ , \tag{5.9}$$

where $\mathrm{BR}^{\mathrm{SM}}$ is the SM branching ratio. It can be seen from the above equation that if r is sizable, the branching ratio could be significantly enhanced from its SM value in the presence of new physics. The direct CP violation parameter (5.6) in the presence of NP becomes

$$A_{CP} = \frac{2r\sin\delta\sin(\phi - \beta)}{1 + r^2 + 2r\cos\delta\cos(\phi - \beta)}\ . \tag{5.10}$$

5.2.1 Effect of a fourth quark generation

We now consider the effect of a sequential fourth generation of quarks [94]. This model is an extension of the SM with the addition of a fourth quark generation. It retains all the features of the SM except that it brings into existence the new members denoted by (t', b'). The fourth up-type quark (t') like u, c, t quarks contributes in the $b \to d$ transition at the loop level and hence will modify the SM result. The effect of the fourth generation of quarks in various B decays is extensively studied in the literature [95, 96].

Due to the additional fourth generation, there will be mixing among the new b' quark and the three down-type quarks of the SM and the resulting mixing matrix will be a 4×4 matrix. Accordingly, the unitarity condition becomes $\lambda_u + \lambda_c + \lambda_t + \lambda_{t'} = 0$ and thus the effective Hamiltonian modifies as

$$\mathcal{H}_{eff} = -\frac{G_F}{\sqrt{2}}\Big[V_{tb}V_{td}^* \sum C_i O_i + V_{t'b}V_{t'd}^* \sum C_i^{t'} O_i\Big], \tag{5.11}$$

where $C_i^{t'}$ are the new Wilson coefficients arising due to the t' quark in the loop. The values of these Wilson coefficients at the M_W scale can be obtained from the corresponding contributions from the t quark by replacing the mass of t quark in the Inami Lim functions [14] by t' mass (here we neglect the renormalization group (RG) evolution of these coefficients from t' mass scale to the weak scale M_W). These values can then be evolved to the m_b scale using RG equation [10], as

$$\vec{C}_i(m_b) = U_5(m_b, M_W, \alpha)\vec{C}(M_W) \ , \tag{5.12}$$

where $\vec{C}$ is the 10×1 column vector of the Wilson coefficients and U_5 is the five flavor 10×10 evolution matrix. The explicit forms of $\vec{C}(M_W)$ and $U_5(m_b, M_W, \alpha)$ are given in [10]. We briefly present the method here. The renormalization group equation for the Wilson coefficients $\vec{C}$ is given as

$$\frac{d}{d \ln \mu}\vec{C} = \gamma^T(g)\vec{C}(\mu) \ , \tag{5.13}$$

which can be solved with the help of the U matrix

$$\vec{C}(\mu) = U(\mu, M_W)\vec{C}(M_W) \ , \tag{5.14}$$

where $\gamma^T(g)$ is the transpose of the anomalous dimension matrix $\gamma(g)$ and g is the QCD coupling. With the help of $dg/d\ln\mu = \beta(g)$, U obeys the same equation as $\vec{C}(\mu)$. We expand $\gamma(g)$ to the first two terms in the perturbative expansion

$$\gamma(\alpha_s) = \gamma^{(0)}\frac{\alpha_s}{4\pi} + \gamma^{(1)}\left(\frac{\alpha_s}{4\pi}\right)^2 . \tag{5.15}$$

To this order the evolution matrix $U(\mu, m)$ is given by

$$U(\mu, m) = \left(1 + \frac{\alpha_s(\mu)}{4\pi}J\right)U^{(0)}(\mu, m)\left(1 - \frac{\alpha_s(m)}{4\pi}J\right) , \tag{5.16}$$

where $U^{(0)}$ is the evolution matrix in leading logarithmic approximation and the matrix J expresses the next-to-leading corrections. We have

$$U^{(0)}(\mu, m) = V\left(\left[\frac{\alpha_s(m)}{\alpha_s(\mu)}\right]^{\frac{\vec{\gamma}^{(0)}}{2\beta_0}}\right)_D V^{-1} , \tag{5.17}$$

where V diagonalizes $\gamma^{(0)T}$, i.e., $\gamma_D^{(0)} = V^{-1}\gamma^{(0)T}V$ and $\vec{\gamma}^{(0)}$ is the vector containing the diagonal elements of the diagonal matrix $\gamma_D^{(0)}$. In terms of $G = V^{-1}\gamma^{(1)T}V$ and a matrix H whose elements are

$$H_{ij} = \delta_{ij}\gamma_i^{(0)}\frac{\beta_1}{2\beta_0^2} - \frac{G_{ij}}{2\beta_0 + \gamma_i^{(0)} - \gamma_j^{(0)}} , \tag{5.18}$$

the matrix J is given by $J = VHV^{-1}$.

Now there are also additional contributions to the RG evolution from QED and therefore the matrix $U(m_1, m_2)$ is substituted by $U(m_1, m_2, \alpha)$ which is the full 10×10 QCD-QED RG evolution matrix. The explicit expressions for the coefficients $C(M_W)$ including $\mathcal{O}(\alpha)$ corrections can be found in [10]. The 10×10 anomalous dimension matrix $\gamma(g^2, \alpha)$ which includes QCD and QED contributions now becomes

$$\gamma(g^2, \alpha) = \gamma_s(g^2) + \frac{\alpha}{4\pi}\Gamma(g^2) , \tag{5.19}$$

where the term due to QED corrections has the following expansion

$$\Gamma(g^2) = \gamma_e^{(0)} + \frac{\alpha_s}{4\pi}\gamma_{se}^{(1)} + ... \tag{5.20}$$

The RG evolution matrix now becomes

$$U(m_1, m_2, \alpha) = U(m_1, m_2) + \frac{\alpha}{4\pi} R(m_1, m_2) \;, \tag{5.21}$$

where $U(m_1, m_2)$ represents pure QCD evolution and $R(m_1, m_2)$ is the additional evolution in the presence of electromagnetic interaction. The expression for $R(m_1, m_2)$ is

$$R(m_1, m_2) \equiv -\frac{2\pi}{\beta_0} V \left(K^{(0)}(m_1, m_2) + \frac{1}{4\pi} \sum_{i=1}^{3} K_i^{(1)}(m_1, m_2) \right) V^{-1} \;. \tag{5.22}$$

The matrix kernels in the above equation are defined by

$$(K^{(0)}(m_1, m_2))_{ij} = \frac{M_{ij}^{(0)}}{a_i - a_j - 1} \left[\left(\frac{\alpha_s(m_2)}{\alpha_s(m_1)} \right)^{a_j} \frac{1}{\alpha_s(m_1)} - \left(\frac{\alpha_s(m_2)}{\alpha_s(m_1)} \right)^{a_i} \frac{1}{\alpha_s(m_2)} \right] , \tag{5.23}$$

$$\left(K_1^{(1)}(m_1, m_2) \right)_{ij} = \begin{cases} \frac{M_{ij}^{(1)}}{a_i - a_j} \left[\left(\frac{\alpha_s(m_2)}{\alpha_s(m_1)} \right)^{a_j} - \left(\frac{\alpha_s(m_2)}{\alpha_s(m_1)} \right)^{a_i} \right] & i \neq j \\ M_{ii}^{(1)} \left(\frac{\alpha_s(m_2)}{\alpha_s(m_1)} \right)^{a_i} \ln \frac{\alpha_s(m_1)}{\alpha_s(m_2)} & i = j \end{cases} , \tag{5.24}$$

$$K_2^{(1)}(m_1, m_2) \;=\; -\alpha_s(m_2)\, K^{(0)}(m_1, m_2)\, H \,, \tag{5.25}$$

$$K_3^{(1)}(m_1, m_2) \;=\; \alpha_s(m_1)\, H\, K^{(0)}(m_1, m_2) \tag{5.26}$$

with

$$\begin{aligned} M^{(0)} &= V^{-1}\, \gamma_e^{(0)T}\, V \,, \\ M^{(1)} &= V^{-1} \left(\gamma_{se}^{(1)T} - \frac{\beta_1}{\beta_0} \gamma_e^{(0)T} + \left[\gamma_e^{(0)T}, J \right] \right) V \,, \end{aligned} \tag{5.27}$$

where the explicit expressions for the 10×10 leading order and next-to-leading order anomalous dimension matrices $\gamma_s^{(0)}$, $\gamma_e^{(0)}$, $\gamma_s^{(1)}$ and $\gamma_{se}^{(1)}$ are given in [10].

Thus, on using this RG evolution, we obtain the new Wilson coefficients $C_i^{t'}$ at the m_b scale and we present their values in Table 5.1, for a representative set of values for $m_{t'} = 400$ GeV.

Table 5.1: Values of the new Wilson coefficients at m_b scale where C_i^{new} represents $C_i^{t'}$ for the fourth quark generation model and $\tilde{C}_i$ for the FCNC mediated Z boson model. The phase $\phi' = (\phi - \beta)$ is the relative weak phase between the NP and SM amplitudes.

Wilson Coefficients	4-Generation ($m_{t'}$=400 GeV)	Z boson model $\left(\kappa = \frac{\lvert U_{bd}\rvert}{\lvert V_{tb}V_{td}^*\rvert}\right)$	Z' model ($\xi_{L,R} = \lvert\xi_{L,R}\rvert e^{i\phi'}$)
$C_3^{new}(m_b)$	0.0195	$0.19\ \kappa e^{i\phi'}$	$0.05\ \xi_L - 0.01\ \xi_R$
$C_4^{new}(m_b)$	-0.0373	$-0.066\ \kappa e^{i\phi'}$	$-0.14\ \xi_L + 0.008\ \xi_R$
$C_5^{new}(m_b)$	0.0101	$0.009\ \kappa e^{i\phi'}$	$0.029\ \xi_L + 0.017\ \xi_R$
$C_6^{new}(m_b)$	-0.0435	$-0.031\ \kappa e^{i\phi'}$	$-0.162\ \xi_L + 0.01\ \xi_R$
$C_7^{new}(m_b)$	0.0044	$0.145\ \kappa e^{i\phi'}$	$0.036\ \xi_L - 3.65\ \xi_R$
$C_8^{new}(m_b)$	0.002	$0.053\ \kappa e^{i\phi'}$	$0.01\ \xi_L - 1.33\ \xi_R$
$C_9^{new}(m_b)$	-0.029	$-0.566\ \kappa e^{i\phi'}$	$-4.41\ \xi_L + 0.04\ \xi_R$
$C_{10}^{new}(m_b)$	0.0062	$0.127\ \kappa e^{i\phi'}$	$0.99\ \xi_L - 0.005\ \xi_R$

After obtaining the values of the new Wilson coefficients at the b quark mass scale, one can directly write the decay amplitude due to the fourth generation of quarks analogous to (5.4) as

$$A^{NP} = \frac{G_F}{\sqrt{2}} 2m_\phi f_\phi (\epsilon^* \cdot p_B) F_+^{B\pi}(0) \lambda_{t'} \left(\alpha_3' - \frac{1}{2}\alpha_{3,EW}'\right), \tag{5.28}$$

where $\alpha'_{3(3,EW)}$'s are the new contributions arising from the t' quark contribution. We parameterize the new CKM elements as $\lambda_{t'} = r_d e^{i\phi}$, where ϕ is the new weak phase associated with λ_t'. Furthermore, since the unitarity condition has now become modified, the elements of the 3×3 upper submatrix of the 4×4 quark mixing matrix will be different from the corresponding values of SM CKM matrix elements. Since V_{tb} and V_{td} are not precisely known (i.e., not directly extracted from the experimental data, but fitted using the unitarity constraint) we use the lower limits from [33] i.e., $|V_{tb}| = 0.78$ and $|V_{td}| = 7.4 \times 10^{-3}$.

In order to study the effect of the fourth generation, we need to know the values of the new parameters $(m_{t'}, r_d, \phi)$. Based on an integrated luminosity of $2.3 fb^{-1}$, CDF collaboration [97] gives the lower bound on $m_{t'}$ as $m_{t'} > 284$ GeV. In [98], it has been shown that the observed pattern of deviations in the CP symmetries of B system can be explained in the fourth quark generation model if $m_{t'} > 700$ GeV . Therefore, in our analysis we consider three representative values for $m_{t'}$, i.e., $m_{t'} = 400,\ 600$ and 800 GeV. The value of r_d can be obtained from the measured mass difference ΔM_{B_d} of $B^0 - \bar{B}^0$ system and the corresponding expression for ΔM_{B_d} in the presence of fourth quark generation can be found in Ref. [96]. Thus, we obtain the values r_d for different m_t', consistent with the unitarity condition of 4×4 matrix as: $r_d \sim -3.8 \times 10^{-3}$ ($m_{t'} = 400$ GeV), $r_d \sim -2.7 \times 10^{-3}$ ($m_{t'} = 600$ GeV) and $r_d \sim -2.1 \times 10^{-3}$ ($m_{t'} = 800$ GeV). Using these values, in Figure 5.1 and Figure 5.2 we show the variation of the branching ratio and the direct CP asymmetry, respectively, with the new weak phase ϕ for three different values of $m_{t'}$. From Figure 5.1, one can see that the branching ratio is significantly enhanced from its SM value and this enhancement is more pronounced for large $m_{t'}$. It should also be noted that nonzero direct CP violation in this mode could be possible in the presence of an additional generation of quarks.

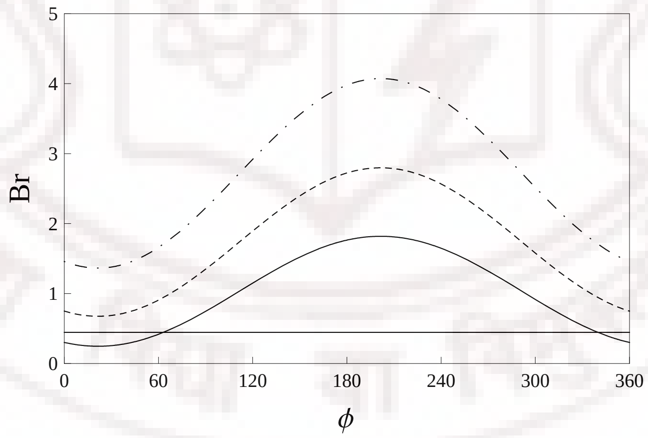

Figure 5.1: Variation of CP averaged branching ratio (5.9) (in units of 10^{-8}) with the new weak phase ϕ, where the solid, dashed and dot-dashed lines correspond to $m_{t'} = 400, 600$ and 800 GeV, respectively. The horizontal line represents the SM value.

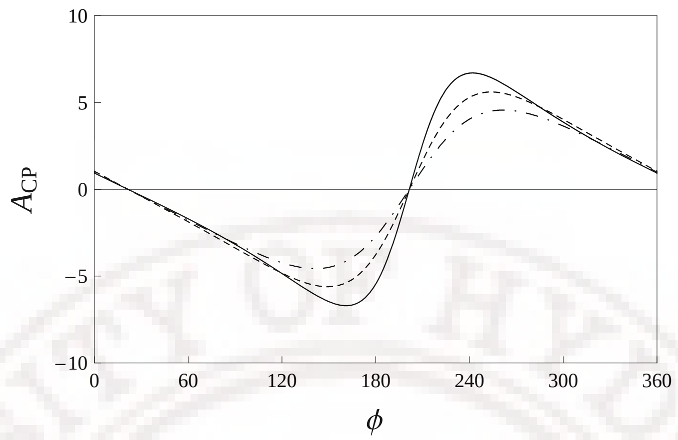

Figure 5.2: Variation of direct CP asymmetry (5.10) (in %) with the new weak phase ϕ, where the solid, dashed and dot-dashed lines correspond to $m_{t'} = 400, 600$ and 800 GeV, respectively.

5.2.2 Effect of the FCNC mediated Z boson

Now we consider another extension of the SM, where the fermion sector is enlarged by an extra down-type singlet quark. Isosinglet quarks appear in many extensions of the SM like the low energy limit of the E_6 GUT models [99]. The mixing of this singlet down-type quark with the three SM down-type quarks provides a framework to study the deviations of the unitarity constraint of the 3×3 CKM matrix. The mixing also induces tree level flavor changing neutral currents, which can thus substantially modify the SM results. In this model, the Z mediated FCNC interaction is given by [100]

$$\mathcal{L} = \frac{g}{2\cos\theta_W}[\bar{d}_{L\alpha} U_{\alpha\beta} \gamma^\mu d_{L\beta}] Z_\mu \ , \qquad (5.29)$$

with

$$U_{\alpha\beta} = \sum_{i=u,c,t} V^\dagger_{\alpha i} V_{i\beta} = \delta_{\alpha\beta} - V^*_{4\alpha} V_{4\beta} \ , \qquad (5.30)$$

where $\alpha,\ \beta$ are generation indices and U is the neutral current mixing matrix for the down quark sector. The non-vanishing component of $U_{\alpha\beta}$ will lead to the presence of FCNC transitions at the tree level. The implications of the

FCNC mediated Z boson effect has been extensively studied in the context of b physics [101, 102, 103].

Because of the new interactions the effective Hamiltonian describing $b \to d\bar{s}s$ process is given as [102]

$$\mathcal{H}_{eff}^{Z} = -\frac{G_F}{\sqrt{2}} V_{tb} V_{td}^{*} [\tilde{C}_3 O_3 + \tilde{C}_7 O_7 + \tilde{C}_9 O_9] \ , \tag{5.31}$$

where the four-quark operators O_3, O_7 and O_9 have the same structure as the SM QCD and electroweak penguin operators and the new Wilson coefficients $\tilde{C}_i$'s at the M_Z scale are given by

$$\begin{aligned} \tilde{C}_3(M_Z) &= \frac{1}{6}\frac{U_{bd}}{V_{tb}V_{td}^{*}}, \\ \tilde{C}_7(M_Z) &= \frac{2}{3}\frac{U_{bd}}{V_{tb}V_{td}^{*}}\sin^2\theta_W, \\ \tilde{C}_9(M_Z) &= -\frac{2}{3}\frac{U_{bd}}{V_{tb}V_{td}^{*}}(1-\sin^2\theta_W). \end{aligned} \tag{5.32}$$

These new Wilson coefficients will be evolved from the M_Z scale to the m_b scale using the renormalization group equation [10] as described earlier. Because of the RG evolution, these three Wilson coefficients generate a new set of Wilson coefficients $\tilde{C}_i (i = 3, \cdots, 10)$ at the low energy regime (i.e., at the m_b scale) as presented in Table 5.1. Thus, one can write the new amplitude due to the tree level FCNC mediated Z boson effect in a straightforward manner from Eq. (5.4) by replacing $\alpha_{3(3,EW)}$ by $\tilde{\alpha}_{3(3,EW)}$, where $\tilde{\alpha}$'s are related to the new Wilson coefficients $\tilde{C}_i(m_b)$'s. In order to see the effect of this FCNC mediated Z boson effect, we have to know the value of the parameter $Z-b-d$ coupling parameter which can be explicitly written as $U_{bd} = |U_{bd}|e^{i\phi}$ and the allowed range of $|U_{bd}|$ is found to be $(2 \times 10^{-4} \le |U_{bd}| \le 1.2 \times 10^{-3})$ [103]. In Figure 5.3 and Figure 5.4, respectively, we present the variation of the CP averaged branching ratio (5.9) with $|U_{bd}|$ and ϕ and the direct CP asymmetry parameter A_{CP} with ϕ, where we have used $\sin^2\theta_W = 0.231$. From the figures, it can be seen that the branching ratio could be significantly enhanced and large CP violation could be possible in this model.

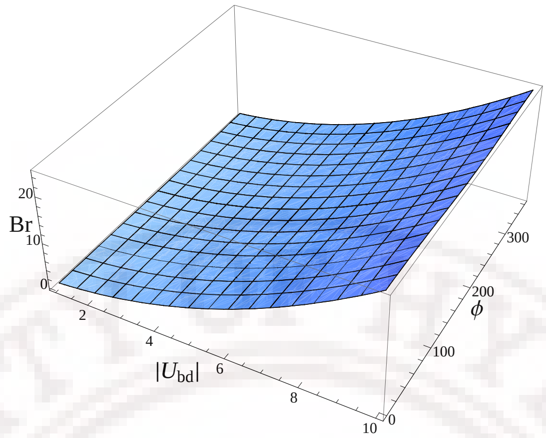

Figure 5.3: Variation of the CP averaged branching ratio (5.9) (in units of 10^{-8}) with $|U_{bd}|$ (in units of 10^{-4}) and the new weak phase ϕ

5.2.3 Effect of the FCNC mediated Z' boson

Now we consider the effect due to an extra $U(1)'$ gauge boson Z'. The existence of an extra Z' boson is a feature of many models addressing physics beyond the SM, e.g., models based on extended gauge groups characterized by additional $U(1)$ factors [104]. In particular, they often occur in grand unified theories (GUTs), superstring theories and theories with large extra dimensions. The new physics models which contain exotic fermions also predict the existence of an additional gauge boson. Flavor mixing can be induced at the tree level in the up-type and/or down-type quark sector after diagonalizing their mass matrices. FCNCs due to Z' exchange can be induced by mixing among the SM quarks and the exotic quark which have different Z' quantum numbers. The search for the extra Z' boson occupies an important place in the experimental programs of the Fermilab Tevatron and CERN LHC [105]. At such hadron colliders, heavy neutral gauge bosons with mass upto around 5 TeV can be produced and detected via two fermion decays $pp(p\bar{p}) \to Z' \to l^+l^- \ (l = e, \mu)$.

Here we consider the model in which the interaction between the Z' boson and fermions are flavor nonuniversal for left-handed couplings and flavor

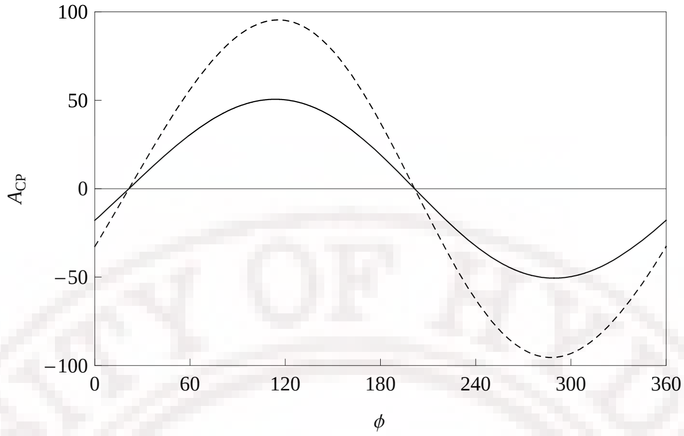

Figure 5.4: Variation of direct CP asymmetry (5.10) (in %) with the new weak phase ϕ where the dashed and solid lines correspond to $|U_{bd}| = 10^{-4}$ and 5×10^{-4}.

diagonal for right-handed couplings. The model can be also be found in Ref. [106, 107], where it has been shown that such a model can successfully explain the deviations of $S_{\phi K}$ and $S_{\eta' K}$ from $S_{\psi K}$ and also can explain the $B \to \pi K$ puzzle. We briefly present the method here. The Lagrangian for the neutral current interaction with the Z' in the gauge basis is

$$\mathcal{L}^{Z'} = -g' J'_\mu Z'^\mu \;, \tag{5.33}$$

where g' is the gauge coupling associated with the $U(1)'$ group at the M_W scale. The renormalization group running between M_W and $M_{Z'}$ is neglected here. The Z' boson is assumed to have no mixing with the SM Z boson. The chiral current is

$$J'_\mu = \sum_{i,j} \overline{\psi}_i^I \gamma_\mu \left[(\epsilon_{\psi_L})_{ij} P_L + (\epsilon_{\psi_R})_{ij} P_R \right] \psi_j^I \;, \tag{5.34}$$

where the sum extends over the flavors of fermion fields, the chirality projection operators are $P_{L,R} \equiv (1 \mp \gamma_5)/2$, the superscript I refers to the gauge interaction eigenstates and ϵ_{ψ_L} (ϵ_{ψ_R}) denote the left-handed (right-handed) chiral couplings. ϵ_{ψ_L} and ϵ_{ψ_R} are hermitian under the requirement of a real

Lagrangian. The fermion Yukawa coupling matrices Y_ψ in the weak basis can be diagonalized as

$$Y_\psi^D = V_{\psi_R} Y_\psi V_{\psi_L}^\dagger \tag{5.35}$$

using the bi-unitary matrices $V_{\psi_{L,R}}$ in $\psi_{L,R} = V_{\psi_{L,R}} \psi_{L,R}^I$, where $\psi_{L,R}^I \equiv P_{L,R}\psi^I$ and $\psi_{L,R}$ are the mass eigenstate fields. The usual CKM matrix is then given by

$$V_{CKM} = V_{u_L} V_{d_L}^\dagger \ . \tag{5.36}$$

The chiral Z' coupling matrices in the physical basis of up-type and down-type quarks are, respectively,

$$B_u^X \equiv V_{u_X} \epsilon_{u_X} V_{u_X}^\dagger \ , \quad B_d^X \equiv V_{d_X} \epsilon_{d_X} V_{d_X}^\dagger \ , \quad (X = L, R) \tag{5.37}$$

where $B_{u(d)}^X$ are hermitian. As long as the ϵ matrices are not proportional to the identity matrix, the B^X matrices will have nonzero off-diagonal elements that induce FCNC interactions at tree level. The assumption of flavor diagonal right-handed couplings demands $B_{u(d)}^R \propto I$. However, the flavor changing left-handed couplings will give new contributions to the SM operators.

The effective Hamiltonian describing the transition $b \to d\bar{s}s$ mediated by the Z' boson is therefore given by [106]

$$\mathcal{H}_{eff}^{Z'} = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{td}^* \left[\left(\frac{g' M_Z}{g_1 M_{Z'}} \right)^2 \frac{B_{db}^L}{V_{tb} V_{td}^*} (B_{ss}^L O_9 + B_{ss}^R O_7) \right] \ , \tag{5.38}$$

where $g_1 = e/(\sin\theta_W \cos\theta_W)$ and $B_{ij}^{L(R)}$ denote the left (right)-handed effective Z' couplings of the quarks i and j at the weak scale. The diagonal elements are real due to the hermiticity of the effective Hamiltonian but the off-diagonal elements may contain an effective weak phase. Therefore, both the terms in (5.38) will have the same weak phase due to B_{db}^L. We can parameterize these coefficients as

$$\xi_L = \left(\frac{g' M_Z}{g_1 M_{Z'}} \right)^2 \left(\frac{B_{db}^L B_{ss}^L}{V_{tb} V_{td}^*} \right) = |\xi_L| e^{i\phi'}, \ \ \xi_R = \left(\frac{g' M_Z}{g_1 M_{Z'}} \right)^2 \left(\frac{B_{db}^L B_{ss}^R}{V_{tb} V_{td}^*} \right) = |\xi_R| e^{i\phi'}, \tag{5.39}$$

where $\phi' = \phi - \beta$ (ϕ is the weak phase associated with B^L_{db}).

In order to see the effect of the Z' boson, we have to know the values of the coefficients ξ_L and ξ_R or equivalently B^L_{db} and $B^{L,R}_{ss}$. Assuming only left-handed couplings are present, the bound on FCNC Z' coupling (B^L_{db}) from $B^0 - \bar{B}^0$ mass difference has been obtained in Ref. [108] as

$$y|\mathrm{Re}(B^L_{db})^2| < 5 \times 10^{-8}, \quad y|\mathrm{Im}(B^L_{db})^2| < 5 \times 10^{-8}\,, \tag{5.40}$$

where $y = (g' M_Z / g_1 M_{Z'})^2$. Generally one expects $g'/g_1 \sim 1$, if both the $U(1)$ gauge groups have the same origin from some grand unified theories, $M_Z/M_{Z'} \sim 0.1$ for a TeV scale neutral Z' boson, which yields $y \sim 10^{-2}$. However in Ref. [108], assuming a small mixing between Z and Z' bosons, the value of y is taken as $y \sim 10^{-3}$. Using $y \sim 10^{-2}$, one can obtain a more stringent bound $|B^L_{db}| < 10^{-3}$. It has been shown in [107] that the mass difference of $B_s - \bar{B}_s$ mixing can be explained if $|B^L_{sb}| \sim |V_{tb}V^*_{ts}|$. Similarly, the CP asymmetry anomaly in $B \to \phi K, \pi K$ can be resolved if $|B^L_{sb} B^{L,R}_{ss}| \sim |V_{tb}V^*_{ts}|$. From these two relations, one can obtain $|B^L_{ss}| \sim 1$. Thus, it is expected that $\xi_{L,R} \sim 10^{-3}$. However, in this analysis we vary their values within the range $(0.01 - 0.001)$.

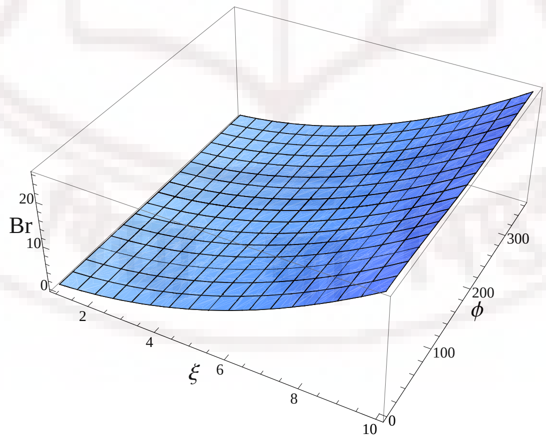

Figure 5.5: Variation of the CP averaged branching ratio (5.9) (in units of 10^{-8}) with ξ (in units of 10^{-3}) and the new weak phase ϕ .

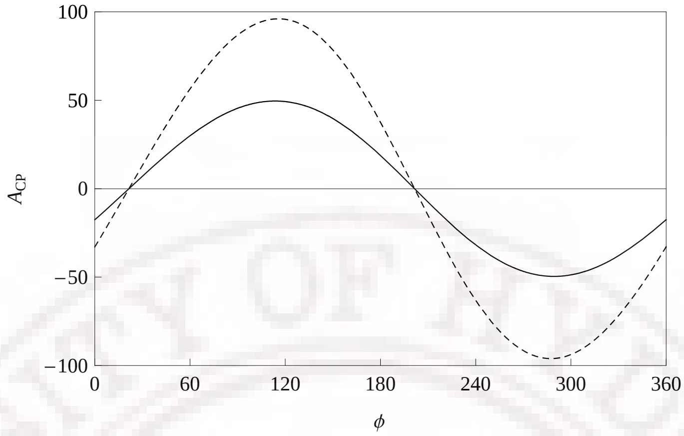

Figure 5.6: Variation of direct CP asymmetry (5.10) (in %) with the new weak phase ϕ where the dashed and solid lines correspond to $\xi = 10^{-3}$ and 5×10^{-3}.

After having an idea about the magnitudes of these new coefficients which are at the M_Z scale, we now evolve them to the b scale using the renormalization group equation [10] as described earlier. The new Wilson coefficients at the m_b scale are presented in Table 5.1. Using the values of these coefficients at b scale, we can analogously obtain the new contribution to the transition amplitude as done in the case of Z boson. Now using $|\xi_L| = |\xi_R| = \xi$, in Figure 5.5 and Figure 5.6, respectively, we show the variation of the CP averaged branching ratio with ξ and the new weak phase ϕ and the direct CP violation with ϕ. In this case also one can have a significant enhancement in the branching ratio for large ξ, or in other words for a lighter Z' boson. Furthermore, the observation of this mode could in turn help us to constrain the Z' mass.

5.3 Conclusion

To conclude, we have studied the $B^- \to \phi\pi^-$ decay mode in the standard model and in some beyond the standard model scenarios. This is a pure penguin rare decay process and proceeds through the quark level transition

$b \to d\bar{s}s$, which occurs at the one-loop level and is therefore expected to be highly suppressed in the SM. The SM prediction of its branching ratio is $\sim \mathcal{O}(10^{-9})$ which is below the experimental upper limit of $\mathcal{O}(10^{-7})$. We have analysed this decay mode in the fourth quark generation model and in the FCNC mediated Z and Z' models. In the fourth quark generation model, we find that the branching ratio enhances from its SM value with increasing $m_{t'}$ and it can have a value of $\sim \mathcal{O}(10^{-8})$. In the Z and Z' models, the branching ratio can be significantly enhanced for sizable new physics couplings $|U_{bd}|$ and ξ, respectively. In these cases it can reach up to $\mathcal{O}(10^{-7})$ level but still within the experimental upper limit. Furthermore, it is found that large direct CP violation could be possible in this decay mode in the presence of the mentioned new physics models. Thus, if this mode could be observed in the upcoming LHCb experiment, it will provide a clear signal of new physics and also can be used to constrain the parameter space of various new physics models. However, it should be noted that it would not be possible to distinguish between these new physics models considering this mode alone.

Chapter 6

Summary

In this thesis, we have studied some decay modes of the B meson which are interesting in the context of CP violation and new physics. We studied some of the rare B decay modes in the SM and in some beyond the SM scenarios also. In such a study, our interest was on computing the two observables, the CP asymmetry parameter and branching ratio of these modes. We also studied some other B decay modes and we came up with methods to extract the weak phase γ and the parameter $2\beta + \gamma$.

We explored the decay modes $B_c^\pm \to (D^0)D_s^\pm \to (K^{*\pm}K^\mp)D_s^\pm$ and $B_c^\pm \to (\bar{D}^0)D_s^\pm \to (K^{*\pm}K^\mp)D_s^\pm$ for the possible extraction of γ. The angle β (or $\sin 2\beta$) has been cleanly determined from the measurement of the time-dependent CP asymmetry in the golden decay mode $B_d^0 \to J/\psi K_S$. The angle α can be measured using the CP asymmetries in $B_d^0 \to \pi^+\pi^-$ but there are theoretical hadronic uncertainties due to the existence of penguin diagrams. It is imperative that the angle γ also be measured independently to understand better the CKM mechanism of CP violation in the SM. There have been many attempts before to formulate methods to extract the angle γ. One has to measure the angle with all possible clean methods available to arrive at a conclusion and thereby reducing the error in γ to a minimum. Therefore, in the continued effort, we came up with another method to determine γ. This method is better suited as the interfering amplitudes of the two decay modes we have considered are roughly of equal sizes. Furthermore, as the decay B meson is a charged meson, we require no tagging nor time-dependent studies for this method. Also since a large number of B_c mesons are expected to be produced at the LHC, it therefore would be very interesting to explore the determination of γ with these modes. We believe that during the first few years of LHC run, we will have a meaningful value of angle γ with reduced errors and emphasize that the strategy we have presented here will be an added asset to our endeavour to measure the angle γ.

With the decay modes $B^0 \to D^0 K^{*0}$ and $B^0 \to \bar{D}^0 K^{*0}$, we developed a similar method for the extraction of γ. The other parameter to be measured is $2\beta+\gamma$. Since the angle β is well measured by now, therefore, the measurement of $2\beta+\gamma$ will be useful in the clean determination of γ. Since, as emphasized before, one needs to have as many clean methods as possible to improve the sensitivity and to resolve the discrete ambiguities, the method presented will be very much helpful in this direction. We presented another important and simple way to extract the weak phase $\gamma/(2\beta+\gamma)$ from the decay modes $\overset{(-)}{B}_d \to \overset{(-)}{D}{}^0 \overset{(-)}{K}{}^{*0}$, $\overset{(-)}{D}{}^{*0} \overset{(-)}{K}{}^{*0}$. These channels are described by color suppressed tree diagrams only and are free from penguin contributions. For the extraction of γ, we considered the decay modes $B^0 \to D^0(\bar{D}^0)K^{0*}$, with subsequent decay of $D^0(\bar{D}^0)$ into the non-CP state $K^{*+}K^-$. The use of the non-CP state allows the two interfering amplitudes to be of the same order and hence one can cleanly extract the CKM angle γ. We then considered the processes $B^0 \to D^{*0}(\bar{D}^{*0})K^{0*}$, where the final states are admixtures of CP-even and CP-odd states. On using the angular distributions of the final decay products, it was shown that it is possible to disentangle them. We considered the longitudinal component of the time-dependent decay rates of these modes and we have shown that $\phi \equiv (2\beta+\gamma)$ can be cleanly obtained. These modes could be very much suited for determining the phase γ $(2\beta+\gamma)$ as they are free from penguin pollution and also as the branching ratios are measurable at hadron factories such as the LHCb.

We then considered the rare hadronic decay modes $B \to f_0 K(\pi)$, involving the scalars $f_0(980, 1370, 1500)$ and a pseudoscalar meson $K(\pi)$ in the final state. We first presented the decay mode $B \to f_0(980)K$. Since the structure of the $f_0(980)$ meson is not well established, we considered it as a $q\bar{q}$ state, comprising of both $s\bar{s}$ and $(u\bar{u} + d\bar{d})/\sqrt{2}$ components with a mixing angle of 138°, which has been experimentally determined from $\phi \to \gamma f_0$ decays. The decay modes under consideration are dominated by the loop induced $b \to s\bar{q}q$ ($q = s,\ u,\ d$) penguins along with a small $b \to u$ tree level transition (for $B^+ \to f_0 K^+$) and annihilation diagrams. Therefore, the standard model expectation of direct CP violation is negligibly small and the mixing-induced CP violation parameter in the mode $B^0 \to f_0 K_S$ is expected to give the same value of $\sin(2\beta)$, as extracted from $B^0 \to J/\psi K_S$ but with opposite sign. Using the generalized factorization approach, we found that the branching ratios in the standard model are below the current experimental values and the direct CP violation in the decay mode $B^+ \to f_0 K^+$ to be of the order of a few percent. We then analyze the decay modes in the minimal supersymmetric standard model (MSSM) with R-parity violation. On using the RPV model, we could show that the direct CP violating asymmetry in $B^+ \to f_0(980)K^+$ could be as large as $\sim 80\%$ and the mixing-induced CP asymmetry in $B^0 \to f_0 K_S$ (i.e., $-S_{f_0 K_S}$) could deviate significantly from that of $\sin(2\beta)_{J/\psi K_S}$. In the second part, we presented the study involving the scalars $f_0(1370, 1500)$ in the final state. We have taken the flavor content of $f_0(1500)$ as $f_0(1500) = n\bar{n}\sin\theta + s\bar{s}\cos\theta$ and the flavor content for $f_0(1370)$ as $f_0(1370) = s\bar{s}\sin\theta + n\bar{n}\cos\theta$. Since the mixing angle θ for these two scalars is not known, we vary the branching ratios for these modes with θ and show the variations as plots. The modes we have considered can be important grounds for looking for new physics beyond the SM and can also could help in the understanding of the nature of the light scalar mesons.

Finally, we look for the possible existence/manifestation of NP in the decay mode $B \to \phi\pi$. One of the ways of searching for NP is by studying the rare decay modes which arises at the one-loop level and are induced by flavor changing neutral current transitions. We presented the investigation of the effect of an extra fourth quark generation and FCNC mediated Z and Z' bosons on the rare decay mode $B^- \to \phi\pi^-$. In the standard model, this mode receives only $b \to d$ penguin contributions and therefore is highly suppressed. The branching ratio obtained is $\sim 5 \times 10^{-9}$ and this makes the mode a very sensitive probe for new physics. With the fourth quark generation model, we found that the branching ratio enhances from its SM value with increasing $m_{t'}$ and it can have a value of $\sim \mathcal{O}(10^{-8})$. With the Z and Z' models, the branching ratio could also be significantly enhanced for sizable new physics couplings $|U_{bd}|$ and ξ, respectively. It can reach up to $\mathcal{O}(10^{-7})$ level in these cases but still within the experimental upper limit. Also, it is found that we can have large direct CP violation in this decay mode in the presence of the considered new physics models.

B physics remains as one of the most exciting and active fields of particle physics, especially in the context of CP violation and new physics. There is anticipation that with the high precision measurement of the B decay modes at the hadron colliders, some of the unresolved issues in B physics and in particle physics in general, could be resolved and possibly new physics effects will be revealed and also we could have more precise measurements of the CP asymmetry parameters and other SM parameters as well.

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