OSCILLATORY PHASE OF NONCLASSICAL INFLATON IN FRW UNIVERSE

A thesis submitted for the degree of

DOCTOR OF PHILOSOPHY IN PHYSICS

BY

K VENKATARATNAM KAMMA

SCHOOL OF PHYSICS
UNIVERSITY OF HYDERABAD
HYDERABAD 500 046
INDIA

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Declaration

I hereby declare that, this thesis entitled **OSCILLATORY PHASE OF NONCLASSICAL INFLATON IN FRW UNIVERSE** is based on the work done by me at the School of Physics, University of Hyderabad under the supervision of Dr. P. K. Suresh. No part of this thesis has been previously submitted for a degree or diploma or any other qualification at this university or any other.

(K Venkataratnam Kamma)

Hyderabad,
Date:

Certificate

This is to certify that, this thesis entitled **OSCILLATORY PHASE OF NONCLASSICAL INFLATON IN FRW UNIVERSE** is based on the work done by Mr.K Venkataratnam Kamma under my supervision at the School of Physics, University of Hyderabad, in fulfillment of the requirements for the award of the degree of **Doctor of Philosophy in Physics**. No part of this thesis has been previously submitted for a degree or diploma or any other qualification at this university or any other.

(Dr. P.K. Suresh)
Thesis Supervisor

(Dean, School of Physics)

Preface

The thesis is dedicated to the study of oscillatory phase of inflaton by representing it in nonclassical state formalisms of quantum optics. The thesis deals with the role of nonclassical and thermal effects of inflaton in semiclassical theory of gravity in the post inflationary era of the universe.

The standard cosmology is spectacularly successful but the model leaves many features of the universe unexplained. The most important of these are horizon problem, singularity problem, flatness problem, homogeneity problem, structure formation problem, monopole problem and so on. All these problems are very difficult and defy solutions within the standard cosmology theory. Most of these problems can be addressed or considerably relaxed in the inflationary scenario. According to which the universe in the past expanded almost exponentially with time, while its energy density was dominated by the effective potential energy density of a homogeneous scalar field, called the inflaton. At the end of inflation the inflaton field started quasi-periodic motion with slowly decreasing amplitude. Right after the inflationary period the universe was devoid of particles, that is, cold. Quasi-periodic evolution of the inflaton field led to creation of particles of various kinds, after thermalization of which due to collisions and decays, the universe became hot again. From then on, it can be described by the usual hot big bang theory. Therefore the oscillatory phase of the inflaton and its related issues are important to understand particle creation and further evolution of the universe.

Most of the inflationary scenario and related issues can be described with a scalar field in the classical Friedmann equations, assuming its validity even at the very early stage of the universe. However, it is believed that quantum effects of matter fields and quantum fluctuations played significant role in this regime, though quantum gravity effects are considered to be negligible. Therefore, the proper description of a cosmological model can be studied in terms of the semiclassical Friedmann equations in which quantized matter field is taken as source and the corresponding background metric as classical. Recently, the study of inflation and its related issues in the semiclassical theory of gravity has received much attention.

In the present work we use the coherent and squeezed state formalisms of quantum optics to study the inflaton in the oscillatory phase, in the semi-classical theory of gravity. We study particle production, density fluctuations and validity of the semiclassical theory of gravity in the flat Friedmann universe. Since we use the representation of the inflaton in the coherent and squeezed states, it would be useful to examine classical or nonclassical nature of the field, in the cosmological context. In quantum optics context such study is carried out by using a parameter known as the Mandel's Q parameter and is studied in the cosmological context with the associated cosmological parameters.

The aim of the present study is to consider the flat Friedmann model of the universe in the semiclassical theory of gravity. However, the solutions of the Friedmann model imply an open and a closed Friedmann models also. Therefore it is interesting to see how the semiclassical gravity and its related phenomenon play role in determining dynamics of the open and closed Friedmann models. Thus the general goal of the present work is to study a massive minimal nonclassical scalar field in the flat, open and closed Friedmann universe by representing it in terms of the coherent and squeezed state formalisms of quantum optics as well as in their thermal counterparts.

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To

My Family

Contents

UNIVERSITY OF HYDERABAD

Chapter 1

Introduction

Cosmology is the scientific study of origin and evolution of the universe. In modern sense, it deals with the study of overall dynamical and physical behaviour of billions of galaxies spread across vast distances and of the evolution of this enormous system over several billion years [1]. Modern cosmology began when Einstein's general theory of relativity applied to understand the universe [2]. A remarkable progress in understanding of the universe has been made over several years since Friedmann obtained the solutions to the Einstein field equations [2]. When combined the Friedmann solutions with the Hubble redshift distance relation, it is interpreted that we live in an expanding universe [3]. Thus extrapolate back in time, investigators deduced that the universe emerged with great explosion, from unbelievable dense, hot region, popularly known as big bang. The hot big bang or standard cosmology [3] is considered as a very successful theory to describe many observed features of the universe [4]. One of the strong predictions of hot big bang model is the existence of a particular kind of radiation that would preserve its black body form even today, known as the cosmic microwave background radiation.

The discovery of the microwave background radiation [5, 6], together with the fact that the abundances of Helium isotopes and Deutirium has led to the wide acceptance of the standard cosmology. The standard cosmology is spectacularly successful. It provides reliable and tested account of the history of the universe from about 0.01 sec after the big bang until today, some 15 billions years later. Thus in short, the primary pieces of evidence that support the standard model of cosmology are the expansion of the universe, the cosmic microwave background radiation and abundance of the light elements.

Despite its success, the hot big bang model leaves many features of the universe unexplained. The most important of these are horizon problem, singularity problem, flatness problem, homogeneity problem, structure formation problem, monopole problem and so on. All these problems are very difficult and defy solutions within the standard cosmology theory. Most of these problems have, in the past couple of decades, been either completely resolved or considerably relaxed in the context of one complete scenario, called the inflationary scenario [7]. At present there are several versions [7, 8, 9] of the inflationary model, called the inflationary paradigm. According to the simplest version of the inflationary scenario [10], the universe in the past expanded almost exponentially with time, while its energy density was dominated by the effective potential energy density of a homogeneous scalar field, called the inflaton. Just after the inflationary period the universe was devoid of particles, that is, cold. Therefore a mechanism is needed to understand that how the universe reheated after the inflation because the temperature of universe was insufficient for the nucleosynthesis and moreover the universe was devoid of particles.

It is believed that at the end of the inflation, the inflaton field started quasi-periodic motion with slowly decreasing amplitude and that led to creation of particles of various kinds, after thermalization of which due to collisions and decays, the universe became hot again. Thus, the temperature of the universe raised to the extent that it was sufficient to trigger thermonuclear reactions. From then on, it can be described by the usual hot big bang theory. Therefore the oscillatory phase of the inflaton and its related issues are important to understand particle creation and further evolution of the universe.

Most of the inflationary scenario and related issues are described on the basis of classical Friedmann equations with scalar field on the Friedmann-Robertson-Walker (FRW) universe, assuming its validity even at very early stage of the universe. However, it is believed that quantum effects of matter fields and quantum fluctuations played significant role in the early universe, though quantum gravity effects were negligible. Therefore, the proper description of a cosmological model can be studied in terms of the semiclassical gravity of the Friedmann equations with quantized matter fields as source for gravity. Recently, study of inflation in the semiclassical theory of gravity has received much attention [11, 12]. There are works in which quantum properties of the inflaton were investigated in the inflationary scenario. In the new inflation scenario quantum effects of the inflaton were partially taken into account by using one-loop effective potential with an initial thermal condition [13]. In the stochastic inflationary [14] scenario the inflaton is studied quantum mechanically by dealing with the phase-space quantum distribution function and the probability distribution [15]. The semiclassical quantum gravity seems to be a viable method throughout the whole non-equilibrium quantum process from the pre-inflation period of hot plasma in thermal equilibrium to the inflation period and finally to the matter dominated era.

The studies aforementioned show that results obtained in classical gravity are quite different from those in semiclassical gravity. Even though both classical and quantum inflaton in the oscillatory phase leads to the same power law expansion, the correction to expansion does not show any oscillatory behaviour in semiclassical gravity in contrast with the oscillatory behaviour seen in classical gravity. It is to be noted that coherently oscillating inflaton suffers from particle production. Such studies revealed that quantum effects played role in the inflation and its related phenomenon. Recently, it has been found that nonclassical state formalisms of quantum optics, such as coherent and squeezed sates, are quite useful to deal with quantum effects in cosmology [16] - [27]. At a glance, it may appear that coherent and squeezed state formalisms and cosmology are two different branches of Physics having no connections. However, the mathematical and physical properties of these states find much use to deal many quantum issues in cosmology. The squeezed and coherent states are being used as probes for studying quantum effects in cosmology such as cosmological particle creation [17], inflationary scenario [23], entropy generation [28], detection of gravitational waves [29], etc., [30]-[32]. It is believed that the relic graviton and other primordial perturbations created from zero-point quantum fluctuations, in the process of cosmological evolution, should be now in a strongly squeezed states [17]. The squeezed vacuum states under consideration are the many particle states and hence the resulting field can be called classical, but the statistical properties of the field differ greatly from those of the corresponding coherent state and from that point of view, the produced field is highly nonclassical. In the present work we have made use of the squeezed state formalism to calculate the expectation value of the energy momentum tensor of the scalar field. The vacuum expectation values of the energy momentum tensor defined prior to any dynamics in the gravitational field give us all the information about the particle production and vacuum polarization and hence it may be argued that the squeezed state representation of the scalar field can account for particle creation.

We examine particle production in the semiclassical Friedmann equation and obtain their solutions in the aforementioned states in the flat, open and closed FRW cosmological models. Also, we study density fluctuations and hence examine validity of the semiclassical theory in the oscillatory phase of inflaton in the coherent and squeezed states. Since, we use the quantum optical states to study the inflaton, it would be useful to study its classical or nonclassical nature in the cosmological context. In quantum optics context such study is carried out by using a quantity known as the Mandel's Q parameter [33]. The Q parameter is a measure to examine classical or nonclassical nature of the field under investigation.

In the semiclassical approach of gravity, the back reaction can play an important role in determining the dynamics of the universe [34]. Thus the back reaction also be taken into account where matter field treated as quantum mechanical, even though the background metric is classical. Many authors have studied the back reaction problem in the semiclassical gravity [35] - [37]. However, we have not included the effects of back reaction in the present work.

1.1 Motivation and Thesis Outline

At the end of inflation the universe became cold. The temperature was not sufficient to trigger thermonuclear reactions or nucleosynthesis. So, we need a mechanism to understand, how the temperature of the universe enhanced to initiate nucleosynthesis process. One of the mechanisms that reheat the universe after the inflation is due to particle production and its related phenomenon. After the inflation substantial amount of particles production occurred that helped to repopulate the universe with matter and radiation. The produced particles moved randomly and collided among themselves. Due to those collisions the temperature raised in such a way that the universe became hot again and nucleosynthesis started. Therefore, end of the inflationary period is crucial for the further evolution of the universe because the temperature attained after the reheating was not too large and not too small. Thus, study of the oscillatory phase of inflaton and its associated phenomenon are important to understand the universe.

Usually, the classical Friedmann equations with unquantized homogeneous scalar field are used to describe the inflation and its related phenomenon. Quantum effects in such model can be investigated in terms of the semiclassical Friedmann equations with the quantized inflaton. Such studies show that corresponding results are quite different from its classical counterpart. One interesting issue related to these kind of studies is particle creation due to quantum effects [38, 39]. Thus, the present work is to study the oscillatory phase of the inflaton after inflation in semiclassical theory of gravity by representing the inflaton in the coherent and squeezed state formalisms of the quantum optics. We examine, particle creation in the coherent and squeezed states, in the semiclassical theory of gravity for the flat FRW universe. We, also study density fluctuations and validity of the semiclassical theory in the quantum optical states, during the oscillatory phase of inflaton. We examine classical or nonclassical behaviour of the inflaton in the coherent and squeezed state formalisms, with a quantity called the cosmological Q parameter. Finally, we study the oscillatory phase of nonclassical inflaton and its related issues in the thermal counterparts of the coherent and squeezed states.

The primary aim of present study is to consider the flat Friedmann model of the universe in the oscillatory phase of the quantized inflaton in the semiclassical theory of gravity. However the solutions to the Friedmann equations imply the open and closed FRW universe also. Therefore it is interesting to see how the semiclassical gravity and its related phenomenon play role in determining dynamics of the open and closed FRW models. Thus, the general goal of the present work is to study a massive minimal nonclassical scalar field in the flat, open and closed FRW universe by representing it in terms of the coherent and squeezed states formalism of quantum optics as well as in their thermal counterparts.

In chapter 2, we present a brief discussion of the Einstein field equations. Scalar field in curved spacetime and the basic mechanism of particle creation for the quantized scalar field is explained briefly. The basic setup of the scalar field in the FRW models is also discussed. In the last section of the chapter 2, quantum optical states such as the coherent and squeezed state formalisms are introduced and their basic properties are also discussed briefly, in view of application of these states in the cosmological context.

Chapter 3, describes the semiclassical Friedmann equations and their solutions in the flat, open and closed FRW universe models in terms of representation of the scalar field in the coherent and squeezed state formalisms. A comparative study of the solutions in the three different models of the universe is also done.

Chapter 4, contains the study of the particle production of the nonclassical inflaton during its oscillatory phase in the flat FRW universes in the coherent and squeezed state formalisms. The chapter contains the study of particle creation in the other two Friedmann models and a comparative discussion of it with the flat FRW model is also carried out.

In chapter 5, we examine validity of the semiclassical Friedmann equation during the oscillatory phase of inflaton. This is done by computing the density fluctuations of energy momentum tensor in the semiclassical theory of gravity, in the coherent and squeezed states with the help of a dimensionless quantity. The chapter also deals with the cosmological Q parameter to examine classical or nonclassical nature of inflaton field, in the coherent and squeezed states.

Chapter 6, studies solutions of the semiclassical Friedman equation in the thermal coherent, squeezed vacuum and squeezed states in the three FRW models of the universe. We compare the solutions of the closed, open and flat FRW models of the universe in the thermal nonclassical states. We study, particle production at the end of inflation in the oscillatory regime in the semiclassical theory of gravity by using the thermal coherent and squeezed state formalisms. Density and quantum fluctuations are also studied in the thermal coherent and squeezed vacuum states. Using the density fluctuations, we examine validity of the semiclassical theory of gravity, in the flat FRW universe. A comparative study of classical gravity with semiclassical gravity in the thermal coherent state is also presented.

In chapter 7, we summarize the results.

Chapter 2

FRW Universe and Scalar Field

The Friedmann models of the universe, which are based on the Einstein's general theory of relativity and cosmological principle, have improved our understanding about the universe. In fact, the standard cosmology is considered as a reliable model and is compatible with most of the observed features of the universe. Understanding of early universe scenario underwent remarkable progress when the Friedmann models are combined with the scalar fields. It is believed that quantum fields played important roles in the dynamical behavior of the early universe [40] - [41]. The effects of quantum field in early universe have been investigated by many authors [40] - [53]. In the present chapter we provide a brief account of the FRW universe and scalar field in curved spcaetime. The basic mechanism of particle creation for a quantized scalar field in a given background is described. The chapter ends with a brief discussion of the basic properties of the coherent and squeezed state formalisms of quantum optics.

2.1 Einstein Field Equations in Cosmology

Einstein's general theory of relativity laid the foundation of modern cosmology. It gives a set of equations, which connect geometry of the spacetime with source for it, known as the Einstein's field equations. The applications of the Einstein's field equations in cosmology are based on important assumption, called the cosmological principle. The principle states that at any given cosmic time, the universe is homogeneous and isotropic. Therefore, to formulate the Einstein field equations in the standard cosmology, we need a metric which incorporate the homogeneous and isotropic properties of the universe. The Friedmann-Robertson-Walker metric is used to describe such a universe and is given by (with $c = 1$)

$$ds^2 = -dt^2 + S^2(t)\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta\, d\phi^2)\right] \tag{2.1}$$

where r, θ and ϕ are referred to as comoving coordinates. In (2.1) $S(t)$ is called the cosmic scale factor and k is the curvature parameter and can take three values $k = 0, +1$ and -1 correspondingly we get three models of the universe respectively known as flat, closed and open FRW universe. Note that in the present work small Greek indices μ and ν are running from 0,1,2, 3 and Einstein summation convention is also assumed.

The Einstein field equations in the general theory relativity can be written as [54]

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi G\, T_{\mu\nu} \tag{2.2}$$

where $R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}\, R = G_{\mu\nu}$ is called the Einstein tensor and $T_{\mu\nu}$ is known as the energy momentum tensor which depends upon the distribution of matter and energy in space. In the standard cosmology, the energy momentum tensor is usually taken as a perfect fluid description of matter and is given by

$$T_{\mu\nu} = pg_{\mu\nu} + (p+\rho)u_\mu u_\nu, \tag{2.3}$$

where p and ρ are, respectively, known as pressure and energy density of the fluid and u_μ is the four velocity. The energy momentum tensor which describes the content of the universe and its pure temporal and spatial components are given by

$$T_{00} = \rho, \tag{2.4}$$

$$T_{11} = T_{22} = T_{33} = -p. \tag{2.5}$$

The energy momentum tensor satisfy the following conservation condition

$$\partial_\mu T^\mu_\nu = 0. \tag{2.6}$$

Where ∂_μ denote a covariant derivative. Thus the nontrival equations that follow from (2.1 -2.6) are

$$\left(\frac{\dot{S}}{S}\right)^2 + \frac{k}{S^2} = \frac{8\pi G}{3}\rho, \tag{2.7}$$

and

$$2\frac{\ddot{S}}{S} + \left(\frac{\dot{S}}{S}\right)^2 + \frac{k}{S^2} = -8\pi G p. \tag{2.8}$$

Combining (2.7 and 2.8) with the Einstein equations, we can write

$$G_{\mu\nu} = 8\pi G \, T_{\mu\nu}. \tag{2.9}$$

These set of equations are called the Einstein field equations. There have been impressive advances in developing solutions to the field equations of general relativity [2]. The first solution is obtained by Schwarzschild for a spherically symmetric mass distribution. The most important solutions of Einstein field equations are the Friedmann solutions which are used to construct different cosmological models, known as the Friedmann models of the universe.

Equations (2.7) and (2.8) appear to be independent but are related. This is because the energy density and pressure satisfy the conservation equation, thus following (2.3 -2.8) we get

$$\dot{\rho} + 3\frac{\dot{S}}{S}(\rho + p) = 0. \tag{2.10}$$

The Friedmann models of the unvierse have been discussed so far without one important aspect known as the cosmological constant. The cosmological constant Λ was introduced to modify the Einstein field equations. The modified field equation with cosmological constant can be written as

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} \, R - \Lambda \, g_{\mu\nu} = 8\pi G \, T_{\mu\nu}. \tag{2.11}$$

Einstein discarded the cosmological constant but later on it has made several comebacks. Currently, the cosmological constant problem has received much attention in theory as well as observations [55]. The recent observations of TyIa supernova [56] indicate a small positive Λ term, however for the sake of simplicity, we consider the FRW models of the universe without the cosmological constant.

2.2 Quantum Field Theory in Curved Spacetime

In this section, we give a brief account of the scalar field and its quantization procedure. The physical mechanism of particle creation for the scalar field in curved spacetime is also discussed briefly. We use units $G = c = \hbar = 1$.

Let us consider the action for a real, massive scalar field ϕ, given by [40]

$$\mathcal{A} = \int \mathcal{L} \, dx^4, \tag{2.12}$$

where $\mathcal{L}$ is the Lagrangian density of the scalar field and the field obey the following wave equation

$$(\Box - m^2)\phi = 0. \tag{2.13}$$

Here $\Box$ denotes the generally covariant d'Alembertian operator. A useful concept in this context is that of the inner product of a pair of solutions of the generally covariant Klein-Gordon equation, (2.13) and is defined as

$$(f_1, f_2) = i \int (f_2^* \overleftrightarrow{\partial}_\mu f_1) \, d\Sigma^\mu, \tag{2.14}$$

where $d\Sigma^\mu = d\Sigma \, n^\mu$, with $d\Sigma$ is volume element in a given spacelike hypersurface, and n^μ is the timelike unit vector normal to this hypersurface. The crucial property of the inner product is that it is independent of the choice of hypersurface. That is, if Σ_1 and Σ_2 are two different non-intersecting hypersurfaces, then

$$(f_1, f_2)_{\Sigma_1} = (f_1, f_2)_{\Sigma_2}. \tag{2.15}$$

The scalar field that we considered so far is classical in nature. The quantization of a scalar field in a curved spacetime can be carried out by canonical quantization methods. Choose a foliation of the spacetime into spacelike hypersurfaces. Let Σ be a particular hypersurface with unit normal vector n^μ labelled by a constant value of the time coordinate t. The derivative of ϕ in the normal direction is $\dot{\phi} = n^\mu \partial_\mu \phi$, and the canonical momentum is defined as

$$\pi = \frac{\delta \mathcal{L}}{\delta \dot{\phi}} \ . \tag{2.16}$$

Next one can impose the following canonical commutation relation

$$[\phi(x,t), \pi(x',t)] = i\delta(x,x'), \tag{2.17}$$

where $\delta(x, x')$ is a delta function in the hypersurface with the property that $\int \delta(x, x')\ d\Sigma = 1$.

Let $\{f_j\}$ be a complete set of positive normal solutions of (2.13). Then $\{f_i^*\}$ will be a complete set of negative norm solutions, and $\{f_j, f_j^*\}$ form a complete set of solutions of the wave equation in terms of which we may expand an arbitrary solution. Express the field operator ϕ as a sum of annihilation and creation operators

$$\phi = \sum_j (a_j f_j + a_j^\dagger f_j^*), \tag{2.18}$$

where $[a_j, a_{j'}^\dagger] = \delta_{jj'}$. This expansion defines a vacuum state $|0>$ such that $a_j|0>$=0. In flat spacetime, we take the positive norm solutions to be positive frequency solutions, $\int_j \propto e^{-i\omega t}$. Regardless of the Lorentz frame in which t is the time coordinate, this procedure defines the same, unique Minkowski vacuum state. In curved spacetime, the situation is quite different. There is, in general, no unique choice of the $\{f_j\}$, and hence no unique notion of the vacuum state. This means that we cannot identify what constitutes a state without particle content, and the notion of particle becomes ambiguous. One possible resolution of this difficulty is to choose some quantities other than particle content to label quantum states. Possible choices might include local expectation values, such as $< \phi >, < \phi^2 >$, etc., in the particular case of an asymptotically flat spacetime, we might use the particle content in an asymptotic region. Even this characterization is not unique. However, this non-uniqueness is an essential feature of the theory with physical consequences, namely the phenomenon of particle creation, which we discuss in the next section.

2.2.1 Particle Creation in Curved Spacetime

Let us consider a spacetime which is asymptotically flat in the past and in the future, but which is non-flat in the intermediate region. Let $\{f_j\}$ be positive frequency solutions in the past (call "in-region"), and let $\{F_j\}$ be positive frequency solutions in the future (call "out-region"). We may choose these sets of solutions to be orthonormal, so that

$$\begin{aligned} (f_j, f_{j'}) &= (F_j, F_{j'}) = \delta_{jj'} \\ (f_j^*, f_{j'}^*) &= (F_j^*, F_{j'}^*) = -\delta_{jj'} \\ (f_j, f_{j'}^*) &= (F_j, F_{j'}^*) = 0. \end{aligned} \tag{2.19}$$

Though, these functions are defined by their asymptotic properties in different regions, they are solutions of the wave equation everywhere in the spacetime. We may expand the in-modes in terms of the out-modes:

$$f_j = \sum_k (\alpha_{jk} F_k + \beta_{jk} F_k^*). \tag{2.20}$$

Inserting this expansion into the orthogonality relations, (2.19), leads to the conditions

$$\sum_k (\alpha_{jk}\alpha^*_{j'k} - \beta_{jk}\beta^*_{j'k}) = \delta_{jj'}, \tag{2.21}$$

and

$$\sum_k (\alpha_{jk}\alpha_{j'k} - \beta_{jk}\beta_{j'k}) = 0. \tag{2.22}$$

The inverse expansion of (2.20) is

$$F_k = \sum_j (\alpha^*_{jk} f_j - \beta_{jk} f^*_j). \tag{2.23}$$

The field operator, ϕ, may be expanded in terms of either the $\{f_j\}$ or the $\{F_j\}$:

$$\phi = \sum_j (a_j f_j + a^\dagger_j f^*_j) = \sum_j (b_j F_j + b^\dagger_j F^*_j) \ . \tag{2.24}$$

The a_j and $a^\dagger_j$ are annihilation and creation operators, respectively, in the in-region, whereas the b_j and $b^\dagger_j$ are the corresponding operators for the out-region. The in-vacuum state is defined by $a_j|0>_{in}= 0$, $\forall\ j$, and describes the situation when no particles are present initially. The out-vacuum state is defined by $b_j|0>_{out}= 0$, $\forall\ j$, and describes the situation when no particles are present at late times. Noting that $a_j = (\phi, f_j)$ and $b_j = (\phi, F_j)$, we may expand the two sets of creation and annihilation operators in terms of one another as

$$a_j = \sum_k (\alpha^*_{jk} b_k - \beta^*_{jk} b^\dagger_k), \tag{2.25}$$

or

$$b_k = \sum_j (\alpha_{jk} a_j + \beta^*_{jk} a^\dagger_j). \tag{2.26}$$

This is known as the Bogoliubov transformation, and the α_{jk} and β_{jk} are called the Bogoliubov coefficients. Next we describe the physical phenomenon of particle creation by a time dependent gravitational field. Let us assume that no particles were present before the gravitational field is turned on. If the Heisenberg picture is adopted to describe the quantum dynamics, then $|0>_{in}$ is the state of the system for all time. However, the physical number operator which counts particles in the out-region is $N_k = b_k^\dagger b_k$. Thus the mean number of particles created into mode k is

$$\langle N_k\rangle =_{in} \langle 0|b_k^\dagger b_k|0\rangle_{in} = \sum_j |\beta_{jk}|^2. \tag{2.27}$$

If any of the β_{jk} coefficients are non-zero, that is if any mixing of positive and negative frequency solutions occurs, then it can be termed that particles created by the gravitational field.

2.3 Coherent States and Squeezed States

Coherent and squeezed states are important classes of quantum states, well known in quantum optics [57] - [61]. Coherent states are considered as most classical, that can be generated from the vacuum state $|0>$ by the action of displacement operator. In the present study, we use single mode coherent and squeezed states only. A single mode coherent state can be defined as [61]

$$|\alpha >= D(\alpha)|0> \tag{2.28}$$

where $D(\alpha)$ is the single mode displacement operator, given by

$$D(\alpha) = \exp(\alpha a^\dagger - \alpha^* a). \tag{2.29}$$

Here, α is a complex number and $a, a^\dagger$ are respectively the annihilation and creation operators, satisfying $[a, a^\dagger] = 1$.

The single mode displacement operator given by (2.29) satisfy the following properties

$$\begin{aligned} D^\dagger a D &= a + \alpha \\ D^\dagger a^\dagger D &= a^\dagger + \alpha^*. \end{aligned} \tag{2.30}$$

A squeezed state is generated by the action of the squeezing operator on any coherent state. Therefore, a single mode squeezed state is defined as

$$|\alpha, \xi >= Z(r, \vartheta) D(\alpha) |0 >, \tag{2.31}$$

with $Z(r, \vartheta)$ the single mode squeezing operator given by

$$Z(r, \vartheta) = \exp \frac{r}{2} \left(e^{-i\vartheta} a^2 - e^{i\vartheta} a^{\dagger 2} \right). \tag{2.32}$$

Here, r is the squeezing parameter, which determines the strength of squeezing and ϑ is the squeezing angle, which determines the distribution between conjugate variable, with $0 \leq r \leq \infty$ and $-\pi \leq \vartheta \leq \pi$.

The squeezing operator satisfy the following properties

$$Z^\dagger a Z = a \cosh r - a^\dagger e^{i\vartheta} \sinh r$$

$$Z^\dagger a^\dagger Z = a^\dagger \cosh r - a e^{-i\vartheta} \sinh r. \tag{2.33}$$

By setting $\alpha = 0$ in (2.31), one obtains the squeezed vacuum state, and is defined as

$$|\xi >= Z(r, \vartheta)|0 > . \tag{2.34}$$

The squeezed vacuum state is considered as many-particle state and hence the resulting field may be called classical. However, the statistical properties of these states greatly differ from the coherent states and therefore, this state is considered as highly nonclassical having no analog in classical physics. In the case of coherent states, variance of the conjugate variables are always equal to each other, while in a squeezed state one component of the noise is always squeezed with respect to the other. Therefore, in (x, p) plane, the noise for the coherent state can be described by a circle and for the squeezed state, it is an ellipse.

Chapter 3

Semiclassical Friedmann Equations

Most of the early universe models are based on the classical gravity of Friedmann equations and scalar field on the FRW metric. To study the scalar field and the Friedmann equations at a deeper level, both background metric and the field are to be treated quantum mechanically. Since a consistent quantum theory of gravity is not available at present, in most of the cosmological models the background metric is considered as classical (not quantized) and matter field as quantum. Such an approximation of the Einstein equation is known as the semiclassical Einstein equation [34]. The semiclassical approximation is also expected to be valid where the quantum gravity effects are negligible. The semiclassical theory of gravity provides a description of the gravitational field of quantum systems with the expectation value of energy momentum tensor as the source of gravity. The present chapter is devoted to the study of a minimally coupled massive inflaton for the FRW universe in the semiclassical theory of gravity by representing the inflaton in coherent state and squeezed state formalisms. The chapter begins with an introduction to the semiclassical Friedmann equation. The chapter also contains canonical quantization method of inflaton and representation scheme of it in the coherent and squeezed state formalisms. The solutions of the semiclassical Friedmann equation in the aforementioned nonclassical states are obtained in the oscillatory phase of the inflaton for the flat FRW universe. The chapter end with solutions for the open and closed FRW models in semiclassical gravity.

3.1 Inflaton in Semiclassical Gravity

In semiclassical theory of gravity the Einstein equations can be written (here onward we use $\hbar = c = 1$ and $G = \frac{1}{m_p^2}$) as

$$G_{\mu\nu} = \frac{8\pi}{m_p^2} \langle T_{\mu\nu} \rangle. \tag{3.1}$$

Here, $G_{\mu\nu}$ is the Einstein tensor and $< T_{\mu\nu} >$ is the expectation value of the energy momentum tensor for the matter field in a suitable quantum state under consideration, with the quantum state satisfying the Schrödinger equation

$$i\frac{\partial}{\partial t}|\psi >= \hat{\mathcal{H}}|\psi > . \tag{3.2}$$

Here, $|\psi >$ denotes a quantum state and $\hat{\mathcal{H}}$ is the Hamiltonian operator governing the quantum state. The semiclassical theory of gravity provides a description of the gravitational field of quantum systems with the expectation value of the energy momentum tensor as the source for gravity.

Consider the Friedmann-Robertson-Walker spacetime with line element

$$ds^2 = -dt^2 + S^2(t)\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right]. \tag{3.3}$$

Since we deal with the semiclassical Einstein equation, the metric is treated as unquantized external field. Then the Friedmann equation in the semiclassical theory can also be written in terms of the Hamiltonian of the quantized source field, as follows

$$\left(\frac{\dot{S}}{S}\right)^2 + \frac{k}{S^2} = \frac{8\pi}{3m_p^2 S^3(t)}\langle\hat{\mathcal{H}}_m\rangle \tag{3.4}$$

where $k = 0, \pm 1$ corresponds to flat, closed and open FRW universes respectively and $< \hat{\mathcal{H}}_m >$ represents the expectation value of the Hamiltonian for the quantized source field under consideration.

Now consider a minimally coupled massive homogeneous scalar field ϕ (inflaton) in a metric, then its Lagrangian density can be written as

$$\mathcal{L} = -\frac{1}{2}\sqrt{(-g)}(g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + m^2\phi^2), \tag{3.5}$$

and the equation of motion of the scalar field is governed by the Klein-Gordon equation

$$\ddot{\phi} + 3\frac{\dot{S}}{S}\dot{\phi} + \frac{dV}{d\phi} = 0. \tag{3.6}$$

Where the overdot represents a derivative with respect to time. Note that in the present study we use single mode inflaton only and hence the notation for the modes index is dropped out in the entire discussions.

Using the Lagrangian density (3.5) and the metric (3.3), the temporal component of the energy momentum tensor for the flat FRW (k=0) universe is given by

$$T_{00-flat} = \frac{1}{2}S^3(t)(\dot{\phi}^2 + m^2\phi^2). \tag{3.7}$$

Similarly the temporal component of the energy momentum tensor for the open FRW ($k = -1$) universe, with $r = \sinh\chi$ is

$$T_{00-open} = \frac{1}{2}S^3(t)\sinh^2\chi(\dot{\phi}^2 + m^2\phi^2), \tag{3.8}$$

and for the closed FRW ($k = +1$) universe, with $r = \sinh\chi$ is

$$T_{00-closed} = \frac{1}{2}\frac{S^3(t)\sinh^2\chi\cosh\chi}{\sqrt{1-\sinh^2\chi}}(\dot{\phi}^2 + m^2\phi^2). \tag{3.9}$$

The scalar field can be quantized by defining momentum conjugate to $\hat{\phi}$ as $\hat{\pi} = \frac{\partial\mathcal{L}}{\partial\dot{\phi}}$ and following the canonical quantization procedure, the Hamiltonian for the scalar field can be written for the flat, open and closed FRW universe respectively as follows

$$\hat{\mathcal{H}}_{m-flat} = \frac{1}{2S^3(t)}\hat{\phi}^2 + \frac{m^2S^3(t)}{2}\hat{\pi}^2, \tag{3.10}$$

$$\hat{\mathcal{H}}_{m-open} = \frac{1}{2S^3(t)\sinh^2\chi}\hat{\phi}^2 + \frac{m^2S^3(t)\sinh^2\chi}{2}\hat{\pi}^2, \tag{3.11}$$

and

$$\hat{\mathcal{H}}_{m-closed} = \frac{\sqrt{1-\sinh^2\chi}}{2S^3(t)\sinh^2\chi\cosh\chi}\hat{\phi}^2 + \frac{m^2S^3(t)\sinh^2\chi\cosh\chi}{2\sqrt{1-\sinh^2\chi}}\hat{\pi}^2. \tag{3.12}$$

Now we are in a position to study the inflaton in the coherent and squeezed states. Since our primary motivation is to study the oscillatory phase of inflaton, we first focus on the inflaton in the flat FRW metric.

3.2 Nonclassical States and Semiclassical Friedmann Equations

We are primarily interested in study of the oscillatory phase of inflaton in nonclassical states. Thus we consider a massive inflaton which is minimally coupled to the flat FRW metric and study its quantum features by representing the inflaton in nonclassical states. Initially, we consider the inflaton in a nonclassical state which is very close to the classical state known as the coherent state. The aim is to obtain a leading solution to the semiclassical Einstein equation in the coherent state representation of the inflaton and then extend the procedure in the case of squeezed state representation.

3.2.1 Flat FRW Universe

Consider a massive inflaton, minimally coupled to a spatially flat FRW universe with $k = 0$ for the metric (3.3). Thus the time-time component of the classical gravity is now the classical Einstein (or Friedmann) equation

$$\left(\frac{\dot{S}}{S}\right)^2 = \frac{8\pi}{3m_p^2}\frac{T_{00}}{S^3(t)} \tag{3.13}$$

where T_{00} is the energy density of the inflaton for the flat FRW metric and can be obtained from (3.5). In the cosmological context, the classical Einstein equation (3.13) means that the Hubble parameter, $H = \frac{\dot{S}}{S}$, is determined by the energy density of the dynamically evolving inflaton as described by (3.6).

The semiclassical Friedmann equation can be written in terms of the Hamiltonian of inflaton as follows

$$\left(\frac{\dot{S}}{S}\right)^2 = \frac{8\pi}{3m_p^2}\frac{1}{S^3(t)}\langle\hat{\mathcal{H}}_m\rangle, \tag{3.14}$$

where $\langle\hat{\mathcal{H}}\rangle$ represent the expectation value of the Hamiltonian of the scalar field in a quantum state under consideration. Thus for the semiclassical Friedmann equation (3.14), we have to find the expectation value of the above Hamiltonian in a given quantum state and therefore

$$\langle\hat{\mathcal{H}}_m\rangle = \frac{1}{2S^3}\langle\hat{\pi}^2\rangle + \frac{m^2S^3}{2}\langle\hat{\phi}^2\rangle. \tag{3.15}$$

Before moving into the consideration of the nonclassical states it would be convenient to discuss the semiclassical Friedmann equation in the number state. The eigenstates of the Hamiltonian are the Fock states

$$\hat{a}^\dagger(t)\hat{a}(t)|n,\phi,t>= n|n,\phi,t>, \tag{3.16}$$

where $\hat{a}$ and $\hat{a}^\dagger$ are the annihilation and creation operators obeying the boson commutation relation $[\hat{a},\hat{a}^\dagger] = 1$, the other combinations being zero. In the present context these operators can be written as follows

$$\begin{aligned}\hat{a}(t) &= \phi^*(t)\hat{\pi} - S^3\dot{\phi}^*(t)\hat{\phi}\\ \hat{a}^\dagger(t) &= \phi(t)\hat{\pi} - S^3\dot{\phi}(t)\hat{\phi}.\end{aligned} \tag{3.17}$$

From (3.17) it follows that

$$\hat{\phi} = \frac{1}{i}(\phi^*\hat{a}^\dagger - \phi\hat{a}) \tag{3.18}$$

$$\hat{\pi} = iS^3(\dot{\phi}\hat{a} - \dot{\phi}^*\hat{a}^\dagger). \tag{3.19}$$

The expectation value of the Hamiltonian (3.10) can be calculated in the number state by using (3.15), (3.16), (3.18) and (3.19) and is obtained as

$$\langle\hat{\mathcal{H}}_m\rangle = S^3\left[(n+\frac{1}{2})(\dot{\phi}^*\dot{\phi} + m^2\phi^*\phi)\right]. \tag{3.20}$$

Hence, the semiclassical Friedmann equation (3.14), in the number state, can be written as

$$\left(\frac{\dot{S}(t)}{S(t)}\right)^2 = \frac{8\pi}{3m_p^2}\left[(n+\frac{1}{2})(\dot{\phi}^*\dot{\phi}+m^2\phi^*\phi)\right]. \tag{3.21}$$

In (3.21), ϕ and ϕ^* satisfy (3.6) and the Wronskian condition

$$S^3(t)\left[\dot{\phi}^*(t)\phi(t)-\phi^*(t)\dot{\phi}(t)\right]=i. \tag{3.22}$$

The Wronskian and the boundary conditions, fix the normalization constants of the two independent solutions.

Our next task is to solve the semiclassical Einstein equation (3.20) for which we transform the solution in the following form

$$\phi(t)=\frac{1}{S^{3/2}}\varphi(t), \tag{3.23}$$

thereby obtaining

$$\ddot{\varphi}(t)+\left(m^2-\frac{3}{4}\left(\frac{\dot{S}(t)}{S(t)}\right)^2-\frac{3}{2}\frac{\ddot{S}(t)}{S(t)}\right)\varphi(t)=0. \tag{3.24}$$

Now, we focus on the oscillatory phase of the inflaton after inflation. In the parameter region satisfying the inequality

$$m^2>\frac{3\dot{S}^2}{4S^2}+\frac{3\ddot{S}}{2S}. \tag{3.25}$$

The inflaton has an oscillatory solution of the form

$$\varphi(t)=\frac{1}{\sqrt{2\omega(t)}}\exp(-i\int\omega(t)dt), \tag{3.26}$$

with

$$\omega^2(t)=m^2-\frac{3}{4}\left(\frac{\dot{S}(t)}{S(t)}\right)^2-\frac{3}{2}\frac{\ddot{S}(t)}{S(t)}+\frac{3}{4}\left(\frac{\dot{\omega}(t)}{\omega(t)}\right)^2-\frac{1}{2}\frac{\ddot{\omega}(t)}{\omega(t)}. \tag{3.27}$$

$\phi, \phi^*, \dot{\phi}, \dot{\phi}^*$ can be evaluated by using (3.23), (3.26) and are respectively obtained as

$$\phi(t) = \frac{1}{S^{3/2}\sqrt{2\omega(t)}} \exp(-i \int \omega(t)dt) \tag{3.28}$$

$$\phi^*(t) = \frac{1}{S^{3/2}\sqrt{2\omega(t)}} \exp(i \int \omega(t)dt) \tag{3.29}$$

$$\dot{\phi}(t) = \frac{\exp(-i\int w(t)dt)}{\sqrt{2w(t)}} \left(-\frac{3}{2}S^{\frac{-5}{2}}(t)\dot{S}(t) - \frac{1}{2}\frac{\dot{w}(t)}{w(t)}S^{\frac{-3}{2}}(t) - iw(t)S^{\frac{-3}{2}}(t)\right) \tag{3.30}$$

$$\dot{\phi}^*(t) = \frac{\exp(i\int w(t)dt)}{\sqrt{2w(t)}} \left(-\frac{3}{2}S^{\frac{-5}{2}}(t)\dot{S}(t) - \frac{1}{2}\frac{\dot{w}(t)}{w(t)}S^{\frac{-3}{2}}(t) + iw(t)S^{\frac{-3}{2}}(t)\right). \tag{3.31}$$

Substituting (3.28 - 3.31) in (3.21) we get

$$\left(\frac{\dot{S}(t)}{S(t)}\right)^2 = \left[\frac{4\pi}{3m_p^2}\left(n+\frac{1}{2}\right)\frac{1}{S^3}\frac{1}{w(t)}\left(w^2(t) + m^2 + \frac{1}{4}\left(\frac{\dot{w}(t)}{w(t)} + \frac{3\dot{S}(t)}{S(t)}\right)^2\right)\right]. \tag{3.32}$$

To solve for the scale factor, rewrite (3.32) as follows

$$S^3(t) = \frac{4\pi}{3m_p^2}\left(n+\frac{1}{2}\right)\frac{1}{w(t)\left(\frac{\dot{S}(t)}{S(t)}\right)^2}\left(w^2(t) + m^2 + \frac{1}{4}\left(\frac{\dot{w}(t)}{w(t)} + \frac{3\dot{S}(t)}{S(t)}\right)^2\right). \tag{3.33}$$

Therefore

$$S(t) = \left[\frac{4\pi}{3m_p^2}\left(n+\frac{1}{2}\right)\frac{1}{w(t)\left(\frac{\dot{S}(t)}{S(t)}\right)^2}\left(w^2(t) + m^2 + \frac{1}{4}\left(\frac{\dot{w}(t)}{w(t)} + \frac{3\dot{S}(t)}{S(t)}\right)^2\right)\right]^{\frac{1}{3}}. \tag{3.34}$$

We solve (3.34) using the following approximation ansatzs

$$w_0(t) = m \tag{3.35}$$

and

$$S_0(t) = S_0 t^{2/3}. \tag{3.36}$$

Thus the next order perturbative solution is obtained in the number state as

$$S_1(t) = \left[\frac{6\pi}{m_p^2}\left(n+\frac{1}{2}\right)mt^2\left(1+\frac{1}{2m^2t^2}\right)\right]^{1/3}. \tag{3.37}$$

As an alternative to the n representation of the inflaton, we next consider the inflaton in the coherent and squeezed state formalisms and hence the semiclassical Einstein equation can be expressed in terms of their respective parameters. First, we consider the coherent state case and then procedure can be extended to the squeezed states.

From (3.15), (2.30), (3.18) and (3.19) we get

$$\langle\hat{\pi}^2\rangle_{cs} = S^6\left[2\left(|\alpha|^2+\frac{1}{2}\right)\dot{\phi}^*\dot{\phi} - \alpha^{*2}\dot{\phi}^{*2} - \alpha^2\dot{\phi}^2\right], \tag{3.38}$$

$$\langle\hat{\phi}^2\rangle_{cs} = 2\left(|\alpha|^2+\frac{1}{2}\right)\phi^*\phi - \alpha^{*2}\phi^{*2} - \alpha^2\phi^2. \tag{3.39}$$

Substituting (3.38) and (3.39) in (3.15), the expectation value of the Hamiltonian obtained for the coherent state as,

$$< \hat{\mathcal{H}}_m >_{cs} = S^3(t)\left[\left(|\alpha|^2+\frac{1}{2}\right)(\dot{\phi}^*\dot{\phi}+m^2\phi^*\phi) - \frac{1}{2}\alpha^{*2}\left[\dot{\phi}^{*2}+m^2\phi^{*2}\right] - \frac{1}{2}\alpha^2(\dot{\phi}^2+m^2\phi^2)\right]. \tag{3.40}$$

Thus using (3.40) in (3.14), the semiclassical Friedmann equation can be written for the coherent state as

$$\left(\frac{\dot{S}(t)}{S(t)}\right)^2 = \frac{8\pi}{3m_p^2}\left[\left(|\alpha|^2+\frac{1}{2}\right)(\dot{\phi}^*\dot{\phi}+m^2\phi^*\phi) - \frac{1}{2}\alpha^{*2}\left[\dot{\phi}^{*2}+m^2\phi^{*2}\right] - \frac{1}{2}\alpha^2(\dot{\phi}^2+m^2\phi^2)\right]. \tag{3.41}$$

Similarly the perturbative solution for the coherent state can be computed by substituting (3.28) - (3.31) in (3.41) and is obtained as

$$\left(\frac{\dot{S}(t)}{S(t)}\right)^2 = \frac{1}{S^3(t)}\frac{8\pi}{3m_p^2}\left[\frac{\left(|\alpha|^2+\frac{1}{2}\right)\left(A+w^2(t)+m^2\right)}{2w(t)}\right.$$
$$-\frac{1}{2}\frac{\alpha^{*2}\exp(2i\int w(t)dt)}{2w(t)}$$
$$\times\left(A-w^2(t)-iw(t)\left[\frac{3\dot{S}(t)}{S(t)}+\frac{\dot{w}(t)}{w(t)}\right]+m^2\right)$$
$$-\frac{1}{2}\frac{\alpha^{2}\exp(-2i\int w(t)dt)}{2w(t)}$$
$$\left.\times\left(A-w^2(t)+iw(t)\left[\frac{3\dot{S}(t)}{S(t)}+\frac{\dot{w}(t)}{w(t)}\right]+m^2\right)\right] \tag{3.42}$$

where

$$A = \frac{1}{4}\left(\frac{\dot{w(t)}}{w(t)}+\frac{3\dot{S(t)}}{S(t)}\right)^2. \tag{3.43}$$

In order to solve, we rewrite (3.42) as

$$S^3(t) = \frac{8\pi}{3m_p^2\left(\frac{\dot{S}(t)}{S(t)}\right)^2}\left[\frac{\left(|\alpha|^2+\frac{1}{2}\right)\left(A+w^2(t)+m^2\right)}{2w(t)}\right.$$
$$-\frac{1}{2}\frac{\alpha^{*2}\exp(2i\int w(t)dt)}{2w(t)}\left(A-w^2(t)-iw(t)\left[\frac{3\dot{S}(t)}{S(t)}+\frac{\dot{w}(t)}{w(t)}\right]+m^2\right)$$
$$\left.-\frac{1}{2}\frac{\alpha^{2}\exp(-2i\int w(t)dt)}{2w(t)}\left(A-w^2(t)+iw(t)\left[\frac{3\dot{S}(t)}{S(t)}+\frac{\dot{w}(t)}{w(t)}\right]+m^2\right)\right]. \tag{3.44}$$

Therefore the scale factor in the coherent state is obtained as

$$S(t) = \left[\frac{8\pi}{3m_p^2\left(\frac{\dot{S}(t)}{S(t)}\right)^2}\left[\frac{\left(|\alpha|^2+\frac{1}{2}\right)\left(A+w^2(t)+m^2\right)}{2w(t)}\right]\right.$$
$$-\frac{1}{2}\frac{\alpha^{*2}\exp(2i\int w(t)dt)}{2w(t)}\left(A-w^2(t)-iw(t)\left[\frac{3\dot{S}(t)}{S(t)}+\frac{\dot{w}(t)}{w(t)}\right]+m^2\right)$$
$$\left.-\frac{1}{2}\frac{\alpha^{2}\exp(-2i\int w(t)dt)}{2w(t)}\left(A-w^2(t)+iw(t)\left[\frac{3\dot{S}(t)}{S(t)}+\frac{\dot{w}(t)}{w(t)}\right]+m^2\right)\right]^{\frac{1}{3}}. \tag{3.45}$$

Applying the approximation ansatzs (3.35) and (3.36) in (3.45), the next order perturbative solution for the coherent state is obtained as

$$S_1(t) = \left[\frac{6\pi t^2}{3m_p^2}\left[\frac{\left(|\alpha|^2+\frac{1}{2}\right)\left(\frac{1}{t^2}+2m^2\right)}{2m} - \frac{1}{2}\frac{\alpha^{*2}\exp(2i\int mdt)}{2m}\left(\frac{1}{t^2}-\frac{2im}{t}\right) - \frac{1}{2}\frac{\alpha^2\exp(-2i\int mdt)}{2m}\left(\frac{1}{t^2}+\frac{2im}{t}\right)\right]\right]^{1/3}. \tag{3.46}$$

Using

$$\alpha = e^{i\theta}\alpha, \tag{3.47}$$

here θ is the phase factor associated with the coherent state, we get

$$S_1(t)_{cs} = \left[\frac{6\pi}{m_p^2}\left(|\alpha|^2+\frac{1}{2}\right)mt^2\left(1+\frac{1}{2m^2t^2}\right) - \frac{3\alpha^2}{m}\frac{\pi t^2}{m_p^2} \times\left[\left(\frac{\cos 2(\theta-mt)}{t^2}\right) - \frac{2m}{t}\sin 2(\theta-mt)\right]\right]^{1/3}. \tag{3.48}$$

Similarly, the expectation values of $\hat{\pi}^2$ and $\hat{\phi}^2$ can be computed in the squeezed vacuum state, by using (3.18), (3.19), (2.33) and (2.34) are obtained as

$$<\hat{\pi}^2>_{svs} = S^6\left[\left(2\sinh^2 r+1\right)\dot{\phi}^*\dot{\phi} + \dot{\phi}^{*2}(e^{-i\vartheta}\sinh r\cosh r) + \dot{\phi}^2(e^{i\vartheta}\sinh r\cosh r)\right], \tag{3.49}$$

$$<\hat{\phi}^2>_{svs} = \left(2\sinh^2 r+1\right)\phi^*\phi + \phi^{*2}(e^{-i\vartheta}\sinh r\cosh r) + \phi^2(e^{i\vartheta}\sinh r\cosh r). \tag{3.50}$$

Substituting (3.49) and (3.50) in (3.15), the expectation value of the Hamiltonian for the squeezed vacuum state is

$$<\hat{\mathcal{H}}_m>_{svs} = S^3\left[\left(\sinh^2 r+\frac{1}{2}\right)\left(\dot{\phi}^*\dot{\phi}+m^2\phi^*\phi\right) + \frac{\sinh 2r}{4}(e^{-i\vartheta}[\dot{\phi}^{*2}+m^2\phi^{*2}] + e^{i\vartheta}[\dot{\phi}^2+m^2\phi^2])\right]. \tag{3.51}$$

Hence, the semiclassical Einstein equation for the squeezed vacuum state, yields

$$\left(\frac{\dot{S}(t)}{S(t)}\right)^2 = \frac{8\pi}{3m_p^2}\left[(\sinh^2 r + \frac{1}{2})(\dot{\phi}^*\dot{\phi} + m^2\phi^*\phi) + \frac{\sinh 2r}{4}\left[e^{-i\vartheta}(\dot{\phi}^{*2} + m^2\phi^{*2}) + e^{i\vartheta}(\dot{\phi}^2 + m^2\phi^2)\right]\right]. \tag{3.52}$$

To obtain the solution for the semiclassical Einstein equation in the squeezed vacuum state, we substitute (3.28) - (3.31) in (3.52), thus

$$\begin{aligned}\left(\frac{\dot{S}(t)}{S(t)}\right)^2 = \frac{1}{S^3(t)}\frac{8\pi}{3m_p^2}\Bigg[&\frac{\left(\sinh r + \frac{1}{2}\right)\left(A + w^2(t) + m^2\right)}{2w(t)} \\ &+ \frac{\sinh 2r}{4}\Bigg[e^{-i\vartheta}\frac{\exp(2i\int w(t)dt)}{2w(t)} \\ &\times\left(A - w^2(t) - iw(t)\left[\frac{3\dot{S}(t)}{S(t)} + \frac{\dot{w}(t)}{w(t)}\right] + m^2\right) \\ &+ e^{i\vartheta}\frac{\exp(-2i\int w(t)dt)}{2w(t)} \\ &\times\left(A - w^2(t) + iw(t)\left[\frac{3\dot{S}(t)}{S(t)} + \frac{\dot{w}(t)}{w(t)}\right] + m^2\right)\Bigg]\Bigg].\end{aligned} \tag{3.53}$$

The equation (3.53) can be rewritten as

$$\begin{aligned}S^3(t) = \frac{8\pi}{3m_p^2\left(\frac{\dot{S}(t)}{S(t)}\right)^2}&\Bigg[\frac{\left(\sinh^2 r + \frac{1}{2}\right)\left(A + w^2(t) + m^2\right)}{2w(t)} \\ &+ \frac{\sinh 2r}{4}\Bigg[e^{-i\vartheta}\frac{\exp(2i\int w(t)dt)}{2w(t)} \\ &\times\left(A - w^2(t) - iw(t)\left[\frac{3\dot{S}(t)}{S(t)} + \frac{\dot{w}(t)}{w(t)}\right] + m^2\right) \\ &+ e^{i\vartheta}\frac{\exp(-2i\int w(t)dt)}{2w(t)}\left(A - w^2(t) + iw(t)\left[\frac{3\dot{S}(t)}{S(t)} + \frac{\dot{w}(t)}{w(t)}\right] + m^2\right)\Bigg]\Bigg].\end{aligned} \tag{3.54}$$

Hence, it follows that

$$S(t) = \left[\frac{8\pi}{3m_p^2 \left(\frac{\dot{S}(t)}{S(t)}\right)^2} \left[\frac{\left(\sinh^2 r + \frac{1}{2}\right)\left(A + w^2(t) + m^2\right)}{2w(t)} \right] \right.$$
$$+ \frac{\sinh 2r}{4} \left[e^{-i\vartheta} \frac{\exp(2i \int w(t)dt)}{2w(t)} \right.$$
$$\times \left(A - w^2(t) - iw(t) \left[\frac{3\dot{S}(t)}{S(t)} + \frac{\dot{w}(t)}{w(t)} \right] + m^2 \right)$$
$$\left. \left. + e^{i\vartheta} \frac{\exp(-2i \int w(t)dt)}{2w(t)} \left(A - w^2(t) + iw(t) \left[\frac{3\dot{S}(t)}{S(t)} + \frac{\dot{w}(t)}{w(t)} \right] + m^2 \right) \right] \right]^{\frac{1}{3}}. \tag{3.55}$$

Using the approximation ansatzs in (3.55), the next order perturbative solution for the squeezed vacuum state becomes

$$S_1(t)_{svs} = \left[\frac{6\pi}{m_p^2} \left(\sinh^2 r + \frac{1}{2} \right) mt^2 \left(1 + \frac{1}{2m^2t^2} \right) \right.$$
$$\left. + \frac{6\pi t^2}{m_p^2} \frac{\sinh 2r}{4} \left[\frac{\cos(\vartheta - 2mt)}{mt^2} - \frac{2}{t} \sin(\vartheta - 2mt) \right] \right]^{\frac{1}{3}}. \tag{3.56}$$

Analogously, for a squeezed state the expectation values of $\hat{\phi}^2$ and $\hat{\pi}^2$ are obtained as

$$\langle \hat{\pi}^2 \rangle_{ss} = S^6(t) \left[(2\sinh^2 r + 1 + 2|\alpha|^2) \dot{\phi}^* \dot{\phi} \right.$$
$$\left. + \dot{\phi}^{*2} (e^{-i\vartheta} \sinh r \cosh r - \alpha^{*2}) + \dot{\phi}^2 (e^{i\vartheta} \sinh r \cosh r - \alpha^2) \right], \tag{3.57}$$
$$\langle \hat{\phi}^2 \rangle_{ss} = (2\sinh^2 r + 1 + 2|\alpha|^2) \phi^* \phi + \phi^{*2} (e^{-i\vartheta} \sinh r \cosh r - \alpha^{*2})$$
$$+ \phi^2 (e^{i\vartheta} \sinh r \cosh r - \alpha^2). \tag{3.58}$$

Thus using the above expressions in (3.15) , we get

$$< \hat{\mathcal{H}}_m >_{ss} = S^3(t) \left[(\sinh^2 r + \frac{1}{2} + |\alpha|^2)(\dot{\phi}^* \dot{\phi} + m^2 \phi^* \phi) \right.$$
$$+ \left(\frac{e^{-i\vartheta} \sinh r \cosh r - \alpha^{*2}}{2} \right) (\dot{\phi}^{*2} + m^2 \phi^{*2})$$
$$\left. + \left(\frac{e^{i\vartheta} \sinh r \cosh r - \alpha^2}{2} \right) (\dot{\phi}^2 + m^2 \phi^2) \right]. \tag{3.59}$$

Hence

$$\left(\frac{\dot{S}(t)}{S(t)}\right)^2 = \frac{8\pi}{3m_p^2}\Bigg[(\sinh^2 r + \frac{1}{2} + |\alpha|^2)(\dot{\phi}^*\dot{\phi} + m^2\phi^*\phi) + \left(\frac{e^{-i\vartheta}\sinh r\cosh r - \alpha^{*2}}{2}\right)(\dot{\phi}^{*2} + m^2\phi^{*2}) + \left(\frac{e^{i\vartheta}\sinh r\cosh r - \alpha^{2}}{2}\right)(\dot{\phi}^{2} + m^2\phi^{2})\Bigg]. \tag{3.60}$$

Similarly substituting (3.28) - (3.31) in (3.60), we get

$$\left(\frac{\dot{S}(t)}{S(t)}\right)^2 = \frac{1}{S^3(t)}\frac{8\pi}{3m_p^2}\Bigg[\frac{\left(\sinh^2 r + \frac{1}{2} + |\alpha|^2\right)\left(A + w^2(t) + m^2\right)}{2w(t)} + \left(\frac{e^{-i\vartheta}\sinh r\cosh r - \alpha^{*2}}{2}\right)\frac{\exp(2i\int w(t)dt)}{2w(t)} \times \left(A - w^2(t) - i\omega(t)\left[\frac{3\dot{S}(t)}{S(t)} + \frac{\dot{w}(t)}{w(t)}\right] + m^2\right) + \left(\frac{e^{i\vartheta}\sinh r\cosh r - \alpha^{2}}{2}\right)\frac{\exp(-2i\int w(t)dt)}{2w(t)} \times \left(A - w^2(t) + iw(t)\left[\frac{3\dot{S}(t)}{S(t)} + \frac{\dot{w}(t)}{w(t)}\right] + m^2\right)\Bigg]. \tag{3.61}$$

Therefore

$$S^3(t) = \frac{8\pi}{3m_p^2\left(\frac{\dot{S}(t)}{S(t)}\right)^2}\Bigg[\frac{\left(\sinh^2 r + \frac{1}{2} + |\alpha|^2\right)\left(A + w^2(t) + m^2\right)}{2w(t)} + \left(\frac{e^{-i\vartheta}\sinh r\cosh r - \alpha^{*2}}{2}\right)\frac{\exp(2i\int w(t)dt)}{2w(t)} \times \left(A - w^2(t) - iw(t)\left[\frac{3\dot{S}(t)}{S(t)} + \frac{\dot{w}(t)}{w(t)}\right] + m^2\right) + \left(\frac{e^{i\vartheta}\sinh r\cosh r - \alpha^{2}}{2}\right)\frac{\exp(-2i\int w(t)dt)}{2w(t)} \times \left(A - w^2(t) + iw(t)\left[\frac{3\dot{S}(t)}{S(t)} + \frac{\dot{w}(t)}{w(t)}\right] + m^2\right)\Bigg]\Bigg], \tag{3.62}$$

and hence,

$$S(t) = \left[\frac{8\pi}{3m_p^2 \left(\frac{\dot{S}(t)}{S(t)}\right)^2} \left[\frac{\left(\sinh^2 r + \frac{1}{2} + |\alpha|^2\right)(A + w^2(t) + m^2)}{2w(t)} \right.\right.$$
$$+ \left(\frac{e^{-i\vartheta}\sinh r \cosh r - \alpha^{*2}}{2}\right) \frac{\exp(2i\int w(t)dt)}{2w(t)}$$
$$\times \left(A - w^2(t) - iw(t)\left[\frac{3\dot{S}(t)}{S(t)} + \frac{\dot{w}(t)}{w(t)}\right] + m^2\right)$$
$$+ \left(\frac{e^{i\vartheta}\sinh r \cosh r - \alpha^2}{2}\right) \frac{\exp(-2i\int w(t)dt)}{2w(t)}$$
$$\left.\left. \times \left(A - w^2(t) + iw(t)\left[\frac{3\dot{S}(t)}{S(t)} + \frac{\dot{w}(t)}{w(t)}\right] + m^2\right)\right]\right]^{1/3}. \tag{3.63}$$

We get the next order perturbative solution for the squeezed state as

$$S_1(t)_{ss} = \left[\frac{6\pi}{m_p^2}\left(\sinh^2 r + \frac{1}{2} + |\alpha|^2\right) mt^2 \left(1 + \frac{1}{2m^2t^2}\right)\right.$$
$$+ \frac{6\pi t^2}{m_p^2}\frac{\sinh 2r}{4}\left[\frac{\cos(\vartheta - 2mt)}{2mt^2} - \frac{2}{t}\sin(\vartheta - 2mt)\right]$$
$$\left. - \frac{3\alpha^2}{m}\frac{\pi t^2}{m_p^2}\left[\frac{\cos 2(\theta - mt)}{t^2} - \frac{2m}{t}\sin 2(\theta - mt)\right]\right]^{1/3}. \tag{3.64}$$

From (3.48), (3.56) and (3.64), it follows that

$$S_1(t)_{cs} \sim t^{2/3}, \quad S_1(t)_{svs} \sim t^{2/3} \quad \text{and} \quad S_1(t)_{ss} \sim t^{2/3}.$$

Therefore, approximate leading solution to the semiclassical Einstein equation, in the oscillating phase of inflaton in the coherent, squeezed vacuum and squeezed states, leads to the same power law expansion $t^{2/3}$. We next solve the Friedmann equation in the coherent, squeezed vacuum and squeezed states for the open and closed FRW models for the nonclassical inflaton, in the semiclassical theory of gravity.

3.2.2 Open FRW Universe

The semiclassical Friedmann equation for the open FRW universe can be written as

$$\left(\frac{\dot{S}}{S}\right)^2 - \frac{1}{S^2} = \frac{8\pi}{3m_p^2}\frac{1}{S^3(t)\sinh^2\chi} < \hat{\mathcal{H}}_{m-open} > . \tag{3.65}$$

Where $< \hat{\mathcal{H}}_{m-open} >$ is expectation value of the Hamiltonian of the inflaton (3.11) in a quantum state under the consideration. Let the eigenstates of the Hamiltonian are the Fock states, then

$$\hat{a}^\dagger(t)\hat{a}(t)|n,\phi,t> = \hat{n}|n,\phi,t>, \tag{3.66}$$

where $\hat{a}$ and $\hat{a}^\dagger$ are annihilation and creation operators obeying boson commutation relation $[\hat{a},\hat{a}^\dagger]$=1 and other are being zero. In the present context these operators can be expressed respectively as

$$\begin{aligned} \hat{a}(t) &= \phi^*(t)\hat{\pi} - S^3(t)\sinh^2\chi\dot{\phi}^*(t)\hat{\phi} \\ \hat{a}^\dagger(t) &= \phi(t)\hat{\pi} - S^3(t)\sinh^2\chi\dot{\phi}(t)\hat{\phi}. \end{aligned} \tag{3.67}$$

Instead of the n representation of the quantized inflaton, we consider it in the coherent state formalism. Thus the semiclassical Einstein equation can be written in terms of the coherent state parameters. By using the definition and properties of the coherent states (2.28 - 2.30) the expectation values of $\hat{\pi}^2$ and $\hat{\phi}^2$ are obtained as

$$< \hat{\pi}^2 >_{cs} = S^6\sinh^4\chi[(2|\alpha|^2+1)\dot{\phi}^*\dot{\phi} - \alpha^{*2}\dot{\phi}^{*2} - \alpha^2\dot{\phi}^2] \tag{3.68}$$

$$< \hat{\phi}^2 >_{cs} = \left(2|\alpha|^2+1\right)\phi^*\phi - \alpha^{*2}\phi^{*2} - \alpha^2\phi^2. \tag{3.69}$$

Using (3.68) and (3.69) the expectation value of Hamiltonian (3.11) in the open FRW universe becomes

$$\begin{aligned} < \hat{\mathcal{H}}_{m-open} >_{cs} = {} & S^3(t)\sinh^2\chi\Big[(|\alpha|^2+\frac{1}{2})(\dot{\phi}^*\dot{\phi} + m^2\phi^*\phi) \\ & -\frac{1}{2}\alpha^{*2}(\dot{\phi}^{*2} + m^2\phi^{*2}) - \frac{1}{2}\alpha^2(\dot{\phi}^2 + m^2\phi^2)\Big]. \end{aligned} \tag{3.70}$$

Similarly, using the properties of the squeezed vacuum state (2.33 - 2.34) and squeezed state (2.31 - 2.33) the expectation values of the corresponding Hamiltonian are respectively obtained as

$$\begin{aligned} < \hat{\mathcal{H}}_{m-open} >_{svs} &= S^3(t)\sinh^2\chi\left[\left(\sinh^2 r + \frac{1}{2}\right)(\dot{\phi}^*(t_0)\dot{\phi}(t_0) \right. \\ &\quad + m^2\phi^*(t_0)\phi(t_0)) + \frac{\sinh 2r}{4} \\ &\quad \left. \times \left(e^{-i\vartheta}(\dot{\phi}^{*2} + m^2\phi^{*2}) + e^{i\vartheta}(\dot{\phi}^2 + m^2\phi^2)\right)\right], \quad (3.71) \end{aligned}$$

and

$$\begin{aligned} < \hat{\mathcal{H}}_{m-open} >_{ss} &= S^3(t)\sinh^2\chi\left[(\sinh^2 r + \frac{1}{2} + |\alpha|^2)(\dot{\phi}^*\dot{\phi} + m^2\phi^*\phi) \right. \\ &\quad + \left(\frac{e^{-i\vartheta}\sinh r\cosh r - \alpha^{*2}}{2}\right)(\dot{\phi}^{*2} + m^2\phi^{*2}) \\ &\quad \left. + \left(\frac{e^{i\vartheta}\sinh r\cosh r - \alpha^2}{2}\right)(\dot{\phi}^2 + m^2\phi^2)\right]. \quad (3.72) \end{aligned}$$

Using the expectation value of the Hamiltonian (3.11), in the coherent state, in (3.65) we get

$$\begin{aligned} \left(\frac{\dot{S}}{S}\right)^2 - \frac{1}{S^2} &= \frac{8\pi}{3m_p^2}\left[\left(|\alpha|^2 + \frac{1}{2}\right)\left(\dot{\phi}^*\dot{\phi} + m^2\phi^*\phi\right) \right. \\ &\quad \left. -\frac{1}{2}\alpha^{*2}\left(\dot{\phi}^{*2} + m^2\phi^{*2}\right) - \frac{1}{2}\alpha^2\left(\dot{\phi}^2 + m^2\phi^2\right)\right]. \quad (3.73) \end{aligned}$$

In (3.73) ϕ and ϕ^* are satisfing the corresponding Klein-Gordon equation and the Wronskian condition

$$S^3(t)\sinh^2\chi\left[\dot{\phi}^*(t)\phi(t) - \phi^*(t)\dot{\phi}(t)\right] = i. \quad (3.74)$$

The Wronskian and boundary conditions, fix the normalization constants of two independent solutions. Transform the solution in the following form

$$\phi(t) = \frac{1}{S^{3/2}(t)\sinh\chi}\varphi(t), \quad (3.75)$$

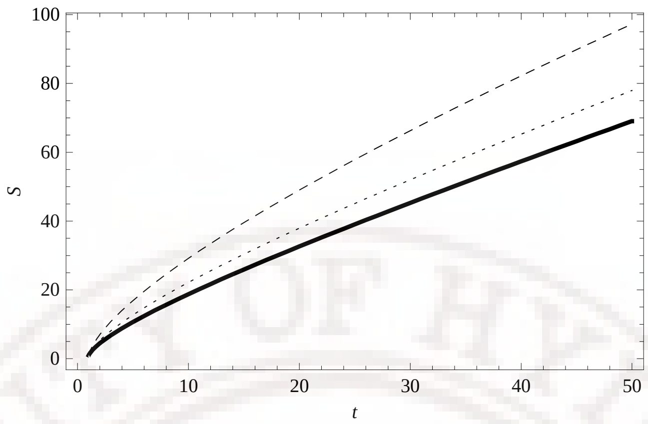

Figure 3.1: Plot for scale factor S and time t for open FRW model in coherent state with different values of coherent state parameter, thick line (α=.25), the dot line (α=.75) and the dash line (α=1.5).

where $\varphi(t)$ is given by (3.26) and substituting the values of $\phi, \dot{\phi}, \dot{\phi}^*, \phi^*$ etc subjected to the Wronskian condition (3.74) and applying the approximation ansatzs, we get

$$S(\dot{S}^2 - 1) = \frac{8\pi}{3m_p^2 \sinh^2 \chi} \left[(\alpha^2 + \frac{1}{2})(\frac{1}{t^2} + 2m^2) - \frac{\alpha^2}{2mt^2} \right]. \quad (3.76)$$

The solutions of (3.76) are obtained numerically for different values of the coherent state parameter and are shown in Figure 3.1. Similarly by adopting same procedure, the semiclassical Friedmann equation for the open FRW universe in the squeezed vacuum state is obtained as

$$S(\dot{S}^2 - 1) = \frac{8\pi}{3m_p^2 \sinh^2 \chi} \left[(\sinh^2 r + \frac{1}{2})(\frac{1}{t^2} + 2m^2) + \frac{\sinh 2r}{4mt^2} \right]. \quad (3.77)$$

The corresponding numerical solutions for different values of the squeezing parameter are shown in Figure 3.2. Following the same procedure, the semi-classical Friedmann equation for the open FRW universe, in the squeezed

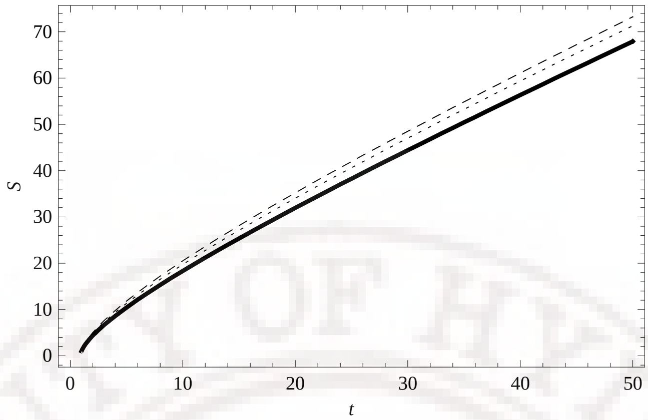

Figure 3.2: Plot for scale factor S and time t for open FRW model in squeezed vacuum state with different values of squeezing parameter, thick line (r=.10), the dot line (r=.40) and the dash line (r=.5).

state is

$$S(\dot{S}^2 - 1) = \frac{8\pi}{3m_p^2 \sinh^2 \chi} \Bigg[(\sinh^2 r + \frac{1}{2} + \alpha^2)(\frac{1}{t^2} + 2m^2) + \frac{\sinh r \cosh r - \alpha^2}{2mt^2} \Bigg] . \tag{3.78}$$

We solve the equation (3.78) numerically for different values of α and r and the results are shown in Figure 3.3.

3.2.3 Closed FRW Universe

The semiclassical Friedmann equation for the closed FRW universe can be written as

$$\left(\frac{\dot{S}}{S}\right)^2 + \frac{1}{S^2} = \frac{8\pi}{3m_p^2} \frac{\sqrt{1 - \sinh^2 \chi}}{S^3(t) \sinh^2 \chi \cosh \chi} < \hat{\mathcal{H}}_{m-closed} > . \tag{3.79}$$

Where $< \hat{\mathcal{H}}_{m-closed} >$ is the expectation value of the Hamiltonian of the inflaton (3.12), in a quantum state under the investigation. Let the eigen states of the Hamiltonian are the Fock states, then

$$\hat{a}^\dagger(t)\hat{a}(t)|n, \phi, t >= \hat{n}|n, \phi, t > . \tag{3.80}$$

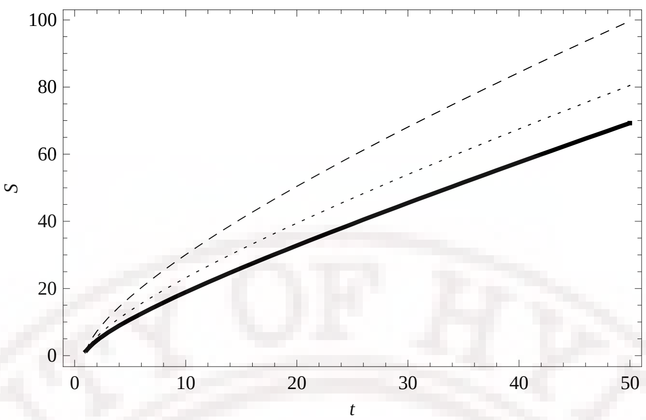

Figure 3.3: Plot for scale factor S and time t for open FRW model in squeezed state with different values of coherent and squeezing parameters,thick line (α=.25,r=.10), the dot line (α=.75,r=.40) and the dash line (α=1.5,r=.5).

where $\hat{a}$ and $\hat{a}^\dagger$ are annihilation and creation operators obeying boson commutation relation $[\hat{a}, \hat{a}^\dagger] = 1$ and the other being zero. The annihilation and creation operators in the closed FRW universe can be expressed as

$$\hat{a}(t) = \phi^*(t)\hat{\pi} - \frac{S^3(t)\sinh^2\chi\cosh\chi}{\sqrt{1-\sinh^2\chi}}\dot{\phi}^*(t)\hat{\phi}$$
$$\hat{a}^\dagger(t) = \phi(t)\hat{\pi} - \frac{S^3(t)\sinh^2\chi\cosh\chi}{\sqrt{1-\sinh^2\chi}}\dot{\phi}(t)\hat{\phi}. \tag{3.81}$$

As an alternative to the n representation the semiclassical Einstein equation can be written in terms of the coherent state in the closed FRW universe. By using the definition and properties of the coherent state (2.28 - 2.30) the expectation values of $\hat{\pi}^2$ and $\hat{\phi}^2$ are obtained as

$$< \hat{\pi} >^2_{cs-closed} = \frac{S^6\sinh^4\chi\cosh^2\chi}{(1-\sinh^2\chi)}\left[(2|\alpha|^2+1)\dot{\phi}^*\dot{\phi} - \alpha^{*2}\dot{\phi}^{*2} - \alpha^2\dot{\phi}^2\right] \tag{3.82}$$
$$< \hat{\phi} >^2_{cs-closed} = (2|\alpha|^2+1)\phi^*\phi - \alpha^{*2}\phi^{*2} - \alpha^2\phi^2. \tag{3.83}$$

The expectation value of the closed FRW unvierse in the coherent state given by

$$\langle \hat{H}_{m-closed}\rangle_{cs} = \frac{S^3(t)\sinh^2\chi\cosh\chi}{\sqrt{1-\sinh^2\chi}}\left[\left(|\alpha|^2+\frac{1}{2}\right)(\dot{\phi}^*\dot{\phi}+m^2\phi^*\phi) -\frac{1}{2}\alpha^{*2}(\dot{\phi}^{*2}+m^2\phi^{*2})-\frac{1}{2}\alpha^2(\dot{\phi}^2+m^2\phi^2)\right]. \quad (3.84)$$

Similarly using the properties of squeezed vacuum state and squeezed state the expectation value of the corresponding Hamiltonians are respectively obtained as

$$< \hat{H}_{m-closed} >_{svs} = \frac{S^3(t)\sinh^2\chi\cosh\chi}{\sqrt{1-\sinh^2\chi}}\left[\left(\sinh^2 r+\frac{1}{2}\right)(\dot{\phi}^*\dot{\phi}+m^2\phi^*\phi) + \frac{\sinh 2r}{4}\left(e^{-i\vartheta}(\dot{\phi}^{*2}+m^2\phi^{*2})+e^{i\vartheta}(\dot{\phi}^2+m^2\phi^2)\right)\right], \quad (3.85)$$

$$< \hat{H}_{m-closed} >_{ss} = \frac{S^3(t)\sinh^2\chi\cosh\chi}{\sqrt{1-\sinh^2\chi}} \times\left[\left(\sinh^2 r+\frac{1}{2}+|\alpha|^2\right)(\dot{\phi}^*\dot{\phi}+m^2\phi^*\phi) + \frac{(e^{-i\vartheta}\sinh r\cosh r-\alpha^2)}{2}(\dot{\phi}^{*2}+m^2\phi^{*2}) + \left(\frac{e^{i\vartheta}\sinh r\cosh r-\alpha^2}{2}\right)(\dot{\phi}^2+m^2\phi^2)\right]. \quad (3.86)$$

The expectation value of $\hat{\mathcal{H}}_{m-closed}$ (3.12) can be calculated in the coherent state and subsequently substitute the result in (3.79), we obtain

$$\left(\frac{\dot{S}}{S}\right)^2+\frac{1}{S^2} = \frac{8\pi}{3m_p^2}\left[\left(|\alpha|^2+\frac{1}{2}\right)\left(\dot{\phi}^*\dot{\phi}+m^2\phi^*\phi\right) -\frac{1}{2}\alpha^{*2}\left(\dot{\phi}^{*2}+m^2\phi^{*2}\right)-\frac{1}{2}\alpha^2\left(\dot{\phi}^2+m^2\phi^2\right)\right]. \quad (3.87)$$

In equation (3.87) ϕ and ϕ^* are satisfying (3.6) and the Wronskian condition

$$|\dot{\phi}^*(t)\phi(t)-\phi^*(t)\dot{\phi}(t)| = i\frac{\sqrt{(1-\sinh^2\chi)}}{S^3(t)\sinh^2\chi\cosh\chi}. \quad (3.88)$$

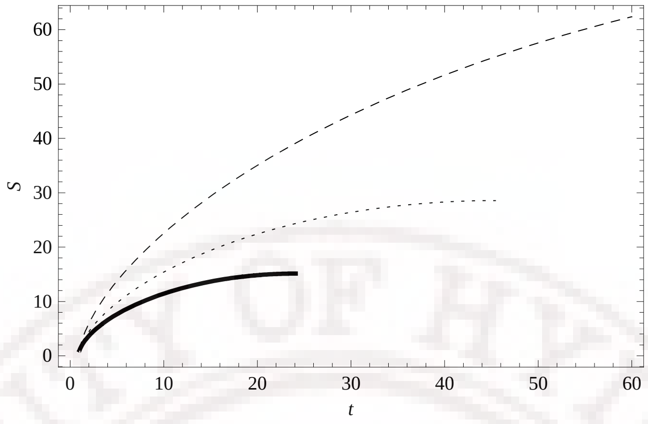

Figure 3.4: Plot for scale factor S and time t for closed FRW universe in coherent state with different values of coherent state parameter, thick line (α=.25), the dot line (α=.75) and the dash line (α=1.5).

Again the Wronskian and the boundary conditions, fix the normalization constants of the two independent solutions. Then, with previously assumed inequality (3.25), we transform the scalar field as follows

$$\phi(t) = \frac{(1-\sinh^2\chi)^{1/4}}{S^{3/2}\sinh\chi\sqrt{\cosh\chi}}\varphi(t), \tag{3.89}$$

and substitute the values of $\phi, \phi^*, \dot{\phi}, \dot{\phi}^* \cdots$ etc., are in (3.87) and then applying the approximation ansatzs that used earlier, we get

$$S\left(\dot{S}^2+1\right) = \frac{8\pi\sqrt{1-\sinh^2\chi}}{3m_p^2\sinh^2\chi\cosh\chi}\left[(\alpha^2+\frac{1}{2})(\frac{1}{t^2}+2m^2)-\frac{\alpha^2}{2mt^2}\right]. \tag{3.90}$$

We solve (3.90) numerically for different values of the coherent state parameter and are shown in Figure 3.4. Following, the similar procedure the semiclassical Friedmann equation in the squeezed vacuum state can be written as

$$S\left(\dot{S}^2+1\right) = \frac{8\pi\sqrt{1-\sinh^2\chi}}{3m_p^2\sinh^2\chi\cosh\chi}\left[(\sinh^2 r+\frac{1}{2})(\frac{1}{t^2}+2m^2)+\frac{\sinh 2r}{4mt^2}\right] \tag{3.91}$$

Again solve equation (3.91) numerically for different values of squeezing parameter and corresponding results are shown in Figure 3.5. Similarly the

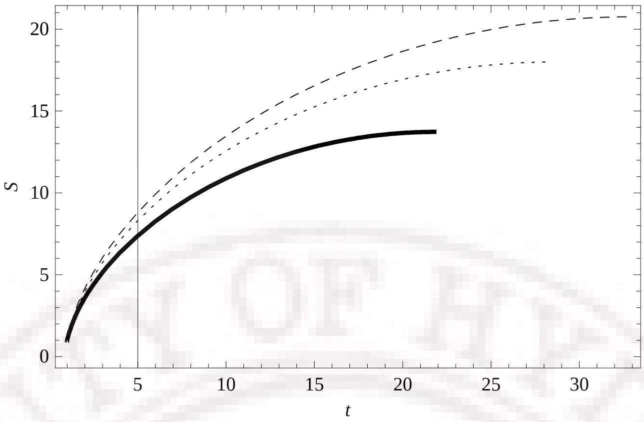

Figure 3.5: Plot for scale factor S and time t for closed FRW universe in squeezed vacuum state with different values of squeezing parameter, thick line (r=.10),the dot line (r=.40) and the dash line (r=.5).

semiclassical Friedmann equation in the squeezed state is obtained as

$$S(\dot{S}^2+1) = \frac{8\pi\sqrt{1-\sinh^2\chi}}{3m_p^2\sinh^2\chi\cosh\chi}\Big[(\sinh^2 r+\alpha^2+\frac{1}{2})(\frac{1}{t^2}+2m^2) + \frac{\sinh 2r}{4mt^2}-\frac{\alpha^2}{2mt^2}\Big]. \quad (3.92)$$

The numerical solutions of (3.92) for different values of α and r are shown in Figure 3.6. We obtained solutions of the semiclassical Friedmann equation in the coherent, squeezed vacuum and squeezed states in the flat, open and closed FRW models of the universe. Thus for the comparative study, solutions of the semiclassical Friedmann equation in the flat, open and closed FRW universes with various values of the coherent and squeezed states parameters are given in Figures 3.7, 3.8 and 3.9.

3.3 Discussions and Conclusions

The inflaton is studied in the semiclassical theory of gravity by representing it in the coherent and squeezed state formalisms of quantum optics. Approximate leading solutions to the semiclassical Friedmann equation are obtained

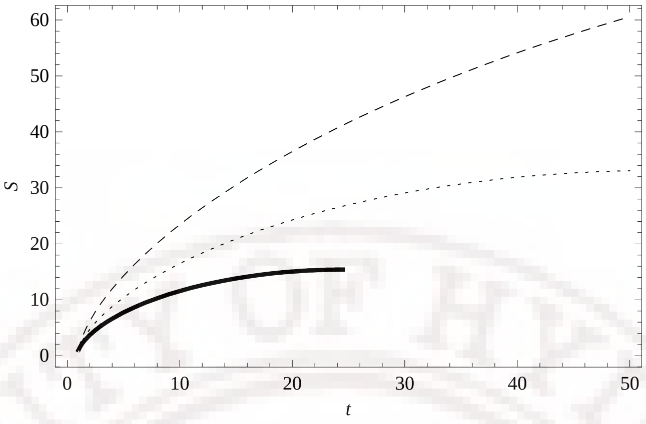

Figure 3.6: Plot for scale factor S and time t for closed FRW universe in squeezed state with different values of coherent and squeezing parameters, thick line (α=.25 ,r=.10), the dot line (α=.75,r=.40) and the dash line (α=1.5,r=.5).

for the flat FRW universe in the coherent and squeezed states in the oscillatory phase of the inflaton. It is observed that the solutions obey $t^{2/3}$ power law expansion. The approximate leading solutions for the closed and open FRW models are obtained numerically in semiclassical theory of gravity. A comparative study of the solutions in the flat, open and closed FRW models of the universe is also carried out for various ranges of the coherent and squeezed state parameters in the semiclassical theory of gravity. The numerical solution of the semiclassical Friedmann equation for various range of the coherent and squeezed state parameters are plotted in Figures 3.1 - 3.6, for the open and closed FRW models. It is clear from the study that the evolution of the scale factor dependent on the associated coherent and squeezed state parameters. In the case of the closed FRW model it is observed that the universe close much faster if the associated coherent and squeezed states parameters take smaller values. While in the case of open FRW model, the scale factor grows much faster when the associated coherent and squeezed parameters take higher values. For a comparative study, solutions of the three FRW models of the universe for various values of the squeezed and

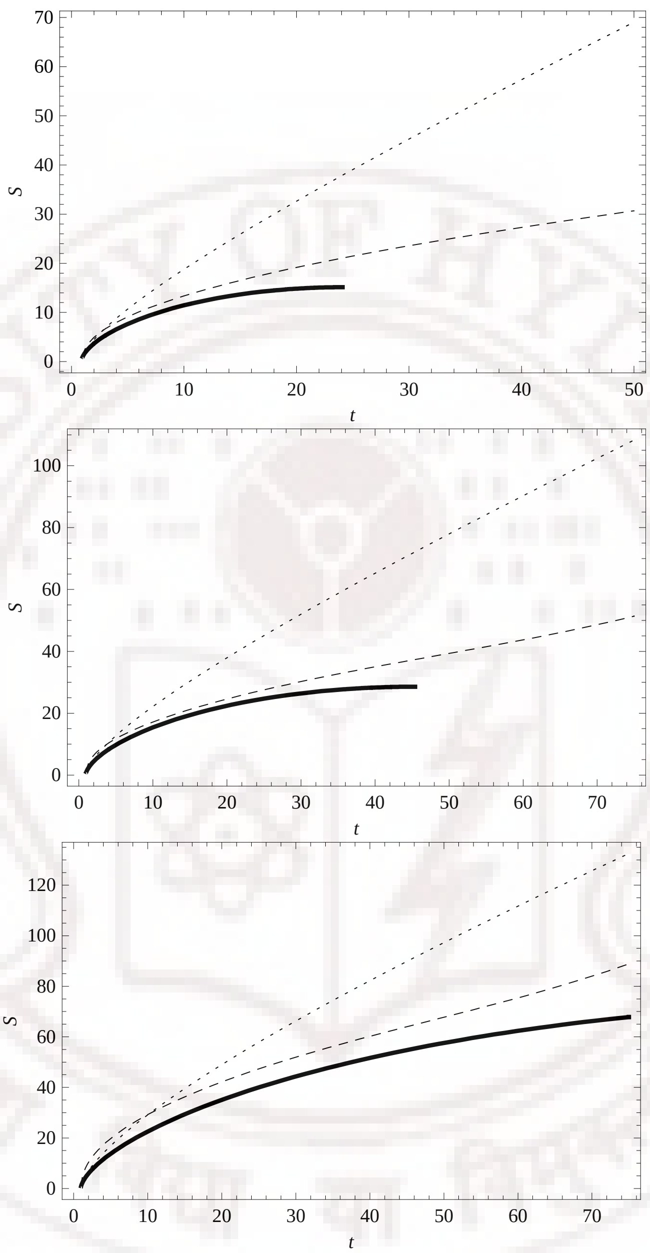

Figure 3.7: Plot for scale factor S and time t for $\alpha = .25, .75$ and 1.5 in coherent state for the semiclassical Friedmann models of the universe, closed (thick line), flat (dash line) and open (dot line).

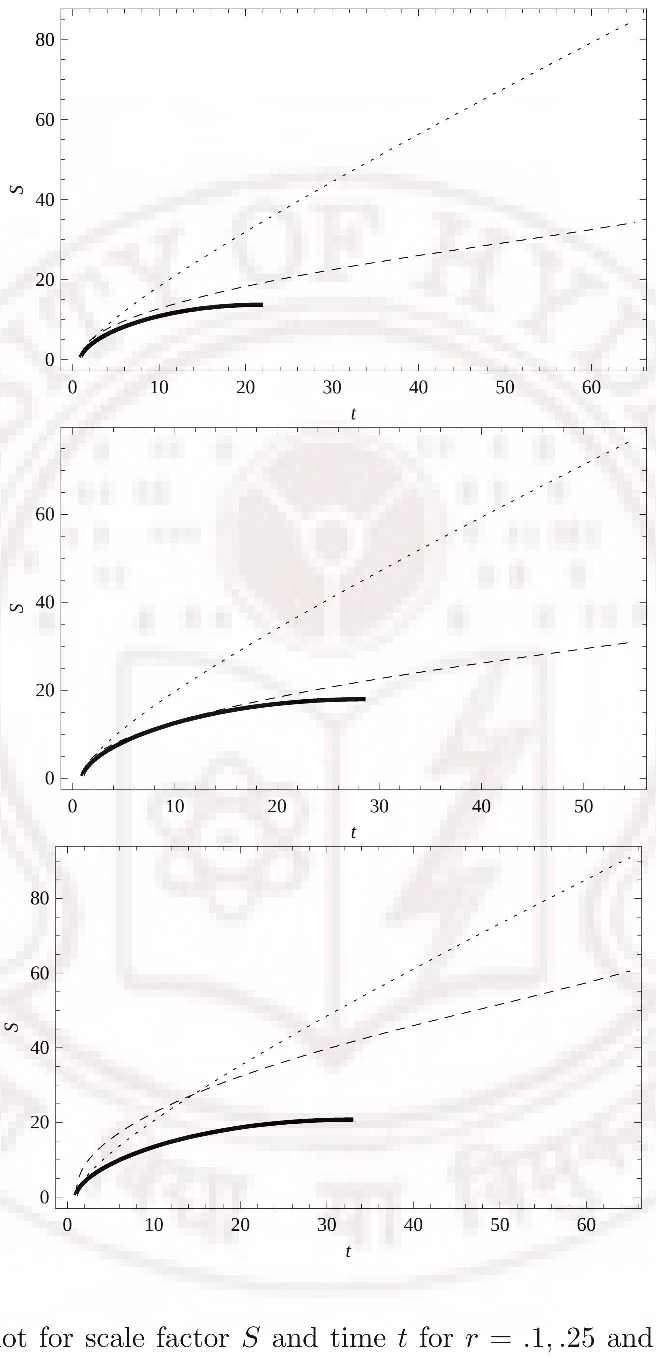

Figure 3.8: Plot for scale factor S and time t for $r = .1, .25$ and $.5$ in squeezed vacuum state for the semiclassical Friedmann models of the universe, closed (thick line), flat (dash line) and open (dot line).

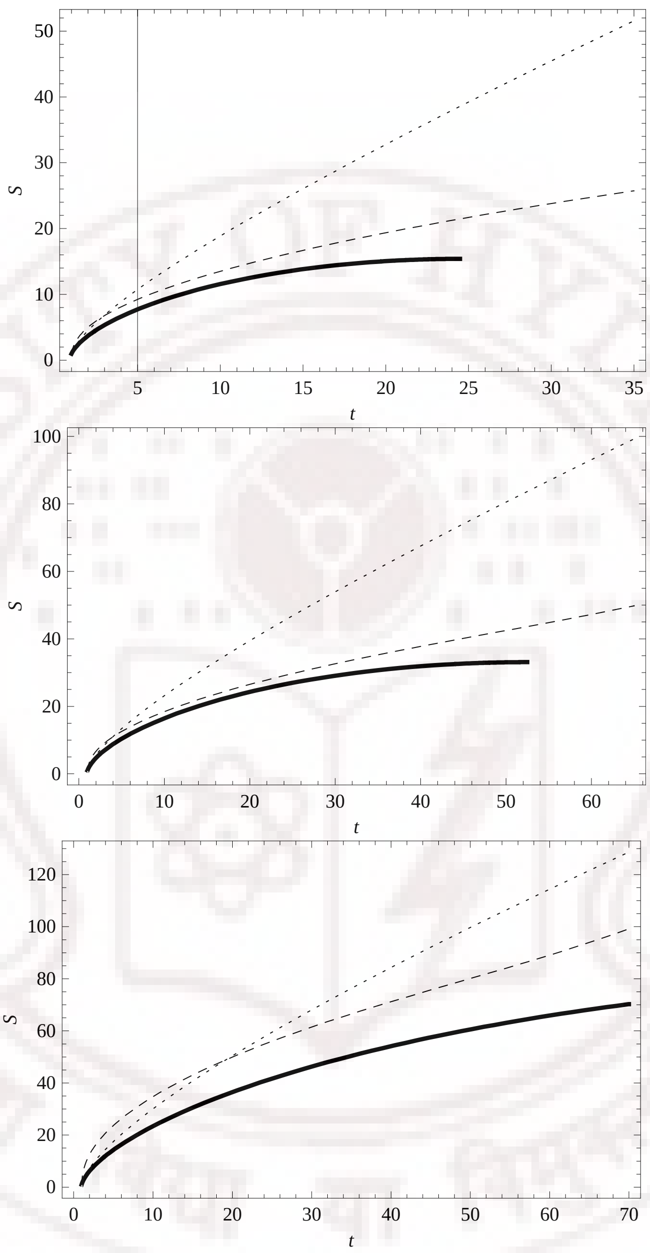

Figure 3.9: Plot for scale factor S and time t for $\alpha = .25, r = .10$, $\alpha = .75, r = .25$ and $\alpha = 1.5, r = .5$ in squeezed state for the closed (thick line), flat (dash line) and open (dot line) universe.

coherent parameters are also discussed, in the semiclassical theory of gravity. Plots for evolution of scale factor with time for the flat, open and closed FRW models in the coherent and squeezed states parameters are also given in Figures 3.7 - 3.9. It is noted that the evolution of the scale factor in all the models are showing the required behaviour in comparison with the standard FRW models.

UNIVERSITY OF HYDERABAD

Chapter 4

Particle Creation in Nonclassical States

The oscillatory phase of the inflaton is considered as one of the important evolution eras of the universe. It is believed that the universe was devoid of particles just after the inflation and particles were created due to coherent oscillations of the inflaton in its oscillatory phase. The particle creation can be better understood in terms of the quantum feature of the inflaton. In this chapter we study the possibility of particle creation due to nonclassical inflaton for a flat FRW universe in the semiclassical theory gravity. The chapter also deals with study of particle creation in the open and closed FRW models of the universe, in the semiclassical theory of gravity.

4.1 Particle Creation in the Oscillatory Phase of Inflaton

In this section, we study the particle production of inflaton in the coherent and squeezed state formalisms for the flat FRW universe, in the semiclassical theory of gravity. The basic setup of the study is as follows. Consider a minimally coupled massive inflaton in a flat FRW metric. Then the quantized inflaton can be expressed in a quantum state under the consideration. The particle creation can be estimated as follows. First, consider a Fock space which has a one parameter dependence on the cosmological time t. Number of particles at a later time t produced from the vacuum at the initial time t_0 is given by [12]

$$N_0(t,t_0) = < 0,\phi,t_0)|\hat{N}(t)|0,\phi,t_0 > . \tag{4.1}$$

Here, $\hat{N}(t) = \hat{a}^\dagger a$ and its expectation value to be calculated in a specific quantum state under the consideration, thus

$$\begin{aligned} < \hat{N}(t) > &= \phi(t)\phi^*(t) < \hat{\pi}^2 > - S^3(t)\phi(t)\dot{\phi}^*(t) < \hat{\pi}\hat{\phi} > \\ &- S^3(t)\dot{\phi}(t)\phi^*(t) < \hat{\phi}\hat{\pi} > + S^6(t)\dot{\phi}(t)\dot{\phi}^*(t) < \hat{\phi}^2 > . \end{aligned} \tag{4.2}$$

Using equations (3.16 - 3.19), $< \hat{\pi}^2 >, < \hat{\pi}\hat{\phi} >$, $< \hat{\phi}\hat{\pi} >$ and $< \hat{\phi}^2 >$ are respectively obtained as

$$\begin{aligned} < \hat{\pi}^2 > = S^6(t)\dot{\phi}^*\dot{\phi} \quad &; \quad < \hat{\pi}\hat{\phi} >= S^3(t)\dot{\phi}\phi^* \\ < \hat{\phi}\hat{\pi} > = S^3(t)\phi\dot{\phi}^* \quad &; \quad < \hat{\phi}^2 >= \phi^*\phi. \end{aligned} \tag{4.3}$$

Substitute (4.3) in (4.2), we get

$$N_0(t,t_0) = S^6(t)|\phi(t)\dot{\phi}(t_0) - \dot{\phi}(t)\phi(t_0)|^2. \tag{4.4}$$

Using the following approximation ansatzs as used earlier, that is

$$w_0(t) = m, \tag{4.5}$$

$$S_0(t) = S_0 t^{2/3}, \tag{4.6}$$

and (3.23), (3.26) number of particles created at a later time t from the vacuum state at the initial time t_0 in the limit mt_0, $mt > 1$ can be computed and is given by [12]

$$\begin{aligned} N_0(t,t_0) &= \frac{1}{4w(t)w(t_0)}\left(\frac{S(t)}{S(t_0)}\right)^3 \\ &\times \left[\frac{1}{4}\left(\frac{3\dot{S}(t)}{S(t)} - \frac{3\dot{S}(t_0)}{S(t_0)} - \frac{\dot{w}(t)}{w(t)} + \frac{\dot{w}(t_0)}{w(t_0)}\right)^2 + (w(t) - w(t_0))^2\right] \\ &\approx \frac{(t-t_0)^2}{4m^2 t_0^2}. \end{aligned} \tag{4.7}$$

The expectation values of $<\hat{\pi}^2>$, $<\hat{\pi}\hat{\phi}>$, $<\hat{\phi}\hat{\pi}>$ and $<\hat{\phi}^2>$ in the coherent state can be calculated by using their properties given in chapter 2, and are obtained as

$$\begin{aligned}
<\hat{\pi}^2>_{cs} &= S^6(t)\left[(2|\alpha|^2+1)\dot{\phi}^*(t_0)\dot{\phi}(t_0) - \alpha^{*2}\dot{\phi}^{*2}(t_0) - \alpha^2\dot{\phi}^2(t_0)\right] \\
<\hat{\pi}\hat{\phi}>_{cs} &= S^3(t)\Big[|\alpha|^2\dot{\phi}^*(t_0)\dot{\phi}(t_0) + (|\alpha|^2+1)\dot{\phi}(t_0)\phi^*(t_0) \\
&\quad -\alpha^2\dot{\phi}(t_0)\phi(t_0) - \alpha^2\dot{\phi}^*(t_0)\phi(t_0)\Big] \\
<\hat{\phi}\hat{\pi}>_{cs} &= S^3(t)\Big[|\alpha|^2\phi^*(t_0)\dot{\phi}(t_0) + (|\alpha|^2+1)\phi(t_0)\dot{\phi}^*(t_0) \\
&\quad -\alpha^2\phi(t_0)\dot{\phi}(t_0) - \alpha^{*2}\dot{\phi}^*(t_0)\Big] \\
<\hat{\phi}^2>_{cs} &= (2|\alpha|^2+1)\phi^*(t_0)\phi(t_0) - \alpha^{*2}\phi^{*2}(t_0) - \alpha^2\phi^2(t_0).
\end{aligned} \tag{4.8}$$

Substituting (4.8) in (4.2) and using approximation ansatzs (4.5) and (4.6), number of particles (at a later time t) produced from the coherent state (at the initial time t_0) is obtained as

$$N_{cs} \simeq (2|\alpha|^2+1)N_0(t,t_0) + |\alpha|^2 - S^6(t)\alpha^{*2}E - S^6(t)\alpha^2 F. \tag{4.9}$$

where $N_0(t,t_0)$ is given by (4.7) and

$$\begin{aligned}
E &= \phi(t)\phi^*(t)\dot{\phi}^{*2}(t_0) - \phi(t)\dot{\phi}^*(t)\dot{\phi}^*(t_0)\phi(t_0) \\
&\quad - \dot{\phi}(t)\phi^*(t)\phi(t_0)\dot{\phi}^*(t_0) + \dot{\phi}\dot{\phi}^*(t)\phi^{*2}(t_0) \\
&= \frac{\exp(2i\int m dt_0)}{4m^2 S_0^6 t_0^2}\left[\frac{1}{t_0^2} + \frac{1}{t^2} - \frac{2im}{t_0}\right] - \frac{1}{4m^2 S_0^6 t^2 t_0^2}\left[\frac{1}{t_0^2} + \frac{1}{t^2} - \frac{2im}{t_0}\right],
\end{aligned} \tag{4.10}$$

$$\begin{aligned}
F &= \phi(t)\phi^*(t)\dot{\phi}^2(t_0) - \phi(t)\dot{\phi}^*(t)\dot{\phi}(t_0)\phi(t_0) \\
&\quad - \dot{\phi}(t)\phi^*(t)\phi(t_0)\dot{\phi}(t_0) + \dot{\phi}\dot{\phi}^*(t)\phi^2(t_0) \\
&= \frac{\exp(-2i\int m dt_0)}{4m^2 S_0^6 t^2 t_0^2}\left[\frac{1}{t_0^2} + \frac{2im}{t_0} - 2m^2 - \frac{2}{tt_0} + \frac{1}{t^2}\right].
\end{aligned} \tag{4.11}$$

Using (4.10), (4.11), (3.23), (3.25), (3.28 - 3.31) and the approximation ansatzs, in (4.9) and dropping imaginary terms, we obtain

$$N_{cs} \simeq (2|\alpha|^2+1)N_0(t,t_0) + |\alpha|^2 - \left(\frac{\alpha^2}{2m^2}\right)\left(\frac{t}{t_0}\right)^2\left(\frac{1}{t_0^2} + \frac{1}{t^2} - m^2\right), \tag{4.12}$$

and substituting $N_0(t, t_0)$ from (4.7) in (4.12), we get

$$N_{cs} \simeq (2|\alpha|^2 + 1) + \frac{(t - t_0)^2}{4m^2} + |\alpha|^2 - \frac{\alpha^2}{2m^2 t_0^4}\left[t^2 + t_0^2 - m^2 t^2 t_0^2\right]. \quad (4.13)$$

Similarly by adopting the same procedure in the case of squeezed vacuum state, number of particles at a later time t produced from the squeezed vacuum state at the initial time t_0 is computed and it leads to the following result

$$N_{svs} \simeq (2\sinh^2 r + 1)\frac{(t - t_0)^2}{4m^2 t_0^4} + \sinh^2 r + \frac{\sinh r \cosh r}{2m^2 t_0^4}[(t - t_0)^2 - m^2 t^2 t_0^2]. \quad (4.14)$$

Analogously by adopting the same procedure in the case of squeezed state number of particles produced is

$$N_{ss} \simeq (2\sinh^2 r + 1 + 2|\alpha|^2)\frac{(t - t_0)^2}{4m^2 t_0^4} + \sinh^2 r + |\alpha|^2 + \frac{\sinh r \cosh r}{2m^2 t_0^4}[(t - t_0)^2 - m^2 t^2 t_0^2] - \frac{\alpha^2}{2m^2 t_0^4}[t^2 + t_0^2 - m^2 t^2 t_0^2]. \quad (4.15)$$

Thus number of particles created in the oscillatory phase of inflaton are obtained in the semiclassical theory of gravity for the various nonclassical states of quantized inflaton in the flat FRW universe.

4.2 Particle Creation in FRW Universe

4.2.1 Open FRW Model

Consider the Fock space which has a one parameter dependence on the cosmological time t. Number of particles at a later time t produced from the vacuum at the initial time t_0 is given by

$$N_0(t, t_0) =< 0, \phi, t_0|\hat{N}(t)|0, \phi, t_0 > . \quad (4.16)$$

Here, $\hat{N}(t) = \hat{a}^{\dagger}\hat{a}$ and its expectation value can be calculated by using (3.67) as

$$\begin{aligned} < \hat{N}(t) > &= \phi(t)\phi^*(t) < \hat{\pi}^2 > -S^3(t)\sinh^2\chi\phi(t)\dot{\phi}^*(t) < \hat{\pi}\hat{\phi} > \\ &\quad -S^3(t)\sinh^2\chi\dot{\phi}(t)\phi^*(t) < \hat{\phi}\hat{\pi} > \\ &\quad +S^6(t)\sinh^4\chi\dot{\phi}(t)\dot{\phi}^*(t) < \hat{\phi}^2 > . \end{aligned} \tag{4.17}$$

Here, $< \hat{\pi}^2 >$, $< \hat{\pi}\hat{\phi} >$, $< \hat{\phi}\hat{\pi} >$ and $< \hat{\phi}^2 >$ are respectively obtained as

$$\begin{aligned} < \hat{\pi}^2 > &= S^6(t)\sinh^4\chi\ \dot{\phi}^*\dot{\phi}; \quad < \hat{\pi}\hat{\phi} >= S^3(t)\sinh^2\chi\ \dot{\phi}\phi^* \\ < \hat{\phi}\hat{\pi} > &= S^3(t)\sinh^2\chi\ \phi\dot{\phi}^*; \quad < \hat{\phi}^2 >= \phi^*\phi. \end{aligned} \tag{4.18}$$

Now substituting (4.18) in (4.17), we get

$$N_0(t, t_0) = S^6(t)\sinh^4\chi|\phi(t)\dot{\phi}(t_0) - \dot{\phi}(t)\phi(t_0)|^2. \tag{4.19}$$

Using the approximation ansatzs, (3.23) and (3.26), number of particles created at a later time t from the vacuum state at the initial time t_0 in the limit mt_0, $mt > 1$ can be calculated. The expectation values of $< \hat{\pi}^2 >$, $< \hat{\pi}\hat{\phi} >$, $< \hat{\phi}\hat{\pi} >$ and $< \hat{\phi}^2 >$ in the coherent state are obtained as

$$\begin{aligned} < \hat{\pi}^2 >_{cs} &= S^6(t)\sinh^4\chi\left[(2|\alpha|^2+1)\dot{\phi}^*(t_0)\dot{\phi}(t_0) - \alpha^{*2}\dot{\phi}^{*2}(t_0) - \alpha^2\dot{\phi}^2(t_0)\right] \\ < \hat{\pi}\hat{\phi} >_{cs} &= S^3(t)\sinh^2\chi\left[|\alpha|^2\dot{\phi}^*(t_0)\phi(t_0) + (|\alpha|^2+1)\dot{\phi}(t_0)\phi^*(t_0)\right. \\ &\quad \left. - \alpha^2\dot{\phi}(t_0)\phi(t_0) - \alpha^2\dot{\phi}^*(t_0)\phi(t_0)\right] \\ < \hat{\phi}\hat{\pi} >_{cs} &= S^3(t)\sinh^2\chi\left[|\alpha|^2\phi^*(t_0)\dot{\phi}(t_0) + (|\alpha|^2+1)\phi(t_0)\dot{\phi}^*(t_0)\right. \\ &\quad \left. - \alpha^2\phi(t_0)\dot{\phi}(t_0) - \alpha^{*2}\phi(t_0)\dot{\phi}^*(t_0)\right] \\ < \hat{\phi}^2 >_{cs} &= (2|\alpha|^2+1)\phi^*(t_0)\phi(t_0) - \alpha^{*2}\phi^{*2}(t_0) - \alpha^2\phi^2(t_0). \end{aligned} \tag{4.20}$$

Substituting (4.20) in (4.17) and using approximation ansatzs (3.35) and (3.36), number of particles (at a later time t) produced from the coherent state (at the initial time t_0) is obtained as

$$N_{cs} = (2|\alpha|^2+1)N_0(t, t_0) + |\alpha|^2 - S^6\sinh^4\chi\alpha^{*2}E - S^6\sinh^4\chi\alpha^2 F. \tag{4.21}$$

Here $N_0(t,t_0)$ is given by (4.7) and

$$
\begin{aligned}
E &= \phi(t)\phi^*(t)\dot{\phi}^{*2}(t_0) - \phi(t)\dot{\phi}^*(t)\dot{\phi}^*(t_0)\phi(t_0) \\
&\quad - \dot{\phi}(t)\phi^*(t)\phi(t_0)\dot{\phi}^*(t_0) + \dot{\phi}(t)\dot{\phi}^*(t)\phi^{*2}(t_0) \\
&= \frac{\exp(2i\int m dt_0)}{4m^2 S_0^6 t^2 t_0^2 \sinh^4\chi}\left[\frac{1}{t_0^2} + \frac{1}{t^2} - \frac{2im}{t_0}\right] \\
&\quad - \frac{1}{4m^2 S_0^6 t^2 t_0^2 \sinh^4\chi}\left[\frac{2}{tt_0} - \frac{2im}{t}\right], \qquad (4.22) \\
F &= \phi(t)\phi^*(t)\dot{\phi}^2(t_0) - \phi(t)\dot{\phi}^*(t)\dot{\phi}(t_0)\phi(t_0) \\
&\quad - \dot{\phi}(t)\phi^*(t)\phi(t_0)\dot{\phi}(t_0) + \dot{\phi}(t)\dot{\phi}^*(t)\phi^2(t_0) \\
&= \frac{\exp(-2i\int m dt_0)}{4m^2 S_0^6 t^2 t_0^2 \sinh^4\chi}\left[\frac{1}{t_0^2} + \frac{2im}{t_0} - 2m^2 - \frac{2}{tt_0} + \frac{1}{t^2}\right]. \qquad (4.23)
\end{aligned}
$$

Using (4.22), (4.23) and the approximation ansatzs, in the expression (4.21) and dropping imaginary terms, we obtain

$$N_{cs} = (2|\alpha|^2+1)N_0(t,t_0) + |\alpha|^2 - \left(\frac{\alpha^2}{2m^2}\right)\left(\frac{t}{t_0}\right)^2\left\{\frac{1}{t_0^2} + \frac{1}{t^2} - m^2\right\}. \quad (4.24)$$

Substituting $N_0(t,t_0)$ in (4.24), we get

$$N_{cs} \simeq (2|\alpha|^2+1)\frac{(t-t_0)^2}{4m^2 t_0^4} + |\alpha|^2 - \frac{\alpha^2}{2m^2 t_0^4}\left[t^2 + t_0^2 - m^2 t^2 t_0^2\right]. \quad (4.25)$$

Analogously, number of particles at a later time t produced from the squeezed vacuum state at the initial time t_0 can be obtained using the following equations

$$
\begin{aligned}
<\hat{\pi}^2>_{svs} &= S^6(t)\sinh^4\chi\Big[(2\sinh^2 r+1)\dot{\phi}^*(t_0)\dot{\phi}(t_0) \\
&\quad + \sinh r\cosh r(e^{-i\vartheta}\dot{\phi}^{*2}(t_0) + e^{i\vartheta}\dot{\phi}^2(t_0))\Big], \\
<\hat{\phi}^2>_{svs} &= (2\sinh^2 r+1)\phi^*(t_0)\phi(t_0) \\
&\quad + \sinh r\cosh r(e^{-i\vartheta}\phi^{*2}(t_0) + e^{i\vartheta}\phi^2(t_0)), \\
<\hat{\pi}\hat{\phi}>_{svs} &= S^3(t)\sinh^2\chi[\sinh^2 r\dot{\phi}^*(t_0)\phi(t_0) + \cosh^2 r\dot{\phi}(t_0)\phi^*(t_0) \\
&\quad + \sinh r\cosh r(e^{-i\vartheta}\dot{\phi}^*(t_0)\phi(t_0) + e^{i\vartheta}\dot{\phi}(t_0)\phi(t_0))], \qquad (4.26) \\
<\hat{\phi}\hat{\pi}>_{svs} &= S^3(t)\sinh^2\chi[\sinh^2 r\phi^*(t_0)\dot{\phi}(t_0) + \cosh^2 r\phi(t_0)\dot{\phi}^*(t_0) \\
&\quad + \sinh r\cosh r(e^{-i\vartheta}\phi(t_0)\dot{\phi}^*(t_0) + e^{i\vartheta}\phi(t_0)\dot{\phi}(t_0))].
\end{aligned}
$$

Substituting (4.26) in (4.17), number of particles produced in the squeezed vacuum state is obtained as

$$N_{svs} = (2\sinh^2 r + 1)N_0(t,t_0) + \sinh^2 r + \frac{\sinh r \cosh r}{2m^2 t_0^4}\left[(t-t_0)^2 - m^2t^2t_0^2\right]. \tag{4.27}$$

Where $N_0(t,t_0)$ is given by (4.7) and substituting it in (4.27), we get

$$N_{svs} \simeq (2\sinh^2 r + 1)\frac{(t-t_0)^2}{4m^2t_0^4} + \sinh^2 r + \frac{\sinh r \cosh r}{2m^2 t_0^4}\left[(t-t_0)^2 - m^2t^2t_0^2\right]. \tag{4.28}$$

Similarly by adopting the same procedure in the case of squeezed state, number of particles at a later time t produced from the squeezed state at the initial time t_0 is computed and it leads to the following result

$$\begin{aligned} N_{ss} \simeq\ & (2\sinh^2 r + 1 + 2|\alpha|^2)\frac{(t-t_0)^2}{4m^2t_0^4} + \sinh^2 r + |\alpha|^2 \\ & + \frac{\sinh r \cosh r}{2m^2 t_0^4}\left[(t-t_0)^2 - m^2t^2t_0^2\right] - \frac{\alpha^2}{2m^2t_0^4}\left[t^2 + t_0^2 - m^2t^2t_0^2\right]. \end{aligned} \tag{4.29}$$

When $r = 0$ the above expression is equal to that of number of particles produced in the coherent state. And for $\alpha = 0$ the expression is same as that of number of particles produced in the squeezed vacuum state.

4.2.2 Closed FRW Model

Consider again, the Fock space which has a one parameter dependence on the cosmological time t. Number of particles at a later time t produced from the vacuum at the initial time t_0 is given by

$$N_0(t,t_0) = \langle 0,\phi,t_0|\hat{N}(t)|0,\phi,t_0\rangle. \tag{4.30}$$

Here $\hat{N}(t) = \hat{a}^\dagger \hat{a}$ and its expectation value can be calculated by using the following equation

$$\langle \hat{N}(t)\rangle = \phi(t)\phi^*(t)\langle\hat{\pi}^2\rangle - \frac{S^3(t)\sinh^2\chi\cosh\chi}{\sqrt{1-\sinh^2\chi}}\phi(t)\dot{\phi}^*(t)\langle\hat{\pi}\hat{\phi}\rangle$$

$$- \frac{S^3(t)\sinh^2\chi\cosh\chi}{\sqrt{1-\sinh^2\chi}}\dot{\phi}(t)\phi^*(t)\langle\hat{\phi}\hat{\pi}\rangle$$
$$+ \frac{S^6(t)\sinh^4\chi\cosh^2\chi}{(1-\sinh^2\chi)}\dot{\phi}(t)\dot{\phi}^*(t)\langle\hat{\phi}^2\rangle. \tag{4.31}$$

By using the definition and properties of the coherent states, the expectation values of $\hat{\pi}^2$ and $\hat{\phi}^2$ are obtained as

$$\langle\hat{\pi}^2\rangle_{cs-closed} = \frac{S^6(t)\sinh^4\chi\cosh^2\chi}{(1-\sinh^2\chi)}\left[\left(2|\alpha|^2+1\right)\dot{\phi}^*\dot{\phi} - \alpha^{*2}\dot{\phi}^{*2} - \alpha^2\dot{\phi}^2\right] \tag{4.32}$$

$$\langle\hat{\phi}^2\rangle_{cs-closed} = (2|\alpha|^2+1)\phi^*\phi - \alpha^{*2}\phi^{*2} - \alpha^2\phi^2. \tag{4.33}$$

The expectation values of $\hat{\pi}\hat{\phi}$ and $\hat{\phi}\hat{\pi}$ in the coherent state are given by

$$\langle\hat{\pi}\hat{\phi}\rangle_{cs-closed} = \frac{S^3(t)\sinh^2\chi\cosh\chi}{\sqrt{(1-\sinh^2\chi)}}\Big[|\alpha|^2\dot{\phi}^*(t_0)\phi(t_0)$$
$$+(|\alpha|^2+1)\dot{\phi}\phi^* - \alpha^2\dot{\phi}(t_0)\phi(t_0) - \alpha^{*2}\dot{\phi}^*(t_0)\phi(t_0)\Big] \tag{4.34}$$

$$\langle\hat{\phi}\hat{\pi}\rangle_{cs-closed} = \frac{S^3(t)\sinh^2\chi\cosh\chi}{\sqrt{(1-\sinh^2\chi)}}\Big[|\alpha|^2\phi^*(t_0)\dot{\phi}(t_0)$$
$$+(|\alpha|^2+1)\phi(t_0)\dot{\phi}^*(t_0) - \alpha^2\phi(t_0)\dot{\phi}(t_0) - \alpha^{*2}\phi(t_0)\dot{\phi}^*(t_0)\Big]. \tag{4.35}$$

The equations (4.32) - (4.35) are substituted in (4.31) and using approximation ansatz, then number of particles produced at a later time t from the coherent state at the initial time t_0 is obtained as

$$N_{cs-closed} = (2|\alpha|^2+1)N_0(t,t_0) + |\alpha|^2$$
$$- \frac{S^6\sinh^4\chi\cosh^2\chi}{(1-\sinh^2\chi)}\alpha^{*2}E - \frac{S^6\sinh^4\chi\cosh^2\chi}{(1-\sinh^2\chi)}\alpha^2 F, \tag{4.36}$$

where E and F are given by

$$E = \phi(t)\phi^*(t)\dot{\phi}^{*2}(t_0) - \phi(t)\dot{\phi}^*(t)\dot{\phi}^*(t_0)\phi(t_0)$$
$$- \dot{\phi}(t)\phi^*(t)\phi(t_0)\dot{\phi}^*(t_0) + \dot{\phi}\dot{\phi}^*(t)\phi^{*2}(t_0)$$

$$= \frac{\exp(2i\int m dt_0)(1-\sinh^2\chi)}{4m^2 S_0^6 t^2 t_0^2 \sinh^4\chi\cosh^2\chi}\left[\frac{1}{t_0^2}+\frac{1}{t^2}-\frac{2im}{t_0}\right]$$
$$-\frac{(1-\sinh^2\chi)}{4m^2 S_0^6 t^2 t_0^2 \sinh^4\chi\cosh^2\chi}\left[\frac{2}{tt_0}-\frac{2im}{t}\right] \quad (4.37)$$
$$F = \frac{\exp(-2i\int m dt_0)(1-\sinh^2\chi)}{4m^2 S_0^6 t^2 t_0^2 \sinh^4\chi\cosh\chi}\left[\frac{1}{t_0^2}+\frac{2im}{t_0}-2m-\frac{2}{tt_0}+\frac{1}{t^2}\right]. \quad (4.38)$$

Dropping the imaginary terms and substitute the value of $N_0(t,t_0)$ from (4.7) we get, number of particles produced at a later time t_0 in the coherent state as

$$N_{cs-closed} \simeq (2|\alpha|^2+1)\frac{(t-t_0)^2}{4m^2t_0^4}+|\alpha|^2-\frac{\alpha^2}{2m^2t_0^4}[t^2+t_0^2-m^2t^2t_0^2]. \quad (4.39)$$

Analogously, number of particles at a later time t produced from the squeezed vacuum state at the initial time t_0 can be obtained using the following equations

$$\begin{aligned}
\langle\hat{\pi}^2\rangle_{svs-closed} &= \frac{S^6(t)\sinh^4\chi\cosh^2\chi}{(1-\sinh^2\chi)}\Big[\left(2\sinh^2 r+1\right)\dot{\phi}^*(t_0)\dot{\phi}(t_0) \\
&\quad +\sinh r\cosh r\left(e^{-i\vartheta}\dot{\phi}^{*2}(t_0)+e^{i\vartheta}\dot{\phi}^2(t_0)\right)\Big] \\
\langle\hat{\phi}^2\rangle_{svs-closed} &= \left(2\sinh^2 r+1\right)\phi^*(t_0)\phi(t_0) \\
&\quad +\sinh r\cosh r\left[e^{-i\vartheta}\dot{\phi}^{*2}(t_0)+e^{i\vartheta}\dot{\phi}^2(t_0)\right] \\
\langle\hat{\pi}\hat{\phi}\rangle_{svs-closed} &= \frac{S^3(t)\sinh^2\chi\cosh\chi}{\sqrt{(1-\sinh^2\chi)}}\Big[\sinh^2 r\dot{\phi}^*(t_0)\phi(t_0) \\
&\quad +\cosh^2 r\dot{\phi}(t_0)\phi^*(t_0) \\
&\quad +\cosh r\sinh r\left(e^{-i\vartheta}\dot{\phi}^*\phi+e^{i\vartheta}\dot{\phi}\phi\right)\Big] \\
\langle\hat{\phi}\hat{\pi}\rangle_{svs-closed} &= \frac{S^3(t)\sinh^2\chi\cosh\chi}{\sqrt{(1-\sinh^2\chi)}}\Big[\sinh^2 r\phi^*\dot{\phi} \\
&\quad +\cosh^2 r\phi\dot{\phi}^* \\
&\quad +\cosh r\sinh r\left(e^{-i\vartheta}\phi\dot{\phi}^*+e^{i\vartheta}\phi\dot{\phi}\right)\Big]\ . \qquad (4.40)
\end{aligned}$$

The equation (4.40) is substituted in (4.31) and applying the approximation ansatzs then simplify, number of particles produced in the squeezed vacuum state and is obtained as

$$N_{svs-closed} \simeq \left(2\sinh^2 r + 1\right) \frac{(t-t_0)^2}{4m^2t_0^4} + \sinh^2 r + \frac{\sinh r \cosh r}{2m^2t_0^4}\left[(t-t_0)^2 - m^2t^2t_0^2\right]. \tag{4.41}$$

Similarly by adopting the same procedure in the case of the squeezed state, number of particles at a later time t produced from the squeezed state at initial time t_0 is computed and it leads to the following result

$$N_{ss-closed} \simeq \left(2\sinh^2 r + 1 + 2|\alpha|^2\right) \frac{(t-t_0)^2}{4m^2t_0^4} + \sinh^2 r + \frac{\sinh r \cosh r}{2m^2t_0^4}\left[(t-t_0)^2 - m^2t^2t_0^2\right] - \frac{\alpha^2}{2m^2t_0^4}[t^2 + t_0^2 - m^2t^2t_0^2]. \tag{4.42}$$

4.3 Discussions and Conclusions

We studied a minimally coupled massive scalar field in the flat, open and closed FRW background metrics, in the semiclassical theory of gravity. Particle production for the flat, open and closed FRW models of the universe is examined in the coherent, squeezed vacuum and squeezed state formalisms. It is observed that particles creation in the coherent state is dependent on the coherent parameter α and in the squeezed vacuum state it is dependent on squeezing parameter r. Particle creation in the closed and open FRW models leads to the same result as that of the flat FRW. Thus it may be concluded that particle production is model independent as far as the semiclassical theory is concerned.

Chapter 5

Density Fluctuations and Semiclassical Theory

The semiclassical theory of gravity seems to be a viable method to understand quantum effects and quantum phenomenon in the early universe where quantum gravity effects are considered to be negligible. However, when the fluctuations of the source field are high then one has to worry about validity of the semicalssical scenario even the background metric is treated as classical. Thus it is reasonable to be assumed that the semiclassical theory of gravity is valid only if the fluctuations in the associated energy momentum tensor are not large. Thus validity of the semiclassical theory can be studied with the help of the energy momentum tensor of the source field in a suitable quantum state under consideration. In this chapter we study validity of the semiclassical Friedmann equation in the oscillatory phase of inflaton in the flat FRW metric for the coherent and squeezed states. Since we use the coherent and squeezed states to represent the inflaton in the semiclassical gravity, it would be useful to examine whether the field exhibit classical or nonclassical feature in the cosmological context. Such study is performed in the quantum optics by means of a quantity known as the Mandel's Q parameter. We examine classical or nonclassical nature of the inflaton in the coherent and squeezed states in terms of the Q parameter computed with the associated cosmological parameters.

5.1 Validity of Semiclassical Theory

In this section, we examine validity of the semiclassical Einstein equation for the flat FRW universe in the coherent and squeezed vacuum states. Validity of the semiclassical Einstein equation can be analyzed with the help of a dimensionless quantity defined in terms of the energy momentum tensor as below

$$\Delta = \left| \frac{\langle : {T^k}^2{}_{\mu\nu} : \rangle - \langle : T^k{}_{\mu\nu} : \rangle^2}{\langle : T^k{}_{\mu\nu} : \rangle^2} \right| \tag{5.1}$$

where $<: {T^k}^2_{\mu\nu} :>$ is the expectation value of the squared energy momentum tensor of a scalar field in a suitable quantum state and $<: T^k{}_{\mu\nu} :>^2$ is its square of the expectation value. The meaning of :: is that the expectation values are to be computed with respect to the normal ordering procedure for the field. If Δ is more than one, then the corresponding semiclassical Einstein equation does not hold in that particular quantum state.

The computation of the fluctuations of all the components of the energy momentum tensor in a given quantum state for all the modes (k) would be cumbersome. Therefore, for the sake of simplicity of the study, we focus on the temporal component of the energy momentum tensor with a single mode of the inflaton and investigate validity of the semiclassical theory in the coherent and squeezed vacuum states.

5.1.1 In Coherent State

Consider the dimensionless quantity in the coherent state and can be written as

$$\Delta_{cs} = \left| \frac{<: T_{00}^2 :>_{cs} - <: T_{00} :>_{cs}^2}{<: T_{00} :>_{cs}^2} \right| \tag{5.2}$$

The first term in (5.2) can be obtained by squaring the temporal component of the energy momentum tensor of the inflaton and taking its expectation value in the coherent state as

$$<: T_{00}^2 :>_{cs} = \frac{1}{4S^6(t)} <: \hat{\pi}^4 :>_{cs} + \frac{m^2}{4} <: \hat{\pi}^2 \hat{\phi}^2 :>_{cs} + \frac{m^2}{4} <: \hat{\phi}^2 \hat{\pi}^2 :>_{cs} + \frac{m^4}{4} S^6(t) <: \hat{\phi}^4 :>_{cs} \quad . \tag{5.3}$$

Then using the properties of the coherent state and applying the approximation ansatzs, $<: \hat{\pi}^4 :>_{cs}$, $<: \hat{\pi}^2\hat{\phi}^2 :>_{cs}$, $<: \hat{\phi}^2\hat{\pi}^2 :>_{cs}$ and $<: \hat{\phi}^4 :>_{cs}$ can be calculated and are respectively obtained as

$$<: \hat{\pi}^4 :>_{cs} = \frac{S_0^6}{4m^2}(4 + 4\alpha^4 + 4\alpha^2) \tag{5.4}$$

$$<: \hat{\pi}^2\hat{\phi}^2 :>_{cs} = \frac{1}{4m^2t^2}(4 + 4\alpha^4 + 4\alpha^2) \tag{5.5}$$

$$<: \hat{\phi}^2\hat{\pi}^2 :>_{cs} = \frac{1}{4m^2t^2}(4 + 4\alpha^4 + 4\alpha^2) \tag{5.6}$$

$$<: \hat{\phi}^4 :>_{cs} = \frac{1}{4m^2S_0^6t^4}(4 + 4\alpha^4 + 4\alpha^2). \tag{5.7}$$

Thus

$$<: T_{00}^2 :>_{cs} = \left(\frac{1}{16m^2t^4} + \frac{1}{8t^2} + \frac{m^2}{16}\right)(4 + 4\alpha^4 + 4\alpha^2). \tag{5.8}$$

Calculating the expectation value of the normal ordered time-time component of energy momentum tensor of the inflaton in the coherent state and then squaring, we get

$$\begin{aligned} <: T_{00} :>_{cs}^2 \;=\; & S^6 \Bigg[\left(|\alpha|^4 + \frac{1}{4} + |\alpha|^2\right) \\ & \times(\dot{\phi}^{*2}\dot{\phi}^2 + m^4\phi^{*2}\phi^2 + 2m^2\dot{\phi}^*\dot{\phi}\phi^*\phi) \\ & + \frac{1}{4}\alpha^{*4}(\dot{\phi}^{*4} + m^4\phi^{*4} + 2m^2\dot{\phi}^{*2}\phi^{*2}) \\ & + \frac{1}{4}\alpha^4(\dot{\phi}^4 + m^4\phi^4 + 2m^2\dot{\phi}^2\phi^2) \\ & - \left(|\alpha|^2\alpha^{*2} + \frac{1}{2}\alpha^{*2}\right) \\ & \times(\dot{\phi}^*\dot{\phi}\dot{\phi}^{*2} + m^2\dot{\phi}^*\dot{\phi}\phi^{*2} + m^2\phi^*\phi\dot{\phi}^{*2} + m^4\phi^*\phi\phi^{*2}) \\ & + \frac{1}{2}\alpha^{*2}\alpha^2(\dot{\phi}^{*2}\dot{\phi}^2 + m^2\dot{\phi}^{*2}\phi^2 + m^2\phi^{*2}\dot{\phi}^2 + m^4\phi^{*2}\phi^2) \\ & - \left(\alpha^2|\alpha|^2 + \frac{1}{2}\alpha^2\right)(\dot{\phi}^2\dot{\phi}^*\dot{\phi} + m^2\dot{\phi}^2\phi^*\phi + m^2\phi^2\dot{\phi}^*\dot{\phi} + m^4\phi^2\phi^*\phi) \Bigg] \end{aligned} \tag{5.9}$$

α	Δ_{cs}	α	Δ_{cs}	α	Δ_{cs}	α	Δ_{cs}	α	Δ_{cs}
1.0	0.7273	2.0	0.9234	3.0	0.9688	4.0	0.9832	5.0	0.9895
1.1	0.7511	2.1	0.9313	3.1	0.9710	4.1	0.9841	5.1	0.9899
1.2	0.7806	2.2	0.9381	3.2	0.9729	4.2	0.9849	5.2	0.9903
1.3	0.8092	2.3	0.9439	3.3	0.9747	4.3	0.9856	5.3	0.9907
1.4	0.8345	2.4	0.9490	3.4	0.9763	4.4	0.9863	5.4	0.9911
1.5	0.8563	2.5	0.9534	3.5	0.9777	4.5	0.9869	5.5	0.9914
1.6	0.8747	2.6	0.9573	3.6	0.9790	4.6	0.9875	5.6	0.9917
1.7	0.8901	2.7	0.9607	3.7	0.9802	4.7	0.9881	5.7	0.9920
1.8	0.9031	2.8	0.9638	3.8	0.9813	4.8	0.9886	5.8	0.9923
1.9	0.9141	2.9	0.9664	3.9	0.9823	4.9	0.9891	5.9	0.9926
2.0	0.9234	3.0	0.9688	4.0	0.9832	5.0	0.9895	6.0	0.9928

Table 5.1: Δ_{cs} for coherent parameter α larger compared to unity.

Substitute the values of $\phi, \dot{\phi}, \dot{\phi}^*, \phi^* \cdots$ etc. and applying the approximation ansatzs, we get

$$<: T_{00} :>_{cs}^2 = \left(\frac{1}{16m^2t^4} + \frac{m^2}{16} + \frac{1}{8t^2} \right) \left[1 + 8\alpha^4 + 2\alpha^2 \right]. \tag{5.10}$$

The equations (5.8) and (5.10) are substituted in (5.2), we get

$$\Delta_{cs} = \left| \frac{3 - 4\alpha^4 + 2\alpha^2}{1 + 8\alpha^4 + 2\alpha^2} \right|. \tag{5.11}$$

In order to examine validity of the semiclassical theory of the Einstein equation, we study Δ_{cs} numerically with the associated coherent state parameter values and are tabulated in Table 5.1. Thus the study shows that the semiclassical theory hold in the coherent state representation of the inflaton in the oscillatory phase.

5.1.2 In Squeezed Vacuum State

Consider the dimensionless quantity in the squeezed vacuum state as

$$\Delta_{svs} = \left| \frac{<: T_{00}^2 :>_{svs} - <: T_{00} :>_{svs}^2}{<: T_{00} :>_{svs}^2} \right|. \tag{5.12}$$

The first term of (5.12) can be obtained by squaring (3.7) and taking the expectation value in the squeezed vacuum state as

$$\begin{aligned} <: T_{00}^2 :>_{svs} &= \frac{1}{4S^6(t)} <: \hat{\pi}^4 :>_{svs} + \frac{m^2}{4} <: \hat{\pi}^2 \hat{\phi}^2 :>_{svs} \\ &+ \frac{m^2}{4} <: \hat{\phi}^2 \hat{\pi}^2 :>_{svs} + \frac{m^4}{4} S^6(t) <: \hat{\phi}^4 :>_{svs} . \end{aligned} \tag{5.13}$$

Using (2.33),(2.34) and applying the approximation ansatzs, we get $<: \hat{\pi}^4 :>_{svs}$, $<: \hat{\pi}^2 \hat{\phi}^2 :>_{svs}$,$<: \hat{\phi}^2 \hat{\pi}^2 :>_{svs}$ and $<: \hat{\phi}^4 :>_{svs}$ can be calculated and are respectively obtained as follows

$$\begin{aligned} <: \hat{\pi}^4 :>_{svs} &= S_0^6 \left[\frac{1}{4m^2} \{ 3 + 6\cosh^2 r \sinh^2 r + 24 \sinh^3 r \cosh r \right. \\ &\left. +12(\sinh^2 r + \sinh^4 r + \cosh r \sinh r) \} \right] \end{aligned} \tag{5.14}$$

$$\begin{aligned} <: \hat{\pi}^2 \hat{\phi}^2 :>_{svs} &= \frac{1}{4m^2 t^2} \{ 3 + 6\cosh^2 r \sinh^2 r + 24 \sinh^3 r \cosh r \\ &+12(\sinh^2 r + \sinh^4 r + \cosh r \sinh r) \} \end{aligned} \tag{5.15}$$

$$\begin{aligned} <: \hat{\phi}^2 \hat{\pi}^2 :>_{svs} &= \frac{1}{4m^2 t^2} \left\{ 3 + 6\cosh^2 r \sinh^2 r + 24 \sinh^3 r \cosh r \right. \\ &\left. +12(\sinh^2 r + \sinh^4 r + \sinh r \cosh r) \right\} \end{aligned} \tag{5.16}$$

$$\begin{aligned} <: \hat{\phi}^4 :>_{svs} &= \frac{1}{4S_0^6 t^4 m^2} \left\{ 3 + 6\cosh^2 r \sinh^2 r + 24 \cosh r \sinh^3 r \right. \\ &\left. +12(\sinh^2 r + \sinh^4 r + \sinh r \cosh r) \right\}. \end{aligned} \tag{5.17}$$

Thus

$$\begin{aligned} <: T_{00}^2 :>_{svs} &= \left(\frac{1}{16m^2 t^4} + \frac{1}{8t^2} + \frac{m^2}{16} \right) \left[3 + 6\cosh^2 r \sinh^2 r + 24 \sinh^3 r \cosh r \right. \\ &\left. + 12 \big(\sinh^2 r + \sinh^4 r + \cosh r \sinh r \big) \right]. \end{aligned} \tag{5.18}$$

By calculating the expectation value of normal ordered time-time component of energy momentum tensor of the inflaton in squeezed vacuum state and then square the result, we get

$$
\begin{aligned}
<: T_{00} :>^2_{svs} = S^6 \Bigg[& (\sinh^4 r + \frac{1}{4} + \sinh^2 r)(\dot{\phi}^{*2}\dot{\phi}^2 + 2m^2 \dot{\phi}^* \dot{\phi} \phi^* \phi + m^4 \phi^{*2} \phi^2) \\
& + \frac{1}{4}(e^{-2i\varphi} \sinh^2 r \cosh^2 r)(\dot{\phi}^{*4} + 2m^2 \dot{\phi}^{*2} \phi^{*2} + m^4 \phi^{*4}) \\
& + \frac{1}{4} e^{2i\varphi} \sinh^2 r \cosh^2 r (\dot{\phi}^4 + m^4 \phi^4 + 2m^2 \dot{\phi}^2 \phi^2) \\
& + \left(\sinh^2 r + \frac{1}{2}\right) e^{-i\varphi} \sinh r \cosh r (\dot{\phi}\dot{\phi}^{*3} + m^2 \phi^* \phi \dot{\phi}^{*2} \\
& + m^2 \dot{\phi}^* \dot{\phi} \phi^{*2} + m^4 \phi \phi^{*3}) \\
& + \frac{1}{2} \sinh^2 r \cosh^2 r (\dot{\phi}^{*2} \dot{\phi}^2 + m^2 \dot{\phi}^{*2} \phi^2 + m^2 \phi^{*2} \dot{\phi}^2 + m^4 \phi^{*2} \phi^2) \\
& + e^{i\varphi} \sinh r \cosh r \left(\sinh^2 r + \frac{1}{2}\right) (\dot{\phi}^2 \dot{\phi}^* \dot{\phi} + m^2 \phi^2 \dot{\phi}^* \dot{\phi} \\
& + m^2 \dot{\phi}^2 \phi^* \phi + m^4 \phi^2 \phi^* \phi) \Bigg]. \qquad (5.19)
\end{aligned}
$$

Using equations (3.23), (3.26), (3.35) and (3.36) in (5.19) we get

$$
\begin{aligned}
<: T_{00} :>^2_{svs} = & \left(\frac{1}{16m^2 t^4} + \frac{1}{8t^2} + \frac{m^2}{4}\right) \Big(1 + 4\sinh^4 r + 4\sinh^2 r \\
& + 4\sinh^2 r \cosh^2 r + 8\sinh^3 r \cosh r + 4\sinh r \cosh r\Big). \qquad (5.20)
\end{aligned}
$$

Substituting equations (5.18) and (5.20) in (5.12), then

$$
\Delta_{svs} = \left| \frac{\begin{matrix} 2 + 2\cosh^2 r \sinh^2 r + 16\sinh^3 r \cosh r \\ +8(\sinh^2 r + \sinh^4 r + \cosh r \sinh r) \end{matrix}}{\begin{matrix} 1 + 4\sinh^4 r + 4\sinh^2 r + 4\sinh^2 r \cosh^2 r \\ +8\sinh^3 r \cosh r + 4\sinh r \cosh r \end{matrix}} \right|. \qquad (5.21)
$$

In order to examine validity of the semiclassical equation, we study Δ_{svs} numerically with the associated squeezing parameter values that are much smaller and larger compared to unity. The corresponding results are tabulated in Table 5.2.

5.2 Cosmological Q Parameter

In this section, we study a quantity which is in the line of the Mandel's Q parameter well known in quantum optics [33]. In quantum optics, it is being

r	Δ_{svs}	r	Δ_{svs}	r	Δ_{svs}	r	Δ_{svs}
0.001	0.5039	0.010	0.5377	0.100	0.7620	1.100	1.3028
0.002	0.5079	0.020	0.5716	0.200	0.9084	1.200	1.3106
0.003	0.5117	0.030	0.6023	0.300	1.0116	1.300	1.3164
0.004	0.5156	0.040	0.6304	0.400	1.0897	1.400	1.3206
0.005	0.5194	0.050	0.6563	0.500	1.1498	1.500	1.3237
0.006	0.5231	0.060	0.6802	0.600	1.1960	1.600	1.3260
0.007	0.5268	0.070	0.7026	0.700	1.2311	1.700	1.3277
0.008	0.5305	0.080	0.7236	0.800	1.2576	1.800	1.3290
0.009	0.5341	0.090	0.7433	0.900	1.2774	1.900	1.3300
0.010	0.5377	0.100	0.7620	1.000	1.2920	2.000	1.3307

Table 5.2: Δ_{svs} for squeezing parameter r smaller and larger compared to unity.

used to examine the deviation of photon statistics. Validity of this parameter has examined for the case where the photon statistics exhibit fluctuations in the photon number [62]. The Q parameter is a natural measure of the departure of the photon number from the variances of a Poissonian process, for which $Q = 0$ and Q is negative for the sub-Poissonian process. The Q parameter which is greater than or equal to zero then the state is said to be classical and for the negative it is called nonclassical. Since we use the squeezed and coherent states to study particle creation during the oscillatory phase of inflaton field, it is interesting to examine the nature of the quantum optical states in the cosmological context with associated cosmological parameters. Thus conveniently, we call the Q parameter as the cosmological Q parameter denoted as Q_{cos}.

The Mandel's Q parameter is defined as [33]

$$Q = \frac{< n^2 > - < n >^2 - < n >}{< n >}, \tag{5.22}$$

where $n = a^\dagger a$, and $<>$ means the expectation value in a quantum state under consideration. First, we compute the Q_{cos} for the coherent states in the flat FRW model. We use the annihilation and creation operators that are defined in the case of flat FRW universe model (3.17). The Q_{cos} in the coherent state can be expressed as

$$Q_{cs} = \frac{< \hat{a}^{\dagger 2}\hat{a}^2 >_{cs} - < \hat{a}^\dagger \hat{a} >_{cs}^2}{< \hat{a}^\dagger \hat{a} >_{cs}}. \tag{5.23}$$

Where $< \hat{a}^\dagger \hat{a} >_{cs}$ is already computed in chapter 4, and the remaining terms can be computed by using the properties of the coherent state with values of $\phi, \dot{\phi}, \dot{\phi}^*, \phi^* \cdots$ etc.,. Then applying the approximation ansatzs, we get

$$\begin{aligned}
< \hat{a}^{\dagger 2}\hat{a}^2 >_{cs} = & \frac{1}{16m^4}\left(\frac{t}{t_0}\right)^4 \Bigg[16m^4\alpha^4 + 28m^4\alpha^2 + \frac{24m^2\alpha^2}{t_0^2} \\
& + \frac{4}{t_0^4} + \frac{11m^2}{t_0^2} + 6m^4 - \frac{3}{tt_0^3} - \frac{3m^2}{tt_0} - \frac{12m^2\alpha^2}{tt_0} \\
& + \frac{3}{t^2t_0^2} + \frac{m^2}{t^2} + \frac{4m^2\alpha^2}{t_0^2} - \frac{2\alpha^4}{t^2t_0^2} - \frac{2m^2\alpha^4}{t^2} - \frac{2m^2\alpha^4}{t_0^2} - 2m^4\alpha^4\Bigg] \\
& - \frac{1}{16m^4}\left(\frac{t_0}{t}\right)^2 \Bigg[\frac{3}{tt_0^3} + \frac{3m^2}{tt_0} + \frac{m^2}{t_0^2} - m^4 + \frac{2\alpha^2}{tt_0^3} \\
& + 14m^2\alpha^2 + \frac{36m^2\alpha^2}{t_0^2} + 11m^4\alpha^2 - \frac{3}{t_0^3} - \frac{3m^2}{t_0} + \frac{3\alpha^2}{t_0^4}\Bigg] \\
& + \frac{1}{16m^4}\Bigg[\frac{-\alpha^4}{t^2t_0^2} - \frac{5m^2\alpha^4}{t^2} - \frac{4m^2\alpha^4}{tt_0} - \frac{5m^2\alpha^4}{t_0^2} + m^4\alpha^4 + \frac{2\alpha^2}{t^2t_0^2} \\
& + \frac{22m^2\alpha^2}{t_0^2} - 2m^4\alpha^2 + \frac{6}{t^2t_0^2} - \frac{m^2}{t^2} - \frac{8m^2}{t_0^2} + 7m^4 + \frac{8m^2}{tt_0} + \frac{6m^2\alpha^2}{t^2} \\
& + 3m^2 - \frac{2m^4\alpha^4}{t_0^2} + \frac{9}{t_0^4}\Bigg] - \frac{1}{16m^4}\left(\frac{t}{t_0}\right)^2 \Bigg[\frac{15}{t_0^4} + \frac{2\alpha^2}{t_0^4} + \frac{16m^2}{t_0^2} \\
& + \frac{4m^2\alpha^2}{t_0^2} + \frac{2m^2\alpha^4}{t_0^2} + 6m^4\alpha^2 + 7m^4\Bigg],
\end{aligned} \tag{5.24}$$

and

$$\begin{aligned}
< \hat{a}^\dagger \hat{a} >_{cs}^2 &= (2|\alpha|^2 + 1)^2 \frac{(t - t_0)^4}{16m^4t_0^8} + |\alpha|^4 - \frac{\alpha^4}{16m^4t_0^8} \\
& \times [2t^2 + t_0^2 - 3t_0^2t^2m^2 - 4tt_0]^2 .
\end{aligned} \tag{5.25}$$

The substituting (5.24) and (5.25) in (5.23), we get

$$
\begin{aligned}
Q_{cs} = \quad & \frac{1}{< \hat{a}^{\dagger}\hat{a} >_{cs}} \Bigg[\frac{1}{16m^4} \bigg(14m^4\alpha^4 + 28m^4\alpha^2 + \frac{16m^2\alpha^2}{t_0^2} + \frac{4}{t_0^4} + \frac{9m^2}{t_0^2} \\
& + 6m^4 - \frac{2\alpha^4}{t_0^4} - \frac{4m^2\alpha^4}{t_0^2} \bigg) \\
& - \frac{1}{16m^4} \bigg(\frac{3}{t_0^4} + \frac{4m^2}{t_0^2} - m^4 + \frac{5\alpha^2}{t_0^4} + 14m^2\alpha^2 + \frac{36m^2\alpha^2}{t_0^2} + \\
& 11m^4\alpha^2 - \frac{3}{t_0^3} - \frac{3m^2}{t_0} \bigg) \\
& + \frac{1}{16m^4} \bigg(-\frac{\alpha^4}{t_0^4} - \frac{14m^2\alpha^4}{t_0^2} + m^4\alpha^4 + \frac{2\alpha^2}{t_0^4} + \frac{28m^2\alpha^2}{t_0^2} \\
& - 2m^4\alpha^2 + \frac{15}{t_0^4} - \frac{m^2}{t_0^2} + 7m^4 + 3m^2 - \frac{2m^4\alpha^4}{t_0^2} \bigg) \\
& - \frac{1}{16m^4} \bigg(\frac{15}{t_0^4} + \frac{2\alpha^2}{t_0^4} + \frac{16m^2}{t_0^2} + \frac{4m^2\alpha^2}{t_0^2} + \frac{2m^2\alpha^4}{t_0^2} + 6m^4\alpha^2 + 7m^4 \bigg) \\
& - \alpha^4 + \frac{\alpha^4}{16m^4 t_0^8} \bigg(-t_0^2 - 3t_0^4 m^2 \bigg)^2 \Bigg]. \qquad (5.26)
\end{aligned}
$$

In equation (5.26) $< \hat{a}^{\dagger}\hat{a} >_{cs}$ is given by (4.13). The expression is studied numerically for small intervals time for a representative value of the coherent state parameter and the results are tabulated in Table 5.3. We can use different values of the coherent state parameter to obtain Q_{cs} but the conclusions remain unaltered.

Next we calculate the Q_{cos} parameter in the squeezed vacuum state and is defined as

$$
Q_{svs} = \frac{< \hat{a}^{\dagger 2}\hat{a}^2 >_{svs} - < \hat{a}^{\dagger}\hat{a} >_{svs}^2}{< \hat{a}^{\dagger}\hat{a} >_{svs}}. \qquad (5.27)
$$

Where $\hat{a}^{\dagger}$ and $\hat{a}$ are respectively the creation and annihilation operators defined in the flat FRW universe. Here, again using the properties of the squeezed vacuum state and the approximation ansatzs, we get

$$\begin{aligned}
< \hat{a}^{\dagger^2}\hat{a}^2 >_{svs} &= \frac{1}{16m^4}\left(\frac{t_0}{t}\right)^4\Bigg[\frac{1}{t_0^4}\big(12\sinh^2 r\cosh^2 r + 3 + 12\sinh^2 r \\
&+12\sinh^4 r + 12\sinh r\cosh r + 24\sinh^3 r\cosh r\big) \\
&+m^4\big(12\sinh^2 r\cosh^2 r + 3 + 12\sinh^2 r + 12\sinh^4 r \\
&-12\sinh r\cosh r - 24\sinh^3 r\cosh r\big) \\
&+\frac{2m^2}{t_0^2}\left(3 + 12\sinh^2 r + 12\sinh^4 r - 12\sinh^2 r\cosh^2 r\right)\Bigg] \\
&-\frac{1}{16m^4}\left(\frac{t_o}{t}\right)^2\Bigg[\frac{1}{tt_0^3}\big(36\sinh^2 r\cosh^2 r + 7 + 26\sinh^2 r \\
&+34\sinh^4 r + 33\sinh r\cosh r + 64\sinh^3 r\cosh r\big) \\
&+\frac{m^2}{tt_0}\big(-39\sinh^2 r\cosh^2 r + 9 + 24\sinh^2 r + 28\sinh^4 r \\
&+9\sinh r\cosh r + 18\sinh^3 r\cosh r + 3\cosh^3 r\sinh r\big) \\
&+\frac{m^2}{t_0^2}\big(3 + 4\sinh^2 r + 2\sinh^4 r - 9\sinh^2 r\cosh^2 r \\
&+3\cosh^3 r\sinh r + \sinh r\cosh r\big) \\
&+m^4\big(-9\sinh^2 r\cosh^2 r - 4\sinh^4 r + 3 - 15\sinh r\cosh r \\
&+3\sinh^3 r\cosh r + 3\cosh^3 r\sinh r\big)\Bigg] \\
&-\frac{1}{16m^4}\Bigg[\frac{1}{tt_0^3}\big(12\sinh^2 r\cosh^2 r + 3 + 12\sinh^2 r \\
&+12\sinh^4 r + 12\sinh r\cosh r + 24\sinh^3 r\cosh r\big) \\
&+\frac{m^2}{tt_0}\big(8\sinh^2 r\cosh^2 r - 5 - 4\sinh^2 r + 4\sinh^4 r \\
&-2\sinh r\cosh r + 8\sinh^3 r\cosh r\big)\Bigg] \\
&-\frac{1}{16m^4}\Bigg[\frac{m^2}{t_0^2}(4 + 2\sinh^2 r - 12\sinh^4 r - 12\sinh^2 r\cosh^2 r \\
&-6\sinh r\cosh r - 14\sinh^3 r\cosh r)\Bigg] \qquad (5.28)
\end{aligned}$$

$$
\begin{aligned}
&-\frac{1}{16m^4}\Big[m^4\big(4\sinh^2 r\cosh^2 r - 4\sinh^4 r - 10 - 6\sinh^2 r\\
&-12\sinh r\cosh r - 6\sinh^3 r\cosh r\big)\\
&-\frac{1}{t^2t_0^2}\big(60\sinh^2 r\cosh^2 r + 60\sinh^4 r + 15 + 60\sinh^2 r\\
&+60\sinh r\cosh r + 118\sinh^3 r\cosh r\big)\\
&-\frac{m^2}{t^2}\big(-20\sinh^2 r\cosh^2 r + 20\sinh^4 r + 5 + 20\sinh^2 r\\
&+6\sinh r\cosh r + 8\sinh^3 r\cosh r\big)\Big]\\
&+\frac{1}{16m^4}\left(\frac{t}{t_0}\right)^4\Big[\frac{1}{t^2t_0^2}\big(10\sinh^2 r\cosh^2 r + 3 + 12\sinh^2 r\\
&+8\sinh^4 r + 15\sinh r\cosh r + 23\sinh^3 r\cosh r\big)\\
&+\frac{m^2}{t^2}\big(18\sinh^2 r\cosh^2 r + 7 + 28\sinh^2 r + 24\sinh^4 r\\
&+21\sinh r\cosh r + 43\sinh^3 r\cosh r\big)\\
&+\frac{m^2}{t_0^2}\big(3 + 12\sinh^2 r + 8\sinh^4 r + 10\sinh^2 r\cosh^2 r\\
&+15\sinh r\cosh r + 23\sinh^3 r\cosh r\big)\\
&+m^4\big(6\sinh^2 r\cosh^2 r + 12\sinh^4 r + 4 + 16\sinh^2 r\\
&+9\sinh r\cosh r + 19\sinh^3 r\cosh r\big)\\
&-\frac{1}{t^4}\big(12\sinh^2 r\cosh^2 r + 12\sinh^4 r + 3 + 12\sinh^2 r\\
&+12\sinh r\cosh r + 24\sinh^3 r\cosh r\big)]\\
&-\frac{1}{16m^4}\left(\frac{t}{t_0}\right)^2\big[\frac{1}{t^3t_0}\big(48\sinh^2 r\cosh^2 r + 9 + 47\sinh^2 r\\
&+48\sinh^4 r + 46\sinh r\cosh r + 96\sinh^3 r\cosh r\big)\\
&+\frac{m^2}{t^2}\big(5 + 17\sinh^2 r + 14\sinh r\cosh r\big)\\
&+\frac{m^2}{tt_0}\big(9 + 47\sinh^2 r + 48\sinh^4 r + 48\sinh^2 r\cosh^2 r\\
&+46\sinh r\cosh r + 96\sinh^3 r\cosh r\big)\\
&+m^4\big(\sinh^2 r(8\cosh^2 r + 12\sinh^2 r + 17 - 3\sinh r\cosh r)\\
&+6\sinh r\cosh r + 7\big)],
\end{aligned}
$$

Δt	Q_{cs}	Δt	Q_{cs}	Δt	Q_{cs}	Δt	Q_{cs}
0.0001	0.0000	0.010	0.0000	0.1	0.0000	1.0	0.0000
0.0002	0.0000	0.020	0.0000	0.2	0.0000	1.1	0.0000
0.0003	0.0000	0.030	0.0000	0.3	0.0000	1.2	0.0000
0.0004	0.0000	0.040	0.0000	0.4	0.0000	1.3	0.0000
0.0005	0.0000	0.050	0.0000	0.5	0.0000	1.4	0.0000
0.0006	0.0000	0.060	0.0000	0.6	0.0000	1.5	0.0000
0.0007	0.0000	0.070	0.0000	0.7	0.0000	1.6	0.0000
0.0008	0.0000	0.080	0.0000	0.8	0.0000	1.7	0.0000
0.0009	0.0000	0.090	0.0000	0.9	0.0000	1.8	0.0000
0.0010	0.0000	0.100	0.0000	1.0	0.0000	1.9	0.0000

Table 5.3: Q_{cs} for various time interval, Δt with $\alpha = 1.2$, $t_0 = m = 1$.

and

$$
\begin{aligned}
< \hat{a}^\dagger \hat{a} >^2_{svs} \;=\; & \frac{(4\sinh^4 r + 1 + 4\sinh^2 r)}{16 m^4 t_0^8}\left[t^4 + t_0^4 + 6t^2 t_0^2 - 4tt_0^3 - 4t^3 t_0\right] \\
& + \sinh^4 r + \frac{\sinh^2 r \cosh^2 r}{4m^4 t_0^8}\left[t^2 + t_0^2 - 2tt_0 - m^2 t^2 t_0^2\right]^2 \\
& + \frac{(2\sinh^4 r + \sinh^2 r)}{2m^2 t_0^4}(t^2 + t_0^2 - 2tt_0) + \frac{\sinh^3 r \cosh r}{m^2 t_0^4} \\
& \times\left[t^2 + t_0^2 - 2tt_0 - m^2 t^2 t_0^2\right] + \frac{(2\sinh^3 r \cosh r + \sinh r \cosh r)}{4m^4 t_0^8} \\
& \times\big(t^4 + 6t_0^2 t^2 - 4t^3 t_0 - m^2 t^4 t_0^2 + t_0^4 - 4tt_0^3 \\
& - m^2 t^2 t_0^4 + 2m^2 t^3 t_0^3\big). \qquad (5.29)
\end{aligned}
$$

Thus, substitute the values $< \hat{a}^{\dagger 2}\hat{a}^2 >_{svs}$, $< \hat{a}^\dagger \hat{a} >^2_{svs}$ and $< \hat{a}^\dagger \hat{a} >_{svs}$ (given in (4.14)) in (5.27) then the resulting expression for Q_{svs} is obtained. This is studied numerically and the corresponding results are tabulated in Tables 5.4-5.8 . We can repeat the same method for obtaining the Q_{cos} parameter of the field in the coherent and squeezed vacuum states for the closed and open FRW models. But one can see that conclusions remain the same as in the case of the flat FRW model.

Δt	Q_{svs}	Δt	Q_{svs}	Δt	Q_{svs}
0.001	-1753.2904	0.0001	-1753.7491	0.000001	-1753.8857
0.002	-1754.4755	0.0002	-1753.6251	0.000002	-1753.8841
0.003	-1757.4436	0.0003	-1753.5205	0.000003	-1753.8826
0.004	-1762.2056	0.0004	-1753.4342	0.000004	-1753.8812
0.005	-1768.7845	0.0005	-1753.3657	0.000005	-1753.8797
0.006	-1777.2136	0.0006	-1753.3149	0.000006	-1753.8782
0.007	-1787.5379	0.0007	-1753.2821	0.000007	-1753.8767
0.008	-1799.8154	0.0008	-1753.2670	0.000008	-1753.8753
0.009	-1814.1165	0.0009	-1753.2692	0.000009	-1753.8739
0.010	-1830.5262	0.0010	-1753.2904	0.000010	-1753.8721

Table 5.4: Q_{svs} for various time interval, Δt with $r = 0.001$, $t_0 = m = 1$.

Δt	Q_{svs}	Δt	Q_{svs}	Δt	Q_{svs}
0.001	-178.8684	0.0001	-178.9839	0.000001	-178.9977
0.002	-178.7606	0.0002	-178.9702	0.000002	-178.9975
0.003	-178.6740	0.0003	-178.9567	0.000003	-178.9974
0.004	-178.6087	0.0004	-178.9435	0.000004	-178.9972
0.005	-178.5643	0.0005	-178.9304	0.000005	-178.9971
0.006	-178.5410	0.0006	-178.9176	0.000006	-178.9969
0.007	-178.5384	0.0007	-178.9050	0.000007	-178.9968
0.008	-178.5566	0.0008	-178.8926	0.000008	-178.9967
0.009	-178.5954	0.0009	-178.8804	0.000009	-178.9965
0.010	-178.5262	0.0010	-178.8684	0.000010	-178.9964

Table 5.5: Q_{svs} for various time interval, Δt with $r = 0.01$, $t_0 = m = 1$.

Δt	Q_{svs}	Δt	Q_{svs}	Δt	Q_{svs}
0.001	-22.8210	0.0001	-22.8258	0.000001	-22.8264
0.002	-22.8163	0.0002	-22.8253	0.000002	-22.8264
0.003	-22.8123	0.0003	-22.8242	0.000003	-22.8264
0.004	-22.8090	0.0004	-22.8241	0.000004	-22.8264
0.005	-22.8063	0.0005	-22.8236	0.000005	-22.8264
0.006	-22.8044	0.0006	-22.8231	0.000006	-22.8264
0.007	-22.8031	0.0007	-22.8225	0.000007	-22.8264
0.008	-22.8025	0.0008	-22.8220	0.000008	-22.8264
0.009	-22.8026	0.0009	-22.8215	0.000009	-22.8264
0.010	-22.8033	0.0010	-22.8210	0.000010	-22.8264

Table 5.6: Q_{svs} for various time interval, Δt with $r = 0.1$, $t_0 = m = 1$.

Δt	Q_{svs}	Δt	Q_{svs}	Δt	Q_{svs}
0.001	-17.3540	0.0001	-17.3605	0.000001	-17.3612
0.002	-17.3474	0.0002	-17.3597	0.000002	-17.3612
0.003	-17.3415	0.0003	-17.3590	0.000003	-17.3612
0.004	-17.3361	0.0004	-17.3583	0.000004	-17.3612
0.005	-17.3313	0.0005	-17.3576	0.000005	-17.3612
0.006	-17.3272	0.0006	-17.3568	0.000006	-17.3612
0.007	-17.3236	0.0007	-17.3561	0.000007	-17.3612
0.008	-17.3205	0.0008	-17.3554	0.000008	-17.3612
0.009	-17.3180	0.0009	-17.3547	0.000009	-17.3612
0.010	-17.3161	0.0010	-17.3540	0.000010	-17.3612

Table 5.7: Q_{svs} for various time interval, Δt with $r = 0.3$, $t_0 = m = 1$.

Δt	Q_{svs}	Δt	Q_{svs}	Δt	Q_{svs}
0.001	-69.5441	0.0001	-70.7463	0.000001	-70.8822
0.002	-68.2762	0.0002	-70.6097	0.000002	-70.8809
0.003	-67.0746	0.0003	-70.4739	0.000003	-70.8795
0.004	-65.9344	0.0004	-70.3388	0.000004	-70.8781
0.005	-64.8512	0.0005	-70.2045	0.000005	-70.8767
0.006	-63.8211	0.0006	-70.0710	0.000006	-70.8754
0.007	-62.8404	0.0007	-69.9382	0.000007	-70.8740
0.008	-61.9059	0.0008	-69.8061	0.000008	-70.8726
0.009	-61.0146	0.0009	-69.6747	0.000009	-70.8712
0.010	-60.1636	0.0010	-69.5441	0.000010	-70.8698

Table 5.8: Q_{svs} for various time interval, Δt with $r = 0.5$, $t_0 = m = 1$.

5.3 Discussions and Conclusions

In this chapter, we studied validity of the semiclassical Einstein equation in the oscillatory phase of the nonclassical inflaton in the coherent and squeezed vacuum state formalisms. The study is performed with a dimensionless quantity in terms of the energy momentum tensor of the inflaton field. We examined the quantity numerically for both the coherent and squeezed vacuum states in the flat FRW metric. The analysis shows that, the dimensionless quantity take values less compared to unity for any value of the coherent state parameter (Table.5.1). Thus, the semiclassical theory holds in the coherent state formalism for inflaton field in the flat FRW model but it is found that the theory holds in the case of squeezed state when the associated squeezing parameter takes much smaller value compared to unity, otherwise the theory does not hold (Table.5.2).

We also examined classical or nonclassical nature of inflaton in the coherent and squeezed vacuum state formalisms of quantum optics in the semiclassical theory of gravity. The study is done with a quantity defined analogous to the Mandel's Q parameter called the cosmological Q parameter. The Q_{cos} is studied in the small interval of time during the oscillatory phase of the inflaton. The cosmological Q_{cos} parameter for the coherent state is studied numerically (Table.5.3). It shows that the coherent state is consistent with its most classical nature with the associated cosmological parameters. The cosmological Q parameter in the squeezed vacuum state show negative values (Tables.5.4-5.8) for the squeezing parameter less than 0.6., but the value of the squeezing parameter is more than 0.55 the corresponding parameter shows switch over behaviour from negative to positive and vice versa. So, the squeezing achieved in the present model is less than sixty percent. Such a switch over behavior is not exhibited in the coherent state. The analysis shows that the cosmological Q parameter for the inflaton field in the coherent and squeezed vacuum states is consistent for their respective nature in the semiclassical theory of gravity. We used the flat FRW universe model to study the Q_{cos} but one can also consider the open and closed FRW models to study the parameter. But conclusions drawn from the flat model are unaltered as far as behaviour of the coherent and squeezed states is concerned, in view of the Q parameter.

Chapter 6

Oscillatory Phase of Inflaton in Thermal States

The interplay between the field theory including finite temperature field theory and classical general relativity has been widely studied in many contexts in cosmology. Various representation schemes of the scalar field have been introduced including the squeezed state and coherent state formalisms of quantum optics and are found useful to understand many problems in the early universe. But most applications of the scalar field in the squeezed and coherent state formalisms in cosmology have not taken into account thermal effects. It seems that finite temperature effects and nonclassical states formalisms may be needed to understand several problems in cosmology. In the earlier studies of nonclassical states in semiclassical theory of gravity [44, 45], we have not taken into account the finite temperature effects. Therefore, it is interesting to see how thermal and quantum effects affecting particle creation, density fluctuations and the related phenomenon in the dynamics of early universe, in the semiclassical theory of gravity.

The present chapter looks at examining the oscillatory phase of the inflaton in the thermal coherent and squeezed states and hence to study particle creation and validity of the semiclassical theory. In the oscillatory phase of inflaton usually it is required to consider only the flat Friedmann model of the universe. Thus emphasis of the present study is to consider the oscillatory phase of nonclassical inflaton and its related issues in the thermal coherent and squeezed states. However, the solutions of the Friedmann model imply the possibility of an open and a closed FRW universe also. Therefore it is interesting to see how the representation of the inflaton in the thermal coherent and squeezed states and related phenomenon play role in determining dynamics of the open and closed FRW models, in the semiclassical theory of gravity. Thus, general goal of the present chapter is to study a massive minimal nonclassical homogeneous scalar field in the flat, open and closed Friedmann-Robertson-Walker universe by representing it in terms of the thermal coherent and squeezed state formalisms.

In the present work we have made use of the thermal squeezed and coherent states formalism to calculate the expectation value of the energy momentum tensor of the scalar field. Since vacuum expectation values of the energy momentum tensor defined prior to any dynamics in a gravitational field can give all information about particle production and vacuum polarization, it may be argued that the representation of the inflaton field in thermal squeezed state can account for enhancing particle creation due to quantum and thermal effects.

6.1 Thermal Coherent and Squeezed States

This part of the chapter discusses some basic properties of the thermal coherent and squeezed states briefly. These states are thermal counterparts of the zero temperature squeezed state and coherent state formalisms that are well known in quantum optics. To incorporate finite temperature effects in the squeezed and coherent state formalisms, the method of thermofield dynamics [63, 64] is required and it has been developed as an extension of zero temperature quantum field theory to finite temperature problems. The formalism of thermofield dynamics can be used to get thermal counterparts of the coherent and squeezed states. In the thermofield dynamics, the thermal coherent states and thermal squeezed states [65] have been defined. The present study employs only the single mode state.

A single mode thermal coherent state (tcs) is defined [65] as

$$|tcs>= D(\alpha, \tilde{\alpha})|0(\beta)> \tag{6.1}$$

where

$$D(\alpha, \tilde{\alpha}) = e^{\alpha a^\dagger + \tilde{\alpha}\tilde{a}^\dagger - \alpha^* a - \tilde{\alpha}^* \tilde{a}} \tag{6.2}$$

and

$$|0(\beta)>= e^{-i\gamma}|0, \tilde{0}> . \tag{6.3}$$

Where

$$\gamma = -i\theta(\beta)(a^\dagger \tilde{a}^\dagger - a\tilde{a}). \tag{6.4}$$

The density matrix approach usually gives a convenient method to incorporate finite temperature effects. Therefore the density operator for the thermal coherent state is defined as

$$\rho(\beta, \alpha)_{tcs} = D^\dagger(\alpha) e^{-\beta w a^\dagger a} D(\alpha) \tag{6.5}$$

where $\beta = \frac{1}{k_B T}$ and w is the energy of the mode, T is the temperature and k_B is Boltzman's constant. In order to proceed further it is useful to introduce a quantity called the characteristic function and is defined as

$$F_{ctcs}(\tau, \tau^*) \equiv \langle \exp(\tau a^\dagger) \exp(-\tau^* a) \rangle. \tag{6.6}$$

Here τ and τ^* are regarded as independent variables so that we have two parameters for each mode. Therefore, the characteristic function for the thermal coherent state is defined [65] as

$$F_{ctcs}(\tau, \tau^*) = \exp\left\{-f(\beta)|\tau|^2 + \tau^* \alpha - \tau \alpha^*\right\} \tag{6.7}$$

where

$$f(\beta) = \frac{1}{e^{\beta w} - 1}. \tag{6.8}$$

Similarly a thermal squeezed state (tss) [65] is defined as

$$|tss>= s(\xi,\tilde{\xi})D(\alpha,\tilde{\alpha})|0(\beta)> \tag{6.9}$$

where

$$s(\xi,\tilde{\xi}) = \exp(\xi a^{\dagger 2} + \tilde{\xi}\tilde{a}^{\dagger 2} - \xi^* a^2 - \tilde{\xi}^*\tilde{a}^2), \tag{6.10}$$
$$\xi = re^{i\phi} \quad , \quad \tilde{\xi} = \tilde{r}e^{i\tilde{\phi}},$$

and $D(\alpha,\tilde{\alpha})$, $|0(\beta)>$ are given by (6.2) and (6.3). The density matrix for the thermal squeezed state is given by

$$\rho_{tss} = D^\dagger(\alpha)s^\dagger(\xi)e^{-\beta w a^\dagger a}s(\xi)D(\alpha), \tag{6.11}$$

and the corresponding characteristic function is

$$\begin{aligned} F_{tss}(\tau,\tau^*) = \exp\Bigg\{ &-|\tau|^2\left(\sinh^2 r\coth\frac{\beta w}{2} + f(\beta)\right) \\ &- \frac{\cosh r\sinh r}{2}\coth\frac{\beta w}{2}(e^{-i\vartheta}\tau^2 + e^{i\vartheta}\tau^{*2}) - \tau\alpha^* + \tau^*\alpha\Bigg\}. \end{aligned} \tag{6.12}$$

By setting $\alpha = \tilde{\alpha} = 0$ and in analogue with zero temperature squeezed state a thermal squeezed vacuum (tsv) can be defined as

$$|tsv>= s^\dagger(\xi)e^{-\beta w a^\dagger a}s(\xi), \tag{6.13}$$

and its characteristic function is

$$\begin{aligned} F_{tsv}(\tau,\tau^*) = \exp\Bigg\{ &-|\tau|^2\left(\sinh^2 r\coth\frac{\beta w}{2} + f(\beta)\right) \\ &- \frac{\cosh r\sinh r}{2}\coth\frac{\beta w}{2}\left(e^{-i\vartheta}\tau^2 + e^{i\vartheta}\tau^{*2}\right)\Bigg\}. \end{aligned} \tag{6.14}$$

The expectation values of $\hat{a}$ and $\hat{a}^\dagger$ in a given state can be calculated by using the corresponding characteristic function with the help of following expressions

$$\langle\hat{a}_i^n\rangle = \left.\frac{\partial^n F_c}{\partial\tau_i^{*n}}\right|_{\tau_i=\tau_i^*=0,} \tag{6.15}$$

and

$$\langle\hat{a}_i^{\dagger n}\rangle = (-1)^n\left.\frac{\partial^n F_c}{\partial\tau_i^n}\right|_{\tau_i=\tau_i^*=0}. \tag{6.16}$$

6.2 Oscillatory Phase of Inflaton in Nonclassical Thermal States

Here, we study the inflaton in the semiclassical Friedmann equations and obtain the solutions in the thermal coherent and squeezed states for the three FRW models. Since our primary motivation is to study the oscillatory phase of inflaton, we first focus the study on the flat FRW metric.

6.2.1 In Flat FRW universe

Consider a minimally coupled massive inflaton in the flat FRW universe for the metric (3.3) with k=0. Therefore, the time-time component of the classical Einstein equation is given by (3.13). In the semiclassical theory, the Friedmann equations can be written in terms of Hamiltonian of the field and is provided in (3.14). Thus the massive minimal inflaton in the flat FRW universe can be described by the time dependent harmonic oscillator with the Hamiltonian (3.10). The eigenstates of the Hamiltonian in the Fock states are described in chapter 3. Hence as an alternate to the n representation of the inflaton, we next consider the inflaton in the thermal coherent and squeezed states formalisms, and hence the semiclassical Einstein equations can be expressed in terms of the thermal coherent and squeezed parameters.

Using (6.1)-(6.8), (6.15), (6.16),(3.14) and (3.17), the semiclassical Einstein equation can be calculated in the thermal coherent state and is obtained as

$$\left(\frac{\dot{S(t)}}{S(t)}\right)^2 = \frac{8\pi}{3m_p^2}\left[\left(f(\beta)+|\alpha|^2+\frac{1}{2}\right)(\dot{\phi}^*\dot{\phi}+m^2\phi^*\phi) -\frac{1}{2}\alpha^{*2}(\dot{\phi}^{*2}+m^2\phi^{*2})-\frac{1}{2}\alpha^2(\dot{\phi}^2+m^2\phi^2)\right], \tag{6.17}$$

where ϕ and ϕ^* satisfy (3.6) and Wronskian condition (3.22). Thus solution for the semiclassical Friedmann equation in the thermal coherent state can be computed subjected to the oscillatory phase of inflaton (3.23) and using the equations (3.25), (3.26), as discussed in chapter 3, we get

$$\left(\frac{\dot{S(t)}}{S(t)}\right)^2 = \frac{1}{S^3(t)}\frac{8\pi}{3m_p^2}\left[\frac{\left(f(\beta)+|\alpha|^2+\frac{1}{2}\right)(A+w^2(t)+m^2)}{2w(t)}\right.$$
$$-\frac{1}{2}\frac{\alpha^{*2}\exp(-2i\int w(t)dt)}{2w(t)}$$
$$\times\left[A-w^2(t)-iw(t)\left[\frac{3\dot{S}(t)}{S(t)}+\frac{\dot{w}(t)}{w(t)}\right]+m^2\right]$$
$$\left.-\frac{1}{2}\frac{\alpha^2\exp(-2i\int w(t)dt)}{2w(t)}\left(A-w^2(t)+iw(t)\left[\frac{3\dot{S}(t)}{S(t)}+\frac{\dot{w}(t)}{w(t)}\right]+m^2\right)\right], \tag{6.18}$$

where w and A are respectively given by (3.27) and (3.43). Using the following approximation ansatzs

$$w_0(t) = m, \tag{6.19}$$

$$S_0(t) = S_0 t^{2/3}, \tag{6.20}$$

in equation (6.18), the next order perturbative solution in the thermal coherent state is obtained as

$$S_{1tcs-flat} = \left[\frac{6\pi}{m_p^2}\left(f(\beta)+\alpha^2+\frac{1}{2}\right)mt^2\left(1+\frac{1}{2m^2t^2}\right)\right.$$
$$\left.-\frac{3\alpha^2}{m}\frac{\pi t^2}{m_p^2}\left[\frac{\cos 2(\theta-mt)}{t^2}-\frac{2m}{t}\sin 2(\theta-mt)\right]\right]^{\frac{1}{3}}. \tag{6.21}$$

Similarly by adopting the same procedure and using (3.3) (3.10) (3.13) (3.14), and (6.13)-(6.16), the semiclassical Einstein equation for the thermal squeezed vacuum state can be written as

$$\left(\frac{\dot{S}(t)}{S(t)}\right)^2 = \frac{8\pi}{3m_p^2}\left[\left(f(\beta)+\sinh^2 r\coth\frac{\beta w}{2}+\frac{1}{2}\right)(\dot{\phi}^*\dot{\phi}+m^2\phi^*\phi)\right.$$
$$\left.+\frac{\sinh 2r}{4}\coth\left(\frac{\beta w}{2}\right)\left[e^{-i\vartheta}(\dot{\phi}^{*2}+m^2\phi^{*2})+e^{i\vartheta}(\dot{\phi}^2+m^2\phi^2)\right]\right]. \tag{6.22}$$

By substituting $\phi, \phi^*, \dot{\phi}, \dot{\phi}^*$ are from (3.23) and (3.26) in the above equation, we get

$$\left(\frac{\dot{S}(t)}{S(t)}\right)^2 = \frac{1}{S^3(t)}\frac{8\pi}{3m_p^2}\left[\frac{\left(f(\beta) + \sinh^2 r \coth(\frac{\beta w}{2}) + \frac{1}{2}\right)(A + w^2(t) + m^2)}{2w(t)}\right.$$
$$+ \frac{\sinh 2r}{4}\coth\left(\frac{\beta w}{2}\right)\left[\frac{e^{-i\vartheta}\exp(2i\int w(t)dt)}{2w(t)}\right.$$
$$\times\left(A - w^2(t) - iw(t)\left[3\frac{\dot{S}(t)}{S(t)} + \frac{\dot{w}(t)}{w(t)}\right] + m^2\right)$$
$$+ \frac{e^{i\vartheta}\exp(-2i\int w(t)dt)}{2w(t)}$$
$$\left.\left.\times\left(A - w^2(t) + iw(t)\left[3\frac{\dot{S}(t)}{S(t)} + \frac{\dot{w}(t)}{w(t)}\right] + m^2\right)\right]\right]. \quad (6.23)$$

Using (6.19) and (6.20), the next order perturbative solution in the thermal squeezed vacuum state is found as

$$S_{1tsvs-flat} = \left[\frac{6\pi}{m_p^2}\left(f(\beta) + \sinh^2 r\coth\left(\frac{\beta w}{2}\right) + \frac{1}{2}\right)mt^2\left(1 + \frac{1}{2m^2t^2}\right)\right.$$
$$\left. + \frac{6\pi t^2}{m_p^2}\frac{\sinh 2r\coth\left(\frac{\beta w}{2}\right)}{4}\left[\frac{\cos(\varphi - 2mt)}{mt^2} - \frac{2}{t}\sin(\varphi - 2mt)\right]\right]^{\frac{1}{3}}. \quad (6.24)$$

Similarly, the semiclassical Einstein equation for the thermal squeezed states can be written as

$$\left(\frac{\dot{S}(t)}{S(t)}\right)^2 = \frac{8\pi}{3m_p^2}\left[\left(f(\beta) + \sinh^2 r\coth(\frac{\beta w}{2}) + \frac{1}{2} + |\alpha|^2\right)\left(\dot{\phi}^*\dot{\phi} + m^2\phi^*\phi\right)\right.$$
$$+ \left(\frac{e^{-i\vartheta}\sinh r\cosh r\coth(\frac{\beta w}{2}) - \alpha^{*2}}{2}\right)(\dot{\phi}^{*2} + m^2\phi^{*2})$$
$$\left. + \left(\frac{e^{i\vartheta}\sinh r\cosh r\coth(\frac{\beta w}{2}) - \alpha^2}{2}\right)(\dot{\phi}^2 + m^2\phi^2)\right]. \quad (6.25)$$

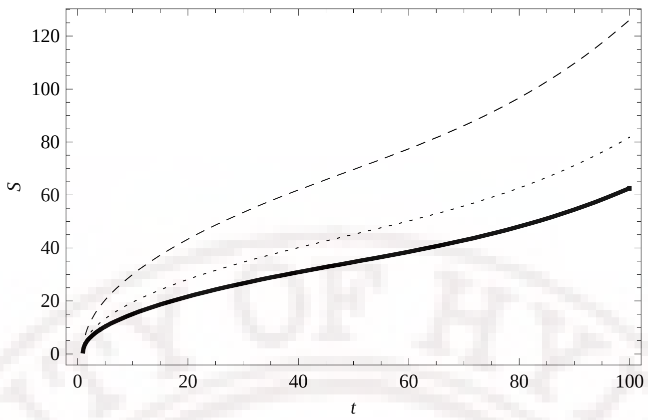

Figure 6.1: Plot for scale factor S and time t for flat FRW model in thermal coherent state with different values of coherent state parameter, thick line ($\alpha = .25$), the dot line ($\alpha = .75$) and the dash line ($\alpha = 1.5$) for temperature $T = 0.5$.

Following same procedure for the thermal squeezed vacuum state, the next order perturbative solution is obtained as

$$S_{1tss-flat} = \left[\frac{6\pi}{m_p^2}\left(f(\beta) + \sinh^2 r \coth\frac{\beta w}{2} + |\alpha|^2 + \frac{1}{2}\right) mt^2 \left(1 + \frac{1}{2mt^2}\right)\right.$$
$$+ \frac{6\pi t^2}{m_p^2}\frac{\sinh 2r}{4}\coth\frac{\beta w}{2}\left[\frac{cos(\vartheta - 2mt)}{mt^2} - \frac{2}{t}\sin(\vartheta - 2mt)\right]$$
$$\left. - \frac{3\alpha^2}{m}\frac{\pi t^2}{m_p^2}\left[\frac{\cos 2(\theta - mt)}{t^2} - \frac{2m}{t}\sin 2(\theta - mt)\right]\right]^{\frac{1}{3}}. \quad (6.26)$$

From the results such as (6.21), (6.24) and (6.26) it follows that $S_{1tcs-flat} \sim S_{1tsvs-flat} \sim S_{1tss-flat} \sim t^{2/3}$. Therefore, the approximate leading solutions of the semiclassical Einstein equation, in the oscillatory phase of the inflaton, in thermal coherent, squeezed vacuum and squeezed states, lead to the same power-law expansion $t^{2/3}$. Plots for the solutions in the flat FRW model for various values of the thermal coherent state parameter with temperatures $T = 0.5$ and $T = 10$ are given, Figures 6.1 and 6.2. Similarly plots for the solutions in the squeezed vacuum and squeezed states are provided with various values of squeezing and coherent states parameters in Figures 6.3 - 6.6.

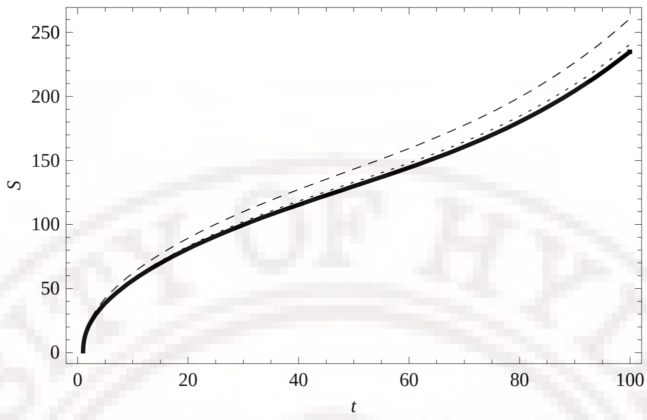

Figure 6.2: Plot for scale factor S and time t for flat FRW model in thermal coherent state with different values of coherent state parameter, thick line ($\alpha = .25$), the dot line ($\alpha = .75$) and the dash line ($\alpha = 1.5$) for temperature $T = 10$.

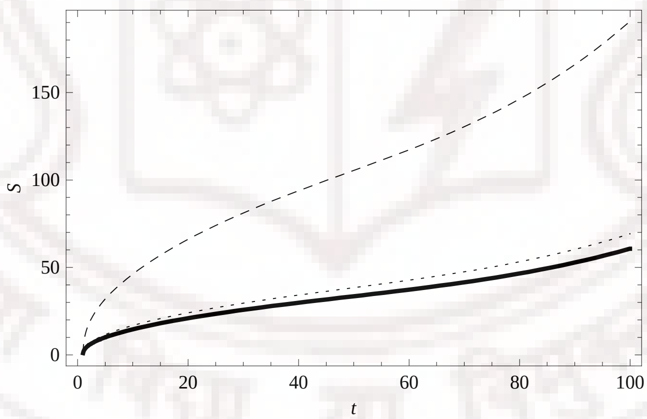

Figure 6.3: Plot for scale factor S and time t for flat FRW model in thermal squeezed vacuum state with different values of squeezing parameter, thick line ($r = .10$), the dot line ($r = .40$) and the dash line ($r = 1.5$) for temperature $T = 0.5$.

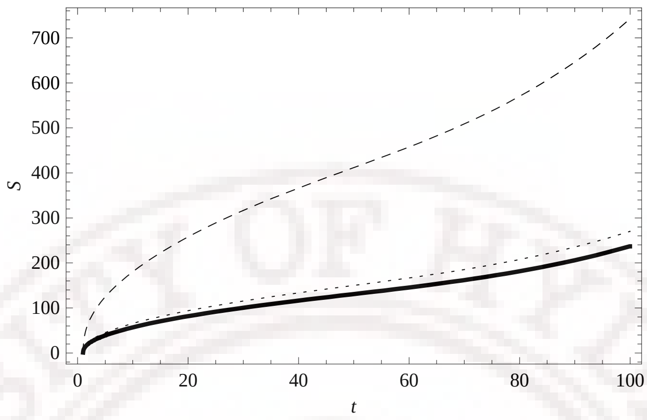

Figure 6.4: Plot for scale factor S and time t for flat FRW model in thermal squeezed vacuum state with different values of squeezing parameter, thick line (r = .10),the dot line (r = .40) and the dash line (r = 1.5) for temperature $T = 10$.

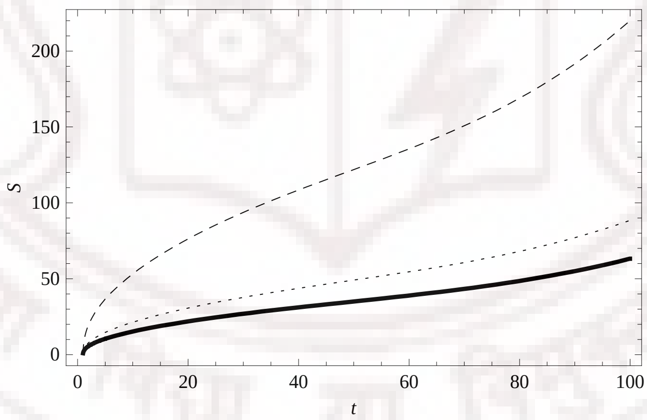

Figure 6.5: Plot for scale factor S and time t for flat FRW model in thermal squeezed state with different values of coherent and squeezing parameters, thick line (α = .25,r = .10),the dot line(α = .75, r = .40) and the dash line (α = 1.5, r = 1.5) for temperature $T = 0.5$.

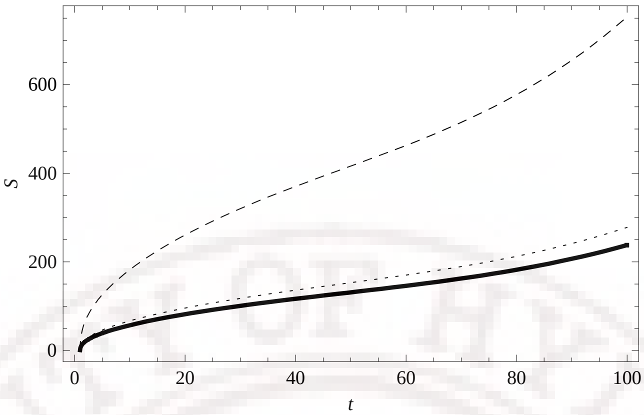

Figure 6.6: Plot for scale factor S and time t for flat FRW model in thermal squeezed state with different values of coherent and squeezing parameters, thick line ($\alpha = .25$, $r = .10$),the dot line ($\alpha = .75$, $r = .40$) and the dash line ($\alpha = 1.5$, $r = 1.5$) for temperature $T = 10$.

Next, we adopt the same method to obtain solutions of the semiclassical Friedmann equations for the open as well as closed FRW universe in the thermal coherent, squeezed vacuum and squeezed states.

6.2.2 In Open FRW Universe

The semiclassical Friedmann equation for the open FRW universe with the metric (3.3), in the thermal coherent state can be computed by using (3.4), (3.11), (6.13) and (6.16), we get

$$\left(\frac{\dot{S}}{S}\right)^2 - \frac{1}{S^2} = \frac{8\pi}{3m_p^2}\left[\left(f(\beta) + |\alpha|^2 + \frac{1}{2}\right)(\dot{\phi}^*\dot{\phi} + m^2\phi^*\phi) - \frac{1}{2}\alpha^{*2}(\dot{\phi}^{*2} + m^2\phi^{*2}) - \frac{1}{2}\alpha^2(\dot{\phi}^2 + m^2\phi^2)\right]. \quad (6.27)$$

Substituting the values of $\phi, \dot{\phi}, \dot{\phi}^*, \phi^*$ etc., subjected to the Wronskian condition (3.74) and applying the approximation ansatzs (6.19) and (6.20) we get

$$S(\dot{S}^2 - 1) = \frac{8\pi}{3m_p^2 \sinh^2\chi}\left[\left(f(\beta) + \alpha^2 + \frac{1}{2}\right)\left(\frac{1}{t^2} + 2m^2\right) - \frac{\alpha^2}{2mt^2}\right]. \quad (6.28)$$

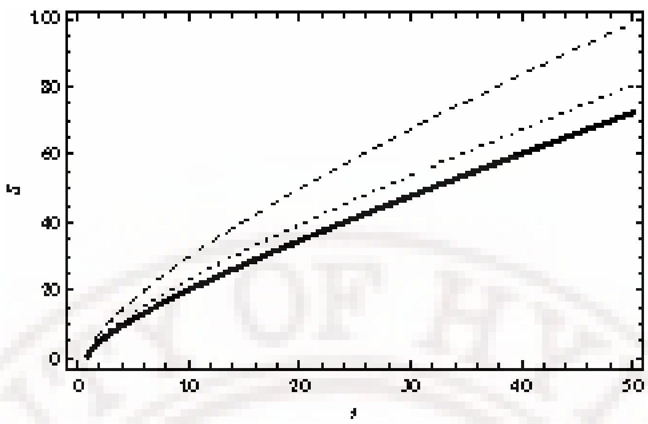

Figure 6.7: Plot for scale factor S and time t for open FRW model in thermal coherent state with different values of coherent state parameter, thick line ($\alpha = .25$),the dot line ($\alpha = .75$) and the dash line ($\alpha = 1.5$) for temperature $T = 0.5$.

We solve (6.28) numerically for different values of the thermal coherent state parameter with various ranges of temperature. The solutions are presented in Figures (6.7,6.8). Similarly, the semiclassical Friedmann equation for the open FRW universe in the thermal squeezed vacuum state is obtained as

$$S(\dot{S}^2 - 1) = \frac{8\pi}{3m_p^2 \sinh^2 \chi}\left[\left(f(\beta) + \sinh^2 r \coth\left(\frac{\beta w}{2}\right) + \frac{1}{2}\right)\right.$$
$$\left.\times \left(\frac{1}{t^2} + 2m^2\right) + \frac{\sinh 2r}{4mt^2}\coth\left(\frac{\beta w}{2}\right)\right]. \quad (6.29)$$

The corresponding numerical solutions for different values of the squeezing parameter are shown in Figures (6.9,6.10). Following the same procedure, the semiclassical Firedmann equation for the open FRW universe, in the thermal squeezed state is

$$S(\dot{S}^2 - 1) = \frac{8\pi}{3m_p^2 \sinh^2 \chi}\left[\left(f(\beta) + \sinh^2 r \coth\left(\frac{\beta w}{2}\right) + \frac{1}{2} + \alpha^2\right)\right.$$
$$\left.\times \left(\frac{1}{t^2} + 2m^2\right) + \frac{\sinh r \cosh r \coth\left(\frac{\beta w}{2}\right) - \alpha^2}{2mt^2}\right]. \quad (6.30)$$

We solve the above equation numerically for different values of α and r and the results are shown in Figures (6.11,6.12).

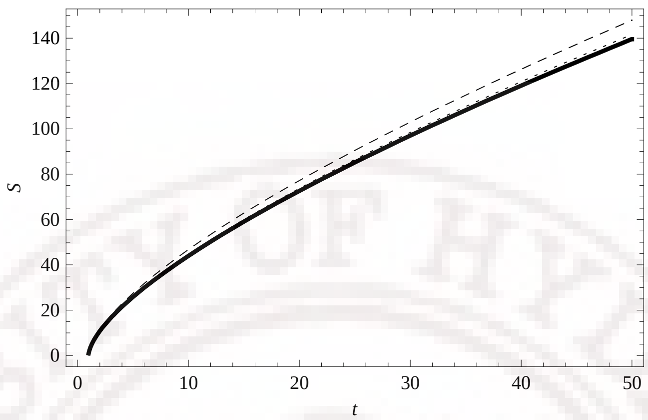

Figure 6.8: Plot for scale factor S and time t for open FRW model in thermal coherent state with different values of coherent state parameter, thick line ($\alpha = .25$),the dot line ($\alpha = .75$) and the dash line ($\alpha = 1.5$) for temperature $T = 10$.

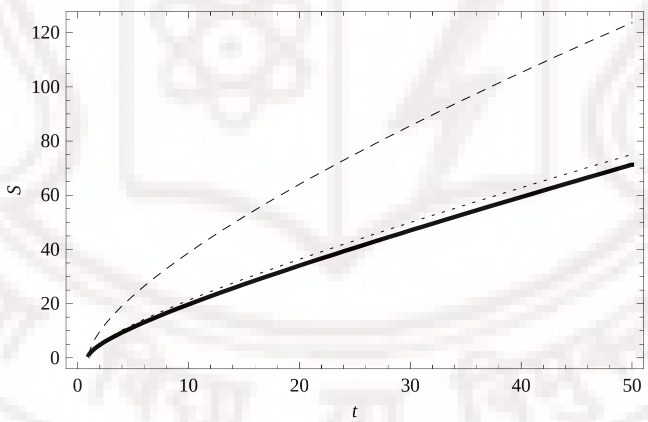

Figure 6.9: Plot for scale factor S and time t for open FRW model in thermal squeezed vacuum state with different values of squeezing parameter, thick line ($r = .10$),the dot line ($r = .40$) and the dash line ($r = 1.5$) for temperature $T = 0.5$.

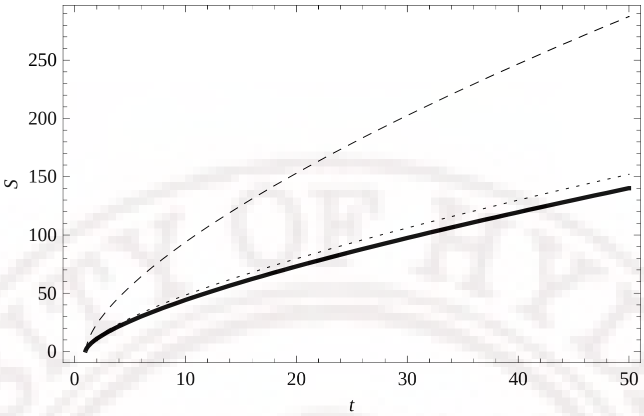

Figure 6.10: Plot for scale factor S and time t for open FRW model in thermal squeezed vacuum state with different values of squeezing parameter, thick line ($r = .10$),the dot line ($r = .40$) and the dash line($r = 1.5$) for temperature $T = 10$.

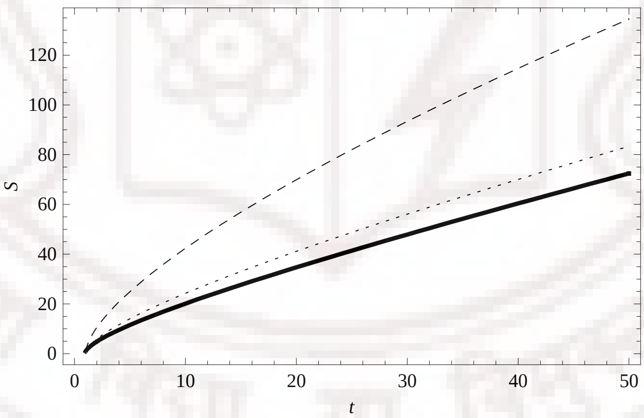

Figure 6.11: Plot for scale factor S and time t for open FRW model in thermal squeezed state with different values of coherent and squeezing parameters, thick line ($\alpha = .25$,$r = .10$),the dot line($\alpha = .75$, $r = .40$) and the dash line ($\alpha = 1.5$, $r = 1.5$) for temperature $T = 0.5$.

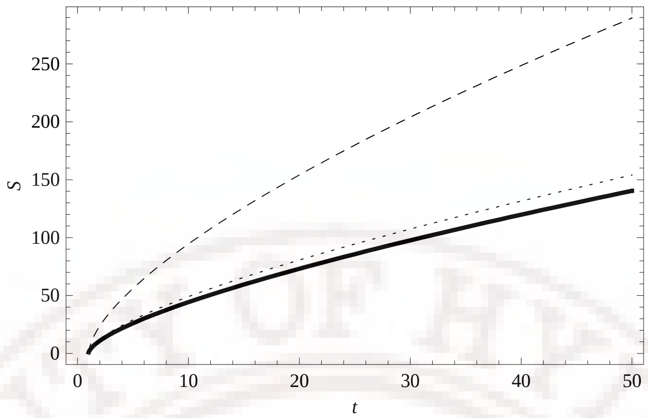

Figure 6.12: Plot for scale factor S and time t for open FRW model in thermal squeezed state with different values of coherent and squeezing parameters, thick line ($\alpha = .25$, $r = .10$),the dot line ($\alpha = .75$, $r = .40$) and the dash line ($\alpha = 1.5$, $r = 1.5$) for temperature $T = 10$.

6.2.3 In Closed FRW Universe

The Friedmann equation for the closed FRW universe with the metric (3.3), in the thermal coherent state, in the semiclassical theory take the following form

$$\left(\frac{\dot{S}}{S}\right)^2 + \frac{1}{S^2} = \frac{8\pi}{3m_p^2}\left[\left(f(\beta) + |\alpha|^2 + \frac{1}{2}\right)(\dot{\phi}^*\dot{\phi} + m^2\phi^*\phi) - \frac{1}{2}\alpha^{*2}(\dot{\phi}^{*2} + m^2\phi^{*2}) - \frac{1}{2}\alpha^2(\dot{\phi}^2 + m^2\phi^2)\right]. \tag{6.31}$$

In (6.31) ϕ and ϕ^* satisfy the equation of motion of the inflaton field and the Wronskian condition (3.88). Substitute the values of $\phi, \phi^*, \dot{\phi}, \dot{\phi}^*$ etc., are in the semiclassical Friedmann equation in the thermal coherent state and then applying the approximation ansatzs, we get

$$S(\dot{S}^2 + 1) = \frac{8\pi\sqrt{1 - \sinh^2\chi}}{3m_p^2 \sinh^2\chi \cosh\chi}\left[\left(f(\beta) + \alpha^2 + \frac{1}{2}\right)\left(\frac{1}{t^2} + 2m^2\right) - \frac{\alpha^2}{2mt^2}\right]. \tag{6.32}$$

We solve (6.32) numerically for different values of the thermal coherent state parameter and are shown in Figures (6.13,6.14). Following the similar

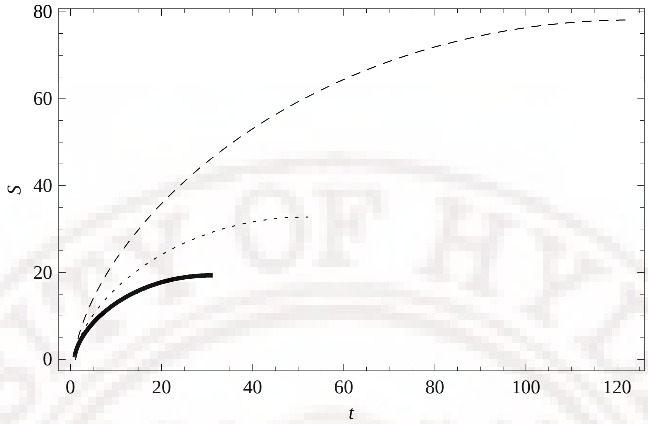

Figure 6.13: Plot for scale factor S and time t for closed FRW model in thermal coherent state with different values of coherent state parameter, thick line (α = .25),the dot line (α = .75) and the dash line (α = 1.5) for temperature $T = 0.5$.

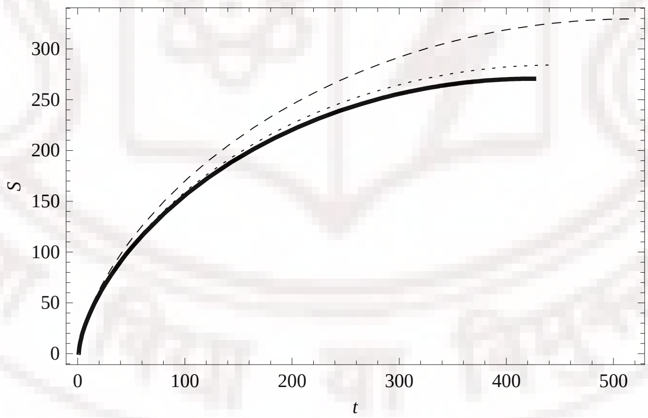

Figure 6.14: Plot for scale factor S and time t for closed FRW model in thermal coherent state with different values of coherent state parameter, thick line (α = .25),the dot line (α = .75) and the dash line (α = 1.5) for temperature $T = 10$.

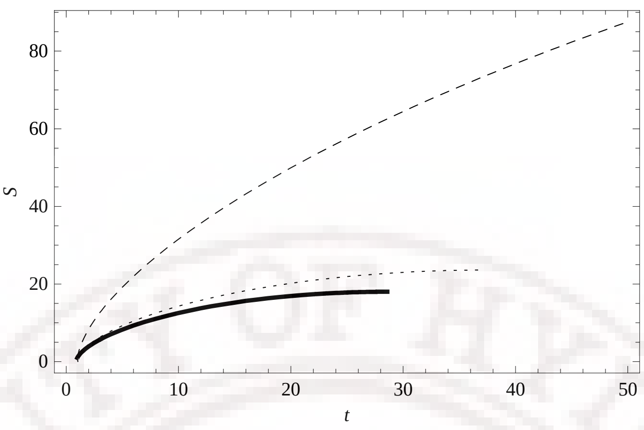

Figure 6.15: Plot for scale factor S and time t for closed FRW model in thermal squeezed vacuum state with different values of squeezing parameter, thick line (r = .10),the dot line (r = .40) and the dash line (r = 1.5) for temperature $T = 0.5$.

procedure, the Friedmann equation in the semiclassical theory in the squeezed vacuum state can be written as

$$S(\dot{S}^2+1) = \frac{8\pi\sqrt{1-\sinh^2\chi}}{3m_p^2\sinh^2\chi\cosh\chi}\left[\left(f(\beta)+\sinh^2 r\coth\left(\frac{\beta w}{2}\right)+\frac{1}{2}\right)\right.$$
$$\left.\times\left(\frac{1}{t^2}+2m^2\right)+\frac{\sinh 2r\coth\left(\frac{\beta w}{2}\right)}{4mt^2}\right], \tag{6.33}$$

we solve (6.33) numerically for different values of the thermal squeezing parameter and are shown in Figures (6.15,6.16). Similarly, by applying the same procedure in the thermal squeezed state, we obtained

$$S(\dot{S}^2+1) = \frac{8\pi\sqrt{1-\sinh^2\chi}}{3m_p^2\sinh^2\chi\cosh\chi}\left[\left(f(\beta)+\sinh^2 r\coth\left(\frac{\beta w}{2}\right)+\alpha^2+\frac{1}{2}\right)\right.$$
$$\left.\times\left(\frac{1}{t^2}+2m^2\right)+\frac{\sinh 2r\coth\left(\frac{\beta w}{2}\right)}{4mt^2}-\frac{\alpha^2}{2mt^2}\right]. \tag{6.34}$$

The numerical solutions of (6.34) for different values of α and r are as shown in Figures (6.17,6.18). Comparative studies of the three FRW models of the universe in the thermal coherent, squeezed vacuum and squeezed states for

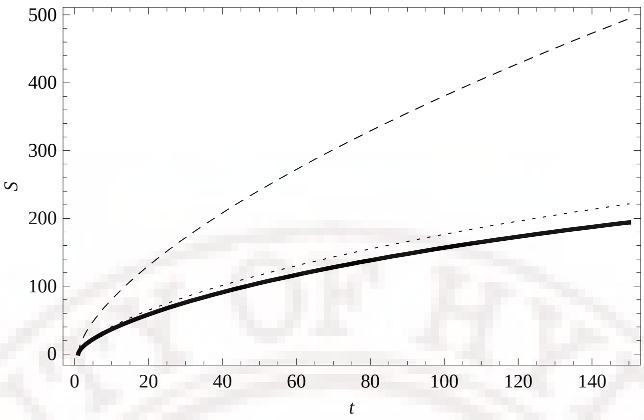

Figure 6.16: Plot for scale factor S and time t for closed FRW model in thermal squeezed vacuum state with different values of squeezing parameter, thick line ($r = .10$),the dot line ($r = .40$) and the dash line ($r = 1.5$) for temperature $T = 10$.

different values of their respective parameters for temperature $T = 0.5$ are shown in Figures (6.19-6.24) and for $T = 10$ are shown Figures (6.25-6.30).

6.3 Particle Production in Nonclassical Thermal States

This section studies the particle production in the oscillatory phase of the inflaton for the flat FRW universe by representing the inflaton in the thermal coherent and squeezed states, in semiclassical theory of gravity. The basic methodology of computing particle creation in the semiclassical theory of gravity is described in chapter 3. Thus, we adopt the same procedure for the thermal counterparts of the quantum optical states.

6.3.1 Flat FRW Universe

Consider the Fock space which has a one parameter dependence on the cosmological time t. Number of particles at a later time t produced from the

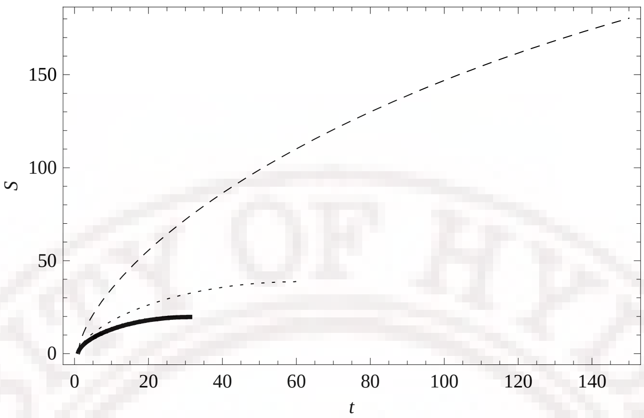

Figure 6.17: Plot for scale factor S and time t for closed FRW model in thermal squeezed state with different values of coherent and squeezing parameters, thick line ($\alpha = .25$,$r = .10$),the dot line ($\alpha = .75$,$r = .40$) and the dash line ($\alpha = 1.5$,$r = 1.5$) for temperature $T = 0.5$.

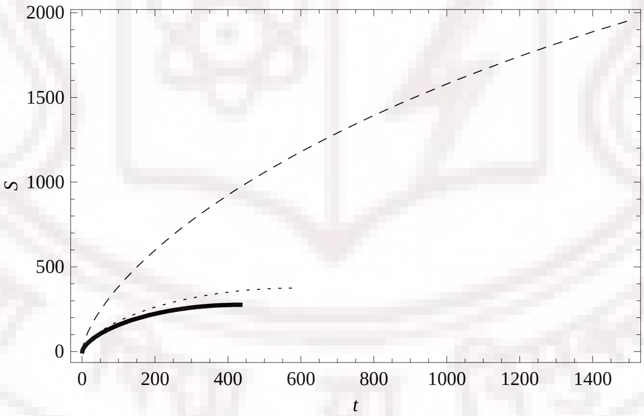

Figure 6.18: Plot for scale factor S and time t for closed FRW model in thermal squeezed state with different values of coherent and squeezing parameters, thick line ($\alpha = .25$,$r = .10$) the dot line ($\alpha = .75$,$r = .40$) and the dash line ($\alpha = 1.5$,$r = 1.5$) for temperature $T = 10$.

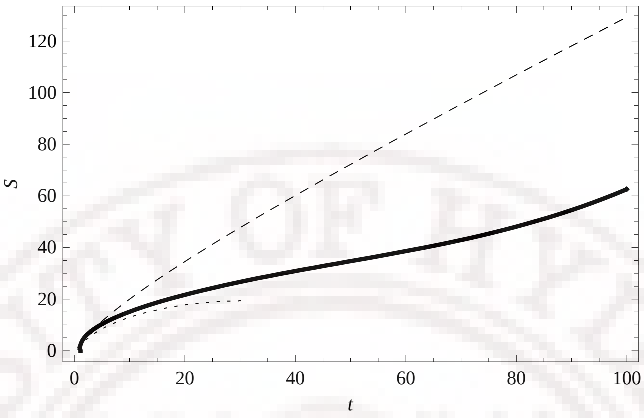

Figure 6.19: Plot for scale factor S and time t for $\alpha = .25$ in thermal coherent state for the semiclassical Friedmann models of the universe, closed(dot line), flat(thick line) and open(dash line) for temperature $T = 0.5$.

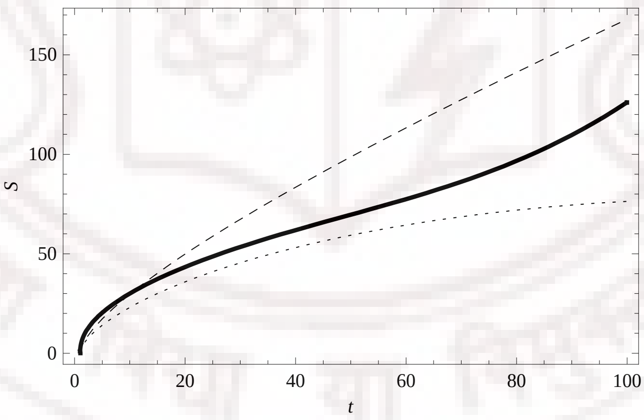

Figure 6.20: Plot for scale factor S and time t for $\alpha = .75$ in thermal coherent state for the semiclassical Friedmann models of the universe, closed(dot line), flat(thick line) and open(dash line) for temperature $T = 0.5$.

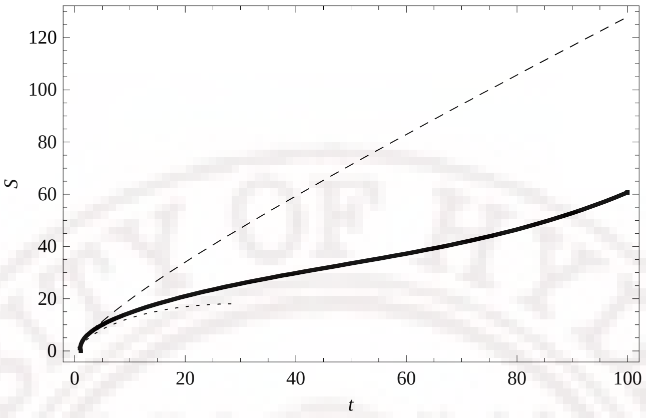

Figure 6.21: Plot for scale factor S and time t for $r = .10$ in thermal squeezed vacuum state for the semiclassical Friedmann models of the universe, closed(dot line), flat(thick line) and open (dash line) for temperature $T = 0.5$.

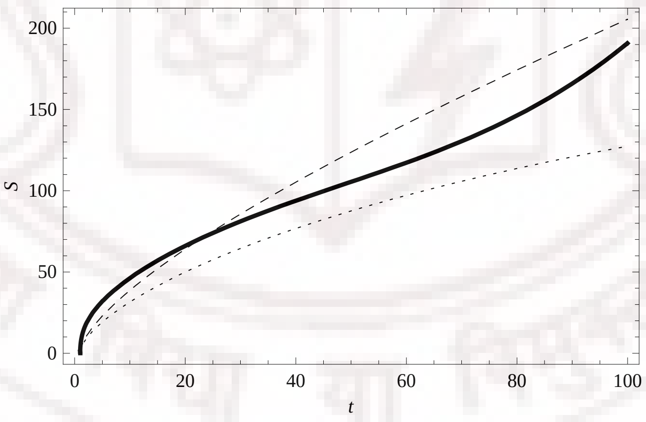

Figure 6.22: Plot for scale factor S and time t for $r = 1.5$ in thermal squeezed vacuum state for the semiclassical Friedmann models of the universe, closed (dot line), flat (thick line) and open (dash line) for temperature $T = 0.5$.

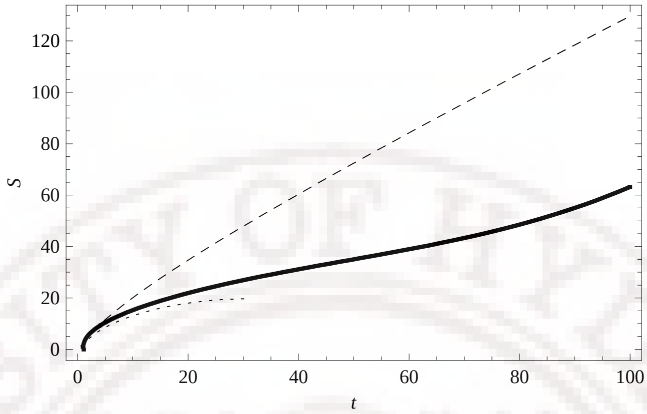

Figure 6.23: Plot for scale factor S and time t for $\alpha = .25$ and $r = .10$ in thermal squeezed state for the semiclassical Friedmann models of the universe, closed (dot line), flat (thick line) and open (dash line) for temperature $T = 0.5$.

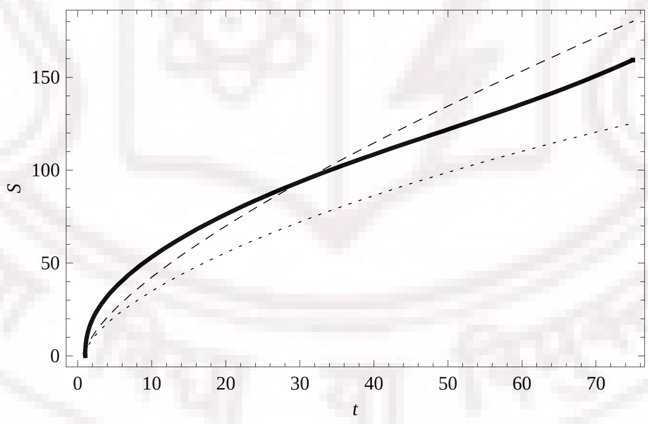

Figure 6.24: Plot for scale factor S and time t for $\alpha = 1.5$ and $r = 1.5$ in thermal squeezed state for the semiclassical Friedmann models of the universe, closed (dot line),flat (thick line) and open (dash line) for temperature $T = 0.5$.

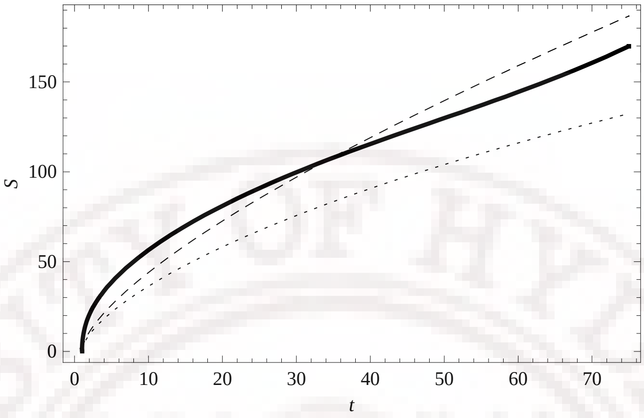

Figure 6.25: Plot for scale factor S and time t for $\alpha = .25$ in thermal coherent state for the semiclassical Friedmann models of the universe, closed (dot line), flat (thick line) and open (dash line) for temperature $T = 10$.

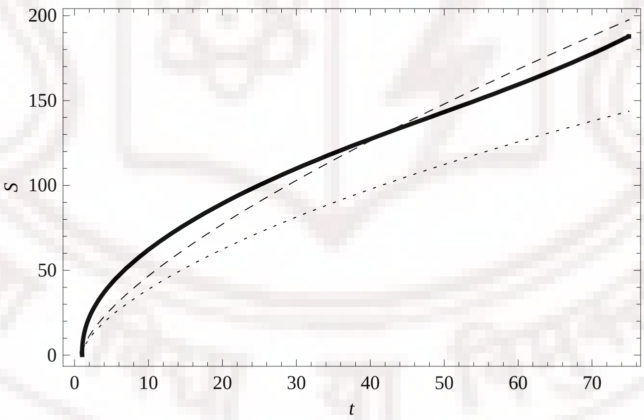

Figure 6.26: Plot for scale factor S and time t for $\alpha = .75$ in thermal coherent state for the semiclassical Friedmann models of the universe, closed (dot line), flat (thick line) and open (dash line) for temperature $T = 10$.

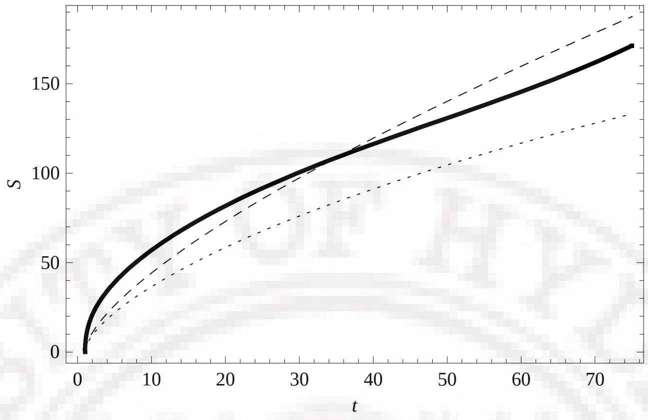

Figure 6.27: Plot for scale factor S and time t for $r = .10$ in thermal squeezed vacuum state for the semiclassical Friedmann models of the universe, closed (dot line), flat (thick line) and open (dash line) for temperature $T = 10$.

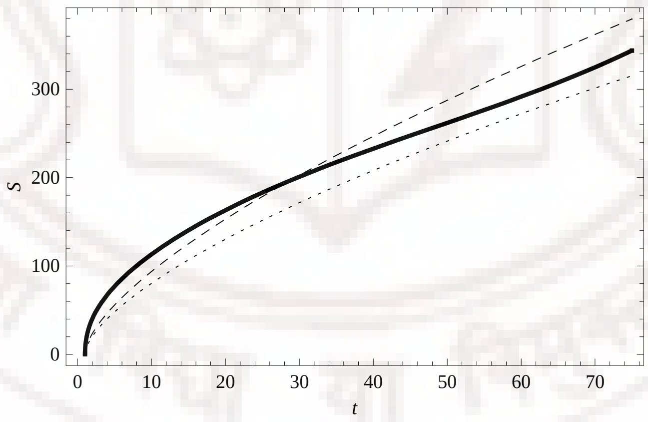

Figure 6.28: Plot for scale factor S and time t for $r = 1.5$ in thermal squeezed vacuum state for the semiclassical Friedmann models of the universe, closed (dot line),flat (thick line) and open (dash line) for temperature $T = 10$.

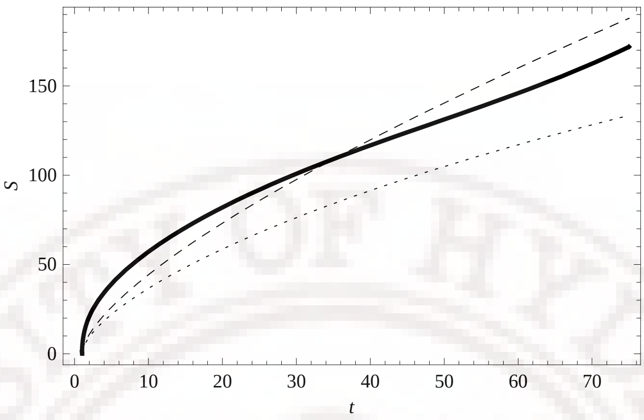

Figure 6.29: Plot for scale factor S and time t for $\alpha = .25$ and $r = .10$ in thermal squeezed state for the semiclassical Friedmann models of the universe, closed (dot line),flat (thick line) and open (dash line) for temperature $T = 10$.

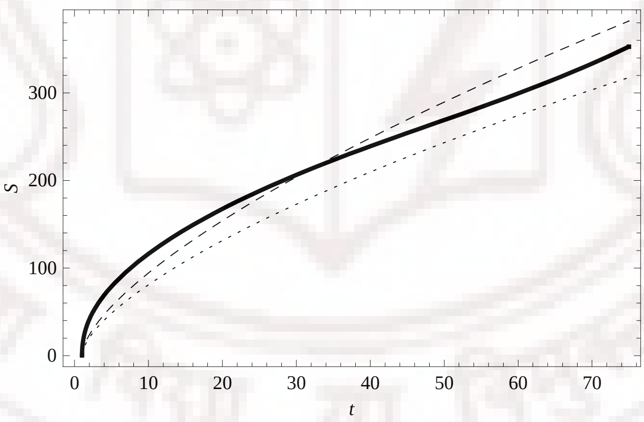

Figure 6.30: Plot for scale factor S and time t for $\alpha = 1.5$ and $r = 1.5$ in thermal squeezed state for the semiclassical Friedmann models of the universe, closed (dot line), flat (thick line) and open (dash line) for temperature $T = 10$.

vacuum at the initial time t_0 is given by

$$N_0(t,t_0) = \langle 0,\phi,t_0|\hat{N}(t)|0,\phi,t_0\rangle. \tag{6.35}$$

Here, $\hat{N}(t) = a^\dagger a$ and its expectation value can be calculated by using (3.17) as

$$\langle \hat{N}(t)\rangle = \phi\phi^*\langle\hat{\pi}^2\rangle - S^3\phi\dot{\phi}^*\langle\hat{\pi}\hat{\phi}\rangle - S^3\dot{\phi}\phi^*\langle\hat{\phi}\hat{\pi}\rangle + S^6\dot{\phi}\dot{\phi}^*\langle\hat{\phi}^2\rangle. \tag{6.36}$$

Here $\langle\hat{\pi}^2\rangle$, $\langle\hat{\pi}\hat{\phi}\rangle$, $\langle\hat{\phi}\hat{\pi}\rangle$ and $\langle\hat{\phi}^2\rangle$ are respectively obtained as.

$$\begin{aligned} \langle\hat{\pi}^2\rangle &= S^6\dot{\phi}^*\dot{\phi} & \langle\hat{\pi}\hat{\phi}\rangle &= S^3\dot{\phi}\phi^* \\ \langle\hat{\phi}\hat{\pi}\rangle &= S^3\phi\dot{\phi}^* & \langle\hat{\phi}^2\rangle &= \phi^*\phi. \end{aligned} \tag{6.37}$$

Therefore, substituting (6.37) in (6.36) we get

$$N_0(t,t_0) = S^6|\phi(t)\dot{\phi}(t_0) - \dot{\phi}(t)\phi(t_0)|^2. \tag{6.38}$$

Using the approximation ansatzs (6.19), (6.20) and (3.23), number of particles created at a later time t from the vacuum state at the initial time t_0 in the limit $mt_0, mt > 1$ can be computed and is given by

$$\begin{aligned} N_0(t,t_0) &= \frac{1}{4w(t)w(t_0)}\left(\frac{S(t)}{S(t_0)}\right)^3 \\ &\times \left[\frac{1}{4}\left(\frac{3\dot{S}(t)}{S(t)} - \frac{3\dot{S}(t_0)}{S(t_0)} - \frac{\dot{w}(t)}{w(t)} + \frac{\dot{w}(t_0)}{w(t_0)}\right)^2 + [w(t)-w(t_0)]^2\right] \\ &\simeq \frac{(t-t_0)^2}{4m^2t_0^4}. \end{aligned} \tag{6.39}$$

Using the properties and definition of the thermal coherent state, the expectation values of $\langle\hat{\pi}^2\rangle$, $\langle\hat{\pi}\hat{\phi}\rangle$, $\langle\hat{\phi}\hat{\pi}\rangle$ and $\langle\hat{\phi}^2\rangle$ in the thermal coherent state are obtained as

$$\begin{aligned} \langle\hat{\pi}^2\rangle_{tcs} &= S^6(t)\left[\left\{2(f(\beta)+|\alpha|^2)+1\right\}\dot{\phi}^*\dot{\phi} - \alpha^{*2}\dot{\phi}^{*2} - \alpha^2\dot{\phi}^2(t_0)\right] \\ \langle\hat{\phi}^2\rangle_{tcs} &= \left[2(f(\beta)+|\alpha|^2)+1\right]\phi^*\phi - \alpha^{*2}\phi^{*2} - \alpha^2\phi^2(t_0) \\ \langle\hat{\pi}\hat{\phi}\rangle_{tcs} &= S^3\left[(f(\beta)+|\alpha|^2)\dot{\phi}^*(t_0)\phi(t_0) + (f(\beta)+|\alpha|^2+1)\mathcal{C}\right] \\ \langle\hat{\phi}\hat{\pi}\rangle_{tcs} &= S^3\left[(f(\beta)+|\alpha|^2)\phi^*(t_0)\dot{\phi}(t_0) + (f(\beta)+|\alpha|^2+1)\mathcal{C}\right], \end{aligned} \tag{6.40}$$

where $\mathcal{C} = \dot{\phi}(t_0)\phi^*(t_0) - \alpha^2\dot{\phi}\phi - \alpha^{*2}\phi\dot{\phi}^*(t_0)$. Substituting (6.40) in (6.36) and using the ansatzs (6.19) and (6.20), number of particles (at a later time t) produced from the coherent state (at the initial time t_0) is obtained as

$$\begin{aligned} N_{tcs} &= (2f(\beta) + 2\alpha^2 + 1)N_0(t,t_0) + f(\beta) + \alpha^2 \\ &\quad - \left(\frac{\alpha^2}{2m^2}\right)\left(\frac{t}{t_0}\right)^2 \left[\frac{1}{t_0^2} + \frac{1}{t^2} - m^2\right]. \end{aligned} \tag{6.41}$$

Substituting the value of $N_0(t,t_0)$ in the above equation, we get

$$N_{tcs} \simeq (2f(\beta) + 2\alpha^2 + 1)\frac{(t-t_0)^2}{4m^2} + f(\beta) + \alpha^2 - \frac{\alpha^2}{2m^2t_0^4}\left[t^2 + t_0^2 - m^2t^2t_0^2\right]. \tag{6.42}$$

Analogously, number of particles at a later time t produced in the thermal squeezed vacuum state at the initial time t_0 can be obtained by using the following equations,

$$\begin{aligned} \langle\hat{\pi}^2\rangle_{tsvs} &= S^6(t)\left[\left\{2f(\beta) + 2\sinh^2 r\coth\left(\frac{\beta w}{2}\right) + 1\right\}\dot{\phi}^*\dot{\phi}\right. \\ &\quad \left. + e^{-i\vartheta}\cosh r\sinh r\coth\left(\frac{\beta w}{2}\right)\dot{\phi}^{*2} + e^{i\vartheta}\cosh r\sinh r\coth\left(\frac{\beta w}{2}\right)\dot{\phi}^2\right] \\ \langle\hat{\phi}^2\rangle_{tsvs} &= \left(2f(\beta) + 2\sinh^2 r\coth\left(\frac{\beta w}{2}\right) + 1\right)\phi^*\phi \\ &\quad + e^{-i\vartheta}\cosh r\sinh r\coth\left(\frac{\beta w}{2}\right)\phi^{*2} + e^{i\vartheta}\cosh r\sinh r\coth\left(\frac{\beta w}{2}\right)\phi^2 \end{aligned}$$

$$\begin{aligned} \langle\hat{\pi}\hat{\phi}\rangle_{tsvs} &= S^3\left[\left(f(\beta) + \sinh^2 r\coth\left(\frac{\beta w}{2}\right)\right)\dot{\phi}^*(t_0)\phi(t_0)\right. \\ &\quad + \left(1 + f(\beta) + \sinh^2 r\coth\left(\frac{\beta w}{2}\right)\right) \\ &\quad \left.\times\,\dot{\phi}(t_0)\phi^*(t_0) + \cosh r\sinh r\coth\left(\frac{\beta w}{2}\right)(e^{-i\vartheta}\dot{\phi}^*\phi + e^{i\vartheta}\dot{\phi}\phi)\right] \\ \langle\hat{\phi}\hat{\pi}\rangle_{tsvs} &= S^3\left[\left(f(\beta) + \sinh^2 r\coth\left(\frac{\beta w}{2}\right)\right)\phi^*\dot{\phi}\right. \\ &\quad + \left(1 + f(\beta) + \sinh^2 r\coth\left(\frac{\beta w}{2}\right)\right) \\ &\quad \left.\times\,\phi\dot{\phi}^* + \cosh r\sinh r\coth\left(\frac{\beta w}{2}\right)(e^{-i\vartheta}\phi\dot{\phi}^* + e^{i\vartheta}\phi\dot{\phi})\right]. \end{aligned} \tag{6.43}$$

Using (6.43) in (6.36) and applying the approximation ansatzs, we get number of particles produced in the thermal squeezed vacuum state as

$$N_{tsvs} \simeq \left(2f(\beta) + 2\sinh^2 r \coth\left(\frac{\beta w}{2}\right) + 1\right)\frac{(t-t_0)^2}{4m^2t_0^4} + f(\beta)$$
$$+ \sinh^2 r \coth\left(\frac{\beta w}{2}\right) + \frac{\sinh r \cosh r \coth\left(\frac{\beta w}{2}\right)}{2m^2t_0^4}\left[(t-t_0)^2 - m^2t^2t_0^2\right]. \tag{6.44}$$

Similarly, number of particles at a later time t produced from the squeezed state at the initial time t_0 is obtained as

$$N_{tss} \simeq \left(2f(\beta) + 2\alpha^2 + 1 + \sinh^2 r \coth\left(\frac{\beta w}{2}\right)\right)\frac{(t-t_0)^2}{4m^2t_0^4} + f(\beta) + \alpha^2$$
$$+ \sinh^2 r \coth\left(\frac{\beta w}{2}\right) + \frac{\sinh r \cosh r \cot\left(\frac{\beta w}{2}\right)}{2m^2t_0^4}\left[(t-t_0)^2 - m^2t^2t_0^2\right]$$
$$- \frac{\alpha^2}{2m^2t_0^4}\left[t^2 + t_0^2 - m^2t^2t_0^2\right]. \tag{6.45}$$

6.3.2 Open FRW Universe

Again, consider the Fock space which has a one parameter dependence on the cosmological time t. Number of particles at a later time t produced from the vacuum at the initial time t_0 is given by

$$N_0(t,t_0) = \langle 0,\phi,t_0|\hat{N}(t)|0,\phi,t_0\rangle. \tag{6.46}$$

Here, $\hat{N}(t) = \hat{a}^\dagger\hat{a}$ and its expectation value can be calculated as

$$\langle\hat{N}(t)\rangle = \phi(t)\phi^*(t)\langle\hat{\pi}^2\rangle - S^3(t)\sinh^2\chi\phi(t)\dot{\phi}^*(t)\langle\hat{\pi}\hat{\phi}\rangle$$
$$- S^3(t)\sinh^2\chi\dot{\phi}(t)\phi^*(t)\langle\hat{\phi}\hat{\pi}\rangle + S^6(t)\sinh^4\chi\dot{\phi}(t)\dot{\phi}^*(t)\langle\hat{\phi}^2\rangle. \tag{6.47}$$

Here, $\langle\hat{\pi}^2\rangle$, $\langle\hat{\pi}\hat{\phi}\rangle$, $\langle\hat{\phi}\hat{\pi}\rangle$ and $\langle\hat{\phi}^2\rangle$ are respectively obtained as

$$\langle\hat{\pi}^2\rangle = S^6(t)\sinh^4\chi\dot{\phi}^*\dot{\phi} \qquad \langle\hat{\pi}\hat{\phi}\rangle = S^3(t)\sinh^2\chi\dot{\phi}\phi^*$$
$$\langle\hat{\phi}\hat{\pi}\rangle = S^3(t)\sinh^2\chi\phi\dot{\phi}^* \qquad \langle\hat{\phi}^2\rangle = \phi^*\phi. \tag{6.48}$$

Using (6.48) in (6.47) we get

$$N_0(t,t_0) = S^6(t)\sinh^4\chi|\phi(t)\dot{\phi}(t_0) - \dot{\phi}(t)\phi(t_0)|^2. \tag{6.49}$$

By using the approximation ansatzs and (6.47), number of particles created at a later time t from the vacuum state at the initial time t_0 in the limit $mt_0, mt > 1$ can be calculated and is given by the same that we get in (6.39).

The expectation values of $< \hat{\pi}^2 >$, $< \hat{\pi}\hat{\phi} >$, $< \hat{\phi}\hat{\pi} >$ and $< \hat{\phi}^2 >$ in the thermal coherent state are obtained as

$$\begin{aligned}
\langle\hat{\pi}^2\rangle_{tcs-open} &= S^6(t)\sinh^4\chi\Big((2|\alpha|^2 + 2f(\beta) + 1)\dot{\phi}^*(t_0)\dot{\phi}(t_0) \\
&\quad - \alpha^{*2}\dot{\phi}^{*2}(t_0) - \alpha^2\dot{\phi}^2(t_0)\Big) \\
\langle\hat{\pi}\hat{\phi}\rangle_{tcs-open} &= S^3(t)\sinh^2\chi\Big[(f(\beta) + |\alpha|^2)\dot{\phi}^*(t_0)\phi(t_0) \\
&\quad +(f(\beta) + |\alpha|^2 + 1)\dot{\phi}(t_0)\phi^*(t_0) - \alpha^2\dot{\phi}(t_0)\phi(t_0) - \alpha^2\dot{\phi}^*(t_0)\phi(t_0)\Big] \\
\langle\hat{\phi}\hat{\pi}\rangle_{tcs-open} &= S^3(t)\sinh^2\chi\Big[(f(\beta) + |\alpha|^2)\phi^*(t_0)\dot{\phi}(t_0) + (f(\beta) + |\alpha|^2 + 1) \\
&\quad \phi(t_0)\dot{\phi}^*(t_0) - \alpha^2\phi(t_0)\dot{\phi}(t_0) - \alpha^{*2}\phi(t_0)\dot{\phi}^*(t_0)\Big] \qquad (6.50) \\
\langle\hat{\phi}^2\rangle_{tcs-open} &= (2|\alpha|^2 + 2f(\beta) + 1)\phi^*(t_0)\phi(t_0) - \alpha^{*2}\phi^{*2}(t_0) - \alpha^2\phi^2(t_0).
\end{aligned}$$

Substituting (6.50) in (6.47) and using the approximation ansatzs and number of particles (at a later time t) produced from the thermal coherent state (at the initial time t_0) is obtained as

$$\begin{aligned}
N_{tcs-open} &= (2f(\beta) + 2|\alpha|^2 + 1)N_0(t,t_0) + f(\beta) + |\alpha|^2 \\
&\quad - S^6(t)\sinh^4\chi\alpha^{*2}E - S^6(t)\sinh^4\chi\alpha^2 F
\end{aligned} \tag{6.51}$$

where

$$\begin{aligned}
E &= \phi(t)\phi^*(t)\dot{\phi}^{*2}(t_0) - \phi(t)\dot{\phi}^*(t)\dot{\phi}^*(t_0)\phi(t_0) \\
&\quad - \dot{\phi}(t)\phi^*(t)\phi(t_0)\dot{\phi}^*(t_0) + \dot{\phi}\dot{\phi}^*(t)\phi^{*2}(t_0) \\
&= \frac{\exp(2i\int m dt_0)}{4m^2 S_0^6 t^2 t_0^2 \sinh^4\chi}\left[\frac{1}{t_0^2} + \frac{1}{t^2} - \frac{2im}{t_0}\right] - \frac{1}{4m^2 S_0^6 t^2 t_0^2 \sinh^4\chi}\left[\frac{2}{tt_0} - \frac{2im}{t}\right]
\end{aligned} \tag{6.52}$$

$$\begin{aligned}
F &= \phi(t)\phi^*(t)\dot{\phi}^2(t_0) - \phi(t)\dot{\phi}^*(t)\dot{\phi}(t_0)\phi(t_0) \\
&\quad - \dot{\phi}(t)\phi^*(t)\phi(t_0)\dot{\phi}(t_0) + \dot{\phi}(t)\dot{\phi}^*(t)\phi^2(t_0) \\
&= \frac{\exp(-2i\int m dt_0)}{4m^2 S_0^6 t^2 t_0^2 \sinh^4\chi}\left[\frac{1}{t_0^2} + \frac{2im}{t_0} - 2m^2 - \frac{2}{tt_0} + \frac{1}{t^2}\right].
\end{aligned} \tag{6.53}$$

Using the approximation ansatzs and dropping the imaginary terms, (6.51) becomes

$$\begin{aligned}
N_{tcs-open} &= (2|\alpha|^2 + 2f(\beta) + 1)N_0(t,t_0) + f(\beta) + |\alpha|^2 \\
&\quad - \left(\frac{\alpha^2}{2m^2}\right)\left(\frac{t}{t_0}\right)^2 \left\{\frac{1}{t_0^2} + \frac{1}{t^2} - m^2\right\},
\end{aligned} \tag{6.54}$$

where $N_0(t, t_0)$ is given (6.39), thus we get

$$\begin{aligned}
N_{tcs-open} &\simeq (2|\alpha|^2 + 2f(\beta) + 1)\frac{(t-t_0)^2}{4m^2 t_0^4} + |\alpha|^2 + f(\beta) \\
&\quad - \frac{\alpha^2}{2m^2}\left(\frac{t}{t_0}\right)^2 \left\{\frac{1}{t_0^2} + \frac{1}{t^2} - m^2\right\}.
\end{aligned} \tag{6.55}$$

Analogously, number of particles at a later time t produced from thermal squeezed vacuum state at the initial time t_0 can be obtained using the following equations

$$\langle\hat{\pi}^2\rangle_{tsvs-open} = S^6(t)\sinh^4\chi\left[\left\{2f(\beta)+2\sinh^2 r\coth\left(\frac{\beta w}{2}\right)+1\right\}\dot{\phi}^*\dot{\phi}\right.$$
$$+\,e^{-i\vartheta}\cosh r\sinh r\coth\left(\frac{\beta w}{2}\right)\dot{\phi}^{*2}$$
$$\left.+e^{i\vartheta}\cosh r\sinh r\coth\left(\frac{\beta w}{2}\right)\dot{\phi}^2\right]$$
$$\langle\hat{\phi}^2\rangle_{tsvs-open} = \left(2f(\beta)+2\sinh^2 r\coth\left(\frac{\beta w}{2}\right)+1\right)\phi^*\phi$$
$$+\cosh r\sinh r\coth\frac{\beta w}{2}\left[e^{-i\vartheta}\phi^{*2}+e^{i\vartheta}\phi^2\right]$$
$$\langle\hat{\pi}\hat{\phi}\rangle_{tsvs-open} = S^3(t)\sinh^4\chi\left[\left(f(\beta)+2\sinh^2 r\coth\left(\frac{\beta w}{2}\right)\right)\dot{\phi}^*(t_0)\phi(t_0)\right.$$
$$+\left(1+f(\beta)+\sinh^2 r\coth\frac{\beta w}{2}\right)\dot{\phi}(t_0)\phi^*(t_0)$$
$$\left.+\cosh r\sinh r\coth\frac{\beta w}{2}\left(e^{-i\vartheta}\dot{\phi}^*\phi+e^{i\vartheta}\dot{\phi}\phi\right)\right]$$

$$\langle\hat{\phi}\hat{\pi}\rangle_{tsvs-open} = S^3(t)\sinh^2\chi\left[\left(f(\beta)+\sinh^2 r\coth\left(\frac{\beta w}{2}\right)\right)\phi^*\dot{\phi}\right.$$
$$+\left(1+f(\beta)+\sinh^2 r\coth\left(\frac{\beta w}{2}\right)\right)\phi\dot{\phi}^*$$
$$\left.+\cosh r\sinh r\coth\frac{\beta w}{2}\left(e^{-i\vartheta}\phi\dot{\phi}^*+e^{i\vartheta}\phi\dot{\phi}\right)\right]. \tag{6.56}$$

The equations (6.56) are substituted in (6.47) and applying the approximation ansatzs then simplify, number of particles produced in the thermal squeezed vacuum state is obtained as

$$N_{tsvs-open} \simeq \left(2f(\beta)+2\sinh^2 r\coth\frac{\beta w}{2}+1\right)\frac{(t-t_0)^2}{4m^2t_0^4}+f(\beta)$$
$$+\sinh^2 r\coth\frac{\beta w}{2}+\frac{\sinh r\cosh r\coth\frac{\beta w}{2}}{2m^2t_0^4}\left[(t-t_0)^2-m^2t^2t_0^2\right]. \tag{6.57}$$

Similarly, number of particles at a later time t produced from thermal squeezed state at the initial time t_0 can be obtained using its corresponding properties and following the same procedure as earlier and using the approximation ansatzs, we get number of particles produced as

$$N_{tss-open} \simeq \left(2f(\beta) + 2|\alpha|^2 + 1 + 2\sinh^2 r \coth\frac{\beta w}{2}\right)\frac{(t-t_0)^2}{4m^2t_0^4} + f(\beta) + |\alpha|^2 + \sinh^2 r \coth\frac{\beta w}{2} + \frac{\sinh r \cosh r \coth\frac{\beta w}{2}}{2m^2t_0^4} \times \left[(t-t_0)^2 - m^2t^2t_0^2\right] - \frac{\alpha^2}{2m^2t_0^4}\left[t^2 + t_0^2 - m^2t^2t_0^2\right]. \quad (6.58)$$

6.3.3 Closed FRW Universe

As it is in the earlier cases, consider the Fock space which has a one parameter dependence on the cosmological time t. Number of particles at a later time t produced from the vacuum at the initial time t_0 is given by

$$N_0(t,t_0) = \langle 0,\phi,t_0|\hat{N}(t)|0,\phi,t_0\rangle. \quad (6.59)$$

Here $\hat{N}(t) = \hat{a}^\dagger\hat{a}$ and its expectation value can be calculated by the following way

$$\langle\hat{N}(t)\rangle = \phi(t)\phi^*(t)\langle\hat{\pi}^2\rangle - \frac{S^3(t)\sinh^2\chi\cosh\chi}{\sqrt{1-\sinh^2\chi}}\phi(t)\dot{\phi}^*(t)\langle\hat{\pi}\hat{\phi}\rangle - \frac{S^3(t)\sinh^2\chi\cosh\chi}{\sqrt{1-\sinh^2\chi}}\dot{\phi}(t)\phi^*(t)\langle\hat{\phi}\hat{\pi}\rangle + \frac{S^6(t)\sinh^4\chi\cosh^2\chi}{(1-\sinh^2\chi)}\dot{\phi}(t)\dot{\phi}^*(t)\langle\hat{\phi}^2\rangle. \quad (6.60)$$

In equation (6.60) the expectation values of $\hat{\pi}^2$ and $\hat{\phi}^2$ are given in the thermal coherent state as

$$\langle\hat{\pi}^2\rangle_{tcs-closed} = \frac{S^6(t)\sinh^4\chi\cosh^2\chi}{(1-\sinh^2\chi)}\Big((2|\alpha|^2 + 2f(\beta) + 1)\dot{\phi}^*(t_0)\dot{\phi}(t_0) - \alpha^{*2}\dot{\phi}^{*2}(t_0) - \alpha^2\dot{\phi}^2(t_0)\Big) \quad (6.61)$$

$$\langle\hat{\phi}^2\rangle_{tcs-closed} = (2|\alpha|^2 + 2f(\beta) + 1)\phi^*(t_0)\phi(t_0) - \alpha^{*2}\phi^{*2}(t_0) - \alpha^2\phi^2(t_0). \quad (6.62)$$

The expectation values of $\hat{\pi}\hat{\phi}$ and $\hat{\phi}\hat{\pi}$ in the thermal coherent state are given by

$$\langle\hat{\pi}\hat{\phi}\rangle_{tcs-closed} = \frac{S^3(t)\sinh^2\chi\cosh\chi}{\sqrt{(1-\sinh^2\chi)}}\Big[(f(\beta)+|\alpha|^2)\dot{\phi}^*(t_0)\phi(t_0) + (f(\beta)+|\alpha|^2+1)\dot{\phi}\phi^* - \alpha^2\dot{\phi}(t_0)\phi(t_0) - \alpha^{*2}\dot{\phi}^*(t_0)\phi(t_0)\Big] \tag{6.63}$$

$$\langle\hat{\phi}\hat{\pi}\rangle_{tcs-closed} = \frac{S^3(t)\sinh^2\chi\cosh\chi}{\sqrt{(1-\sinh^2\chi)}}\Big[(f(\beta)+|\alpha|^2)\phi^*(t_0)\dot{\phi}(t_0) + (f(\beta) + |\alpha|^2+1)\phi(t_0)\dot{\phi}^*(t_0) - \alpha^2\phi(t_0)\dot{\phi}(t_0) - \alpha^{*2}\phi(t_0)\dot{\phi}(t_0)\Big] \tag{6.64}$$

By substituting equations (6.61), (6.62), (6.63), (6.64) in (6.60) and using approximation ansatzs, number of particles produced at a later time t in the thermal coherent state at the initial time t_0 is obtained as

$$N_{tcs-closed} = (2|\alpha|^2 + 2f(\beta)+1)N_0(t,t_0) + f(\beta) + |\alpha|^2 - \frac{S^6\sinh^4\chi\cosh^2\chi}{(1-\sinh^2\chi)}\alpha^{*2}E - \frac{S^6\sinh^4\chi\cosh^2\chi}{(1-\sinh^2\chi)}\alpha^2 F \tag{6.65}$$

where E and F are given by

$$\begin{aligned} E &= \phi(t)\phi^*(t)\dot{\phi}^{*2}(t_0) - \phi(t)\dot{\phi}^*(t)\dot{\phi}^*(t_0)\phi(t_0) \\ &\quad - \dot{\phi}(t)\phi^*(t)\phi(t_0)\dot{\phi}^*(t_0) + \dot{\phi}\dot{\phi}^*(t)\phi^{*2}(t_0) \\ &= \frac{\exp(2i\int m dt_0)(1-\sinh^2\chi)}{4m^2S_0^6t^2t_0^2\sinh^4\chi\cosh^2\chi}\left[\frac{1}{t_0^2}+\frac{1}{t^2}-\frac{2im}{t_0}\right] \\ &\quad - \frac{(1-\sinh^2\chi)}{4m^2S_0^6t^2t_0^2\sinh^4\chi\cosh^2\chi}\left[\frac{2}{tt_0}-\frac{2im}{t}\right] \end{aligned} \tag{6.66}$$

$$F = \frac{\exp(-2i\int m dt_0)(1-\sinh^2\chi)}{4m^2S_0^6t^2t_0^2\sinh^4\chi\cosh\chi}\left[\frac{1}{t_0^2}+\frac{2im}{t_0}-2m-\frac{2}{tt_0}+\frac{1}{t^2}\right]. \tag{6.67}$$

Using the approximation ansatzs and dropping the imaginary terms then substituting the value of $N_0(t,t_0)$, number of particles produced at a later time t_0 in the thermal coherent state in the closed FRW universe can be written as

$$N_{tcs-closed} \simeq (2|\alpha|^2+2f(\beta)+1)\frac{(t-t_0)^2}{4m^2t_0^4}+|\alpha|^2+f(\beta)-\frac{\alpha^2}{2m^2t_0^4}[t^2+t_0^2-m^2t^2t_0^2]. \tag{6.68}$$

Analogously, number of particles at a later time t produced from squeezed vacuum state at the initial time t_0 can be obtained using the following equations

$$\begin{aligned}
\langle\hat{\pi}^2\rangle_{tsvs-closed} &= \frac{S^6(t)\sinh^4\chi\cosh^2\chi}{(1-\sinh^2\chi)} \\
&\quad \left[\left(2f(\beta)+2\sinh^2 r\coth\frac{\beta w}{2}+1\right)\dot{\phi}^*(t_0)\dot{\phi}(t_0)\right. \\
&\quad \left.+\sinh r\cosh r\coth\frac{\beta w}{2}\left(e^{-i\vartheta}\dot{\phi}^{*2}(t_0)+e^{i\vartheta}\dot{\phi}^2(t_o)\right)\right] \\
\langle\hat{\phi}^2\rangle_{tsvs-closed} &= \left(2f(\beta)+2\sinh^2 r\coth\frac{\beta w}{2}+1\right)\phi^*(t_0)\phi(t_0) \\
&\quad +\sinh r\cosh r\coth\frac{\beta w}{2}\left[e^{-i\vartheta}\dot{\phi}^{*2}(t_0)+e^{i\vartheta}\dot{\phi}^2(t_0)\right]
\end{aligned}$$

$$\begin{aligned}
\langle\hat{\pi}\hat{\phi}\rangle_{tsvs-closed} &= \frac{S^3(t)\sinh^2\chi\cosh\chi}{\sqrt{(1-\sinh^2\chi)}}\left[\left(f(\beta)+\sinh^2 r\coth\frac{\beta w}{2}\right)\dot{\phi}^*(t_0)\phi(t_0)\right. \\
&+\left(1+f(\beta)+\sinh^2 r\coth\frac{\beta w}{2}\right)\dot{\phi}(t_0)\phi^*(t_0) \\
&\left.+\cosh r\sinh r\coth\frac{\beta w}{2}\left(e^{-i\vartheta}\dot{\phi}^*\phi+e^{i\vartheta}\dot{\phi}\phi\right)\right] \\
\langle\hat{\phi}\hat{\pi}\rangle_{tsvs-closed} &= \frac{S^3(t)\sinh^2\chi\cosh\chi}{\sqrt{(1-\sinh^2\chi)}}\left[\left(f(\beta)+\sinh^2 r\coth\frac{\beta w}{2}\right)\phi^*\dot{\phi}\right. \\
&+\left(1+f(\beta)+\sinh^2 r\coth\frac{\beta w}{2}\right)\phi\dot{\phi}* \\
&\left.+\cosh r\sinh r\coth\frac{\beta w}{2}\left(e^{-i\vartheta}\phi\dot{\phi}^*+e^{i\vartheta}\phi\dot{\phi}\right)\right].
\end{aligned} \tag{6.69}$$

By substituting (6.69) in (6.60) and using the approximation ansatzs then simplifying, we obtain number of particles produced in the thermal squeezed vacuum state as

$$N_{tsvs-closed} \simeq \left(2f(\beta) + 2\sinh^2 r \coth\frac{\beta w}{2} + 1\right)\frac{(t-t_0)^2}{4m^2t_0^4} + f(\beta) + \sinh^2 r \coth\frac{\beta w}{2} + \frac{\sinh r \cosh r \coth\frac{\beta w}{2}}{2m^2t_0^4}\left[(t-t_0)^2 - m^2t^2t_0^2\right]. \tag{6.70}$$

Similarly by adopting the same procedure in the case of the thermal squeezed state, number of particles at a later time t produced from the thermal squeezed state at initial time t_0 is computed and it leads to the following result

$$N_{tss-closed} \simeq \left(2f(\beta) + 2\sinh^2 r \coth\frac{\beta w}{2} + 1 + 2|\alpha|^2\right)\frac{(t-t_0)^2}{4m^2t_0^4} + f(\beta) + \sinh^2 r \coth\frac{\beta w}{2} + \frac{\sinh r \cosh r \coth\frac{\beta w}{2}}{2m^2t_0^4}\left[(t-t_0)^2 - m^2t^2t_0^2\right] - \frac{\alpha^2}{2m^2t_0^4}[t^2 + t_0^2 - m^2t^2t_0^2]. \tag{6.71}$$

6.4 Validity of Semiclassical Theory in Thermal States

In this part of the chapter, we examine validity of the semiclassical Einstein equation for the flat FRW universe in the thermal coherent and squeezed vacuum states. Since the thermal coherent and squeezed vacuum states exhibit quantum and thermal effects, it would be much useful to examine validity of the semiclassical theory in the oscillatory phase of inflaton. Validity of semiclassical Einstein equation can be examined with the help of a dimensionless quantity (5.1) defined in terms of the energy momentum tensor, as in chapter 5. Again, computation of the fluctuations of the energy momentum tensor in a given quantum state for all the modes (k) would be cumbersome. Therefore for the sake of simplicity of study, we focus on the temporal component of the energy momentum tensor with a single mode of the inflaton and investigate validity of the semiclassical theory, in the thermal coherent and squeezed vacuum states.

6.4.1 In Thermal Coherent State

The density fluctuations in the thermal coherent state for the flat FRW mode in the oscillatory phase of inflaton can be studied with the help of the dimensionless quantity obtained from (5.1), given by

$$\Delta_{tcs} = \left| \frac{<: T_{00}^2 :>_{tcs} - <: T_{00} :>_{tcs}^2}{<: T_{00} :>_{tcs}^2} \right| . \tag{6.72}$$

The first term in the above equation can be obtained by squaring the temporal component of energy momentum tensor and taking the expectation value in thermal coherent state as

$$\begin{aligned} <: T_{00}^2 :>_{tcs} = & \frac{1}{4S^6(t)} <: \hat{\pi}^4 :>_{tcs} + \frac{m^2}{4} <: \hat{\pi}^2 \hat{\phi}^2 :>_{tcs} \\ & + \frac{m^2}{4} <: \hat{\phi}^2 \hat{\pi}^2 :>_{tcs} + \frac{m^4}{4} S^6(t) <: \hat{\phi}^4 :>_{tcs} . \end{aligned} \tag{6.73}$$

Then using the properties of the thermal coherent state and applying the approximation ansatzs, $<: \hat{\pi}^4 :>_{tcs}$, $<: \hat{\pi}^2 \hat{\phi}^2 :>_{tcs}$, $<: \hat{\phi}^2 \hat{\pi}^2 :>_{tcs}$ and $<: \hat{\phi}^4 :>_{tcs}$ can be calculated and are respectively obtained as

$$<: \hat{\pi}^4 :>_{tcs} = \frac{S_0^6}{4m^2}(4 + 12f(\beta) + 12f^2(\beta) + 4\alpha^4 + 4\alpha^2) \tag{6.74}$$

$$<: \hat{\pi}^2 \hat{\phi}^2 :>_{tcs} = \frac{1}{4m^2t^2}(4 + 12f(\beta) + 12f^2(\beta) + 4\alpha^4 + 4\alpha^2) \tag{6.75}$$

$$<: \hat{\phi}^2 \hat{\pi}^2 :>_{tcs} = \frac{1}{4m^2t^2}(4 + 12f^2(\beta) + 12f(\beta) + 4\alpha^4 + 4\alpha^2) \tag{6.76}$$

$$<: \hat{\phi}^4 :>_{tcs} = \frac{1}{4m^2S_0^6t^4}(4 + 12f^2(\beta) + 12f(\beta) + 4\alpha^4 + 4\alpha^2). \tag{6.77}$$

Thus

$$<: T_{00}^2 :>_{tcs} = \left(\frac{1}{16m^2t^4} + \frac{1}{8t^2} + \frac{m^2}{16} \right) \left(4 + 12f(\beta) + 12f^2(\beta) + 4\alpha^4 + 4\alpha^2 \right) . \tag{6.78}$$

The expectation value of the normal ordered time-time component of energy momentum tensor of the inflaton in the thermal coherent state is computed and then squaring the result leads to

$$\begin{aligned}<:T_{00}:>^2_{tcs} = S^6 \Bigg[& \left(|\alpha|^4 + f^2(\beta) + \frac{1}{4} + 2|\alpha|^2 f(\beta) + f(\beta) + |\alpha|^2 \right) \\ & \times (\dot{\phi}^{*2}\dot{\phi}^2 + m^4\phi^{*2}\phi^2 + 2m^2\dot{\phi}^*\dot{\phi}\phi^*\phi) \\ & + \frac{1}{4}\alpha^{*4}(\dot{\phi}^{*4} + m^4\phi^{*4} + 2m^2\dot{\phi}^{*2}\phi^{*2}) \\ & + \frac{1}{4}\alpha^4(\dot{\phi}^4 + m^4\phi^4 + 2m^2\dot{\phi}^2\phi^2) \\ & - \left(|\alpha|^2\alpha^{*2} + f(\beta)\alpha^{*2} + \frac{1}{2}\alpha^{*2} \right) \\ & \times (\dot{\phi}^*\dot{\phi}\dot{\phi}^{*2} + m^2\dot{\phi}^*\dot{\phi}\phi^{*2} + m^2\phi^*\phi\dot{\phi}^{*2} + m^4\phi^*\phi\phi^{*2}) \\ & + \frac{1}{2}\alpha^{*2}\alpha^2(\dot{\phi}^{*2}\dot{\phi}^2 + m^2\dot{\phi}^{*2}\phi^2 + m^2\phi^{*2}\dot{\phi}^2 + m^4\phi^{*2}\phi^2) \\ & - \left(\alpha^2|\alpha|^2 + \frac{1}{2}\alpha^2 \right)(\dot{\phi}^2\dot{\phi}^*\dot{\phi} + m^2\dot{\phi}^2\phi^*\phi + m^2\phi^2\dot{\phi}^*\dot{\phi} + m^4\phi^2\phi^*\phi) \Bigg]. \end{aligned} \tag{6.79}$$

Substitute the values of $\phi, \dot{\phi}, \dot{\phi}^*, \phi^* \cdots$ etc., and applying the approximation ansatzs, we get

$$\begin{aligned} <:T_{00}:>^2_{tcs} = & \left(\frac{1}{16m^2t^4} + \frac{m^2}{16} + \frac{1}{8t^2} \right) \left[4f^2(\beta) + 1 + 4\alpha^2 f(\beta) \right. \\ & \left. + 4f(\beta) + 8\alpha^4 + 2\alpha^2 \right]. \end{aligned} \tag{6.80}$$

By substituting (6.78) and (6.80) in (6.72) we get

$$\Delta_{tcs} = \left| \frac{8f^2(\beta) + 8f(\beta) - 4\alpha^2 f(\beta) - 4\alpha^4 + 2\alpha^2 + 3}{4f^2(\beta) + 4f(\beta) + 4\alpha^2 f(\beta) + 8\alpha^4 + 2\alpha^2 + 1} \right|. \tag{6.81}$$

The equation (6.81) can be examined for validity of the semiclassical Einstein equation, so we study Δ_{tcs} numerically with the associated coherent state parameter for a range of temperature and the results are tabulated in Tables 6.1 and 6.2.

From the numerical study of the dimensionless quantity, for the thermal coherent state in the oscillatory phase of the inflaton, for the flat FRW

T=0.5		T=1.5		T=2.5		T=3	
α	Δ_{tcs}	α	Δ_{tcs}	α	Δ_{tcs}	α	Δ_{tcs}
1.0	0.1475	1.0	0.5912	1.0	0.9635	1.0	1.0959
1.1	0.0153	1.1	0.4083	1.1	0.7868	1.1	0.9307
1.2	0.0845	1.2	0.2514	1.2	0.6194	1.2	0.7690
1.3	0.1604	1.3	0.1201	1.3	0.4654	1.3	0.6152
1.4	0.2187	1.4	0.0120	1.4	0.3272	1.4	0.4726
1.5	0.2641	1.5	0.0763	1.5	0.2056	1.5	0.3432
1.6	0.2998	1.6	0.1480	1.6	0.1002	1.6	0.2278
1.7	0.3282	1.7	0.2062	1.7	0.0099	1.7	0.1263
1.8	0.3512	1.8	0.2534	1.8	0.0668	1.8	0.0379
1.9	0.3700	1.9	0.2918	1.9	0.1317	1.9	0.0383
2.0	0.3855	2.0	0.3232	2.0	0.1864	2.0	0.1037
2.1	0.3984	2.1	0.3489	2.1	0.2324	2.1	0.1597
2.2	0.4093	2.2	0.3702	2.2	0.2711	2.2	0.2074
2.3	0.4185	2.3	0.3878	2.3	0.3036	2.3	0.2480
2.4	0.4264	2.4	0.4025	2.4	0.3311	2.4	0.2826
2.5	0.4331	2.5	0.4148	2.5	0.3543	2.5	0.3121
2.6	0.4390	2.6	0.4252	2.6	0.3739	2.6	0.3373
2.7	0.4442	2.7	0.4340	2.7	0.3906	2.7	0.3588
2.8	0.4487	2.8	0.4415	2.8	0.4048	2.8	0.3772
2.9	0.4527	2.9	0.4480	2.9	0.4169	2.9	0.3930
3.0	0.4562	3.0	0.4535	3.0	0.4273	3.0	0.4065

Table 6.1: Density fluctuations in thermal coherent states with $k_B = \omega = 1$, for various range of coherent state parameter and low temperature.

T=5		T=10		T=15		T=25	
α	Δ_{tcs}	α	Δ_{tcs}	α	Δ_{tcs}	α	Δ_{tcs}
1.0	1.4159	1.0	1.6989	1.0	1.7986	1.0	1.8792
1.1	1.2955	1.1	1.6338	1.1	1.7550	1.1	1.8532
1.2	1.1694	1.2	1.5627	1.2	1.7071	1.2	1.8246
1.3	1.0402	1.3	1.4863	1.3	1.6549	1.3	1.7933
1.4	0.9107	1.4	1.4050	1.4	1.5988	1.4	1.7594
1.5	0.7833	1.5	1.3196	1.5	1.5388	1.5	1.7230
1.6	0.6602	1.6	1.2309	1.6	1.4752	1.6	1.6839
1.7	0.5433	1.7	1.1399	1.7	1.4085	1.7	1.6424
1.8	0.4337	1.8	1.0476	1.8	1.3390	1.8	1.5984
1.9	0.3324	1.9	0.9549	1.9	1.2672	1.9	1.5521
2.0	0.2398	2.0	0.8627	2.0	1.1935	2.0	1.5037
2.1	0.1557	2.1	0.7719	2.1	1.1185	2.1	1.4532
2.2	0.0801	2.2	0.6833	2.2	1.0427	2.2	1.4009
2.3	0.0126	2.3	0.5976	2.3	0.9667	2.3	1.3469
2.4	0.0476	2.4	0.5153	2.4	0.8909	2.4	1.2915
2.5	0.1009	2.5	0.4369	2.5	0.8159	2.5	1.2349
2.6	0.1480	2.6	0.3626	2.6	0.7421	2.6	1.1773
2.7	0.1895	2.7	0.2927	2.7	0.6699	2.7	1.1190
2.8	0.2261	2.8	0.2271	2.8	0.5997	2.8	1.0601
2.9	0.2582	2.9	0.1661	2.9	0.5318	2.9	1.0011
3.0	0.2865	3.0	0.1093	3.0	0.4663	3.0	0.9420

Table 6.2: Density fluctuations in thermal coherent states with $k_B = \omega = 1$, for various range of coherent state parameter and higher temperature.

universe show that the dimensionless quantity is less than unity. Thus, the semiclassical theory holds for the various ranges of the coherent state parameter. However, it is further observed that even if the coherent state parameter is small, the corresponding fluctuations are more than unity for higher temperatures. In such cases, the semiclassical theory does not valid and is due to the fact that the temperature effects take over the situation rather than the coherent state parameter values. Nevertheless, it can be seen that when coherent state parameter takes higher values in the higher temperature cases the corresponding density fluctuations are still less than unity. Therefore, it can be concluded that in general the semiclassical theory holds in the thermal coherent state representation of the inflaton in the oscillatory phase of the inflaton provided the associated temperature effects are less.

6.4.2 In Thermal Squeezed Vacuum State

Next, we examine validity of the semiclassical Einstein equation for the inflaton in the thermal squeezed vacuum for the flat FRW in the oscillatory phase of inflaton. For this the corresponding dimensionless quantity for the thermal squeezed vacuum case can be obtained from (5.1) and is given by

$$\Delta_{tsvs} = \left| \frac{\langle : T_{00}^2 : \rangle_{tsvs} - \langle : T_{00} : \rangle_{tsvs}^2}{\langle : T_{00} : \rangle_{tsvs}^2} \right| . \tag{6.82}$$

The first term in the above equation can be obtained by squaring the temporal component of energy momentum tensor and taking the expectation value in the thermal squeezed vacuum state as

$$\begin{aligned} \langle : T_{00}^2 : \rangle_{tsvs} = & \frac{1}{4S^6(t)} \langle : \hat{\pi}^4 : \rangle_{tsvs} + \frac{m^2}{4} <: \hat{\pi}^2 \hat{\phi}^2 : \rangle_{tsvs} \\ & + \frac{m^2}{4} \langle : \hat{\phi}^2 \hat{\pi}^2 : \rangle_{tsvs} + \frac{m^4}{4} S^6(t) \langle : \hat{\phi}^4 : \rangle_{tsvs}. \end{aligned} \tag{6.83}$$

Then, using the properties of thermal squeezed vacuum state and applying the approximation ansatzs, $<: \hat{\pi}^4 :>_{tsvs}$, $<: \hat{\pi}^2 \hat{\phi}^2 :>_{tsvs}$, $<: \hat{\phi}^2 \hat{\pi}^2 :>_{tsvs}$ and

$<: \hat{\phi}^4 :>_{tsvs}$ can be calculated are respectively obtained as

$$\langle : \hat{\pi}^4 :\rangle_{tsvs} = S_0^6 \left[\frac{1}{4m^2} \left\{ 3 + 6\cosh^2 r \sinh^2 r \coth^2 \frac{\beta w}{2} + 24\sinh^3 r \cosh r \coth \frac{\beta w}{2} + 12 \left(\sinh^2 r \coth \frac{\beta w}{2} + f(\beta) + \sinh^4 r \coth^2 \frac{\beta w}{2} + \cosh r \sinh r \coth \frac{\beta w}{2} \right) \right\} \right] \tag{6.84}$$

$$\langle : \hat{\pi}^2 \hat{\phi}^2 :\rangle_{tsvs} = \frac{1}{4m^2 t^2} \left\{ 3 + 6\cosh^2 r \sinh^2 r \coth^2 \frac{\beta w}{2} + 24\sinh^3 r \cosh r \coth \frac{\beta w}{2} + 12 \left(\sinh^2 r \coth \frac{\beta w}{2} + f(\beta) + \sinh^4 r \coth^2 \frac{\beta w}{2} + \cosh r \sinh r \coth \frac{\beta w}{2} \right) \right\} \tag{6.85}$$

$$\langle : \hat{\phi}^2 \hat{\pi}^2 :\rangle_{tsvs} = \frac{1}{4m^2 t^2} \left\{ 3 + 6\cosh^2 r \sinh^2 r \coth^2 \frac{\beta w}{2} + 24\sinh^3 r \cosh r \coth \frac{\beta w}{2} + 12 \left(\sinh^2 r \coth \frac{\beta w}{2} + f(\beta) + \sinh^4 r \coth^2 \frac{\beta w}{2} + \sinh r \cosh r \coth \frac{\beta w}{2} \right) \right\}. \tag{6.86}$$

$$\langle : \hat{\phi}^4 :\rangle_{tsvs} = \frac{1}{4S_0^6 t^4 m^2} \left\{ \left(3 + 6\cosh^2 r \sinh^2 r \coth^2 \frac{\beta w}{2} + 24\sinh^3 r \cosh r \coth \frac{\beta w}{2} \right) + 12 \left(\sinh^2 r \coth \frac{\beta w}{2} + f(\beta) + \sinh^4 r \coth^2 \frac{\beta w}{2} + \cosh r \sinh r \coth \frac{\beta w}{2} \right) \right\}. \tag{6.87}$$

Thus

$$\langle : T_{00}^2 :\rangle_{tsvs} = \left(\frac{1}{16m^6 t^4} + \frac{1}{8t^2} + \frac{m^2}{16} \right) \left[3 + 6\cosh^2 r \sinh^2 r \coth^2 \frac{\beta w}{2} + 24\sinh^3 r \cosh r \coth \frac{\beta w}{2} + 12 \left(\sinh^2 r \coth \frac{\beta w}{2} + f(\beta) + \sinh^4 r \coth^2 \frac{\beta w}{2} + \cosh r \sinh r \coth \frac{\beta w}{2} \right) \right]. \tag{6.88}$$

Calculating the expectation value of the normal ordered time-time component of energy momentum tensor of the inflaton in the thermal squeezed vacuum state and then squaring, we get

$$
\begin{aligned}
\langle : T_{00} : \rangle^2_{tsvs} = S^6 \Bigg[& \left(\sinh^4 r \coth^2 \frac{\beta w}{2} + \frac{1}{4} + \sinh^2 r \coth \frac{\beta w}{2} + f(\beta) \right) \\
\times & \left(\dot{\phi}^{*2} \dot{\phi}^2 + 2m^2 \dot{\phi}^* \dot{\phi} \phi^* \phi + m^4 \phi^{*2} \phi^2 \right) \\
+ & \frac{1}{4} \left(e^{-2i\vartheta} \sinh^2 r \cosh^2 r \coth^2 \left(\frac{\beta w}{2} \right) \right) (\dot{\phi}^{*4} + 2m^2 \dot{\phi}^{*2} \phi^{*2} + m^4 \phi^{*4}) \\
+ & \frac{1}{4} \left(e^{2i\vartheta} \sinh^2 r \cosh^2 r \coth^2 \left(\frac{\beta w}{2} \right) \right) (\dot{\phi}^4 + m^4 \phi^4 + 2m^2 \dot{\phi}^2 \phi^2) \\
+ & \left(\sinh^2 r \coth \frac{\beta w}{2} + f(\beta) + \frac{1}{2} \right) e^{-i\vartheta} \sinh r \cosh r \coth \frac{\beta w}{2} \\
\times & \ (\dot{\phi} \dot{\phi}^{*3} + m^2 \phi^* \phi \dot{\phi}^{*2} + m^2 \dot{\phi}^* \dot{\phi} \phi^{*2} + m^4 \phi \phi^{*3}) \\
+ & \frac{1}{2} \sinh^2 r \cosh^2 r \coth^2 \frac{\beta w}{2} \\
\times & \ (\dot{\phi}^{*2} \dot{\phi}^2 + m^2 \dot{\phi}^{*2} \phi^2 + m^2 \phi^{*2} \dot{\phi}^2 + m^4 \phi^{*2} \phi^2) \\
+ & e^{i\vartheta} \sinh r \cosh r \coth \frac{\beta w}{2} \left(\sinh^2 r \coth \frac{\beta w}{2} + f(\beta) + \frac{1}{2} \right) \\
\times & \ (\dot{\phi}^2 \dot{\phi}^* \dot{\phi} + m^2 \phi^2 \dot{\phi}^* \dot{\phi} + m^2 \dot{\phi}^2 \phi^* \phi + m^4 \phi^2 \phi^* \phi) \Bigg] .
\end{aligned}
\tag{6.89}
$$

Substitute the values of ϕ, $\dot{\phi}$, $\dot{\phi}^*$, $\phi^* \cdots$ etc., and applying the approximation ansatzs, we get

$$
\begin{aligned}
\langle : T_{00} : \rangle^2_{tsvs} = & \left(\frac{1}{16m^2t^4} + \frac{1}{8t^2} + \frac{m^2}{16} \right) \left(1 + 4\sinh^4 r \coth^2 \left(\frac{\beta w}{2} \right) \right. \\
& + 4\sinh^2 r \coth \frac{\beta w}{2} + 4f(\beta) + 4\sinh^2 r \cosh^2 r \coth^2 \frac{\beta w}{2} \\
& \left. + 8\sinh^3 r \cosh r \coth \frac{\beta w}{2} + 4\sinh r \cosh r \coth \frac{\beta w}{2} \right) .
\end{aligned}
\tag{6.90}
$$

T=0.5							
r	Δ_{tsvs}	r	Δ_{tsvs}	r	Δ_{tsvs}	r	Δ_{tsvs}
0.001	0.6202	0.01	0.6292	0.1	0.7224	1.1	1.3466
0.002	0.6212	0.02	0.6393	0.2	0.8249	1.2	1.3723
0.003	0.6222	0.03	0.6496	0.3	0.9192	1.3	1.3941
0.004	0.6232	0.04	0.6598	0.4	1.0026	1.4	1.4125
0.005	0.6242	0.05	0.6702	0.5	1.0754	1.5	1.4280
0.006	0.6252	0.06	0.6806	0.6	1.1385	1.6	1.4410
0.007	0.6262	0.07	0.6910	0.7	1.1932	1.7	1.4519
0.008	0.6272	0.08	0.7014	0.8	1.2405	1.8	1.4609
0.009	0.6282	0.09	0.7194	0.9	1.2814	1.9	1.4684
0.010	0.6292	0.10	0.7224	1.0	1.3165	2.0	1.4746

Table 6.3: Density fluctuations in thermal squeezed vacuum state with $k_B = \omega = 1$, for various range of squeezing parameter and low temperature.

Equations (6.88) and (6.90) can be substituted in (6.82), we get

$$\Delta_{tsvs} = \left| \frac{\begin{array}{c} 2 + 2\cosh^2 r \sinh^2 r \coth^2 \frac{\beta\omega}{2} + 16 \sinh^3 r \cosh r \coth \frac{\beta\omega}{2} \\ +8 \left[\sinh^2 r \coth \frac{\beta\omega}{2} + f(\beta) \right. \\ \left. + \sinh^4 r \coth^2 \frac{\beta\omega}{2} + \cosh r \sinh r \coth \frac{\beta\omega}{2} \right] \end{array}}{\begin{array}{c} 1 + 4\sinh^4 r \coth^2 \frac{\beta\omega}{2} + 4\sinh^2 r \coth \frac{\beta\omega}{2} + 4f(\beta) \\ +4\sinh^2 r \cosh^2 r \coth^2 \frac{\beta\omega}{2} \\ +8\sinh^3 r \cosh r \coth \frac{\beta\omega}{2} + 4\sinh r \cosh r \coth \frac{\beta\omega}{2} \end{array}} \right|. \quad (6.91)$$

Thus validity of the semiclassical Einstein equation in terms of density fluctuations in the thermal squeezed vacuum state for the oscillatory phase of inflaton is examined numerically with the associated squeezing parameter for a range of temperature and are tabulated in Tables 6.3-6.6. The numerical study of Δ_{tsvs} for various values of squeezing parameter in the oscillatory phase of the inflaton, in the flat FRW metric show that the semiclassical theory does hold only for much smaller values of the associated squeezing parameter compared to unity for any temperature.

T=1							
r	Δ_{tsvs}	r	Δ_{tsvs}	r	Δ_{tsvs}	r	Δ_{tsvs}
0.001	0.7699	0.01	0.7788	0.1	0.8609	1.1	1.2562
0.002	0.7709	0.02	0.7885	0.2	0.9405	1.2	1.2631
0.003	0.7719	0.03	0.7981	0.3	1.0100	1.3	1.2681
0.004	0.7729	0.04	0.8075	0.4	1.0697	1.4	1.2717
0.005	0.7739	0.05	0.8167	0.5	1.1195	1.5	1.2742
0.006	0.7749	0.06	0.8258	0.6	1.1598	1.6	1.2761
0.007	0.7759	0.07	0.8348	0.7	1.1913	1.7	1.2775
0.008	0.7769	0.08	0.8436	0.8	1.2154	1.8	1.2785
0.009	0.7779	0.09	0.8523	0.9	1.2334	1.9	1.2793
0.010	0.7789	0.10	0.8609	1.0	1.2466	2.0	1.2798

Table 6.4: Density fluctuations in thermal squeezed vacuum state with $k_B = \omega = 1$, for various range of squeezing parameter and low temperature.

T=3							
r	Δ_{tsvs}	r	Δ_{tsvs}	r	Δ_{tsvs}	r	Δ_{tsvs}
0.001	0.9184	0.01	0.9273	0.1	1.0147	1.1	1.7189
0.002	0.9194	0.02	0.9372	0.2	1.1110	1.2	1.7368
0.003	0.9204	0.03	0.9470	0.3	1.2079	1.3	1.7492
0.004	0.9214	0.04	0.9567	0.4	1.3038	1.4	1.7574
0.005	0.9224	0.05	0.9664	0.5	1.3955	1.5	1.7629
0.006	0.9234	0.06	0.9761	0.6	1.4792	1.6	1.7664
0.007	0.9244	0.07	0.9858	0.7	1.5518	1.7	1.7686
0.008	0.9253	0.08	0.9954	0.8	1.6115	1.8	1.7699
0.009	0.9263	0.09	1.0051	0.9	1.6582	1.9	1.7707
0.010	0.9273	0.10	1.0147	1.0	1.6934	2.0	1.7711

Table 6.5: Density fluctuations in thermal squeezed state with $k_B = \omega = 1$, for various range of squeezing parameter and higher temperature.

T=5							
r	Δ_{tsvs}	r	Δ_{tsvs}	r	Δ_{tsvs}	r	Δ_{tsvs}
0.001	0.9511	0.01	0.9601	0.1	1.0492	1.1	1.8346
0.002	0.9521	0.02	0.9700	0.2	1.1497	1.2	1.8574
0.003	0.9531	0.03	0.9799	0.3	1.2527	1.3	1.8735
0.004	0.9541	0.04	0.9898	0.4	1.3565	1.4	1.8846
0.005	0.9551	0.05	0.9997	0.5	1.4577	1.5	1.8921
0.006	0.9561	0.06	1.0026	0.6	1.5518	1.6	1.8972
0.007	0.9571	0.07	1.0194	0.7	1.6349	1.7	1.9006
0.008	0.9581	0.08	1.0293	0.8	1.7046	1.8	1.9029
0.009	0.9591	0.09	1.0392	0.9	1.7603	1.9	1.9043
0.010	0.9601	0.10	1.0492	1.0	1.8030	2.0	1.9052

Table 6.6: Density fluctuations in thermal squeezed state with $k_B = \omega = 1$, for various values of squeezing parameter and higher temperature.

6.5 Quantum Fluctuations

In this section, we discuss quantum fluctuations for the inflaton in the semi-classical theory of gravity. The quantum fluctuations in the thermal coherent and thermal squeezed state representations of the inflaton can be studied by using dispersion relations of ϕ and π, which are respectively given by

$$(\Delta\phi)^2 =< \hat{\phi}^2 > - < \hat{\phi} >^2 \tag{6.92}$$

$$(\Delta\pi)^2 =< \hat{\pi}^2 > - < \hat{\pi} >^2 . \tag{6.93}$$

Here, $< \hat{\phi}^2 >, < \hat{\pi}^2 >, < \hat{\phi} >$ and $< \hat{\pi} >$ are respectively the squared expectation values and the expectation values of $\hat{\phi}$ and $\hat{\pi}$ in a given quantum state under consideration.

First, we consider the quantum fluctuations of the inflaton in the thermal coherent state. Using the properties of thermal coherent state and (3.17), it follows that

$$(\Delta\phi)_{tcs} = \sqrt{(2f(\beta) + 1)\phi^*\phi} , \tag{6.94}$$

$$(\Delta\pi)_{tcs} = S^3\sqrt{(2f(\beta) + 1)\dot{\phi}\dot{\phi}^*} , \tag{6.95}$$

substitute the values of $\phi, \phi^*, \dot{\phi}, \dot{\phi}^*$ in the above equations respectively and then applying the approximation ansatzs, we get the dispersion relations in the thermal coherent state

$$(\Delta\phi)_{tcs} = \sqrt{(2f(\beta)+1)\frac{1}{S_0^3 t^2}\frac{1}{2m}}, \tag{6.96}$$

and

$$(\Delta\pi)_{tcs} = S_0^3 t^2 \sqrt{(2f(\beta)+1)\frac{1}{S_0^3 t^2}\frac{1}{2m}\left(\frac{1}{t^2}+m^2\right)}. \tag{6.97}$$

In the limit $mt \gg 1$, the above two expressions give the dispersion relation in the thermal coherent state

$$(\Delta\phi)_{tcs}(\Delta\pi)_{tcs} = (2f(\beta)+1)\frac{1}{2}\sqrt{1+\frac{1}{m^2t^2}}. \tag{6.98}$$

That is

$$(\Delta\phi)_{tcs}(\Delta\pi)_{tcs} = (2f(\beta)+1)(\Delta\phi)_{cs}(\Delta\pi)_{cs} \tag{6.99}$$

where $(\Delta\phi)_{cs}(\Delta\pi)_{cs} = \frac{1}{2}\sqrt{1+\frac{1}{m^2t^2}}$ which is given by [45]. The dispersion relations in the squeezed vacuum state can be calculated by the same procedure and are obtained in the limit $mt \gg 1$ as

$$\begin{aligned}(\Delta\phi)_{tsvs}(\Delta\pi)_{tsvs} = \frac{1}{2m}\Bigg[&\left(2\sinh^2 r\coth\frac{\beta\omega}{2}+1+2f(\beta)+\sinh 2r\coth\frac{\beta\omega}{2}\right)\\ &\times\left\{\left(2\sinh^2 r\coth\frac{\beta\omega}{2}+1+2f(\beta)\right)\left(\frac{1}{t^2}+m^2\right)\right.\\ &\left.+S_0^3t^2\sinh 2r\coth\frac{\beta\omega}{2}\left(\frac{1}{t^2}-m^2\right)\right\}\Bigg]^{1/2}.\end{aligned} \tag{6.100}$$

Similarly for the squeezed state, we get

$$\begin{aligned}(\Delta\phi)_{tss}(\Delta\pi)_{tss} = \frac{1}{2m}\Bigg[&\left(2\sinh^2 r\coth\frac{\beta\omega}{2}+1+2f(\beta)+\sinh 2r\coth\frac{\beta\omega}{2}\right)\\ &\times\left\{\left(2\sinh^2 r\coth\frac{\beta\omega}{2}+1+2f(\beta)\right)\left(\frac{1}{t^2}+m^2\right)\right.\\ &\left.+S_0^3t^2\sinh 2r\coth\frac{\beta\omega}{2}\left(\frac{1}{t^2}-m^2\right)\right\}\Bigg]^{1/2}.\end{aligned} \tag{6.101}$$

The quantum fluctuations of the inflaton is studied in terms of dispersion relations of ϕ and π and for the thermal coherent state case it shows that the dispersion relations of the inflaton are inversely proportional to t. It also noted that the uncertainty relation in thermal coherent state is equivalent to the $(2f(\beta)+1)$ times of the coherent state uncertainty relation. The uncertainty relation calculated for the thermal squeezed vacuum state shows that the relation depends on the assocaited squeezing parameter and temperature.

6.6 Classical and Semiclassical Gravity

Here, we briefly discuss some aspects of the inflaton in classical gravity with semiclassical gravity in the oscillatory regime. In the present context, we compare classical gravity with semiclassical gravity in the thermal coherent states for the Einstein equations.

Consider the oscillatory phase of the inflaton, the initial values of the inflaton can be incorporated with the amplitude and the phase of the real inflaton

$$\phi_r(t) = \frac{\mathcal{B}_0}{S^{3/2}(t)\sqrt{\omega(t)}} \sin(\int \omega(t)dt + \delta) \tag{6.102}$$

where $\mathcal{B}_0$ is the amplitude of the classical inflaton.

The Hamiltonian for the classical inflaton is given by

$$\mathcal{H}_m = \frac{1}{2S^3}\pi_{\phi r}^2 + \frac{m^2 S^3}{2}\phi_r^2, \tag{6.103}$$

where $\pi_{\phi r}^2$ and ϕ_r^2 can be evaluated and these values are substituted in (6.103). The classical energy density for the inflaton in classical gravity becomes

$$\mathcal{H}_m = \frac{\mathcal{B}_0^2}{2}\frac{1}{2\omega}\left[m^2 + \omega^2 + \frac{1}{4}\left(\frac{\dot{\omega}}{\omega} + 3\frac{\dot{S}}{S}\right)^2 \right.$$
$$\left. + \left(\omega^2 - \frac{1}{4}\left(\frac{\dot{\omega}}{\omega} + \frac{3\dot{S}}{S}\right)^2\right) \cos\left(2\int \omega(t)dt + \delta\right) \right.$$

$$-\left(\frac{\dot{\omega}}{\omega}+\frac{3\dot{S}}{S}\right)\sin 2\left(\int\omega(t)dt+\delta\right)-m^2\cos 2\left(\int\omega(t)dt+\delta\right)\Bigg]. \quad (6.104)$$

The result of the Hamiltonian (6.104) shows the oscillatory behaviour of the classical energy density. The oscillating terms determine, in a significant way, the evolution of the geometric invariants through the higher derivatives of the scale factor $S(t)$. The time average over several oscillations of the above Hamiltonian (6.104) is computed by assuming that the expansion of the universe can be neglected during each period of coherent oscillation and is obtained as

$$<\mathcal{H}_m>_{tavg}=\frac{\mathcal{B}_0^2}{2}\frac{1}{2\omega}\left[\frac{1}{4}\left(\frac{\dot{\omega}}{\omega}+\frac{3\dot{S}}{S}\right)^2+m^2+\omega^2\right]. \quad (6.105)$$

Next goal is to compare the result (6.105) with that of the semiclassical theory gravity in the thermal coherent state. Hence, consider the Hamiltonian of the inflaton in the thermal coherent state as

$$\begin{aligned}<\hat{\mathcal{H}}>_{tcs}=S^3\Bigg[&\left(f(\beta)+|\alpha|^2+\frac{1}{2}\right)(\dot{\phi}^*\dot{\phi}+m^2\phi^*\phi)\\ &-\frac{1}{2}\alpha^{*2}(\dot{\phi}^{*2}+m^2\phi^{*2})-\frac{1}{2}\alpha^2(\dot{\phi}^2+m^2\phi^2)\Bigg]. \quad (6.106)\end{aligned}$$

Substituting (3.23) in (6.106) we get, the Hamiltonian in the thermal coherent state as

$$\begin{aligned}<\hat{\mathcal{H}}_m>_{tcs}=\frac{1}{2\omega(t)}\Bigg[&\left(f(\beta)+|\alpha|^2+\frac{1}{2}\right)\left(\frac{1}{4}\left[\frac{3\dot{S}(t)}{S(t)}+\frac{\dot{\omega}(t)}{\omega(t)}\right]^2+\omega^2(t)+m^2\right)\\ &-\frac{1}{2}\alpha^{*2}\exp(2i\int\omega(t)dt)\Bigg\{\frac{1}{4}\left[\frac{3\dot{S}(t)}{S(t)}+\frac{\dot{\omega}(t)}{\omega(t)}\right]^2\\ &-\omega^2(t)-i\omega(t)\left[\frac{3\dot{S}(t)}{S(t)}+\frac{\dot{\omega}(t)}{\omega(t)}\right]+m^2\Bigg\}\\ &-\frac{1}{2}\alpha^2\exp(-2i\int\omega(t)dt)\Bigg\{\frac{1}{4}\left[\frac{3\dot{s}(t)}{S(t)}+\frac{\dot{\omega}(t)}{\omega(t)}\right]^2\\ &-\omega^2(t)+i\omega(t)\left[\frac{3\dot{S}(t)}{S(t)}+\frac{\dot{\omega}(t)}{\omega(t)}\right]+m^2\Bigg\}\Bigg]. \quad (6.107)\end{aligned}$$

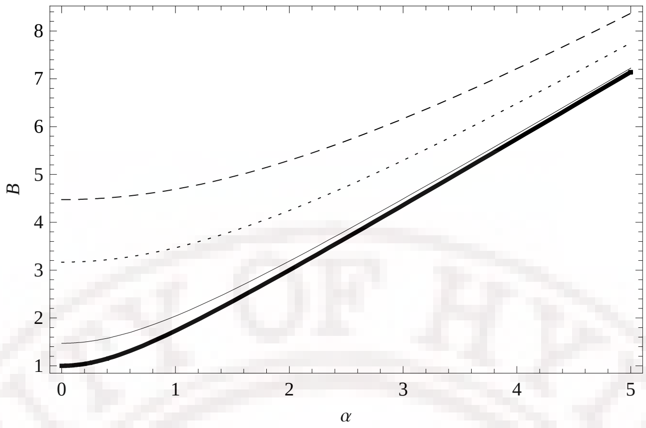

Figure 6.31: Plot for the amplitude B versus coherent state parameter α for various temperatures, T=0 (solid line),T=1 (thin line),T=5 (dotted line) and T=10 (dashed line).

As in the case of the classical energy density, the time average over several oscillations of the energy density of the inflaton in the thermal coherent state is computed by assuming that the expansion of the universe can be neglected during each period of oscillation of the inflaton, and is obtained as

$$< \hat{\mathcal{H}}_m >_{tcs-tavg}= \left(f(\beta)+|\alpha|^2+\frac{1}{2}\right)\frac{1}{2\omega(t)}\left[\frac{1}{4}\left(\frac{\dot{\omega}}{\omega}+\frac{3\dot{S}}{S}\right)^2+\omega^2(t)+m^2\right]. \tag{6.108}$$

Comparing (6.108) with (6.105), we obtained the amplitude as

$$\mathcal{B}_0=\sqrt{2f(\beta)+2|\alpha|^2+1}. \tag{6.109}$$

Thus the classical Einstein equation in the oscillatory phase of inflaton in the flat FRW metric is different from the case of the semiclassical Einstein equation in the thermal coherent state, only by amplitude. This is due to the fact that, the coherent states are analogous to classical behaviour compared to the squeezed vacuum states, but extra effects are coming from the additional thermal property. Plot for the amplitude for different values of coherent state parameter with a range of temperature are given in Figure

6.31. When $T = 0$ the amplitude is reduced to that of zero temperature coherent state result. Hence, it is concluded that one can recover the classical nature of the Einstein equation, from the semiclassical Einstein equation, in the thermal coherent state, by a suitable limiting procedure.

6.7 Discussions and Conclusions

Oscillatory phase of the inflaton and its related phenomenon are very important to understand evolution of the early universe. Since the universe was devoid of particles after inflation, a reheating mechanism is required for the thermonuclear reaction to start. One of the possible candidates for such mechanisms is particle creations due to the coherently oscillating inflaton. Usually the inflation and related issues are studied by using the classical Friedmann equations with classical inflaton as the source for gravity. In the present work, we studied the oscillatory phase of the inflaton in the semiclassical theory of gravity by representing it in the thermal coherent and squeezed state formalisms.

We considered a minimally coupled homogeneous scalar field in the flat, open and closed FRW models of the universe in the semiclassical theory of gravity and obtained their solutions in the thermal coherent, squeezed vacuum and squeezed state formalisms. The solutions for the various cases are given in Figures 1-9. The solutions of three FRW models are also given for the same set of parameters of the thermal coherent and squeezed state cases are provided.

Particle production of the coherently oscillating inflaton, after the inflation in the thermal coherent and squeezed state formalisms, in flat FRW universe is studied. It is observed that particle production is dependent on the thermal coherent parameter and temperature. Particle production in the thermal squeezed vacuum state is dependent on the squeezing parameter and temperature. Particle production in the thermal squeezed state is dependent on parameters of the coherent state, squeezed state and temperature. Particle creation in the flat, open and closed FRW models lead to the same result, from this we can conclude that particle production in the thermal coherent and squeezed states is model independent as far as the semiclassical theory is concerned.

We also studied validity of the semiclassical Einstein equation by computing the density fluctuations in thermal squeezed vacuum state formalism for the flat FRW model in the oscillatory phase of the inflaton. An expression for the density fluctuations is obtained analytically and further it is examined numerically. It is found that the semiclassical theory holds in the thermal coherent state formalism provided that the corresponding thermal effects are less compared to the coherent state parameter values. This is due to the fact that, when the coherent state parameter take less value compared to unity, in the high temperature limit, the thermal effect takes over the situation rather than the coherent state. But on the other hand, the coherent state parameter takes much higher value then the semiclassical theory holds. While in the case of thermal squeezed vacuum case the semiclassical theory holds only when the associated squeezing parameter take much less values compared to unity otherwise the theory breaks down. This is because the thermal and quantum fluctuations are larger due to particles production in the oscillatory phase of the inflaton.

The quantum fluctuations of the inflaton is studied in terms of dispersion relations for the coherent state and it shows that the dispersion relations are inversely proportional to t. Also it is observed that the uncertainty relation in the thermal coherent state is equivalent to the $(2f(\beta)+1)$ times that of the coherent state uncertainty relation. The uncertainty relation calculated for the squeezed vacuum state shows that the relation depends on the associated squeezing parameter and temperature.

A comparative study of classical Einstein equation with the semiclassical Einstein equation in the thermal coherent state is carried out for the flat FRW metric. It is showed that the classical Einstein equation differ from the case of the semiclassical Einstein equation in the thermal coherent state only by amplitude. Plots for the amplitude for different values of the coherent state parameter with a range of temperature are given in Figure 6.31. When $T = 0$ the amplitude is reduced to that of zero temperature coherent state result. Hence, it is concluded that one can recover classical nature of the Einstein equation from semiclassical Einstein equation in the thermal coherent state by a suitable limiting procedure. The present study can account for particle creation due to quantum and thermal effects in the oscillatory phase of the inflaton in the semiclassical theory of gravity.

Chapter 7

Summary and Conclusions

The standard cosmology or the hot big bang model is considered as reliable and successful theory to explain most of the observed features of the universe. Later, the inflationary scenario is introduced to accommodate some of the problems faced in the standard cosmological model. According to which the universe expanded almost exponentially for a brief period of time, in the early stage of its evolution. The candidate which was responsible for the inflation is assumed to be homogeneous scalar field known as inflaton. At the end of the inflationary period the universe was empty of particles. It is believed that particles were created due to the coherent oscillations of the inflaton. Thus the universe reheated after the inflation due to the collisions of created particles and hence triggered the thermonuclear reactions. Thus the oscillatory phase of the inflaton and its related issues are very crucial to understand the subsequent evolution of the universe after inflation. The inflation and its related issues can be described in the standard cosmology with the classical Friedmann equations. In which the background metric is treated as classical and the corresponding source of gravity as unquantized scalar field. But at deeper level both matter field and gravity are to be treated as quantum mechanically. Since a proper quantum theory of gravity is not available, one can describe the early universe model with the semiclassical theory of gravity. In this, the gravity is treated as unquantized external field with quantized matter field as the source. It has been realized that the semiclassical theory of gravity is much useful to incorporate quantum features where quantum gravity effects are considered to be negligible. Moreover, it has been found that the results, in the oscillatory phase of inflaton, in classical gravity are quite different from that of semiclassical gravity.

Recently, it has been observed that the coherent and squeezed state formalisms which are well known in quantum optics are found useful to deal with the quantum effects in the early universe. In the present study, we have employed the quantum optical states to study the oscillatory phase of the inflaton. Our basic setup that we studied is, a minimally coupled massive inflaton in the Friedmann-Robertson-Walker universe in the semiclassical theory of gravity. The approximate leading solutions for a flat FRW metric is obtained in the oscillatory phase of inflaton in the coherent and squeezed states. Particle creation and validity of semiclassical theory is studied in the coherent and squeezed states for the flat FRW model. Classical or nonclassical nature of the inflaton is also studied in the coherent and squeezed states in terms of a quantity which is analogous to a quantity in quantum optics called the Mandel' s Q parameter. We also studied the oscillatory phase of inflaton in the thermal counterparts of the quantum optical states known as the thermal coherent and squeezed states. The solutions of the semiclassical Friedmann equations, particle creation, validity of the semiclassical theory are carried out in the thermal coherent and squeezed states, in the flat FRW universe. The main objective of the present study that we carried out is to understand the oscillatory phase of the nonclassical inflaton in the flat FRW universe. But the solutions of the Friedmann equations imply the open and closed FRW cosmological models also. Therefore we extended the study to the case of closed and open FRW models for both zero temperature coherent and squeezed states as well as their thermal counterparts and are compared with the standard FRW models.

The approximate leading solutions that are obtained in the case of flat

FRW universe, in the oscillatory phase of inflaton, for the coherent and squeezed states and their thermal counterparts, exhibit the same power law of expansion. The solutions for the open and closed FRW models are shown graphically. A comparative study of the solutions in all the FRW models is also done. Thus, it can be concluded that the corresponding solutions show their required behaviour in comparison with the FRW models, in the coherent and squeezed states as well as their thermal counterparts.

Particle production in the oscillatory phase of the inflaton in the coherent and squeezed states in semiclassical theory of gravity is studied in the flat FRW universe. It shows that the quantum effects can play important role in particle creation mechanism. Further, the studies show that particle creation can be enhanced due to quantum and thermal effects. A comparative study of particle creation for the open and closed FRW models is also done in the thermal and zero temperature coherent and squeezed sates. The results show that particle creation is model independent as far as the representation of the inflaton in the coherent and squeezed states as well as the respective thermal states.

Validity of the semiclassical Friedmann equation is examined in the coherent and squeezed vacuum states for the flat FRW universe for the oscillatory phase of inflaton. The validity is checked with the help of a dimensionless quantity in terms of the energy momentum tensor. It shows that the semiclassical theory of gravity hold in the coherent state formalism. In the case of squeezed vacuum state, the theory hold only for the small values of the squeezing parameter compared to unity otherwise it does not. Validity of the semiclassical theory in the thermal coherent state holds provided the associated temperature is not high compared to the coherent state parameter. On the other hand the theory does not hold in the thermal squeezed vacuum state irrespective of the associated temperature which is either low or high.

Classical or nonclassical nature of the inflaton field, is examined in the coherent and squeezed vacuum states, for the flat FRW universe in terms of a quantity in the line of Mandel's Q parameter with associated cosmological parameters. The analysis shows that nature of the squeezed vacuum and coherent states preserve their nature even in the cosmological context. The conclusions remain unaltered in the closed and open FRW models also.

A comparative study of the Friedmann equations in classical gravity with semiclassical gravity in the thermal coherent state is conducted and it shows that both differ only by an amplitude factor. If thermal effects are negligible the result is reduced that of the coherent state.

It is to be noted that, the back reaction effects are not taken into account in the present work and can be considered in future. The present study can account for the enhancement of particle production in the oscillatory phase of inflaton due to quantum and thermal effects. Also it is to be noted that, all the results in the coherent and squeezed states can be recovered from their thermal counterparts by suitable limiting procedure of the temperature effects. It can be considered that the coherent and squeezed states and their thermal counterparts are viable candidates to deal with quantum and thermal effects, in the oscillatory phase of inflaton. The study can be explored further with observations and other nonclassical states, and is beyond the scope of present study.

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List of Publications

1. **K.K.Venkataratnam** and P.K.Suresh,"**Particle Production of Coherently Oscillating Nonclassical inflaton in FRW Universe**", *Int. J. Mod. Phys.* **D13** 239-252 (2004).

2. **K.K.Venkataratnam** and P.K.Suresh "**Density Fluctuations in the Oscillatory phase of a Nonclassical Inflaton in the FRW Universe** ", *Int. J. Mod. Phys.* **D17** 1991-2005 (2008).

3. **K.K.Venkataratnam** and P.K.Suresh " **Non-classical Scalar field in the FRW Universe**", (Accepted in *Int. J. Mod. Phys.***D** (2009))

4. **K.K.Venkataratnam** and P.K.Suresh "**Oscillatory Phase of Nonclassical Thermal Inflaton in FRW Universe** ", *(communicated)*