Aspects of Baryon Asymmetry and Non-equilibrium Particle Production Processes.

Thesis submitted in partial fulfillment of the
requirements for the award of the degree of
Doctor of Philosophy
by

K. V. S. Shiv Chaitanya

School of Physics
University of Hyderabad
Hyderabad 500 046
India

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DEDICATION

To

My

Mother and Father

DECLARATION

I hereby declare that the research work embodied in this thesis entitled **"Aspects of Baryogenesis and Non-equilibrium Particle Production Processes"** has been carried out by me under the supervision of **Prof. Bindu A Bambah**, School of Physics, University of Hyderabad, has not been submitted for any other degree or diploma either in part or full to this or any other university or institution.

Place: Hyderabad

Date: **K. V. S. Shiv Chaitanya**

CERTIFICATE

This is to certify that the research work embodied in this thesis entitled **"Aspects of Baryogenesis and Non-equilibrium Particle Production Processes"** has been carried out by **Mr. K. V. S. Shiv Chaitanya** under my supervision and the same has not been submitted for any other degree or diploma either in part or full to this or any other university or institution.

Place: Hyderabad

Date: **Prof. Bindu A Bambah,**

(Thesis supervisor)

Dean

School of Physics

University of Hyderabad

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Contents

CHAPTER 1

Introduction

Ever since the theoretical prediction of the anti-particle of the electron, the positron, by P.A.M Dirac [1] in 1928 and its subsequent detection by Carl. D Anderson in 1932 [2], anti-particles of all existing particles have been shown to exist. However, particles are much more stable and abundant compared to their anti-particles and it remains an unsolved problem as to why there is a predominance of matter over anti-matter in the universe. Reactions such as $e^+ + e^- \to 2\gamma$ and $p + \bar{p} \to 2\gamma$ etc, tell us that if there are equal numbers of particles and anti-particles, then the universe would be radiation dominated and infact, matter would not exist. The matter in the universe is largely baryonic and this problem is thus called the problem of the baryon asymmetry of the universe (BAU). This asymmetry is characterised by the ratio $\eta = \frac{n_b}{n_\gamma}$, where n_b is the number of baryons and n_γ is the number of photons in the universe. The present value of the asymmetry is $\eta \simeq 3 \times 10^{-10}$. Much progress has been made in the last 80 years since the discovery of anti-matter and many major breaktroughs and milestones have been established, but a viable theory of BAU is still wanting. Thus, the problem of the BAU is an outstanding challenging problem and any insight, however small, will contribute to its understanding. This thesis represents a contribution to this understanding.

It is known that the laws of relativistic quantum mechanics are symmetric, when particles are swapped with antiparticles in a given process. The relevant quantum numbers corresponding to this symmetry are charge conjugation C, parity P, and time reversal T. The combined effect of all three of these symmetries is canonized in the CPT theorem, which states that any Lorentz invariant local quantum field theory must have CPT symmetry. A consequence of the CPT theorem is that the particle and its anti-particle have equal mass and lifetime, but opposite charge. The CPT theorem implies that the universe is matter-antimatter symmetric. However, it is evident that the present universe is matter dominant, and the question naturally arises as to how this has come about?

A major theoretical breakthrough in the understanding of this problem came in 1967, when A. D. Sakharov [3] postulated three conditions that are sufficient to guarantee baryon asymmetry. These are (i) Baryon number (B) violation, (ii) Charge (C) and Parity (CP) violation and (iii) the presence of non-equilibrium processes. Now it has become a standard dictum that Sakharov's requirements are the starting point for building models to characterize BAU. The standard model of particle physics is insufficient to explain BAU as baryon number conservation is built into it. Therefore, one has to look beyond the standard model of particle physics to construct viable models of BAU. The models that have been attempted are GUT(Grand Unified Theories) baryogenesis [4, 5, 6, 7, 8, 9, 10], Electroweak baryogenesis [4, 5, 7, 8, 9], Affeck-Dine baryogenesis [4, 5, 11], Baryogenesis through Leptogenesis [4, 12] and some more models in the context of brane universes [13] and string theory [14]. Most of these models are inadequate in explaining the value of η.

One of the major developments in the last century has been the union of the standard model of particle physics with the standard model of cosmology [6, 15, 16], which deals with the large scale structure of the universe, leading to the exploration of new physics. The energy scales required in the search of new fundamental particles are close to the order of the those of the early universe. Hence, the present and future goals of experiments at the particle accelerators is to reproduce the conditions of the early universe, which itself can be treated as a particle accelerator. The study of early universe problems gives an insight into the fundamental particle interactions and allows a search for new physics beyond the standard model. Hence, it is in this scenario that a solution of the BAU problem may be found. In the standard model of cosmology, the universe began with a big explosion called big bang and the energy of the universe was $\geq 10^{19}$ Gev. The early universe was radiation dominated. As the universe expanded, it cooled, went through many phase transitions and is homogeneous and isotropic. It traversed through the following epochs: the Planck epoch, the inflationary epoch,the reheating epoch, the grand unification epoch and the electroweak epoch. The radiation decoupled from matter to give rise to CMBR (Cosmic microwave background radiation)[17]. The standard model of cosmology has several problems, as shown by the present observations, which require that the universe started from very fine tuned initial conditions. To resolve these problems, the inflationary model was proposed, in which a scalar field called an inflaton field drives the universe through an exponential expansion called inflation. After the inflation, the large potential energy of the inflaton field is converted into particles and radiation in a process called reheating. Now, it is fairly well recognised that earlier than the GUT scale, particle production in the universe took place in the reheating and preheating [18, 19, 20, 21, 22] phases after inflation [23, 24], through the quantum fluctuations of the inflaton field.

In this thesis, we examine two possible mechanisms of baryon asymmetry generation of the universe during the reheating phase after inflation. These are extensions of previous models of N. J. Papastamatiou and L. Parker [25] (modified by R. Rangarajan, D. V. Nanopoulous [26]) and Affleck and Dine [11]. Adapting their work to generate the asymmetry that might be created in a more realistic universe that is initially inflating and then enters a rapid preheating phase, we develop a general formalism based on squeezed rotated states to calculate the baryon asymmetry of the universe in the wake of inflation, in the reheating phase, through a parametric amplification mechanism [18]. Quantum optical methods are used to show a differential amplification of particle and anti-particle modes giving rise to baryon asymmetry. A Bogoliubov transformation, followed by a rotation, is used to diagonalize the Hamiltonian obtained in these models [27]. The diagonalization of the Hamiltonian is carried out in an exact fashion allowing us to derive the evolution equations for particle/anti-particle modes, in an expanding FRW metric, in the mean field approximation. The evolution equations for particle/anti-particle modes are exact and the role played by the expansion factor and the inflaton field on their evolution is shown. In particular, in the Affleck-Dine mechanism, means to reduce the classically calculated value of BAU to a realistic value due to quantum fluctuations and the expansion of the universe are systematically studied. These equations can be solved for any type of inflationary reheating models.

The methods we have developed to calculate BAU are, in general, applicable to many non-equilibrium particle production processes. Therefore, as an illustration of our method, particle production in non equilibrium processes in ultra-high energy hadronic collisions such as a rapidly expanding hot quark gluon plasma (QGP) are studied. Since the goal of future collider experiments is to recreate the conditions of the early universe in the laboratory, it is valuable to test our mechanism for non equilibrium processes in such systems. Non-equilibrium processes during the chiral symmetry breaking phase transition of the QGP may produce a disoriented chiral condensate [28, 29, 30] which decays to produce pions. It is in this non-equilibrium particle production process that testable signals, such as the asymmetry between charged and neutral pions, are derived using the formalism developed in this thesis.

The structure of the thesis is as follows:

In the first chapter, a detailed description of why Sakharov conditions are essential for

BAU is given. The estimation of the baryon asymmetry parameter from nucelosynthesis and cosmic microwave background radiation (CMBR) is outlined.

In the second chapter, the salient features of various models of baryogenesis are discussed and an assessment of the weak and strong points of these models is given.

Since we are interested in BAU during the reheating epoch of the early universe, in the third chapter, a brief introduction to the standard model of cosmology and the problems with it that warrant the inflationary scenario are discussed. A review of the reheating mechanism and the connection between particle creation and the squeezed states formalism is discussed.

In the fourth chapter, the discussion starts with a simple model proposed by N. J. Papastamatiou and L. Parker [25] for BAU, followed by the modification of the model by R.Rangarajan and D.V.Nanopoulous [26]. We then continue with a discussion of our general formalism of squeezed rotated states to calculate baryon asymmetry in the process of reheating through parametric amplification in this model.

In the fifth chapter, the quantum fluctuations in the post inflationary Affleck-Dine baryogenesis model in the reheating epoch are studied. Again, the squeezed states formalism is used to give evolution equations for the particle and anti-particle modes. The role of rapid and slow expansion of the universe and parametric amplification of the quantum fluctuations on the baryon asymmetry produced in the Affleck Dine model is presented.

In the sixth chapter, particle production in non-equilibrium processes using the same methods as in the previous chapters is discussed. As an exemplary case of a non-equilibrium particle production situation, a quantum field theoretical model for the dynamics of the disoriented chiral condensate is presented. A unified approach to relate the quantum field theory directly to the formation, decay, and signals of the DCC and its evolution is undertaken. An evolution of the quantum fluctuations in an external, expanding metric which simulates the expansion of the quark gluon plasma undergoing a chiral phase transition is carried out. The analogy to the baryon asymmetry process is the asymmetry between the charged and neutral pions produced in the decay of the DCC. This is a measurable signal at future accelerators that reproduce the conditions of the early universe and can be used to test the formalism developed in this thesis.

In the seventh chapter, a summary of the work done in this thesis and emerging ideas for future work are given.

1.1 The Sakharov Conditions for BAU.

Since the Sakharov conditions are the starting point of most models of BAU, we will discuss them in some detail in this section.

1.1.1 Baryon number violation.

Particles and antiparticles are distinguished by certain quantum numbers which are $+1$ for particles and -1 for antiparticles. For the baryons, this quantum number is called the baryon number B and for leptons, it is called the lepton quantum number L. The conservation law for baryon number states that in a given interaction the total number of baryons before and after the process are equal. Till date, no interaction which violates the baryon number has been discovered. Therefore, unless there is baryon number violation, an initially baryon symmetric universe cannot evolve into a universe which has a baryon asymmetry. Baryon number violation leads to the decay of the proton. Thus, stability of the proton is a result of baryon number conservation. Hence, to explain the BAU from an initially symmetric universe, we must have a theory in which the proton would decay. The Grand unified theories (GUT) which predict proton decay and give rise to baryon number violation are valid only at the energies of the order of 10^{16} Gev. Therefore, baryon number violation can be accommodated at very high energies. The energy of the universe was of the order of 10^{19}Gev, which is more than the GUT energy, hence, the baryon number conservation can be violated in the early universe. Until and unless the baryon number is violated, the present asymmetry can only be treated as an initial condition.

1.1.2 CP Violation.

Even in the presence of baryon number nonconserving interactions, baryon asymmetry will not be developed unless CP (charge conjugation combined with parity) is violated. Under CP, an anti-particle is converted into a particle. For example, the action of CP on the Dirac field is

$$\begin{aligned} CP\psi(x,t) &= C\eta_P\gamma_0\psi(-x,t) \\ &= \eta_C\eta_P\gamma_0(\gamma_0\psi^*(-x,t)) \\ &= -\eta_C\eta_P\psi^*(-x,t). \end{aligned} \tag{1.1}$$

Now, consider a process $\psi \rightarrow \psi_1^* + \psi_2$. The baryon number for this process is $\Delta B = -1$ and the decay rate is given by $\Gamma(\psi \rightarrow \psi_1^* + \psi_2) = \pi_f |M_{fi}|^2$, where π_f is phase space and $|M_{fi}|^2$ is the transition matrix. Under the action of CP this processes goes to $\psi^* \rightarrow \psi_1 + \psi_2^*$, with the baryon number given by $\Delta B = +1$. The decay rate is $\Gamma(\psi \rightarrow \psi_1^* + \psi_2) = \pi_f |\overline{M}_{fi}|^2$. Under the action of CP $|M_{fi}|^2 = |\overline{M}_{fi}|^2$. If CP is conserved, then every reaction which produces an excess of baryon number is compensated by a reaction that produces a deficit of baryon number at the same rate. If CP is violated, then $|M_{fi}|^2 \neq |\overline{M}_{fi}|^2$ and we can have a baryon number violation. Therefore, to get a net baryon asymmetry it is necessary that CP is violated.

1.1.3 Departure from thermal equilibrium.

The Sakharov's third condition is a consequence of the CPT theorem [31] .

The Baryon number operator is defined as the difference between the number of baryons and antibaryons. $\widehat{B} = n_b - n_{\overline{b}}$, where n_b is the number of baryons and $n_{\overline{b}}$ is the number of antibaryons. Under CPT symmetry, a particle goes to an antiparticle via,

$$CPT(\widehat{B})(CPT)^{-1} = CPT(n_b)(CPT)^{-1} - CPT(n_{\bar{b}})(CPT)^{-1} = n_{\bar{b}} - n_b = -\widehat{B}.$$

At thermal equilibrium, the thermal average of $< B >= Tr(e^{\beta H}\widehat{B})$, with $\beta = \frac{1}{k_B T}$, where k_B is the Boltzmann constant, T is the temperature and H is the Hamiltonian. As result of CPT invariance $[H, CPT] = 0$. Applying CPT operator to $< B >$, we get

$$\begin{aligned} < B > &= Tr(CPT(e^{\beta H}\widehat{B}(CPT)^{-1})) \\ &= Tr(CPT(e^{\beta H})(CPT)^{-1})CPT(\widehat{B})(CPT)^{-1}) \\ &= Tr((e^{\beta H})(-\widehat{B})) \\ &= -Tr((e^{\beta H})(\widehat{B})) \\ &= - < B > . \end{aligned} \tag{1.2}$$

So the thermal average of the baryon number $< B >= 0$. Thus, in equilibrium, whatever baryon number is generated, will be wiped out. Therefore, to generate baryon asymmetry, departure from thermal equilibrium is necessary.

1.2 Estimation of Baryon Asymmetry Parameter η.

1.2.1 Heuristic Estimation of η

We present first a rough estimation of the baryon asymmetry parameter η, followed by a description of more accurate estimates. The first step is to estimate n_B, the matter of the universe. We know that there are about 10^{57} atoms in the sun, about 400 billion stars in our galaxy and roughly 125 billion galaxies in the universe. The total number of atoms in the universe is therefore $\sim 10^{78}$. The observable radius of the universe is 4.4×10^{26}, therefore assuming a spherical universe, the volume of the universe is 3.6×10^{80}. Thus the approximate matter (baryonic) density of the universe is given by $n_B = \frac{10^{78}}{10^{80}} = 10^{-2}$. In the big bang model, the radiation decoupled from matter at around $3000^0 K$ and filled the entire universe, this radiation is called CMBR. The total number of CMBR photons is constant after decoupling. The temperature dependence of the photon density at equilibrium s given by

$$n_\gamma = \frac{1}{\pi^2}\left(\frac{k_B T}{\hbar c}\right)^3 \int_0^\infty \frac{x^2}{\exp(x) - 1} dx \simeq 20.3T^3 \tag{1.3}$$

where k_B is the Boltzmann constant, $\hbar$ is the Planck constant and c is the speed of light in vacuum. The photon density at the time of the CMBR is given by $n_\gamma \simeq 20.3 \times (3000)^3 = 10^{11}$. From this we get a rough estimate BAU of the universe $\eta = \frac{n_B}{n_\gamma} \simeq \frac{10^{-2}}{20.3\times(3000)^3} \approx 10^{-10}$, which is very close to the value calculated by more detailed estimates.

A more accurate determination of η is given by from nucleosynthesis data [31, 6] and CMBR[35, 33] data and is presented below.

1.2.2 Estimation of η from Big-Bang Nucleosynthesis.

One of the successes of the standard model of cosmology is its prediction of the correct abundance of the light elements D, ^3He, ^4He, and ^7Li, through the big bang nucleosynthesis (BBN) [32, 6, 33], in the first few minutes of the universe. An important feature of the BBN is that it depends on T and η and therefore can be used in its estimation.

After the big bang, when the temperature was $T > 1\ Gev$, the protons and neutrons started forming. Then, because of its expansion, the universe cooled down sufficiently (temperature of the order of $T \sim 1\ MeV$) and stable protons and neutrons formed. The relative abundance of the protons and neutrons follow from simple thermodynamic arguments combined with the way the mean temperature of the universe changes with the cosmic expansion. Assuming nuclear statistical equilibrium among the light nuclear species, the abundance of light species with atomic number A is given by [6]

$$X_A = \frac{n_A A}{n_N} = g_A[\zeta(3)^{A-1}\pi^{(1-A)/2}2^{3A-5)/2}]A^{5/2}(\frac{T}{m_N})^{3(A-1)/2)}\eta^{A-1}n_p^Z n_n^{A-Z} exp\frac{B}{T}, \tag{1.4}$$

where g_A internal degrees of freedom, n_N total nucleon density, B is the binding energy of nuclear species, ζ Riemann zeta function and $\eta = \frac{n_N}{n_\gamma}$. This fraction is in favour of the proton because the high mass of neutron causes it to have finite life time. As the universe expands, free neutrons and protons are less stable than the the Helium nuclei. The protons and neutrons have a strong tendency to form ^4He (Helium). However, ^4He requires an intermediate step of deuterium formation. We notice that since $X_A \propto \eta^{A-1}$, around the time of nucleosynthesis $T = 0.1MeV$, the neutron to proton ratio is only slowly decreasing and the sensitivity of ^4He to η is low. The sensitivity to η is much more for D and ^3He. Detailed computer calculations developed by D. N. Schramm and M. S. Turner [34] have predicted the abundance of Helium on a logarithmic scale given by

$$Y_P = 0.245 + log(\frac{\eta}{5 \times 10^{-10}}) + 0.014\Delta N_\nu + 0.002(\tau_n - 8887sec) - \xi_{\nu_e} \tag{1.5}$$

where τ_n time scale, ξ_{ν_e} is the chemical potential and $\Delta N_\nu \equiv N_\nu - 3$, N_ν is the number of generations of neutrinos. The primordial abundances of all the light elements as a function of η have been calculated in this model. For observed abundance it can be shown that $4 \times 10^{-10} < \eta < 7 \times 10^{-10}$. This is how η is extracted from nucleosynthesis.

1.2.3 Estimation of η from CMBR Data.

The cosmic microwave background radiation (CMBR) is the cool remnant of the hot big bang radiation that fills the universe isotropically as blackbody radiation and can be observed at the present temperature of 2.7^0K. Its detection by Arno Penzias and Robert Wilson [17] in 1965 gives a direct evidence of the standard model of cosmology. Extrapolating the CMB data back in time, one can conclude that the early universe was a hot ideal gas, a hypothesis also supported nucleosynthesis data. As the universe expanded, the radiation cooled adiabatically below the threshold value of $T \sim m_e$ and electron-positron pairs in form of a hot plasma were created. As the temperature fell to 3000^0K, below the photoionization energy for hydrogen, photons decoupled from matter and moved unscattered throughout the universe. As the universe expanded, the temperature of the radiation cooled to the present value of 2.7^0K, but, because the photons are decoupled from matter, the spectrum remains the same. CMBR was originally thought to be a smooth blackbody radiation with an isotropic temperature.

Later, the COBE (Cosmic Background Explorer) and WMAP (Wilkinson Microwave Anisotropy Probe) experiments have probed the temperature fluctuations of the very early universe. Acoustic, Doppler, gravitational redshift, and photon diffusion effects combine to form a complicated spectrum of primary anisotropies [35, 36]. The component contributions individually reveal the sensitivity of the spectrum to a variety of cosmological parameters, including the baryon density, the dark matter density, the cosmological constant, the Hubble constant, and the curvature of the universe.

Before redshift $z_* \backsimeq 10^3$, CMB photons are hot enough to ionize hydrogen. Compton scattering tightly couples the photons to the electrons which are in turn coupled to the baryons by electromagnetic interactions. The system can thus be dynamically described as a photon-baryon fluid. Photon pressure resists the gravitational compression of the fluid and sets up acoustic oscillations. At z_*, neutral hydrogen forms and the photons last scatter. Regions of compression and rarefaction at this epoch represent hot and cold spots respectively. The resultant fluctuations appear to the observer as acoustic peaks. Performing a Fourier analysis of the isotropic temperature perturbation $\Theta = \frac{\delta T}{T}$, each mode k evolves almost as a simple harmonic oscillator before recombination,

$$m_{ef}\ddot{\Theta} + k^2 c^2 \frac{\Theta}{3} \backsimeq m_{ef} g. \tag{1.6}$$

The effective dimensionless mass of the oscillator is $m_{ef} = 1 + R$. Here, $R = \frac{(\rho_b + p_b)}{(\rho_\gamma + p_\gamma)} \backsimeq \frac{3\rho_b}{4\rho_\gamma}$ is the baryon-photon momentum density ratio. The oscillation frequency obeys the dispersion relation $\omega = \frac{kc}{3m_{ef}} = kc_s$, where c_s is the sound speed. Gravity provides an effective acceleration of $g = -k^2c^2\psi/3 - \ddot{\phi}$, where ψ is the Newtonian gravitational potential and $\phi \backsimeq -\psi$ is curvature perturbation. The baryons contribute to the mass of the fluid in the form $m_{ef} = 1 + R$, where, $R \propto \Omega_B h^2$. It can be shown that the difference in relative peak amplitudes of the even and odd peaks is $2R\psi$. Thus, the relative peak heights of the acoustic oscillation spectrum probe $\Omega_B h^2$ through R and the amplitude of potential perturbations at last scattering through ψ.

The critical density of baryons Ω_B in terms of η is,

$$\Omega_B h_0^2 = 366 \times 10^5 \eta = (0.019 \pm 0.001) \quad (1.7)$$

$$\Omega_B = 0.045 \pm 0.01, \quad (1.8)$$

here, $\Omega_B = \frac{\rho_B}{\rho_c}$, $\rho_B = m_N n_B$, and $\rho_c = \frac{3H_0^2}{8\pi G_N} = 1.054 \times 10^{-5} h_0^2 GeV cm^3$, where H_0 is the present Hubble parameter given by $H_0 = 100 h_0 km s^{-1} Mpc^{-1}$ and $h_0 = 0.65 \pm 0.05$. Thus the estimation of η can be done from CMBR.

In this chapter we have described the Sakharov conditions and the determination of the baryon asymmetry factor η. These provide a theoretical and empirical basis for various models of baryon asymmetry, which we shall describe in the next chapter.

CHAPTER 2

Models for BAU

As we have seen in chapter 1, the starting point for any theory of BAU must be the Sakharov conditions. Since the standard model of particle physics does not violate, by construction, Sakharov's first condition (baryon number violation), several models that go beyond the standard model have been proposed to get the baryon asymmetry dynamically from an initally baryon symmetric universe. In this chapter, we give a brief description of some of the models which have features that have stood the test of time and which have been useful in clarifying the essence of baryogenesis models. These are GUT(Grand Unified Theory) baryogenesis [37, 38], electroweak baryogenesis [39], Affeck-Dine baryogenesis [11], leptogenesis[40, 31], and recent models based on string theories [41]. The history of these models follows the history of particle physics and cosmology, as everytime a new theory comes into existence, attempts are made to accommodate baryogenesis in the theory.

2.1 GUT Baryogenesis.

The oldest model of baryogenesis was the GUT baryogenesis [37] based on grand unified theories (GUTS). Later, these baryogenesis models have been re-examined in the context of the supersymmetric (SUSY) GUTs [42]. Initially, grand unified theories were proposed to unify strong, weak and electromagnetic interactions within the frame work of gauge theories based on symmetry groups such as $SU(5)$ and $SO(10)$ at the energy scale of the order 10^{16} GeV. These theories predict the violation of baryon number leading to the proton decay. In GUTS, there are heavy gauge and Higgs bosons whose interactions violate the baryon number, thus the first Sakharov condition is satisfied. Typically in GUTS, there exist heavy X bosons that have two decay channels

$$X \rightarrow ql : B_1 = 1/3, \tag{2.1}$$

$$X \rightarrow qq : B_2 = +2/3. \tag{2.2}$$

Similarly the anti-particle $\bar{X}$ has the decay channels

$$\bar{X} \rightarrow \bar{q}\bar{l} : \bar{B}_1 = -1/3 \tag{2.3}$$

$$\bar{X} \rightarrow \bar{q}\bar{q} : \bar{B}_2 = -2/3, \tag{2.4}$$

where, q, $(\bar{q})$ and l $(\bar{l})$ represent the quarks(antiquarks) and leptons(antileptons) which carry baryon number 1/3 (-1/3) and 0 respectively. If one assigns the X boson a baryon number based on one decay mode then the other decay mode violates baryon number.

Let r be the decay rate for each process, then $\Gamma(X \rightarrow ql) \rightarrow r, \Gamma(X \rightarrow qq) \rightarrow 1-r, \Gamma(\bar{X} \rightarrow \bar{q}\bar{l}) \rightarrow \bar{r}, \Gamma(\bar{X} \rightarrow \bar{q}\bar{q}) \rightarrow 1-\bar{r}$. The mean baryon number per decay for X and $\bar{X}$ bosons is given by

$$\begin{aligned} B_X &= r\, B_1 + (1-r)\, B_2 \\ B_{\bar{X}} &= \bar{r}\, \bar{B}_1 + (1-\bar{r})\, \bar{B}_2. \end{aligned} \tag{2.5}$$

Then ΔB, the baryon asymmetry, in X-decays is given by

$$\Delta B = B_X + B_{\bar{X}} = \frac{1}{2}\left(r \ - \bar{r}\right) = \sum_f B_f \frac{\left[\Gamma\left(X \to f\right) - \Gamma\left(\bar{X} \to \bar{f}\right)\right]}{\Gamma_{tot}\left(X\right)}. \tag{2.6}$$

the baryon number produced vanishes when C or CP is conserved ($r = \bar{r}$. If there are no further baryon violating reaction then a net baryon asymmetry will persist after all the X and $\bar{X}$ decay. To explain the present baryon number $\eta = n_B/n_\gamma \approx \epsilon/g_* 10^{-10}$, one requires $\Delta B \sim 10^{-8}$, where ϵ is CP violating parameter and g_* are the effective degrees of freedom at $T \sim m_X$.

The problem with the GUT baryogenesis is that the temperature for processes with B-non conservation is extremely high, close to the Planck scale, $M_{GUT} = 10^{16} GeV$, thus, it would be kinematically difficult to produce these bosons in a thermal environment. At temperatures higher than the GUT scale, the rate of their production would be smaller than that of the expansion of the universe and any generated baryon asymmetry will be wiped out by this expansion. If the temperature of the universe could always be smaller than the GUT scale, the corresponding, thermally produced X-bosons would never have been abundant and their role in baryogenesis would be negligible. Also, one still has problems with proton decay, which is predicted by these theories at a rate that is accessible to experiments and this has not been observed. Nevertheless, the philosophy of GUT baryogenesis survives in many recent models of baryogenesis. Some of them consider the production of bosonic scalar fields with non -zero baryon number which decay into fermions to give rise to baryon asymmetry. In particular the models based on supersymmetry such the Affleck Dine model consider such a scenario.

2.2 Electroweak Baryogenesis.

In the electroweak sector of the standard Model of particle physics, the structure of the vacuum in non-Abelian gauge theories can give rise to baryon number violation. Non-Abelian gauge theories such as $SU(2)_L$ have degenerate vacua, which are topologically distinct. What distinguishes them is the winding number of the gauge field configuration. When one makes a transition from one vacuum to another vacuum, 3 quarks and a lepton are created which violate the baryon number. This baryon number violating transition, violates $B+L$ not $B-L$ (where L is the lepton number). In electroweak theory, both the baryonic current J_B^μ and leptonic current J_L^μ are conserved at the classical level, but not at the quantum level.

$$\begin{aligned} \partial_\mu J_B^\mu &= \partial_\mu J_L^\mu \\ &= N_g \left(\frac{\alpha_2}{\pi} W_a^{\mu\nu} \tilde{W}_{a\mu\nu} - \frac{\alpha'}{8\pi} F_{\mu\nu} \tilde{F}_{\mu\nu} \right), \end{aligned} \tag{2.7}$$

where N_g is the number of generations, $\alpha_2 = \frac{g_2^2}{4\pi}$, $\alpha' = \frac{g'^2}{4\pi}$ are couplings corresponding to $SU_L(2) \times U(1)$; $\tilde{W}_{\mu\nu} = \frac{1}{2}\varepsilon^{\mu\nu\alpha\beta} W_{\alpha\beta}$. This happens because of the chiral nature of fermions and is called the electroweak anomaly. Then from the fact that

$$\begin{aligned} \partial_\mu \left(J_B^\mu - J_L^\mu \right) &= \partial_\mu J_{B-L}^\mu = 0, \\ \partial_\mu \left(J_B^\mu + J_L^\mu \right) &\neq 0 \end{aligned} \tag{2.8}$$

and

$$\Delta \left(B+L \right) = 2 \frac{N_g \alpha_2}{8\pi} \int d^4x W_a^{\mu\nu} \tilde{W}_{a\mu\nu}. \tag{2.9}$$

we see that the electroweak anomaly preserves $B-L$ but not $B+L$.

There exist classical solutions for the Euclidean field equations called instantons which allow tunneling between different vacua (called θ vacua with the parameter θ). The gauge vacuum is then given by a superposition of θ vacua

$$|\theta\rangle = \sum_i e^{-in\theta} |n\rangle . \tag{2.10}$$

It has been shown by t'Hooft[43] that the transition probability between two adjacent vacua is given by $\sim e^{-8\pi^2/g^2}$ for $\nu = 1$. This gives the Euclidean action for a zero temperature solution as $S_E \sim \frac{8\pi^2}{g^2}$.

Thus $(B+L)$ violating amplitude for is $A \sim e^{(-2\pi/\alpha_2)\nu}$, where $\frac{g^2}{4\pi} = \alpha_2$. This is extremely small $\sim 10^{-90}$, hence, the probability of B-violating processes is highly suppressed at zero temperature.

The situation is quite different at the high temperatures of the early universe. Here, one goes from one vacuum to another vacuum by thermal fluctuations rather than tunneling. The transition probability P is given by $P \sim e^{-V_0(T)/T}$, where $V_0(T)$ is the height of the barrier. If the system is able to perform a transition from one vacuum to the next vacuum, then $\Delta(B+L) = 2N_g = 6$. The adjacent vacua of the Electroweak (EW) theory are separated by a ridge with energies larger than that of the vacua. The lowest energy point on this ridge is a saddle point solution to the equations of motion, and is referred to as the Sphaleron [44]. The electroweak phase transition is assumed to be first order and proceeds via bubble nucleation. This gives Sakharov's third condition. The sphaleron processes are in equilibrium outside the bubble wall and they are out of equilibrium inside the bubble, which give rise to B violation, thus Sakharov's first condition is satisfied. Sakharov's second condition, CP violation in the standard model of particle physics is given by Cabibbo-Kobayashi-Maskawa (CKM) matrix [45]. The baryon asymmetry parameter η is given by $\eta \simeq \alpha_2^4 \epsilon_{cp}$, where ϵ_{cp} the magnitude of CP violation.

The problems with electroweak baryogenesis are that one cannot get a first order transition unless the Higgs is light, $m_H < 60$ GeV, which is ruled out by LEP, $(m_H \geq 114)$ GeV. Another problem is the amount of CP-violation required is 8 orders less in magnitude than that of given in the standard model.

2.3 Affleck-Dine Baryogenesis.

The Affleck-Dine mechanism [11] is based on the supersymmetric flat directions [46]. In supersymmetric theories all particles have super partners called sparticles. In the minimal supersymmetric standard model(MSSM) [47], the number of degrees of freedom are increased by virtue of the fact that the bosons and the fermions have supersymmetric counterparts. The increase in the number of degrees of freedom results in directions in field space which have virtually no potential. These are known as flat directions and are made up of squarks or sleptons, so they carry the baryon or the lepton number. During the reheating after inflation, the squarks and sleptons are free to fluctuate along these directions as it costs little energy and can form condensates with a large baryon or lepton number. These condensates, through the interaction with the inflaton field, spontaneously break the supersymmetry. Supersymmetry breaking lifts these flat directions and sets a scale for the potential. Supersymmetry breaking can introduce terms in the Lagrangian that violate baryon number and CP.

It is known that bosons obey Bose-Einstein statistics and they commute and fermions obey Fermi-Dirac statistics and they anticommute. In order to combine them into a single algebra, it requires an introduction of a Z_2 grading under which the bosons are the even elements and the fermions are the odd elements. Such an algebra is called a Lie superalgebra. The simplest supersymmetric extension of the Poincare algebra contains two Weyl spinors with the following anti-commutation relation is defined by

$$\{Q_a, Q_b\} = P^{\mu}(\sigma_{\mu})_{ab}, \tag{2.11}$$

where the Q_a is the fermonic generator, P_μ is the four momentum and $\sigma_\mu = (1, -\sigma_i)$, where σ_i are Pauli matrices.

To illustrate the effects of supersymmetry breaking consider a simple supersymmetric model with one scalar field and a fermonic field. The Lagrangian is given by

$$L = \partial_\mu \phi^\dagger \partial^\mu \phi + \psi^\dagger i\sigma^\mu \partial_\mu \psi + F^\dagger F \tag{2.12}$$

where ϕ is a complex scalar field with two degrees of freedom and ψ is a fermonic field with four degrees of freedom. Therefore, we need to add another scalar field F with two degrees of freedom to ensure supersymmetry. This scalar field F is called an auxiliary field. This field does not contribute to the equation of motion since there is no kinetic term for this field.

Under an infinitesimal supersymmetric transformation, ϕ, ψ, and F transform as follows,

$$\begin{aligned} \delta\psi_l &= \sqrt{2}\partial_\mu\phi\gamma^\mu\alpha_l + \sqrt{2}F\alpha_l \qquad (2.13) \\ \delta\phi &= \sqrt{2}\alpha_l\psi_l \\ \delta F &= \sqrt{2}(\overline{\alpha}_l\gamma^\mu\partial_\mu\psi_l). \qquad (2.14) \end{aligned}$$

where α (spinor) is an infinitesimal paramater. We can see from the above transformations that if we break supersymmetry spontaneously, we cannot give vacuum expectation value to ϕ and ψ since they transform into each other. Therefore the only field left is the auxiliary field F. When supersymmetry is exact $< 0|F|0 >= 0$ and when supersymmetry is broken $< 0|F|0 >\neq 0$. When supersymmetry is exact, the fields along F have zero vacuum expectation value so they are called the flat directions. Upon supersymmetry breaking, the degeneracy along the auxiliary field F direction is lifted, and a potential along this direction is generated.

To introduce interactions by keeping the invariance of the Lagrangian under the infinitesimal supersymmetric transformations and renormalizability in mind, a general supersymmetry-breaking potential was introduce by Wess and Zumino. The Wess-zumino Lagrangian contains an analytic function of the scalar fields W(ϕ), known as the superpotential, given by

$$W(\phi) = \frac{1}{2}m_{ij}\phi_i\phi_j + \lambda_{ijk}\phi_i\phi_j\phi_k. \qquad (2.15)$$

The form of the scalar potential is

$$V(\phi) = \sum_i |\frac{\partial W}{\partial \phi_i}|^2 + \sum_a g^{a2}(\sum_i \phi_i^* T^a \phi_i)^2. \qquad (2.16)$$

The scalar potential can be written in terms of auxiliary fields F and D as

$$V(\phi) = |F|^2 + |D|^2, \qquad (2.17)$$

where $F = \frac{\partial W}{\partial \phi_i}$ and $D = g^a(\sum_i \phi_i^* T^a \phi_i)$. We can see that when SUSY is exact the F and the D terms vanish. This give rise to the F and the D flat directions.

The basic mechanism of Affleck-Dine baryogenesis is as follows.:

In the Affleck Dine model, the cosmological constant in the early universe breaks supersymmetry spontaneously and can give rise to an inflaton potential that violates $B - L$, thus satisfying Sakharov's first condition. The scalar fields, through their interaction with the inflaton field, generate C and CP violation, thus satisfying Sakharov's second condition. The expansion of the universe satisfies Sakharov's third condition.

One can understand the Affleck-Dine mechanism in the context of a toy model. Consider a potential for a complex scalar field ϕ given by

$$V(\phi) = -m_\phi^2 \phi^* \phi + \frac{\lambda}{4}(\phi^4 + \phi^{*4}), \tag{2.18}$$

here the coupling constant $\lambda \approx \epsilon M_S^2 / M_G^2$, where, M_S is the supersymmetry scale breaking, ϵ is a real parameter which characterises CP violation and M_G is the grand unification scale. One can assign ϕ with a baryon number of 1 based on its interactions with the standard model particles. For example, if ϕ is like a squark then a squark decays to a quark and a photino, one can assign the baryon number of the quark to the squark (photinos carry no baryon number). Then the last term in $V(\phi)$ violates baryon number and also violates CP. The baryon current associated with ϕ is

$$j_\mu = -i\phi^* \overleftrightarrow{\partial}_\mu \phi. \tag{2.19}$$

BAU in this model is given by $n_B = \int J_0 d^3x$. Ignoring spatial variations in ϕ, which is a good assumption after inflation, as the field ϕ rolls down its potential, this quantity increases. Solving the equations of motion for ϕ and ϕ^* one finds out the value of n_B. The equations of motion for ϕ and ϕ^* are given by

$$\ddot{\phi} + m^2\phi + 3H\dot{\phi} = \lambda\phi^{*3}, \tag{2.20}$$

and

$$\ddot{\phi}^* + m^2\phi^* + 3H\dot{\phi}^* = -\lambda\phi^{*3}, \tag{2.21}$$

where H is the Hubble's constant. The initial conditions at $t = t_0$ are

$$\phi|_{t=t_0} = i\phi_0, \;\; \dot{\phi}|_{t=t_0} = 0. \tag{2.22}$$

In this linearised approximation the solutin of these equations can be solved for radiation and matter dominated eras. For the radiation dominated era ($H = \frac{1}{2t}$),the solution is

$$\phi_R = \frac{a_r \lambda \phi_0^3}{m^2 (mt)^{3/4}} sin(mt + \delta_r), \tag{2.23}$$

$$\phi_I = \frac{\phi_0}{(mt)^{3/4}} sin(mt - \frac{3}{8}\pi). \tag{2.24}$$

constants a and δ are independent of λ and ϕ_0. The baryon number for the radiation dominated era is given by

$$n_B = \frac{2a_r \lambda \phi_0^4}{m(mt)^2} sin(\delta_r + \frac{1}{8}\pi). \tag{2.25}$$

Similarly for the matter dominated era $H = \frac{2}{3t}$, the solution is

$$\phi_R = \frac{a_r \lambda \phi_0^3}{m^2 (mt)} sin(mt + \delta_m) \tag{2.26}$$

$$\phi_I = \frac{\phi_0}{(mt)} sin(mt)(1 + \bigcirc(\frac{\lambda \phi_0^2}{m^2})), \tag{2.27}$$

The baryon number for the matter dominated era is given by

$$n_B = \frac{2a_r \lambda \phi_0^4}{m(mt)^2} sin(\delta_m). \tag{2.28}$$

The baryon number per particle for large times in this model is given by

$$n_B = r \approx \frac{\lambda \phi_0^2}{m_\phi^2}. \tag{2.29}$$

In this mechanism, one can get a baryon asymmetry as large as 10^{-2}. These models are also popular in the context of superstring theories. Clearly a problem with this model is that the amount of BAU produced is too large. We will be using this model in chapter 5 of this thesis.

2.4 Leptogenesis.

Leptogenesis is a popular mechanism for baryogenesis. In this model, the asymmetry is generated via a lepton asymmetry in the universe, which is partially converted to a baryon asymmetry through sphaleron interactions. The masses for the neutrinos one must use the Majorana mechanism. For this to be allowed one has to go beyond the standard model (as the $B - L$ symmetry of the standard model does not allow Majorana mass generation). This entails adding a non-renoramlizable terms to get lepton-number violating Lagrangian of the form $L = \frac{Z_{ij}}{\Lambda} L_i L_j \phi\phi$, where L_i is lepton doublets, ϕ is the Higgs field, Z_{ij} is a dimensionless complex symmetric matrix of couplings and Λ is the scale where the standard model description breaks. These terms lead to light neutrino masses $m_\nu = \frac{Z<\phi>}{\Lambda}$ by simply taking into account that the SM is an effective theory that is valid only up to some high scale $\Lambda \gg \phi$, This gives an understanding why neutrino masses are much lighter than the charged fermions. The full high energy theory that leads to the non-renormalizable terms needs heavy singlet neutrinos N_α. These are new fermions that are neutral under standard model gauge group, but they do not have any gauge interactions. There are two types of interaction that can be added to the Lagrangian, they are $L_N = M_\alpha N_\alpha N_\alpha + \lambda_{\alpha i} N_\alpha L_i \phi$, where M_α is a Majorana mass matrix for the singlet neutrinos, and $\lambda_{\alpha i}$ is a Yukawa matrix that couples them to the light lepton doublets.

The introduction of singlet neutrinos with Majorana masses and the Yukawa couplings to the doublet leptons fulfills Sakharov conditions. The Majorana nature of the singlet neutrino masses implies that any single heavy mass eigenstates can decay to both $L + \phi$ and $L + \phi^\dagger$. If we assign the N mass eigenstates a lepton number zero, the first mode is $\Delta L = +1$ while the second is $\Delta L = -1$. Thus, lepton number is violated in these decays.

The decay $L + \phi$ is dominated by the tree diagram. There are, however, corrections coming from the one loop diagrams, the interference of the tree and the loop diagram gives rise to a relative phase, which is the source of CP-violation given by

$$\epsilon = \frac{\left[\Gamma\left(N_\alpha \to lH\right) - \Gamma\left(N_\alpha \to \bar{l}\bar{H}\right)\right]}{\left[\Gamma\left(N_\alpha \to lH\right) + \Gamma\left(N_\alpha \to \bar{l}\bar{H}\right)\right]}. \tag{2.30}$$

The decay is out of equilibrium if the decay rate is slower than the expansion rate of the universe when the temperature is of the order of the mass of the decaying singlet neutrino

$\Gamma \lesssim H(T \sim M_\alpha)$. For $M_1 < 10^{14}$ GeV, the final baryon asymmetry is given by

$$\eta = -1.4 \times 10{-}3 \sum_{\alpha,\beta} \epsilon \eta_{\alpha\beta} \tag{2.31}$$

where $\eta_{\alpha\beta}$ paramatrises the washout of ϵ. The leptogenesis mechanism is popular because the parameter space in which it is viable has not yet been constrained, as is the case for the standard model of electroweak baryogenesis. The presence of non-renormalizable terms is a shortcoming of the model.

2.5 Baryogenesis in the Brane-World Scenario.

Brane models are motivated by string theories. The brane model was proposed to explain the weakness of gravity relative to the other fundamental forces of nature. In Randall-Sundrum [48] model, the four-dimensional universe in which we live is restricted to a brane inside a higher-dimensional space, called the bulk. The additional dimensions are compactified. The three fundamental forces, the electromagnetism, the weak and the strong forces are localized on the brane but gravity can propagate in bulk also. In M-theory five-branes can be included in the compactification from 11 to five dimensions. For the remaning six dimensions, the two-dimensions are wrapped into a curve in the Calabi-Yau space and stretch across the remaining four uncompactified dimensions and are parallel to the orbifold planes. Hence, they appear as the three branes in the five dimensional brane world theory which are located somewhere in the bulk between the two orbifold planes. A small-instanton transition process occurs when one of the three-branes in the five-dimensional brane-world model collides with an orbifold fixed plane. The baryogenesis in the context of brane-world models is generated at a small-instanton phase transition which is initiated by a moving brane colliding with the observable boundary. Reasonable values for the baryon asymmetry can be obtained [41]. The generated baryon asymmetry depends on characteristics of the brane-collision, such as the impact of the colliding brane. There is no conclusive evidence for brane world scenario, this is a speculative model

2.6 Gravitational Baryogenesis.

The gravitational interaction between the Ricci scalar curvature and the matter fields dynamically breaks CP in an expanding universe and combined with baryon number violating interactions, can drive the universe towards a baryon asymmetry universe. This model is considered more detail in the chapter 4.

There are several other models proposed apart from the models presented here. In literature there are there about 42 models of baryogenesis, they are as follows, 1) GUT baryogenesis, 2) GUT baryogenesis after preheating, 3) Baryogenesis from primordial black holes, 4) String scale baryogenesis, 5) Affleck-Dine (AD) baryogenesis, 6) Hybridized AD baryogenesis, 7) No-scale AD baryogenesis, 8) Single field baryogenesis, 9) Electroweak (EW) baryogenesis, 10) Local EW baryogenesis, 11) Non-local EW baryogenesis, 12) EW baryogenesis at preheating, 13) SUSY EW baryogenesis, 14) String mediated EW baryogenesis, 15) Baryogenesis via leptogenesis, 16) Inflationary baryogenesis, 17) Resonant baryogenesis, 18) Spontaneous baryogenesis, 19) Coherent baryogenesis, 20) Gravitational baryogenesis, 21) Defect mediated baryogenesis, 22) Baryogenesis from long cosmic strings, 23) Baryogenesis from short cosmic strings, 24) Baryogenesis from collapsing loops, 25) Baryogenesis through collapse of vortons, 26) Baryogenesis through axion domain walls, 27) Baryogenesis through QCD domain walls, 28) Baryogenesis through unstable domain walls, 29) Baryogenesis from classical force, 30) Baryogenesis from electrogenesis, 31) B-ball baryogenesis, 32) Baryogenesis from CPT breaking, 33) Baryogenesis through quantum gravity, 34) Baryogenesis via neutrino oscillations, 35) Monopole baryogenesis, 36) Axino induced baryogenesis, 37) Gravitino induced baryogenesis, 38) Radion induced baryogenesis, 39) Baryogenesis in large extra dimensions, 40) Baryogenesis by brane collision, 41) Baryogenesis via density fluctuations,and 42) Baryogenesis from hadronic jets.

Yet, inspite of this plethora of models, an acceptable theory of Baryogenesis is yet to be found.

CHAPTER 3

Standard Model of Cosmology, Inflation and Particle Production

One of the aims of this thesis is to estimate baryon asymmetry in the wake of inflationary models of reheating in the early universe. To make the thesis self contained, in this chapter, we introduce the concept of inflation, reheating and the methods we use to calculate particle production in the early universe. All these concepts will be useful in the subsequent chapters. Modern cosmology is based on the standard model of cosmology, which is well supported by experimental evidence such as CMBR (COBE, WMAP), nucleosynthesis and the Hubble expansion of the universe. It also has some serious shortcomings which need attention. The inflationary model was proposed to overcome these shortcomings. Inflation is nothing but exponential expansion, which involves the introduction of a scalar field which is displaced from its vacuum value. As this field rolls slowly into its vacuum state, determined by the inflationary potential, exponential expansion results and solves some of the problems of the

Standard model of cosmology. Single or multi scalar fields can be involved in inflation, and we shall see that they are useful in models for baryogenesis. The reheating process occurs after the termination of inflation due to the quantum fluctuations of the inflaton field. Particle production at this stage can be described by parametric resonance and squeezed states. We shall briefly describe each of these processes in this chapter.

3.1 Standard Model of Cosmology.

The standard model of cosmology (standard big bang model) is based on the cosmological principle which states that the universe is isotropic and homogeneous. Starting with Einsteins equations [49]

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G T_{\mu\nu}, \tag{3.1}$$

where $R_{\mu\nu}$ and R are the Ricci tensor and scalar, $T_{\mu\nu}$ is the energy momentum tensor and G is the gravitational constant and assuming a perfect fluid description of the matter

$$T_{\mu\nu} = -p g_{\mu\nu} R + (P + \rho) u_\mu u_\nu, \tag{3.2}$$

the dynamics of the universe is given by the Friedman-Robertson-Walker (FRW) solution [50]

$$ds^2 = dt^2 - a(t)^2 \left[\frac{dr^2}{1 - kr^2} + r^2 (d\theta^2 + sin^2\theta d\phi^2) \right]. \tag{3.3}$$

The coordinates r, θ and ϕ are comoving spherical coordinates obtained by rescaling the radial coordinate. k, is the scalar curvature of 3-space which has values -1, 0 or $+1$, corresponding to open, spatially flat or closed universes respectively and $a(t)$ is the expansion factor of the universe. The physical distance is given by $R_p = a(t) \int \frac{dr}{\sqrt{1-kr^2}}$ and for a flat ($k = 0$) universe $R_p = a(t)r$. The velocity is given by $V = HR_p$ ($\frac{dr}{dt} = 0$), where H is the Hubble parameter given by $H = \frac{\dot{a}}{a}$.

If we assume the $T_{\mu\nu} = diag[-\rho, P, P, P]$ where ρ is energy density and P is the pressure, then from the continuity equation we get

$$\dot{\rho} = -3H(\rho + P). \tag{3.4}$$

Combining this with Einstein's equation (3.1) we get the Friedman equation :

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2} = \frac{8\pi}{3}G\rho, . \tag{3.5}$$

In terms of Hubble's constant we have:

$$\frac{k}{H^2a^2} = \frac{\rho}{3H^2/8\pi G} - 1 = \Omega - 1, \tag{3.6}$$

where $\Omega = \frac{\rho}{\rho_c}$ is the ratio between the density and the critical density required to close the universe and this critical density is given by $\rho_c = \frac{3H^2}{8\pi G}$. $\Omega > 1$, $\Omega = 1$ and $\Omega < 1$ correspond to open, flat and closed universes respectively.

Another important assumption of the standard model of cosmology is that the matter is described by a classical ideal gas, with an equation of state $P = w\rho$, where w is constant of proportionality independent of time. The energy density ρ goes as $\rho \propto a^{-3(1+w)}$. For the radiation dominated era $P = \frac{1}{3}\rho$, which gives $\rho \propto a^{-4}$. For the matter dominated era $P = 0$ gives $\rho \propto a^{-3}$. For the vacuum energy dominated era $P = -\rho$, which gives $\rho \propto const$.

The problems of standard model of cosmology are

- The Horizon Problem:

  The horizon problem arises from the fact that the transfer of information is limited by the speed of light. This results in a particle horizon which limits the separation of two regions that are in causal contact. The CMBR shows that the universe is isotropic if the universe is radiation dominated or matter dominated since the big bang. The particle horizon corresponds to approximately R_p which is 2^0 of the sky, so they would be no mechanism for distant objects to have same temperature through out the universe to make it homogenous and isotropic.

- The Flatness Problem:

  In the equation (3.6), the observations from CMBR give a value of $\Omega \approx 1$. Therefore, since $\Omega = \frac{\rho}{\rho_c}$, the total energy density of the early universe must be approximately equal to the present critical density. The flatness problem is the question of why the initial conditions correspond to the universe being close to spatial flatness? The problem is that any small perturbation will cause a fluctuation of the energy density to grow exponentially. Thus, it requires extreme fine tuning to maintain the density of the early universe to this critical density value.

- The Entropy Problem:

  The entropy problem is related to the flatness problem in the following way [51]. It is known that the entropy in a comoving volume stays constant in an adiabatic expansion. The entropy of the universe is given by $S_U = T^3 H^{-3}$, where H is the Hubble's constant. Rewriting the Friedman equation in terms of entropy we get $\Omega - 1 = \frac{\kappa m_p^2}{S_U^{2/3} T^2}$. Since we have seen from the flatness problem that $\Omega \approx 1$, the value of the entropy $S_U \approx 10^{60}$, which is very very large. This reliance on fine tuned initial conditions is called flatness problem.

3.2 Inflation.

The idea of inflation [23] is proposed to overcome the problems of standard model of cosmology. Inflation is defined to be any epoch of the universe dominated by the negative pressure $\rho = -P$ vacuum density. With this condition, the Friedman equation (3.5) becomes

$$H = \frac{\dot{a}}{a} = constant, \tag{3.7}$$

therefore

$$a = e^{Ht}. \tag{3.8}$$

Thus inflation corresponds to an exponential expansion of the universe. With this hypothesis, the problems in the standard model of cosmology are solved. Substituting into eq (3.6), we see that with the large value of the expansion parameter, we automatically get $\Omega \approx 1$. Thus the flatness problem is solved. We have seen that the entropy problem is related to the flatness problem, thus the entropy problem is also solved. Inflation solves the Horizon problem by postulating that prior to the inflation, in a small radius of 10^{-23}, cm corresponding to a time scale of 10^{-36} sec, before inflation, the entire universe is causally connected and achieves equilibrium to become isotropic. Inflation then expanded the universe rapidly, freezing in these properties, so that the universe evolved to an almost homogeneous, isotropic state. The information needed to change it from that state is not in causal contact, thus the homogeneity and isotropy is maintained. In these ways, inflation is a solution to the problems of standard model of cosmology.

In order to drive the universe into an inflationary phase, scalar field(s) known as inflaton(s) were proposed. The initial energy density of the universe is stored in the potential energy of the inflaton field. The inflaton is initially at its highest energy state. It rolls down to its minimum energy state via random fluctuations, which trigger a phase transition, and cause the inflaton releases its potential energy in the form of radiation and matter in a process known as reheating. Various inflationary scenarios have been proposed. Most of these vary with in the form of the inflaton potential, the number of inflaton fields and the order of phase transition.

In inflationary models, the inflaton field ϕ has a potential energy $V(\phi)$ which is flat near $\phi = 0$ and steep near $\phi = \phi_{min}$. At high cosmic temperatures $V(\phi)$ is flat due to temperature corrections. As the inflaton field rolls slowly down the potential to the state $\phi = 0$, during which the energy density is nearly constant, the scale factor grows exponentially. As the temperature drops, the effective potential $V(\phi)$ develops a small barrier seperating the local minimum at $\phi = 0$ and the vacua at $\phi = \phi_{min}$. The tunneling of ϕ from $\phi = 0$ to $\phi = \phi_{min}$ gives rise to a phase transition. The first inflationary model proposed was based on first order phase transition and called old inflation and was later on modified to be a second order phase transition model [52].

An example of a theoretical model for inflation is one with a Lagrangian density of the form [6, 33]

$$L = g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi), \tag{3.9}$$

where V(ϕ) is the potential of the scalar field. These contributes to the energy momentum tensor by

$$T^{\mu\nu} = \frac{\partial L}{\partial(\partial_\mu\phi)}\partial^\nu\phi - g^{\mu\nu}L. \tag{3.10}$$

In the homogeneous state, the energy momentum tensor becomes a perfect fluid, which is given in eq (3.2). The value of ρ and P in terms of scalar field ϕ are given by

$$\rho = \frac{1}{2}\dot{\phi}^2 + V(\phi), \tag{3.11}$$

$$P = \frac{1}{2}\dot{\phi}^2 - V(\phi). \tag{3.12}$$

Putting eq (3.11) and (3.12) in the Friedman equation (3.5) we get

$$H^2 = \frac{8\pi G}{2}(\frac{1}{2}\dot{\phi}^2 + V(\phi)). \tag{3.13}$$

The equation for an expanding universe containing a homogeneous scalar field is given by

$$\ddot{\phi} + 3H\dot{\phi} + \frac{d}{d\phi}V(\phi) = 0. \tag{3.14}$$

To produce enough inflation and rapid reheating after the inflation, the potential should be flat initially and steep afterwards. The scalar field would then be expected to roll slowly on the flat potential and the $\ddot{\phi}$ can be neglected. Then the equation of motion becomes

$$3H\dot{\phi} + \frac{d}{d\phi}V(\phi) = 0, \tag{3.15}$$

and the Friedman equation can be approximated as

$$H^2 = \frac{8\pi G}{2}V(\phi). \tag{3.16}$$

The slow-roll [33] parameters are defined as

$$\epsilon = -\frac{\dot{H}}{H^2} = 4\pi G\frac{\dot{\phi}^2}{H^2}, = \frac{1}{16\pi G}(\frac{V'}{V})^2, \tag{3.17}$$

and

$$\eta(\phi) = \frac{1}{8\pi G}\frac{V''}{V}, , \tag{3.18}$$

where, the first measures the slope of the potential and the second, the curvature. The necessary conditions for the slow-roll approximation to hold are

$$\epsilon << 1; |\eta| << 1. \tag{3.19}$$

These slow-roll condition are sufficient for inflation to take place.

As stated earlier, the original inflationary model proposed by A. Guth[23] in 1981 was based on an inflationary potential that under goes first order phase transition. The early universe is supercooled and goes into a metastable state which is called the false vacuum. The equation of state in the false vacuum approaches p = $-\rho$, and inflation occurs. After inflation, the scalar field tunnels through the false vacuum to the true vacuum. The tunneling produces a small bubble of true vacuum in the sea of false vacuum, expanding at the speed of light. The problem with the old inflationary model is that it requires a large cosmological constant. If the bubble walls expand with the speed of light, the space between the bubbles expands exponentially, and the bubbles at the present time will be much smaller than our apparent horizon. This is in contradiction with the observed isotropy of the CMBR. This model also has the problem that it will not reheat properly. When the bubbles nucleated, they do not generate any radiation. Radiation could only be generated in collisions between bubble walls.

To overcome these limitations, the new inflationary model was proposed independently by A. Linde [52], and by A. Albrecht and P. Steinhardt [53]. This model is based on a double well inflaton potential, which undergoes second order phase transition. In one version of the model the Coleman-Weinberg potential is used,

$$V(\phi) = \frac{1}{4}(\phi^2 - \sigma^2)^2. \tag{3.20}$$

At $T < T_c$ (T_c being the critical temperature), thermal fluctuation trigger the instability of field ϕ at $\phi(x) = 0$ and then $\phi(x)$ goes to a global minimum $\phi(x) = +\sigma$ or $\phi(x) = -\sigma$. Within the fluctuations region $\phi(x)$ is homogeneous, neglecting spatial gradient. The slow rolling conditions (3.17) and (3.18) are satisfied in this region as required for inflation.

Because of fine tuning problems, to satisfy the slow roll conditions, the inflaton must have a very small mass. Other inflationary models have been proposed, but the basic philosophy is the same. A few of these models are summarized below.

The chaotic inflationary model, proposed by A. Linde [52] in 1986, uses a potential $V(\phi) = \frac{m^2}{2}\phi^2$. This potential has a minimum at $\phi = 0$. ϕ oscillates at minimum if universe does not expand and the equation of motion is given by

$$\ddot{\phi} + 3H\dot{\phi} + m^2\Phi = 0, \tag{3.21}$$

the above equation is like a harmonic oscillator, with a friction term $3H\dot{\phi}$. Initially the scalar field ϕ and H are too large, therefore the friction term in the equation (3.21) is large and the scalar field moves very slowly as a ball in a viscous liquid. At this stage, the energy density of scalar field, unlike the density of matter, remains almost constant, and the expansion of universe is continued with much greater speed. Due to the rapid growth of the scale factor of the universe and slow motion of the field ϕ, we can make the following approximations $\ddot{\phi} << 3H\dot{\phi}$, $H^2 >> \frac{K}{a^2}$, $\dot{\phi}^2 << m^2\Phi$. The equation of motion reduces to

$$3\frac{\dot{a}}{a}\dot{\phi} = -m^2\Phi. \tag{3.22}$$

The Friedman equation is given by

$$(\frac{\dot{a}}{a})^2 = 2\pi G(\dot{\phi}^2.), \tag{3.23}$$

leading to,

$$\frac{\dot{a}}{a} = 3m\Phi G(\sqrt{\frac{\pi}{3}}), \tag{3.24}$$

and gives

$$a = e^{Ht}, \tag{3.25}$$

where $H = 3m\Phi G(\sqrt{\frac{\pi}{3}})$. Thus it gives rise to inflation.

The disadvantage of the above inflationary models is that they require tiny parameters in order to reproduce the present observations. To overcome this A. Linde proposed hybrid inflation with two scalar fields ϕ and χ. The field ϕ provides the vacuum energy density which drives inflation and field χ is the slowly varying field during inflation. The effective potential is given by

$$V(\phi,\chi) = k^2(M^2 - \frac{\phi}{4}) + \frac{\lambda^2\phi^2\chi^2}{4} + \frac{m^2\chi^2}{2}, \tag{3.26}$$

where $k, \lambda > 0$ are dimensionless constants and $M, m > 0$ are mass parameters. The vacua lie at $\phi = \pm 2M, \phi = 0$. For $m = 0$, V has a flat direction at $\phi = 0$, where $V = k^2M^4$ and the mass of ϕ is $m^2 = -k^2M^2 + \lambda^2\chi^2/2$. So, for $\phi = 0$ and $|\chi| > \chi_c \equiv \frac{2kM}{\lambda}$, we obtain a flat valley of minima. For $m \neq 0$, the valley acquires a slope and the system can inflate as the field χ slowly rolls down this valley. The slowroll conditions are satisfied and inflation sets in and continues till χ reaches χ_c, then the inflation terminates. It is followed by a waterfall i.e. a sudden entrance into an oscillatory phase about a global minimum at which point reheating starts.

3.3 Reheating.

During inflation, the universe expanded exponentially and the temperature of the universe dropped very low, leaving it devoid of particles. For matter and radiation to form, the universe must have gone through a period of reheating after the inflation. Successful inflation must transfer energy in the inflaton field to radiation and reheat the universe to at least 1 MeV for Nucleosynthesis to occur. Quasiperiodic evolution of the inflaton field leads to the creation of particles. The process by which the large potential energy density present during inflation gets converted into radiation and matter at the end of inflation is known as reheating. This is an important concept for the ideas presented in this thesis, since matter and antimatter must be generated in the reheating epoch. The first theory of particle production during reheating [18] was proposed using first order Born approximation for an oscillating inflaton field. This approach had some shortcomings and did not give sufficient particle production. In order to rectify this situation, parametric resonance effects were included. Parametric resonance effects during reheating can enhance the rate of particle production considerably.

Particle creation or annihilation generally occurs in a time-dependent background, the number of created particles grows exponentially, either when the background is periodic in time, or, when the coupling constant changes suddenly. This results in parametric amplification of the long wavelength modes, which in turn give rise to far from equilibrium particle production.

The first reheating model in which particle production is obtained through parametric resonance phenomenon, was proposed by Y. Shtanov, J. Traschen and R. H. Brandenberger [18]. They considered particle production using the interaction Lagrangian

$$L_{int} = -f\phi\bar{\psi}\psi - (\sigma\phi + h\phi^2)\chi^2. \tag{3.27}$$

Here $\bar{\psi}$ and ψ describe spinor particles and χ describes scalar particles with masses given by m_ψ and m_χ, respectively. The coupling constants f and g are dimensionless, while σ has the dimensions of mass. The inflaton field ϕ is treated as a classical external field. The scalar field ϕ undergoes quasi-periodic evolution, with a mode decomposition

$$\phi^2 = \bar{\phi}^2 + \sum_{n=1}^{\infty} \zeta_n cos(n\omega t), \tag{3.28}$$

where, ϕ_n and ζ_n are slowly varying amplitudes with time and $\bar{\phi}^2 \approx \frac{\phi_0^2}{2}$. The evolution of a particular mode χ_k of the quantum scalar field χ, in the presence of the rapidly oscillating classical scalar field ϕ is given by

$$\ddot{\chi}_k + 3H\dot{\chi}_k + (k^2 + m_\chi^2 + 2\sigma\phi + 2h\phi^2)\chi_k = 0, \tag{3.29}$$

where $k = k/a$ is comoving wave number. Performing the transformation $\chi_k = \frac{Y_k}{a^{\frac{3}{2}}}$, the above equation becomes

$$\ddot{Y}_k + (\omega_k^2 + g(\omega t))Y_k = 0, \tag{3.30}$$

where,

$$\omega_k^2 = k^2 + m_\chi^2 - \frac{9}{4}H^2 - \frac{3}{2}\dot{H} + 2h\bar{\phi}^2 \tag{3.31}$$

and

$$g(\omega t) = 2\sigma\phi + 2h(\phi^2 - \bar{\phi}^2). \tag{3.32}$$

$g(\omega t)$ is(to a good approximation), a periodic function of time t

$$g(x) = \sum_{n=-\infty}^{\infty} g_n e^{inx}, \tag{3.33}$$

where, g_n is the amplitude satisfying $g_n^* = g_{-n}$. It is convenient to introduce the following quantities $g_n = |g_n|e^{i\alpha_n}$ and $\omega_0 = (\frac{p}{q}\omega)^2 + \epsilon\Delta$, where α_n is the phase and p and q are integers. Then 3.29 is the equation of motion for a time dependent harmonic oscillator and can be approximated by the Mathieu's equation. This equation has parametric resonance for certain values of $\omega_k - (\frac{n}{2}\omega)^2 \equiv \Delta_n < |g_n|$. This is valid in the case when the frequencies of ω_k and ω change slowly with time. From this condition it follows that $\omega_k^2 - k^2 \equiv m_\chi^2 - \frac{9}{4}H^2 - \frac{3}{2}\dot{H} + 2h\bar{\phi}^2 \ll \omega^2$ and $k^2 \gg m_\chi^2 - \frac{9}{4}H^2 - \frac{3}{2}\dot{H}$ for the resonance values of k^2.

Using the time dependence of the coefficients in the equations (3.29) and (3.30) and the Bogoliubov transformation (squeezed states), leading to χ particle production, ref [18] found the mean occupation number for the generated χ particles, per mode to be

$$N_k = |\beta_k|^2 \backsimeq sinh^2(\int (\frac{1}{n\omega}\sqrt{|g_n|^2 - \Delta_n dt}). \tag{3.34}$$

As long as $\Delta_n < |g_n|$, the growth is exponential and a large population of particles can be produced by the oscillating coherent source.

To see the connection of squeezed states to particle production through the parametric resonance phenomenon in the presence of the background time dependent oscillator, will be considered in the next section.

3.4 Squeezed States and Particle Production.

To generalize the connection between particle production and squeezed states with relevance to this thesis, we consider the following simple action (which admits spontaneous symmetry breaking)as an example [54],

$$S = \int d^4x \frac{1}{2}[\partial_\mu \phi^a \partial_\nu \phi^a - \frac{1}{4}\lambda(\phi^a\phi^a - v^2)^2], \tag{3.35}$$

For simplicity and illustrative purposes, we use the Hartree-Fock approximation and $\phi^a\phi^a$ term is replaced by its spatial average $< \phi >^2 (t)$, to get terms upto quadratic order in ϕ^a. Quantizing the field ϕ^a (a are internal indices, which we shall not consider in the discussion) by carrying out a mode expansion

$$\phi^a(x,t) = \int \sqrt{\frac{1}{2\omega(k)}} \frac{d^3k}{(2\pi)^3}(a_k e^{ik\cdot x} + a_k^\dagger e^{-ik\cdot x}), \tag{3.36}$$

we get the Hamiltonian

$$H = \int d^3k[\frac{\omega(k)}{2}(1+\frac{\Omega^2(k,t)}{\omega(k)}(a_k^\dagger a_k + a_k a_k^\dagger) - \frac{\omega^2(k)-\Omega^2(k,t)}{4\omega(k)}[a_k^\dagger a_{-k}^\dagger + a_{-k}a_k]] \tag{3.37}$$

where, $\Omega^2(k,t) = k^2 + \lambda(< \phi >^2 (t) - v^2)$, $\omega^2(k) = \Omega(k, t \to \infty) = \sqrt{k^2+m^2}$ and $m^2 = \frac{1}{2}\lambda v^2$.

The above Hamiltonian can be diagonalized using squeezing transformations given by

$$A_k(t) = \mu a_k(t) + e^{i\theta}\nu a_k^\dagger(t) = cosh(r)a_k(t) + e^{i\theta}sinh(r)a_k^\dagger(t) = S^{-1}(\xi,t)a_k S(\xi,t), \tag{3.38}$$

$$A_k^\dagger(t) = e^{-i\theta}\mu a_k^\dagger(t) + \nu a_k(t) = e^{-i\theta}cosh(r)a_k^\dagger(t) + sinh(r)a_k(t) = S^{-1}(\xi,t)a_k^\dagger S(\xi,t), \tag{3.39}$$

where the unitary transformation for squeezing $S(\xi,t)$ is given by

$$S(\xi,t) = exp[\frac{1}{\sqrt{2}}\int d^3k[\xi(a_k^\dagger)^2 - \xi^*(a_k)^2]. \tag{3.40}$$

and $\xi = re^{i\theta}$ is a squeezing parameter [55]. The diagonalized Hamiltonian is

$$H = \int d^3k[\Omega(k,t)(A_k^\dagger(t)A_k(t) + \frac{1}{2})]. \tag{3.41}$$

For a each mode $a_k(t)$, the initial vacuum is given by $a_k(t)|0, 0 >= 0$ and for each mode $A_k(t)$, the final vacuum is given by $A_k(t)|0, t >= 0$. The initial vacuum is related to the final vacuum at a later time t by

$$|0, t\rangle = exp[\frac{1}{\sqrt{2}} \int d^3k[\xi(a_k^\dagger)^2 - \xi^*(a_k)^2]|0, 0\rangle. \quad (3.42)$$

This shows us that the final vacuum is populated by the following number of free quanta. The average number of quanta for each mode is realted to the squeezing parameter by

$$N_k = \langle 0, 0|A_k^\dagger(t)A_k(t)|0, 0\rangle = |\nu(t)|^2, \quad (3.43)$$

the vacuum is populated at later time by physical particles.

To calculate the squeezing parameter , we go over to the "co-ordinate representation" by defining the operators

$$A_k(t) = \frac{1}{2\sqrt{\Omega(k,t)}}(\Omega(t)\pi(k, t) + ip(k, t)), \quad (3.44)$$

$$A_k^\dagger(t) = \frac{1}{2\sqrt{\Omega(k,t)}}(\Omega(t)\pi(k, t) - ip(k, t)), \quad (3.45)$$

where π and p represents co-ordinate and momentum. Putting eq (3.45) in (3.41), the diagonalised Hamiltonian reduces to the following

$$H(t) = \int \frac{d^3k}{(2\pi)^3}\frac{1}{2}\left[(\Omega(k, t))^2\pi^2(k, t) + p^2(k, t)\right] = \int \frac{d^3k}{(2\pi)^3}\frac{1}{2}H_k(t), \quad (3.46)$$

which resembles a time dependent harmonic oscillator. In the mean field approximation this Hamiltonian operator acts on a wave function ψ, which is mode decomposed into

$$|\psi >= \prod_k |\psi >_k . \quad (3.47)$$

by

$$-i\frac{\partial}{\partial t}|\psi >_k= H_k(t)\psi >_k= \frac{1}{2}\left[(\Omega(k, t))^2\pi^2(k, t) + p^2(k, t)\right]|\psi >_k . \quad (3.48)$$

The solution of the eq(3.48)in the coordinate representation is a Gaussian

$$< \pi|\psi > (t) = C(t)exp[-B(t)\pi^2], \quad (3.49)$$

which is a squeezed state in the co-ordinate representation.The parameters r and ξ are related to B by $cosh(r) = \frac{\omega^2+4|B|^2}{4\omega ReB}$ and $sin(2\xi) = \frac{ImB}{sinh(r)ReB}$, this gives $B(t) = \frac{i\dot{\psi}}{\psi}$. The equation of motion of ψ is given by

$$-\frac{d^2\psi}{dt^2} + (k^2 + \lambda(< \phi^2 > -v^2))\psi = 0. \tag{3.50}$$

This equation can be is formally viewed as the stationary Schrodinger equation for the wave function $\psi(t)$ of a particle with mass $m = 1/2$, on a line t, having energy ω^2 and moving through a potential $V(t) = \lambda(< \phi^2 > -v^2)$.

$$\frac{d^2\psi}{dt^2} + (E - V(t))\psi = 0. \tag{3.51}$$

Consider a non-equilibrium scenario such that $< \phi^2 >= 0$ in the region $t < t_1$ (symmetry is restored), i.e $V = -\lambda v^2$ and $< \phi^2 >= v^2$ in the region $t > t_1$ (symmetry is broken) i.e $V = 0$, then this equation (3.51) becomes equivalent to a potential barrier problem [56, 29].

The usual solutions to the barrier are given by

$$\psi(t)_- = e^{-i\omega_i t} + R^* e^{i\omega_i t},\ t < t_1 \tag{3.52}$$

$$\psi(t)_+ = Te^{-i\omega_f t}\ t > t_1. \tag{3.53}$$

where R and T are reflection and transmission coefficients across the barrier and ω_i and ω_f are the initial and final frequencies (Ω and ω),defined by

$$T = \frac{|\psi_+|^2 e^{-i\omega_f t}}{|\psi_-|^2 e^{-i\omega_i t}}, \tag{3.54}$$

and $|R|^2 + |T|^2 = 1$.

Now considering the particle creation problem modelled by a time dependent harmonic oscillator, the solution is

$$\psi(t)_- = \frac{e^{-i\omega_i t}}{\sqrt{2\omega_i}},\ t < t_1, \tag{3.55}$$

$$\psi(t)_+ = \mu\frac{e^{-i\omega_f t}}{\sqrt{2\omega_f}} + \nu\frac{e^{i\omega_f t}}{\sqrt{2\omega_f}}\ t > t_1, \tag{3.56}$$

Comparing the two solutions the transmission coefficient (T) can be related to squeezing parameter (r) through $sinh^2(r) = |\nu|^2 = \frac{R}{T}$. Therefore from the calculation of reflection and transmission coefficient one can get the Bogoliubov coefficients and number of particles generated. This method is a particularly method of calculating the Bogoliubov coefficients and we shall be using it in later chapters.

We emphasize that the analogy with Schrodinger wave equation of eqn(3.48) is purely mathematical. When we consider a quantum field, the modes ϕ_k^a are not particles moving in real space and time t. The mode functions do not represent reflected or transmitted waves. The quantum-mechanical analogy is used only to visualize the qualitative behavior of the mode functions.

The same analogy can be taken over to the expanding space times, where the usual effective frequency is given by $\omega^2 = \frac{k^2}{a^2} + m^2 - \frac{a''}{a}$. Extension of this method to curved space-times is used to explain the Unruh effect and Hawking radiation.

One cannot define the usual vacuum for these situation, as the modes do not oscillate but behave as growing and decaying exponents, so the analogy with a harmonic oscillator breaks down. There one can only define the vacuum in the space time where the expansion is constant ans spacetime is flat in the remote past and the distant future. It can be shown that the two vacua in these regions are related by Bogoliubov transformations, giving rise to presence of particles in the later times.

In the next chapter we will explore these techniques to generate the baryon asymmetry in the context of minimally coupled gravitational baryogenesis.

CHAPTER 4

Baryon Asymmetry in the Inflationary Reheating Scenario : Model 1

We have seen from the previous chapters that baryon number violation, CP violation and non equilibrium processes are necessary conditions for BAU. In the aftermath of the establishment of the inflationary cosmology, it seemed necessary that baryon number violation may have to be imposed as an initial state condition after inflation, as all matter is diluted away in the inflationary process. Aesthetically and practically this was unsatisfactory, for, even if there is a baryon number violation at high temperatures, when thermal equilibrium is established, the asymmetry is wiped out. Thus, models were proposed to account for the separation of matter and antimatter on a scale large enough to account for the local baryon asymmetry of the universe. In the context of particle production in expanding universe, the problem was first addressed by B. Touissant et al [57] and refined by N. J. Papastamatiou and L. Parker [25] who showed that baryon asymmetry can arise if there are B, C and T violating processes which result from the interaction between matter and gravitational fields. These interactions result from the minimal coupling of gravity and matter and can violate B and CP symmetry. In the context of Hawking radiation from black holes, which would have a net baryonic flux, the C and CP violating Lagrangian introduced by B. Touissant et al [57] was of the form

$$L = \sqrt{-g}[g^{\mu\nu}\partial_\mu\phi_i^*\partial_\nu\phi_i + -m_i^2\phi_i^*\phi - \phi_i^* V_{ij}\phi_i R_{\alpha\beta\gamma\delta}R_{\alpha\beta\gamma\delta}], \tag{4.1}$$

where, $R_{\alpha\beta\gamma\delta}$ is the Riemann curvature tensor, ϕ_i are the scalar fields and V_{ij} is a matrix which contains the baryon number, C and CP violating interactions. In the strong gravitational limit the $V_{ij}(x)$ is a function of space. The particles interact with the gravitational field via $V_{ij}(x)$ and antiparticles interact via $V_{ji}(x) = V_{ij}^*(x)$. With this Lagrangian [57] calculated the asymmetry in a strong gravitational field in the vicinity of black holes.

N. J. Papastamatiou and L. Parker [25] modified the above model for the generation of the BAU in the context of an isotropically expanding universe. They considered the following Lagrangian with two complex scalar fields, ϕ which has a non zero baryon number and ψ which has a zero baryon number given by

$$\begin{aligned} L = {} & \sqrt{-g}[g^{\mu\nu}\partial_\mu\phi^*\partial_\nu\phi + g^{\mu\nu}\partial_\mu\psi^*\partial_\nu\psi - (m_1^2+\xi_1 R)\phi^*\phi - (m_2^2+\xi_2 R)\psi^*\psi - \\ & \lambda R(\phi^*\Lambda\psi + \psi^*\Lambda^*\phi)], \end{aligned} \tag{4.2}$$

where $m_{1,2}$ are masses of fields ϕ and ψ respectively. In the presence of expansion, the spacetime dependent potential $V_{ij}(x)$ becomes a function of time $\Lambda(t)$ and the gravitational field $R_{\alpha\beta\gamma\delta}R_{\alpha\beta\gamma\delta}$ is replaced with the Ricci scalar R and is treated as a classical background. The interaction term $\lambda R(\phi^*\Lambda\psi + \psi^*\Lambda^*\phi)$ may be regarded as representing an effective two particle interaction with the graviton that violates baryon number conservation as well as CP invariance (since Λ is complex), thus satisfying the first and second Sakharov's conditions. The fields participating in the interaction are quantized and their time development is found by perturbation theory. The BAU is calculated for the cosmological model in which universe contracts to minimum and then expands to present time. In such a model $a(t)$ (expansion parameter) is treated as constant at early and late times. All the particles present at the late times are generated by the gravitational field and by the asymmetric interaction. They found the created baryon-number density has no explicit dependence on the parameters which characterize the purely gravitational pair production. BAU in this model is calculated perturbatively and first appears in the order λ^2 as

$$\Delta N^{(out)}_{\phi\vec{k}} = \langle N^{(out)}_{\phi\vec{k}} \rangle - \langle \bar{N}^{(out)}_{\phi\vec{k}} \rangle = \lambda^2(|I_2|^2 - |I_3|^2), \tag{4.3}$$

where $N^{(out)}_{\phi\vec{k}}$ are number of particles and $\bar{N}^{(out)}_{\phi\vec{k}}$ are number of anti-particles produced at late times,

$$I_2 = \int_{-\infty}^{\infty} dt R(t)\Lambda(t)\chi^*_{\phi k}(t)\chi^*_{\psi k}(t) \tag{4.4}$$

and

$$I_3 = \int_{-\infty}^{\infty} dt R(t)\Lambda(t)\chi_{\phi k}(t)\chi_{\psi k}(t) \tag{4.5}$$

where $\chi_{\phi k}(t)$ and $\chi_{\psi k}(t)$ are the c-number solutions for equation of motion for the fields ϕ and ψ in Fourier space. They found that $\Delta N^{(out)}_{\phi\vec{k}} \neq 0$ even when $m = 0$. This implies that gravity acts an external field which induces asymmetric pair production. To be consistent with Einstein equations with zero cosmological constant at $t \geq G^{\frac{1}{2}}$, one requires $\lambda \sim .8$. With such a high value of λ, the perturbation results are innaccurate. For small values of λ one cannot generate sufficient asymmetry in this model. This model also has the ad-hoc approximation of taking R to replace the graviton field (requiring justification). This model was proposed prior to the inflationary scenario. The time scales in this model correspond to the time scales for inflation, hence it was natural to consider this model in the context of reheating after inflation.

R. Rangarajan and D. V. Nanopoulus [26] studied the possibility of creating BAU by particle production during the reheating phase by the decay of complex inflaton field, with a suitable modification of the above model. The modified Lagrangian is given by

$$\begin{aligned} L &= \sqrt{-g}[g^{\mu\nu}\partial_\mu\phi^*\partial_\nu\phi + g^{\mu\nu}\partial_\mu\psi^*\partial_\nu\psi - (m_\phi^2+\xi_\phi R)\phi^*\phi - (m_\psi^2+\xi_\psi R)\psi^*\psi - \\ & \quad +g^{\mu\nu}\partial_\mu\eta^*\partial_\nu\eta - (m_\eta^2+\xi_\eta R)\eta^*\eta - V(\eta) - \lambda(\eta^2\phi^*\psi + \eta^{*2}\psi^*\phi)]. \end{aligned} \tag{4.6}$$

A minimally coupled complex inflaton field η^2 replaces the term $R\Lambda$ in the interaction in (4.2). The initial velocity of the η field and the shape of the inflaton potential ensures that its phase varies as the inflaton rolls down its potential. This gives rise to dynamic CP violation.

To calculate the BAU they followed the same perturbative methods of N. J. Papastamatiou and L. Parker [25], with a slight modification. In their calculation, they use the fact that the annihilation and creation operators are not same during the phase immedeately after inflation and late times after reheating. By using the Bogoliubov transformations, which relates the annihilation and creation operators of early times to that of late times, BAU in this model is found to be

$$\Delta N^{(out)}_{\phi\vec{k}} = \langle N^{(out)}_{\phi\vec{k}}\rangle - \langle \bar{N}^{(out)}_{\phi\vec{k}}\rangle = \lambda^2(|I_2|^2 - |I_3|^2), \tag{4.7}$$

where $N^{(out)}_{\phi\vec{k}}$ are number of particles and $\bar{N}^{(out)}_{\phi\vec{k}}$ are number of anti-particles produced at late times.

$$I_2 = \int_{-\infty}^{\infty} dt\eta^2\chi_k^{\phi*}(t)\chi_k^{\psi*}(t) \tag{4.8}$$

and

$$I_3 = \int_{-\infty}^{\infty} dt\eta^2\chi_k^{\phi}(t)\chi_k^{\psi}(t) \tag{4.9}$$

where $\chi_k^\phi(t)$ and $\chi_k^\psi(t)$ are the complex functions that solve the equation of motion for the ϕ and ψ, with $\lambda = 0$ in Fourier space. They considered chaotic and natural inflationary reheating scenarios. In the former case, the complex decaying field is the inflaton itself and, in the latter case, the phase of the complex field is the inflaton.

Even though the resultant BAU in all models is insufficient, the value of this work is that one should not ignore the inflationary and reheating era while calculating the asymmetry.

4.1 The Formalism.

We now consider the Lagrangian of R. Rangarajan and D. V. Nanopoulus [26] and calculate particle production, via parametric resonance during reheating, using general formalism of squeezed rotated states. This method allows us to calculate the BAU without resorting to perturbation theory, which may be the reason why insufficeint BAU is generated in the above models. We give the details of the method in the following sections.

Considering the Lagrangian of (4.6), we will treat the inflaton field η as a background classical field. Its temporal evolution is determined by a Euler Lagrange equation of motion, with the inflaton potential $V(\eta)$ in the Friedman-Robertson-Walker (FRW) metric $ds^2 = dt^2 - a^2(t)dx^2$,

$$\ddot{\eta} + 3\frac{\dot{a}}{a}\dot{\eta} + \frac{\partial V}{\partial \eta} = 0. \tag{4.10}$$

In our analysis, the background gravitational field is coupled minimally i.e. $(\xi_\phi = \xi_\psi = \xi_\eta = 0)$ and the action is

$$\begin{aligned} S &= \int d^3x dt a^3(t)(\frac{1}{2}(\dot{\phi^*})(\dot{\phi}) - \frac{1}{2a^2(t)}(\nabla\phi^*)(\nabla\phi) + \frac{1}{2}(\dot{\psi^*})(\dot{\psi}) - \frac{1}{2a^2(t)}(\nabla\psi^*)(\nabla\psi) \\ &\quad -m_\phi\phi^*\phi - m_\psi\psi^*\psi - \lambda(\eta^2\phi^*\psi + \eta^{*2}\psi^*\phi)). \end{aligned} \tag{4.11}$$

Applying a Legendre transformation

$$H = \sum \pi_i \dot{\phi}_i - L, \tag{4.12}$$

where, π_ϕ and π_ψ are the canonical momenta of the fields ϕ and ψ, given by

$$\pi_i = \frac{\partial L}{\partial_0 \phi_i}, \tag{4.13}$$

the Hamiltonian is

$$\begin{aligned} H &= \int d^3x dt a^3[\frac{1}{a^6}\pi^*_\phi\pi_\phi + \frac{1}{a^2}\nabla\phi^*\nabla\phi + \frac{1}{a^6}\pi^*_\psi\pi_\psi + \frac{1}{a^2}\nabla\psi^*\nabla\psi + m^2_\phi\phi^*\phi + m^2_\psi\psi^*\psi \\ &\quad +\lambda(\eta^2\phi^*\psi + \eta^{*2}\psi^*\phi)]. \end{aligned} \tag{4.14}$$

We quantize the the fields ϕ and ψ, using the mode expnasion

$$\psi(x^\mu) = \int d\tilde{k}[a_k^\psi e^{-ik\cdot x} + b_k^{\dagger\psi} e^{ik\cdot x}], \tag{4.15}$$

$$\psi^*(x^\mu) = \int d\tilde{k}[a_k^{\dagger\psi} e^{ik\cdot x} + b_k^\psi e^{-ik\cdot x}], \tag{4.16}$$

$$\phi(x^\mu) = \int d\tilde{k}[a_k^\phi e^{-ik\cdot x} + b_k^{\dagger\phi} e^{ik\cdot x}], \tag{4.17}$$

$$\phi^*(x^\mu) = \int d\tilde{k}[a_k^{\dagger\phi} e^{ik\cdot x} + b_k^\phi e^{-ik\cdot x}], \tag{4.18}$$

with

$$k \cdot x = k_\mu x^\mu = \omega t - k_i x_i, \tag{4.19}$$

$$d\tilde{k} = \frac{d^3k dt}{[(2\pi)^3 2\omega_k]^{\frac{1}{2}}}. \tag{4.20}$$

The quantized Hamiltonian is then

$$\begin{aligned} H = & \int d^3k[\frac{\omega_\psi}{a^3} a_k^{\dagger\psi} a_k^\psi + \frac{\omega_\phi}{a^3} a_k^{\dagger\phi} a_k^\phi + \frac{\lambda\eta^2 a^3}{2\sqrt{\omega_\phi\omega_\psi}} a_k^{\dagger\psi} a_k^\phi + \frac{\lambda\eta^{*2} a^3}{2\sqrt{\omega_\phi\omega_\psi}} a_k^{\dagger\phi} a_k^\psi] \\ & +[\frac{\omega_\psi}{a^3} b_{-k}^\psi b_{-k}^{\dagger\psi} + \frac{\omega_\phi}{a^3} b_{-k}^\phi b_{-k}^{\dagger\phi} + \frac{\lambda\eta^2 a^3}{2\sqrt{\omega_\phi\omega_\psi}} b_{-k}^\phi b_{-k}^{\dagger\psi} + \frac{\lambda\eta^{*2} a^3}{2\sqrt{\omega_\phi\omega_\psi}} b_{-k}^\psi b_{-k}^{\dagger\phi}] \\ & + \frac{\lambda\eta^2 a^3}{2\sqrt{\omega_\phi\omega_\psi}}[a_k^{\dagger\psi} b_{-k}^{\dagger\phi} + b_k^\psi a_{-k}^\phi] + \frac{\lambda\eta^{*2} a^3}{2\sqrt{\omega_\phi\omega_\psi}}[a_k^{\dagger\phi} b_{-k}^{\dagger\psi} + b_k^\phi a_{-k}^\psi], \end{aligned} \tag{4.21}$$

where

$$\frac{\omega_\phi^2}{a^6} = \underline{k}^2 + m_\phi^2, \tag{4.22}$$

$$\frac{\omega_\psi^2}{a^6} = \underline{k}^2 + m_\psi^2,. \tag{4.23}$$

Here, $\underline{k} = \frac{k}{a}$ is the physical wave (co-wave) number of the mode k. By defining the following

$$\frac{\Omega_\psi^2(t)}{a^6} = \frac{\omega_\psi^2}{a^6} + \sqrt{\frac{\omega_\psi}{\omega_\phi}}\lambda\eta^2(t), \tag{4.24}$$

$$\frac{\Omega_\phi^2(t)}{a^6} = \frac{\omega_\phi^2}{a^6} + \sqrt{\frac{\omega_\phi}{\omega_\psi}}\lambda\eta^{*2}(t), \tag{4.25}$$

the Hamiltonian is written as follows

$$H = \int d^3k[\frac{\omega_\psi}{a^3}a_k^{\dagger\psi}a_k^\psi + \frac{\omega_\phi}{a^3}a_k^{\dagger\phi}a_k^\phi + \frac{\omega_\psi}{a^3}\left[\frac{\Omega_\psi^2(t)}{\omega_\psi^2}-1\right]a_k^{\dagger\psi}a_k^\phi + \frac{\omega_\phi}{a^3}\left[\frac{\Omega_\phi^2(t)}{\omega_\phi^2}-1\right]a_k^{\dagger\phi}a_k^\psi]$$
$$+[\frac{\omega_\psi}{a^3}b_k^\psi b_k^{\dagger\psi} + \frac{\omega_\phi}{a^3}b_k^\phi b_k^{\dagger\phi} + \frac{\omega_\psi}{a^3}\left[\frac{\Omega_\psi^2(t)}{\omega_\psi^2}-1\right]b_k^\phi b_k^{\dagger\psi} + \frac{\omega_\phi}{a^3}\left[\frac{\Omega_\phi^2(t)}{\omega_\phi^2}-1\right]b_k^\psi b_k^{\dagger\phi}]$$
$$+\frac{\omega_\psi}{a^3}\left[\frac{\Omega_\psi^2(t)}{\omega_\psi^2}-1\right][a_k^{\dagger\psi}b_{-k}^{\dagger\phi} + b_{-k}^\psi a_k^\phi] + \frac{\omega_\phi}{a^3}\left[\frac{\Omega_\phi^2(t)}{\omega_\phi^2}-1\right][a_k^{\dagger\phi}b_{-k}^{\dagger\psi} + b_{-k}^\phi a_k^\psi]. \quad (4.26)$$

This Hamiltonian has two symmetries $su(2)$ and $su(1,1)$. To illustrate these symmetries, the following number operators are defined

$$N_1 = a_k^{\dagger\psi}a_k^\psi, \;\; N_2 = a_k^{\dagger\phi}a_k^\phi, \;\; N_3 = b_k^{\dagger\psi}b_k^\psi, \;\; N_4 = b_k^{\dagger\phi}b_k^\phi. \quad (4.27)$$

Then, the operators

$$J_+ = a_k^{\dagger\psi}a_k^\phi, \; J_- = a_k^{\dagger\phi}a_k^\psi, \; J_0 = \frac{1}{2}(N_1 - N_2), \quad (4.28)$$
$$M_+ = b_k^{\dagger\psi}b_k^\phi, \; M_- = b_k^{\dagger\phi}b_k^\psi, \; M_0 = \frac{1}{2}(N_3 - N_4),$$

satisfy the su(2) algebras

$$[J_+, J_-] = -2J_0, \; [J_+, J_0] = -J_+, \; [J_-, J_0] = J_-, \quad (4.29)$$

$$[M_+, M_-] = -2M_0, \; [M_+, M_0] = -M_+, \; [M_-, M_0] = M_-. \quad (4.30)$$

Another set of operators are :

$$K_+ = a_k^{\dagger\psi}b_{-k}^{\dagger\phi}, \; K_- = b_{-k}^\phi a_k^\psi, \; K_0 = \frac{1}{2}(N_1 + N_3 + 1),$$
$$L_+ = a_k^{\dagger\phi}b_{-k}^{\dagger\psi}, \; L_- = b_{-k}^\psi a_k^\phi, \; L_0 = \frac{1}{2}(N_2 + N_4 + 1). \quad (4.31)$$

These satisfy the su(1,1) algebras

$$[K_+, K_-] = 2K_0, \; [K_+, K_0] = K_+, \; [K_-, K_0] = -K_-, \quad (4.32)$$

$$[L_+, L_-] = 2L_0,\ [L_+, L_0] = L_+,\ [L_-, L_0] = -L_-. \tag{4.33}$$

We can write the Hamiltonian as

$$\begin{aligned} H &= \int d^3k[\frac{2\omega_\psi}{a^3}K_0 + \frac{2\omega_\phi}{a^3}L_0 + \frac{\omega_\psi(t)}{a^3}\left[\frac{\Omega^2_\psi(t)}{\omega^2_\psi} - 1\right](J_+ + M_+) \\ &+ \frac{\omega_\phi}{a^3}\left[\frac{\Omega^2_\phi(t)}{\omega^2_\phi} - 1\right](J_- + M_-) + \frac{\omega_\psi}{a^3}\left[\frac{\Omega^2_\psi(t)}{\omega^2_\psi} - 1\right](K_+ + L_-) \\ &+ \frac{\omega_\phi}{a^3}\left[\frac{\Omega^2_\phi(t)}{\omega^2_\phi} - 1\right](L_+ + K_-)]. \end{aligned} \tag{4.34}$$

The su(1,1) and su(2) symmetries of the Hamiltonian are now manifestly evident.

These symmetries of the Hamiltonian will aid us to diagonalise it, so that one can go over to the parametric amplification picture. This Hamiltonian has resemblance to quantum optical Hamiltonians in beam splitting and parametric amplification processes [58]. Thus, we will use quantum optical methods to diagonalize it. The Hamiltonian can be written in terms of new creation and annihilation operators using the following unitary transformation

$$H' = U^\dagger(R_2)U^\dagger(R_1)HU(R_1)U(R_2), \tag{4.35}$$

where

$$U(R_1)U(R_2) = exp[\theta(J_+e^{2i\xi} + J_-e^{2i\xi})]exp[\theta(M_+e^{2i\xi} + M_-e^{2i\xi})], \tag{4.36}$$

and operator $U(R_1)$ provides the well known(beam splitting) transformation relations:

$$U^\dagger(R_1)\begin{pmatrix} a^\psi_k \\ a^\phi_k \end{pmatrix}U(R_1) = \begin{pmatrix} cos(\theta) & e^{2i\xi}sin(\theta) \\ -e^{-2i\xi}sin(\theta) & cos(\theta) \end{pmatrix}\begin{pmatrix} a^\psi_k \\ a^\phi_k \end{pmatrix} = \begin{pmatrix} A_k \\ B_k \end{pmatrix}, \tag{4.37}$$

while $U(R_2)$ provides the relation

$$U^\dagger(R_2)\begin{pmatrix} b^\phi_{-k} \\ b^\psi_{-k} \end{pmatrix}U(R_2) = \begin{pmatrix} cos(\theta) & e^{2i\xi}sin(\theta) \\ -e^{-2i\xi}sin(\theta) & cos(\theta) \end{pmatrix}\begin{pmatrix} b^\phi_k \\ b^\psi_k \end{pmatrix} = \begin{pmatrix} C_k \\ D_k \end{pmatrix}, \tag{4.38}$$

The angle θ is determined from the relation $tan(\theta) = \sqrt{\omega_\phi/\omega_\psi}$, so that $cos(\theta) = \sqrt{\omega_\psi/(\omega_\phi + \omega_\psi)}$ and $sin(\theta) = \sqrt{\omega_\phi/(\omega_\phi + \omega_\psi)}$. Then the Hamiltonian assumes the form

$$H = H_1 + H_2, \tag{4.39}$$

where H_1 and H_2 are given by

$$H_1 = \int d^3k \frac{(\omega_\phi + \omega_\psi)}{a^3}[\beta[A_k^\dagger A_k + C_k C_k^\dagger] + \frac{\lambda\eta^2}{2\omega_\phi\omega_\psi} A_k^\dagger C_{-k}^\dagger + \frac{\lambda\eta^{*2}}{2\omega_\phi\omega_\psi} C_{-k}A_k], \quad (4.40)$$

$$H_2 = \int d^3k \frac{(\omega_\phi + \omega_\psi)}{a^3}[\alpha[B_k^\dagger B_k + D_K D_k^\dagger] + \frac{\lambda\eta^2}{2\omega_\phi\omega_\psi} D_{-k}B_k + \frac{\lambda\eta^{*2}}{2\omega_\phi\omega_\psi} B_k^\dagger D_{-k}^\dagger]. \quad (4.41)$$

Here

$$2\alpha = 1 - \frac{\lambda|\eta|^2 a^6}{2\omega_\phi\omega_\psi}, \quad (4.42)$$

$$2\beta = 1 + \frac{\lambda|\eta|^2 a^6}{2\omega_\phi\omega_\psi}. \quad (4.43)$$

The complexity of the η field gives rise to the CP violation. After the $su(2)$ rotations we get

$$\begin{aligned} H' = \int d^3k \frac{(\omega_\phi + \omega_\psi)}{a^3}&[\beta(t)[A_k^\dagger A_k + C_{-k}C_{-k}^\dagger] \\ &+\alpha(t)[B_{-k}^\dagger B_{-k} + D_k D_k^\dagger] \\ &+\frac{\omega_\psi}{a^3}\left[\frac{\Omega_\psi^2(t)}{\omega_\psi^2} - 1\right][A_k^\dagger C_{-k}^\dagger + D_k B_{-k}] \\ &+\frac{\omega_\phi}{a^3}\left[\frac{\Omega_\phi^2(t)}{\omega_\phi^2} - 1\right][B_{-k}^\dagger D_k^\dagger + C_{-k}A_k]]. \end{aligned} \quad (4.44)$$

Now define operators:

$$D_{1+} = A_k^\dagger C_{-k}^\dagger,\ D_{1-} = C_{-k}A_k,\ D_{10} = \frac{1}{2}(A_k^\dagger A_k + C_{-k}^\dagger C_{-k} + 1), \quad (4.45)$$

$$D_{2+} = B_{-k}^\dagger D_k^\dagger,\ D_{2-} = D_k B_{-k}\ D_{20} = \frac{1}{2}(B_{-k}^\dagger B_{-k} + D_k^\dagger D_k + 1), \quad (4.46)$$

satisfying the su(1,1) algebras

$$[D_{1+}, D_{1-}] = 2D_{10},\ [D_{1+}, D_{10}] = D_{1+},\ [D_{1-}, D_{10}] = -D_{1-}, \quad (4.47)$$

$$[D_{2+}, D_{2-}] = 2D_{20},\ [D_{2+}, D_{20}] = D_{2+},\ [D_{2-}, D_{20}] = -D_{2-}. \quad (4.48)$$

Then in terms of the su(1,1) generators

$$\begin{aligned} H' = \int d^3k \frac{2(\omega_\phi + \omega_\psi)}{a^3}&[[\beta(t)D_{10} + \alpha(t)D_{20}] \\ &+\frac{\omega_\psi}{a^3}\left[\frac{\Omega_\psi^2(t)}{\omega_\psi^2} - 1\right][D_{1+} + D_{2-}] + \frac{\omega_\phi}{a^3}\left[\frac{\Omega_\phi^2(t)}{\omega_\phi^2} - 1\right](D_{2+} + D_{1-})]. \end{aligned} \quad (4.49)$$

This su(1,1) symmetry implies diagonalizability through a squeezing (Bogoliubov) transformation from the quantum optical analogies that we are applying, we see here that a product of two squeezing transformations will provide us with a diagonal Hamiltonian. The squeezing transformations we use are given by

$$S(\zeta_1)S(\zeta_2) = exp[\zeta_1 D_{1+} - \zeta_1^* D_{1-}]exp[\zeta_2 D_{2+} - \zeta_2^* D_{2-}], \tag{4.50}$$

where, $\zeta_1 = r_1 exp[i\gamma_1]$ and $\zeta_2 = r_2 exp[i\gamma_2]$ are the squeezing parameters. The operators $S(\zeta_1)$and $S(\zeta_2)$ provide the relevant Bogoliubov transformations for the Hamiltonian. The creation and annihilation operators A_k, B_k, C_k and D_k in terms of new creation and annihilation operators are given by

$$A_s(k,t) = \mu_1 A_k + \nu_1 C_{-k}^\dagger, \tag{4.51}$$

$$A_s^\dagger(k,t) = \mu_1^* A_k^\dagger + \nu_1^* C_{-k}, \tag{4.52}$$

$$B_s(k,t) = \mu_2 B_{-k} + \nu_2 D_k^\dagger, \tag{4.53}$$

$$B_s^\dagger(k,t) = \mu_2^* B_{-k}^\dagger + \nu_2^* D_k, \tag{4.54}$$

The Bogoliubov coefficients can be read off as follows

$$\mu_1 = Cosh(r) = \frac{2\omega_\phi\omega_\psi - \lambda\eta^2 a^6}{2\sqrt{\omega_\phi\omega_\psi}(\omega_\phi\omega_\psi - \lambda\eta^2 a^6)^{\frac{1}{2}}} = \frac{2\alpha(t)}{(4\alpha(t)-1)^{\frac{1}{2}}}, \tag{4.55}$$

$$\nu_1 = Sinh(r) = \frac{\lambda r^2 a^6}{2\sqrt{\omega_\phi\omega_\psi}(\omega_\phi\omega_\psi - \lambda\eta^2 a^6)^{\frac{1}{2}}} = e^{-i\gamma_1}\frac{2\alpha(t)-1}{(4\alpha(t)-1)^{\frac{1}{2}}}, \tag{4.56}$$

$$\mu_2 = Cosh(r) = \frac{2\omega_\phi\omega_\psi + \lambda\eta^2 a^6}{2\sqrt{\omega_\phi\omega_\psi}(\omega_\phi\omega_\psi + \lambda\eta^2 a^6)^{\frac{1}{2}}} = \frac{2\beta(t)}{(4\beta(t)-1)^{\frac{1}{2}}}, \tag{4.57}$$

$$\nu_2 = Sinh(r) = \frac{\lambda\eta^2 a^6}{2\sqrt{\omega_\phi\omega_\psi}(\omega_\phi\omega_\psi + \lambda r^2 a^6)^{\frac{1}{2}}} = e^{-i\gamma_2}\frac{2\beta(t)-1}{(4\beta(t)-1)^{\frac{1}{2}}}, \tag{4.58}$$

where,

$$|\mu_1|^2 - |\nu_1|^2 = 1, \quad |\mu_2|^2 - |\nu_2|^2 = 1. \tag{4.59}$$

Thus the final diagonalized Hamiltonian is

$$
\begin{aligned}
H_f &= S(\zeta_2)^\dagger S(\zeta_1)^\dagger U^\dagger(R_2) U^\dagger(R_1)[H] U(R_1) U(R_2) S(\zeta_1) S(\zeta_2) \\
&= \int d^3k \frac{(\omega_\phi + \omega_\psi)}{2a^3}(4\alpha(t)-1)^{\frac{1}{2}}[A_s^\dagger(k,t)A_s(k,t)+1] \\
&\quad + \frac{(\omega_\phi + \omega_\psi)}{2a^3}(4\beta(t)-1)^{\frac{1}{2}}[B_s^\dagger(k,t)B_s(k,t)+1]. \qquad (4.60)
\end{aligned}
$$

We use the methods presented at the end of chapter 3 of the importance of the Bogoliubov transformations and the consequent population of a vacuum at a later time with particles of another vacuum at an earlier time to relate the squeezed rotated vacuum state $|0(k,t), 0(k,t)>$ of $H_f(t)$ to $|0(k), 0(k)>_{H'}$ the rotated vacuum state of H' and the original vacuum for H. $|0(k), 0(k)>_H$ is defined as $a_k^\phi|0(k), 0(k)>_H = 0$, $a_k^\psi|0(k), 0(k)>_H = 0$, $b_{-k}^\phi|0(k), 0(k)>_H = 0$, and $b_{-k}^\psi|0(k), 0(k)>_H = 0$). The vacuum of H_f is related to Vacuum of H' by

$$|0(k,t), 0(k,t)> = e^{\int \frac{d^3k}{(2\pi)^3}\zeta_1(D_{1+}-D_{1-})+\zeta_2(D_{2+}-D_{2-})}|0(k), 0(k)>_{H'}. \qquad (4.61)$$

In turn the rotated vacuum of H' is related to the original vacuum of H by

$$|0(k), 0(k)>_{H'} = e^{\int \frac{d^3k}{(2\pi)^3}[\theta(J_+e^{2i\xi}+J_-e^{2i\xi})]+[\theta(M_+e^{2i\xi}+M_-e^{2i\xi})]}|0(k), 0(k)>, \qquad (4.62)$$

Thus, the vacuum of H_f is related to H by

$$|0(t), 0(t)> = e^{\int \frac{d^3k}{(2\pi)^3}\zeta_1(D_{1+}-D_{1-})+\zeta_2(D_{2+}-D_{2-})} e^{\int \frac{d^3k}{(2\pi)^3}[\theta(J_+e^{2i\xi}+J_-e^{2i\xi})]+[\theta(M_+e^{2i\xi}+M_-e^{2i\xi})]}|0(k), 0(k)>, \qquad (4.63)$$

and is a squeezed rotated vacuum. The number of particles and anti-particles can be calculated by the relationship between the creation and annihilation operators of the initial freequanta a_k^ψ, b_k^ψ, a_k^ϕ and b_k^ϕ to the final creation and annihilation operators A_s and B_s by

$$
\begin{aligned}
A_s(k,t) &= (\mu_1 cos(\theta))a_k^\psi + (\nu_1 sin\theta e^{2i\xi})b_{-k}^{\dagger\phi} + (\mu_1 sin\theta e^{2i\xi})a_k^\phi + (\nu_1 cos(\theta))b_{-k}^{\dagger\psi}, \qquad (4.64) \\
B_s(k,t) &= (\mu_2 cos(\theta))a_{-k}^\psi + (\nu_2 sin\theta e^{-2i\xi})b_k^{\dagger\phi} + (\mu_2 sin\theta e^{-2i\xi})a_{-k}^\phi + (\nu_2 cos(\theta))b_k^{\dagger\psi} \qquad (4.65)
\end{aligned}
$$

Note that it is the ϕ field which carries the baryon number. This gives us the number of baryons generated at time t as

$$N_B(t) = \sum_k \langle A_s^\dagger(k,t) A_s(k,t) \rangle = \sum_k |\nu_{k1}|^2, \tag{4.66}$$

while the number of anti-baryons is

$$N_{\overline{B}}(t) = \sum_k \langle B_s^\dagger(k,t) B_s(k,t) \rangle = \sum_k |\nu_{k2}|^2. \tag{4.67}$$

This shows clearly that the vacuum $|0(k,t), 0(k,t) >$ is populated with particles and anti-particles with respect to vacuum $|0(k), 0(k) >$. So we have a resultant baryon asymmetry given by

$$\begin{aligned} N_B(t) - N_{\overline{B}}(t) &= \sum_k (|\nu_{k1}|^2 - |\nu_{k2}|^2), \\ &= \sum_k \left[\frac{(\lambda a^6 |\eta|^2)^2}{(\omega_\phi \omega_\psi - \lambda |\eta|^2 a^6)} \right] - \left[\frac{(\lambda a^6 |\eta|^2)^2}{(\omega_\phi \omega_\psi + \lambda |\eta|^2 a^6)} \right] \end{aligned} \tag{4.68}$$

where the k dependence is through ω_ϕ, ω_ψ. The time dependence of the asymmetry parameter comes from the inflaton field $\eta(t)$ and the expansion parameter $a(t)$. To get a proper dynamical estimation of the parameter in an expanding universe, we will study the time evolution the number of particles and anti-particles produced.

4.2 Evolution Of the Asymmetry Parameter:

To obtain the time evolution equations for the wave functions of the particles (baryon number 1) and the anti-particles (baryon number -1), in a realistic expanding universe, we go over to the co-ordinate representation by defining the following operators

$$A_s(k,t) = \frac{e^{i\int \frac{\Omega_+(t)}{a^3}dt}}{2\sqrt{\frac{\Omega_+(t)}{a^3}}}(\frac{\Omega_+(t)}{a^3}\Pi_A(k,t) + iP_{\Pi_A}(k,t)), \tag{4.69}$$

$$A_s^\dagger(k,t) = \frac{e^{-i\int \frac{\Omega_+(t)}{a^3}dt}}{2\sqrt{\frac{\Omega_+(t)}{a^3}}}(\frac{\Omega_+(t)}{a^3}\Pi_A(k,t) - iP_{\Pi_A}(k,t)), \tag{4.70}$$

$$B_s(k,t) = \frac{e^{i\int \frac{\Omega_-(t)}{a^3}dt}}{2\sqrt{\frac{\Omega_-(t)}{a^3}}}(\frac{\Omega_-(t)}{a^3}\Pi_B(k,t) + iP_{\Pi_B}(k,t)), \tag{4.71}$$

$$B_s^\dagger(k,t) = \frac{e^{-i\int \frac{\Omega_-(t)}{a^3}dt}}{2\sqrt{\frac{\Omega_-(t)}{a^3}}}(\frac{\Omega_-(t)}{a^3}\Pi_A(k,t) - iP_{\Pi_B}(k,t)). \tag{4.72}$$

The Hamiltonian H_f in the coordinate representation is:

$$H_f(t) = \int \frac{d^3k}{(2\pi)^3}\frac{1}{2}\left[(\frac{\Omega_+}{a^6})^2\Pi_A^2(k,t) + P_{\Pi_A}^2(k,t) + (\frac{\Omega_-}{a^6})^2\Pi_B^2(k,t) + P_{\Pi_B}^2(k,t)\right], \tag{4.73}$$

where

$$(\frac{\Omega_+}{a^6})^2 = \frac{(\omega_\phi + \omega_\psi)^2}{4a^6}(4\alpha(t) - 1), \tag{4.74}$$

$$(\frac{\Omega_-}{a^6})^2 = \frac{(\omega_\phi + \omega_\psi)^2}{4a^6}(4\beta(t) - 1). \tag{4.75}$$

Since the $\frac{\Omega_+}{a^6}$ and $\frac{\Omega_-}{a^6}$ are time dependent, the Hamiltonian H_f in (4.60) represents a time dependent harmonic oscillator.

The time evolution of a wave function $\chi(t)$ under the action of a Hamiltonian H(t) is simply

$$H(t)\chi(t) = i\frac{d}{dt}\chi(t),. \tag{4.76}$$

From the form of H_f given in eq(4.73), it is clear that it is the direct sum of two independent Hamiltonians $H_A(t)$ and $H_B(t)$ for each of the A_s and B_s modes. Therefore, the wave function $\chi(t)$ for the Hamiltonian H_f is just the sum of two wavefunctions $\chi_A(t)$ and $\chi_B(t)$ where they evolve independently. From here on we consider only $H_A(t)$. Similar expression will follow for $H_B(t)$.

Fourier decomposing the wave function $\chi_A(t)$ and the Hamiltonian $H_A(t)$, we get

$$H_A(k,t)\chi_A(k,t) = i\frac{d}{dt}\chi_A(k,t). \tag{4.77}$$

In the coordinate space representation (Π_A, P_A), the wave functions $\chi_A(k,t)$ can be represented by Gaussian wave function

$$\chi_A(k,t) = L_A(t)e^{[-W_i(t)\Pi_A^2]}\chi_A(k,0). \tag{4.78}$$

A time derivative gives

$$i\frac{\partial}{\partial t}\chi_A(k,t) = \left(i\frac{\dot{L}_A}{L_A} - i\Pi_A^2\dot{W}_A(k,t)\right)\chi_A(k,t), \tag{4.79}$$

while the eq (4.76) gives

$$i\frac{\partial}{\partial t}\chi_A(k,t) = \frac{1}{2a^6}\left[\Omega_A^2\Pi_A^2 - \frac{\partial^2}{\partial\Pi_A^2}\right]\chi_A(k,t). \tag{4.80}$$

From these two equations we obtain

$$\frac{\dot{L}_A}{L_A} = \frac{\dot{\chi}_A(k,t)}{\chi_A(k,t)} + \Pi_A^2\dot{W}_A(k,t) \tag{4.81}$$

and

$$W_A(t) = -ia^3/2\frac{\dot{\chi}_A(k,t)}{\chi_A(k,t)}. \tag{4.82}$$

Thus the evolution equation is

$$\ddot{\chi}_A(k,t) + 3\frac{\dot{a}}{a}\dot{\chi}_A(k,t) + \frac{\Omega_{\Pi_A}^2}{a^6}\chi_A(k,t) = 0. \tag{4.83}$$

In a similar fashion the evolution equation for $\chi_B(k,t)$ is

$$\ddot{\chi}_B(k,t) + 3\frac{\dot{a}}{a}\dot{\chi}_B(k,t) + \frac{\Omega_-^2}{a^6}\chi_B(k,t) = 0. \tag{4.84}$$

We consider a conformally flat FRW metric

$$ds^2 = a(\tau)^2(d\tau^2 - dx^2), \tag{4.85}$$

by employing the scaled time

$$d\tau = a(t)^{-1}dt. \tag{4.86}$$

The equations of motion for the wave functions $\chi_A(k,t)$ and $\chi_B(k,t)$ given above can be transformed into ones that resemble damped harmonic oscillators with time dependent frequencies.

$$\frac{1}{a^2(\tau)}\frac{d^2}{d\tau^2}\chi_A(k,\tau) + \frac{2}{a^3(\tau)}\frac{da}{d\tau}\frac{d}{d\tau}\chi_A(k,\tau) + \frac{\Omega_-}{a^6}\chi_A(k,\tau) = 0, \tag{4.87}$$

$$\frac{1}{a^2(\tau)}\frac{d^2}{d\tau^2}\chi_B(k,\tau) + \frac{2}{a^3(\tau)}\frac{da}{d\tau}\frac{d}{d\tau}\chi_B(k,\tau) + \frac{\Omega_+}{a^6}\chi_B(k,\tau) = 0. \tag{4.88}$$

A change of variable

$$u_A = a\chi_A, \tag{4.89}$$

and similarly for $\chi_B(k,\tau)$ transforms (4.87)and (4.88) to

$$-u_A'' + (E - V_1(\eta,\tau))u_A = 0, \tag{4.90}$$

and

$$-u_B'' + (E + V_2(\eta,\tau))u_B = 0, \tag{4.91}$$

where prime denotes differential with respect to τ and

$$E = -\frac{(\omega_\phi + \omega_\psi)^2}{4a^6}, \tag{4.92}$$

$$V_1(\eta,\tau) = -\frac{1}{a(\tau)}\frac{d^2a}{d\tau^2} - [A|\eta|^2], \tag{4.93}$$

$$V_2(\eta,\tau) = \frac{1}{a(\tau)}\frac{d^2a}{d\tau^2} - [A|\eta|^2], \tag{4.94}$$

where $A = \frac{\lambda(\omega_\phi+\omega_\psi)^2}{8\omega_\phi\omega_\psi}$, ω_ϕ and ω_ψ are given by the equations (4.22) and (4.23) respectively. We now have time evolution equations in the Schrodinger like form described in chapter 3. Thus, we know that by calculating the reflection "R" and the transmission coefficients "T" across the potentials $V_1(\eta,\tau)$ and $V_2(\eta,\tau)$, we can get the number of particles and anti-particles produced by $sinh^2(r) = |\nu|^2 = \frac{R}{T}$. We see from the equations (4.90) and (4.91) that the baryons encounter a potential barrier whereas the anti-baryons encounter a potential well.

We assume an oscillating inflaton background

$$\eta = \Lambda(\tau) sin(m\tau), \tag{4.95}$$

where $\Lambda = |\Lambda|e^{i\xi}$ is complex and m is the inflaton mass. In the general case, the amplitude Λ would be a function of time, since the expansion of the universe would cause the oscillating inflaton to lose energy and hence decrease its amplitude and frequency, giving a phenomenological approximation $\Lambda = \Lambda_0 e^{\tau/\tau_1}$, where τ_1 is the damping scale. In this analysis, we shall assume Λ is almost constant allowing the inflaton oscillations to fall in a resonance band causing a parametric amplification/suppression of the particle/antiparticle modes.

With this assumption of the form of the classical background inflaton field, the evolution equations (4.90) and (4.91) are given by

$$u_A'' + (-\frac{a''}{a} + E - \frac{B}{2} + \frac{B}{2} cos(2m\tau))u_A = 0, \tag{4.96}$$

and

$$u_A'' + (-\frac{a''}{a} + E + \frac{B}{2} - \frac{B}{2} cos(2m\tau))u_A = 0, \tag{4.97}$$

where $B = |\Lambda|^2 A$. These are exact equations for the evolution of the baryon and anti-baryon fields in an expanding universe.

We first consider the case of constant expansion, to reduce (4.96) and (4.97) to the Mathieu equations

$$u_A'' + (E - \frac{B}{2} + \frac{B}{2} cos(2m\tau))u_A = 0, \tag{4.98}$$

and

$$u_A'' + (E + \frac{B}{2} - \frac{B}{2} cos(2m\tau))u_A = 0, \tag{4.99}$$

From the theory of Mathieu equations and parametric resonance we see that the frequency

$$\omega_k^2 = \frac{2\underline{k}^2 + m_\phi^2 + m_\psi^2 + 2\sqrt{\underline{k}^2 + m_\phi^2}\sqrt{\underline{k}^2 + m_\phi^2}}{4} = (\frac{n}{2}\omega)^2, \tag{4.100}$$

must be half integer multiples of a lowest frequency ω. By applying the theory of parametric resonance from the chapter 3, we get $\omega \propto \underline{k}$. Now we make another approximation, by assuming that $m_\phi^2, m_\psi^2 \ll \omega_k^2$, we find

$$u_A'' + (E - Bsin^2(m\tau))u_A = 0, \tag{4.101}$$

and

$$u_B'' + (E + Bsin^2(m\tau))u_B = 0, \tag{4.102}$$

where

$$E = \underline{k}^2,\ B = \lambda|\Lambda|^2. \tag{4.103}$$

In the region of broad resonance, we replace the oscillating potential near its zeros with an asymptotically flat potential of the form

$$|\eta|^2 = |\Lambda|^2 sin^2(m\tau) \simeq 2|\Lambda|^2 tanh^2(m\frac{(\tau - \tau_i)}{\sqrt{2}}). \tag{4.104}$$

In each instability region, we have

$$u_A'' + (\underline{k}^2 - 2\lambda|\Lambda|^2 tanh^2(\frac{m(\tau - \tau_i)}{\sqrt{2}}))u_A = 0, \tag{4.105}$$

and

$$u_B'' + (\underline{k}^2 + 2\lambda|\Lambda|^2 tanh^2(\frac{m(\tau - \tau_i)}{\sqrt{2}}))u_B = 0. \tag{4.106}$$

We will calculate the transmission and reflection coefficients of the above equations, which will then provide the amount of particle production for each case.

First we will solve the differential equation (4.106) governing the evolution of the anti particle wave function [59]. We define:

$$\kappa_2^2 = \frac{\underline{k}^2}{\rho^2}, \tag{4.107}$$

and a change of variable as before

$$y = \rho(\tau - \tau_i), \tag{4.108}$$

gives

$$\frac{d^2u_B}{dy^2} + \left[\kappa_2^2 + \frac{(2\lambda|\Lambda|^2)}{\rho^2} tanh^2(y)\right] u_B = 0. \tag{4.109}$$

The solution to the above equation is given by

$$u_B = \frac{1}{\sqrt{(\frac{2\lambda|\Lambda|^2}{\rho^2} + \kappa_2^2)}} (e^x + e^{-x})^{-i\sqrt{(\frac{2\lambda|\Lambda|^2}{\rho^2}+\kappa_2^2)}} F(a, b, c, \frac{1}{1+e^{-2x}}), \tag{4.110}$$

where a, b and c are given by

$$a = -4i\left(\sqrt{(\frac{2\lambda|\Lambda|^2}{\rho^2} + \kappa_2^2)}. - \sqrt{\frac{2\lambda|\Lambda|^2}{\rho^2} - \frac{1}{4}}\right) + \frac{1}{2} \tag{4.111}$$

$$b = -4i\left(\sqrt{(\frac{2\lambda|\Lambda|^2}{\rho^2} + \kappa_2^2)}. + \sqrt{\frac{2\lambda|\Lambda|^2}{\rho^2} - \frac{1}{4}}\right) + \frac{1}{2} \tag{4.112}$$

$$c = -4i\sqrt{(\frac{2\lambda|\Lambda|^2}{\rho^2} + \kappa_2^2)}. + 1. \tag{4.113}$$

By using the properties of hypergeometric function $(\tau - \tau_i) \to \infty$ we get

$$u_B = \frac{\mu_2}{\sqrt{(\frac{2\lambda|\Lambda|^2}{\rho^2} + \kappa_2^2)}} e^{-(\sqrt{(\frac{2\lambda|\Lambda|^2}{\rho^2}+\kappa_2^2)})(\tau-\tau_i)} + \frac{\nu_2}{\sqrt{(\frac{2\lambda|\Lambda|^2}{\rho^2} + \kappa_2^2)}} e^{(\sqrt{(\frac{2\lambda|\Lambda|^2}{\rho^2}+\kappa_2^2)})(\tau-\tau_i)} \tag{4.114}$$

The amount of particle production for $B_s(k,t)$ modes is given by

$$n_{2k} = |\nu_{2k}|^2 = \frac{\left(cosh(\pi\sqrt{\frac{2\lambda|\Lambda|^2}{\rho^2} - \frac{1}{4}})\right)^2}{sinh^2\left(\pi\sqrt{(\frac{2\lambda|\Lambda|^2}{\rho^2} + \kappa_2^2)}\right)}. \tag{4.115}$$

In the similar fashion, the equation (4.105) can be solved using the following definition

$$\kappa_1^2 = \frac{k^2 - (2\lambda|\Lambda|^2)}{\rho^2}, \; \rho^2 = \frac{m^2}{2} \tag{4.116}$$

along with a change of variable given by:

$$y = \rho(\tau - \tau_i). \tag{4.117}$$

Hence, we get the evolution equation for u_A is

$$\frac{d^2u_A}{dy^2} + \left[\kappa_1^2 + \frac{(2\lambda|\Lambda|^2)}{\rho^2} sech^2(y)\right] u_A = 0. \tag{4.118}$$

The transmission cofficient [60] for this potential barrier is given by

$$|T|^2 = \frac{sinh^2(\pi\kappa_1)}{\left(cos(\pi\sqrt{\frac{2(\lambda|\Lambda|^2)}{\rho}+\frac{1}{4}})\right)^2 + sinh^2(\pi\kappa_1)}. \tag{4.119}$$

The amount of particle production for $A_s(k,t)$ modes is given by

$$n_{1k} = |\nu_{1k}|^2 = \frac{\left(cos^2(\pi\sqrt{\frac{2(\lambda|\Lambda|^2)}{\rho}+\frac{1}{4}})\right)^2}{(sinh^2(\pi\kappa_1))^2}. \tag{4.120}$$

The total integrated baryon asymmetry of the universe is given by

$$N_B - N_{\overline{B}} = \int_0^\infty dkk^2 n_{1k} - \int_0^\infty dkk^2 n_{2k}. \tag{4.121}$$

4.3 Results and Discussions.

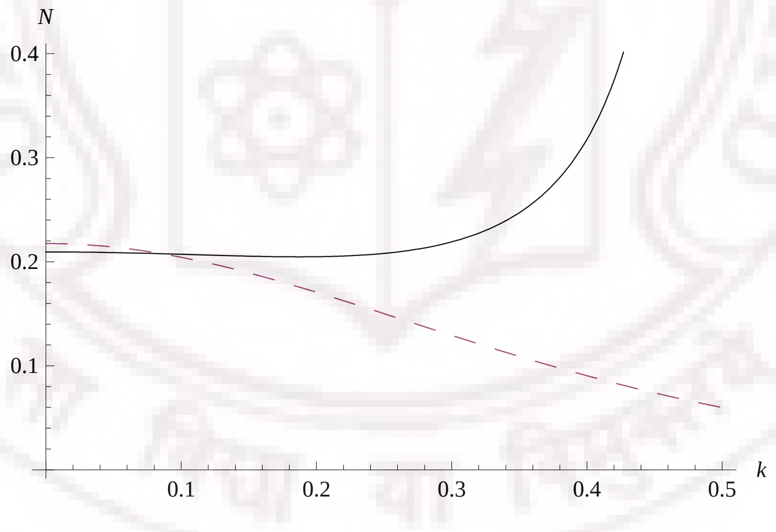


Figure 4.1: Fig. 1. shows the variation of particles (dashed line) and antiparticles (solid lines) for $\frac{2\lambda|\Lambda|^2}{m^2} = .25$ as a function of comoving wave number k

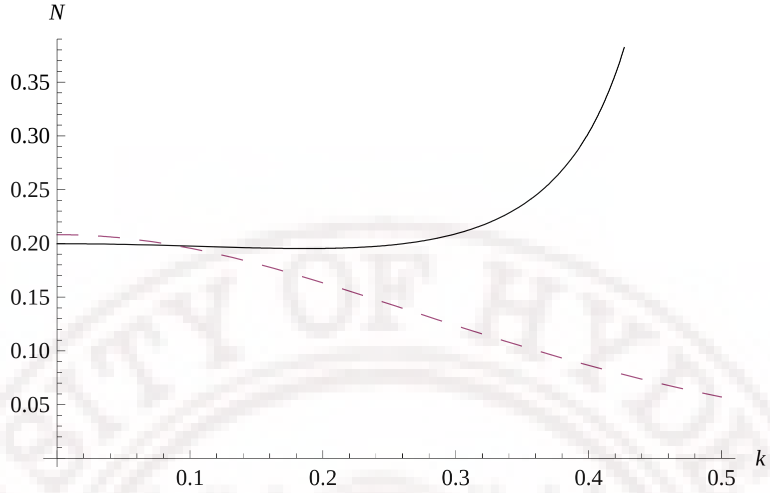


Figure 4.2: Fig. 1. shows the variation of particles (dashed line) and antiparticles (solid lines) for $\frac{2\lambda|\Lambda|^2}{m^2} = .26$ as a function of comoving wave number k

We plot the co-wave number dependent number of baryons and anti-baryons in figure 1. and figure 2. for various values of the parameter $\frac{2\lambda|\Lambda|^2}{m^2}$. As illustrated in the figures, the baryon asymmetry depends on the parameter $\frac{2\lambda|\Lambda|^2}{m^2}$. We see that as the value of $\frac{2\lambda|\Lambda|^2}{m^2}$ decreases the value of asymmetry decreases. This can be explained as in a rapidly expanding universe, Λ decreases and makes $\frac{2\lambda|\Lambda|^2}{m^2}$ smaller, while, in a slowly expanding universe, Λ changes very slowly, therefore, the inflaton field stays in the instability bands for a longer time and thus gives rise to sufficent asymmetry.

On the other hand, for large amplitude oscillations, the parameter $\frac{2\lambda|\Lambda|^2}{m^2}$ can be very large. In this case, resonance occurs for a broad range of values of the momentum and the amplification of the baryon modes and the suppression of the anti-baryon modes can become more efficient. Thus this parameter $\frac{2\lambda|\Lambda|^2}{m^2}$ governs the range of momentum modes undergoing parametric amplification. From our model we have generated a baryon asymmetry in an entirely non-perturbative fashion. The approximations made are only at the end of the derivations and have been used to illustrate the methodology. It is to be noticed that the equations (4.96-4.97) are exact for an oscillating inflaton field in an FRW Universe. It is entirely possible to solve these equations to incorporate the effects of the expansion of the Universe on the generation of baryon asymmetry. Since we are relying on parametric resonance for enhancement/suppression of the particle/anti-particle creation, the restriction to the lowest instability band of the Mathieu equation is a reasonable one.

CHAPTER 5

Post Inflationary Affleck-Dine Baryogenesis: Model II

Efficient inflation requires the inflaton potential to be initially flat and later steep, as discussed in chapter 3. Such potentials are natural in the supersymmetric theories. When supersymmetry is exact, a number of flat directions exist, which upon breaking of the supersymmetry give rise to potentials in the field space. Supersymmetric theories also naturally satisfy the criterion of having B-violating and CP violating interactions. Therefore, supersymmetric theories becomes a natural choice when considering models for the generation of the baryon asymmetry. One such model is the Affleck Dine [11] model, discussed in chapter 2. The mechanism of baryon asymmetry generation, using classical arguments, in this model is too efficient and produces a baryon asymmetry that is too large. Various attempts have been made to dilute the asymmetry. In one model, inflation is incorporated and entropy is released after baryogenesis through decay of the inflaton field [61, 62], again using classical arguments. Other models introduce nonrenormalizable terms [63, 64]. In [65], a preliminary perturbative analysis of out of equilibrium quantum fluctuations using non-equilibrium finite temperature field theory in the AD model, led to some amount of reduction in the asymmetry. In our analysis, we give a more comprehensive quantum field theoretic treatment of the quantum fluctuations in the AD model, using a background field analysis, and calculate the resultant baryon asymmetry in a variety of inflationary reheating scenarios. The squeezed state formalism, that we have developed in the previous chapter, allows the analysis of these quantum effects in a transparent fashion.

The superpotential in the supersymmetric theories is given by eq(2.15). From this a general superpotential can be constructed by incorporating B and CP violation required to satisfy the Sakharov conditions. This is generally of the form [64]

$$V(\phi) = (m^2 - cH^2)|\phi|^2 + (\frac{a\lambda\phi^4}{M} + h.c.) + \lambda^2\frac{|\phi|^2}{M^2}, \tag{5.1}$$

where ϕ is the superfield, m is the gravity mediated soft SUSY breaking scalar mass, H is the Hubble's constant, M is the mass corresponding to the GUT or Planck scale, c and a are constants and λ is a coupling constant. The first term is the soft supersymmetry breaking term, the second and third term give the CP and the baryon number violation. Keeping the terms up to the first order in λ, Affleck and Dine constructed a model, which incooperates all the features of the SUSY and satisfies the Sakharov conditions, to calculate BAU.

5.1 Quantum Flucutations in The Affeck Dine Model

Using the above arguments we consider a model having the action [65],

$$S = \int d^4x\sqrt{-g}[g_{\mu\nu}(\partial^\mu\phi^\dagger)(\partial^\nu\phi) - m_\phi^2\phi^\dagger\phi + \frac{i\lambda}{4}(\phi^4 - \phi^{\dagger 4})], \tag{5.2}$$

where ϕ is a complex scalar field, m_ϕ is the mass of the scalar field and λ is a dimensional less coupling constant. The baryon number violation comes from the coupling $\lambda \approx \epsilon M_S^2/M_G^2$, where, M_S is the supersymmetry breaking scale, ϵ is a real parameter which characterises CP violation and M_G is the grand unification scale. The background metric is the FRW metric

$$ds^2 = dt^2 - a^2(t)dx^2,$$

where a(t) is expansion parameter. Going over to conformal time η, such that $d\eta = a^{-1}dt$, the action can be written as

$$S = \int d\eta d^3x a^4 [\frac{1}{a^2}((\frac{\partial \phi^\dagger}{\partial \eta})(\frac{\partial \phi}{\partial \eta}) - (\nabla \phi^\dagger)(\nabla \phi)) - m_\phi^2 \phi^\dagger \phi + \frac{i\lambda}{4}(\phi^4 - \phi^{\dagger 4})]. \tag{5.3}$$

Defining $\chi = a\phi$, we get

$$S = \int d\eta d^3x [((\frac{\partial \chi^\dagger}{\partial \eta})(\frac{\partial \chi}{\partial \eta}) - (\nabla \chi^\dagger)(\nabla \chi)) - (m_\chi^2 a^2 - \frac{a''}{a})\chi^\dagger \chi + \frac{i\lambda}{4}(\chi^4 - \chi^{\dagger 4})]. \tag{5.4}$$

Decomposing the field χ into its real and imaginary parts,

$$\chi = \frac{1}{\sqrt{2}}(\chi_1 + i\chi_2), \tag{5.5}$$

$$\chi^\dagger = \frac{1}{\sqrt{2}}(\chi_1 - i\chi_2), \tag{5.6}$$

and substituting equations (5.5) and (5.6) in equation (5.4), the action reduces to,

$$S = \int d\eta d^3x [\frac{1}{2}(\chi_1') - \frac{1}{2}(\nabla\chi_1)^2 + \frac{1}{2}(\chi_2') - \frac{1}{2}(\nabla\chi_2)^2 - \frac{m_\eta^2}{2}(\chi_1 + \chi_2^2) - 2\lambda\chi_1\chi_2[\chi_1^2 - \chi_2^2]], \tag{5.7}$$

where prime denotes derivative with respect to conformal time η and $m_\eta^2 = m_\chi^2 - \frac{a''}{a}$.

We use the background field analysis to study the quantum fluctuations [28, 29, 30] around a classical solution. We decompose the superfield χ_i into a background classical component χ_{icl} and the quantum fluctuation $\widehat{\chi}_i$ as

$$\chi_i = \chi_{icl} + \widehat{\chi}_i, \tag{5.8}$$

where χ_{icl} classical component satisfies the classical equation of motion

$$\frac{\delta S}{\delta \chi_i}|_{\chi_i=\chi_{icl}} = 0. \tag{5.9}$$

Expanding the action in a Taylor series around χ_{icl}

$$S[\chi_i, \chi_j] = s[\chi_i, \chi_j] + \frac{\delta S[\chi_i, \chi_j]}{\delta \chi_i}|_{\chi_i=\chi_{icl}} + \frac{1}{2}(\widehat{\chi}_i|\frac{\delta^2 S[\chi_i, \chi_j]}{\delta \chi_i \delta \chi_i}|\widehat{\chi}_i + \widehat{\chi}_i|\frac{\delta^2 S[\chi_i, \chi_j]}{\delta \chi_i \delta \chi_j}|\widehat{\chi}_j) + ..., \tag{5.10}$$

we see that the second term in the expansion is zero form the eq(5.9) and the contributions for quantum fluctuations comes from the quadratic and the higher order terms.

Restricting ourselves to the quadratic fluctuations, in a mean field type approach, the action is

$$\begin{aligned} S &= \int d\eta d^3x[\frac{1}{2}(\widehat{\chi}_1') - \frac{1}{2}(\nabla\widehat{\chi}_1)^2 + \frac{1}{2}(\widehat{\chi}_2') - \frac{1}{2}(\nabla\widehat{\chi}_2)^2 - \frac{m_\eta^2}{2}(\widehat{\chi}_1 + \widehat{\chi}_2^2) \\ &\quad -\frac{1}{2}(\widehat{\chi}_i|\frac{\partial^2 V[\chi]}{\delta\chi_i\delta\chi_i}|\widehat{\chi}_i + \widehat{\chi}_i|\frac{\partial^2 V[\chi]}{\delta\chi_i\delta\chi_j}|\widehat{\chi}_j)], \end{aligned} \tag{5.11}$$

where

$$V(\chi) = 2\lambda\chi_1\chi_2(\chi_1^2 - \chi_2^2). \tag{5.12}$$

Using Legendre transformation, the Hamiltonian is

$$\begin{aligned} H &= \int d\eta d^3x[\frac{\widehat{p}_1^2}{2} + \frac{(\nabla\widehat{\chi}_1)^2}{2} + \frac{m_\eta^2}{2}\widehat{\chi}_1^2 + \frac{\widehat{p}_2^2}{2} + \frac{(\nabla\widehat{\chi}_2)^2}{2} \\ &\quad + \frac{m_\eta^2}{2}\widehat{\chi}_2^2 + 2\lambda(\rho\widehat{\chi}_1\widehat{\chi}_2 + \delta(\widehat{\chi}_1^2 - \widehat{\chi}_2^2))], \end{aligned} \tag{5.13}$$

where, $\widehat{p}_i$ are the canonical momenta of the $\widehat{\chi}_i$ fields and $\delta = 2(\chi_{10}^2 - \chi_{20}^2)$ and $\rho = 3\chi_{10}\chi_{20}$.

We quantize the fields by

$$\widehat{\chi}_1 = \int dk[a_k^\dagger e^{ik\cdot x} + a_k e^{-ik\cdot x}], \tag{5.14}$$

$$\widehat{\chi}_2 = \int dk[b_k^\dagger e^{ik\cdot x} + b_k e^{-ik\cdot x}], \tag{5.15}$$

where,

$$k \cdot x = k_\mu x^\mu = \omega t - k_i x_i, \tag{5.16}$$

$$d\tilde{k} = \frac{d^3k dt}{[(2\pi)^3 2\omega_k]^{\frac{1}{2}}}. \tag{5.17}$$

The Hamiltonian is then

$$
\begin{aligned}
H &= \int \frac{d^3k}{(2\pi)^3}[[\frac{\omega}{2}+\frac{\lambda\delta}{2\omega}](a_k^\dagger a_k + a_{-k}a_{-k}^\dagger) \\
&\quad +[\frac{\omega}{2}-\frac{\lambda\delta}{2\omega}](b_k^\dagger b_k + b_{-k}b_{-k}^\dagger)] \\
&\quad +(\frac{\lambda\rho}{2\omega}(a_k^\dagger b_k + a_{-k}b_{-k}^\dagger + a_k^\dagger b_{-k}^\dagger + a_k b_{-k}) \\
&\quad +\frac{\lambda\delta}{2\omega}(a_k^\dagger a_{-k}^\dagger + a_k a_{-k}) - \frac{\lambda\delta}{2\omega}(b_k^\dagger b_{-k}^\dagger + b_k b_{-k})]],
\end{aligned}
\tag{5.18}
$$

where

$$\omega = k^2 + m_\eta^2. \tag{5.19}$$

The Hamiltonian has su(1,1) and su(2) symmetries, as can be seen by defining

$$N_1 = \frac{1}{2}(a_k^\dagger a_k + a_{-k}a_{-k}^\dagger),\ \ N_2 = \frac{1}{2}(b_k^\dagger b_k + b_{-k}b_{-k}^\dagger), \tag{5.20}$$

$$J_+ = \frac{1}{2}(a_k^\dagger b_k + a_{-k}^\dagger b_{-k}),\ J_- = \frac{1}{2}(b_k^\dagger a_k + b_{-k}^\dagger a_{-k}),\ J_0 = \frac{1}{2}(N_1 - N_2), \tag{5.21}$$

$$
\begin{aligned}
K_+ = a_k b_{-k},\ K_- = b_{-k}^\dagger a_k^\dagger,\ K_0 = \frac{1}{2}(N_1 + N_2 + 1), \\
L_{1-} = a_k^\dagger a_{-k}^\dagger,\ L_{1+} = a_{-k}a_k,\ L_{10} = \frac{1}{2}(N_1 + 1), \\
L_{2-} = b_k^\dagger b_{-k}^\dagger,\ L_{2+} = b_{-k}b_k,\ L_{20} = \frac{1}{2}(N_2 + 1).
\end{aligned}
\tag{5.22}
$$

It can be seen that

$$[J_+, J_-] = -2J_0,\ [J_+, J_0] = -J_+,\ [J_-, J_0] = J_-, \tag{5.23}$$

defines an su(2) algebra. While,

$$[K_+, K_-] = 2K_0,\ [K_+, K_0] = K_+,\ [K_-, K_0] = -K_-, \tag{5.24}$$

$$[L_{1+}, L_{1-}] = 2L_{10},\ [L_{1+}, L_{10}] = L_{1+},\ [L_{1-}, L_{10}] = -L_{1-}, \tag{5.25}$$

$$[L_{2+}, L_{2-}] = 2L_{20},\ [L_{2+}, L_{20}] = L_{2+},\ [L_{2-}, L_{20}] = -L_{2-}. \tag{5.26}$$

satisfy su(1,1) algebras. In terms of these generators the Hamiltonian is

$$\begin{aligned} H &= \int \frac{d^3k}{(2\pi)^3}[(\frac{\omega}{2} + \frac{\lambda\delta}{2\omega})N_1 + (\frac{\omega}{2} - \frac{\lambda\delta}{2\omega})N_2 \\ &+ \frac{\lambda\rho}{2\omega}(J_+ + J_- + K_+ + K_-) \\ &+ \frac{\lambda\delta}{2\omega}(L_{1+} + L_{1-}) - \frac{\lambda\delta}{2\omega}(L_{2+} + L_{2-})]. \end{aligned} \tag{5.27}$$

This Hamiltonian can diagonalized by using a series of unitary transformations. Since again it bares a resemblence to quantum optical Hamiltonians in beam splitting and parametric down conservation processes. The first transformation is a rotation given by the operator

$$U(R_1) = exp[\theta(J_+ e^{2i\xi} + J_- e^{2i\xi})]. \tag{5.28}$$

$U(R_1)$ provides the transformation relations:

$$U^\dagger(R_1)\begin{pmatrix} a_k \\ b_k \end{pmatrix} U(R_1) = \begin{pmatrix} Cos(\theta) & e^{2i\xi}Sin(\theta) \\ -e^{-2i\xi}Sin(\theta) & Cos(\theta) \end{pmatrix}\begin{pmatrix} a_k \\ b_k \end{pmatrix} = \begin{pmatrix} A_k \\ B_k \end{pmatrix}. \tag{5.29}$$

This leads us to a Hamiltonian H_1 in terms of the new creation and annihilation operators $A_k^\dagger$, $B_k^\dagger$, A_k and B_k given by

$$\begin{aligned} U^\dagger(R_1)HU(R_1) = H_1 &= \int \omega^2 \frac{d^3k}{(2\pi)^3}(m_1[A_k^\dagger A_k + A_{-k}^\dagger A_{-k}] \\ &+ m_2[B_k^\dagger B_k + B_{-k}^\dagger B_{-k}] + n_1[A_k A_{-k} + A_{-k}^\dagger A_k^\dagger] \\ &+ n_2[B_k B_{-k} + B_{-k}^\dagger B_k^\dagger]), \end{aligned} \tag{5.30}$$

where m_1, m_2, n_1 and n_2 are given by

$$m_1 = \frac{\omega^2 - \Omega^2}{\omega^2},\ m_2 = \frac{\omega^2 + \Omega^2}{\omega^2},\ n_1 = -\frac{\Omega^2}{\omega^2},\ n_2 = \frac{\Omega^2}{\omega^2}. \tag{5.31}$$

This rotated Hamiltonian has an $su(1,1)$ symmetry seen by defining

$$D_{1+} = A_k^\dagger A_{-k}^\dagger,\ D_{1-} = A_{-k}A_k,\ D_{10} = \frac{1}{2}(A_k^\dagger A_k + A_{-k}^\dagger A_{-k} + 1), \tag{5.32}$$

$$D_{2+} = B_{-k}^\dagger B_k^\dagger,\ D_{2-} = B_k B_{-k}\ D_{20} = \frac{1}{2}(B_{-k}^\dagger B_{-k} + B_k^\dagger B_k + 1), \tag{5.33}$$

which satisfy the $su(1,1)$ algebras,

$$[D_{1+}, D_{1-}] = 2D_{10},\ [D_{1+}, D_{10}] = D_{1+},\ [D_{1-}, D_{10}] = -D_{1-}, \tag{5.34}$$

$$[D_{2+}, D_{2-}] = 2D_{20},\ [D_{2+}, D_{20}] = D_{2+},\ [D_{2-}, D_{20}] = -D_{2-}. \tag{5.35}$$

The Hamiltonian in terms of the new generators is written as

$$H_1 = \int \frac{d^3k}{(2\pi)^3}(\omega_{\chi_1} + \omega_{\chi_2})[[m_1 D_{10} + m_2 D_{20}] + n_1[D_{1+} + D_{2-}] + n_2(D_{2+} + D_{1-})]. \tag{5.36}$$

Since an su(1,1) symmetry implies diagonalizability through a squeezing (Bogoliubov) transformations, we see here that a product of two squeezing transformations will provide us with a diagonal Hamiltonian. The squeezing transformations are given by

$$S(\zeta_1)S(\zeta_2) = exp[\zeta_1 D_{1+} - \zeta^* D_{1-}]exp[\zeta_2 D_{2+} - \zeta_2^* D_{2-}], \tag{5.37}$$

where, $\zeta_1 = r_1 exp[i\gamma]$ and $\zeta_2 = r_2 exp[i\gamma]$ are the squeezing parameters. Then the new creation and annihilation operators in terms of A_k and B_k are given by

$$A_s(k,t) = \mu_1 A_k + \nu_1 A^\dagger_{-k}, \tag{5.38}$$
$$A^\dagger_s(k,t) = \mu_1^* A^\dagger_k + \nu_1^* A_{-k}, \tag{5.39}$$
$$B_s(k,t) = \mu_2 B_{-k} + \nu_2 B^\dagger_k, \tag{5.40}$$
$$B^\dagger_s(k,t) = \mu_2^* B^\dagger_{-k} + \nu_2^* B_k. \tag{5.41}$$

The Bogoliubov coefficients are

$$\mu_1 = Cosh(r) = \frac{m_1}{\sqrt{m_1^2 - n_1^2}}, \tag{5.42}$$
$$\nu_1 = Sinh(r) = e^{-i\gamma}\frac{n_1}{\sqrt{m_1^2 - n_1^2}}, \tag{5.43}$$
$$\mu_2 = Cosh(r) = \frac{m_2}{\sqrt{m_2^2 - n_2^2}}, \tag{5.44}$$
$$\nu_2 = Sinh(r) = e^{-i\gamma}\frac{n_2}{\sqrt{m_2^2 - n_2^2}}, \tag{5.45}$$

which satisfy

$$|\mu_1|^2 - |\nu_1|^2 = 1, \quad |\mu_2|^2 - |\nu_2|^2 = 1. \tag{5.46}$$

Thus, the final diagonalized Hamiltonian after three unitary transformations is

$$\begin{aligned} H_f &= S(\zeta_2)^\dagger S(\zeta_1)^\dagger U^\dagger(R_1)[H]U(R_1)S(\zeta_1)S(\zeta_2) \\ &= \int \frac{d^3k}{(2\pi)^3}\Omega_+[A_s^\dagger(k,\eta)A_s(k,\eta)+1] \\ &\quad +\Omega_-[B_s^\dagger(k,\eta)B_s(k,\eta)+1]], \end{aligned} \tag{5.47}$$

where $\Omega_+ = \sqrt{m_1^2 - n_1^2} = \sqrt{\omega^2 - 2\Omega^2}$ and $\Omega_- = \sqrt{m_2^2 - n_2^2} = \sqrt{\omega^2 + 2\Omega^2}$.

The vacuum state of H_f at time η is given by $|0(\eta), 0(\eta)>$ and vacuum state of H at initial time is given by $|0,0>$. They are related by

$$|0(t),0(t)>= e^{\int \frac{d^3k}{(2\pi)^3}[\zeta_1 D_{1+} - \zeta_1^* D_{2-}] + [\zeta_1 D_{1+} - \zeta_1^* D_{2-}]} e^{\int \frac{d^3k}{(2\pi)^3}[\theta(J_+ e^{2i\xi} + J_- e^{2i\xi})]}|0,0> . \tag{5.48}$$

It is seen that the vacuum at later times is populated with particles and anti-particles with respect to vacuum state at initial time. The number of particles and anti-particles can be estimated from the relationship between the creation and annihilation operators at initial time given by a_k, and b_k, and the final creation and annihilation at later time given by operators A_s and B_s.

$$\begin{aligned} A_s(k,\eta) &= (\mu_1 Cos(\theta))a_k + (\nu_1 Sin\theta e^{2i\xi})b_{-k}^\dagger \\ &\quad +(\mu_1 Sin\theta e^{2i\xi})b_k + (\nu_1 Cos(\theta))a_{-k}^\dagger, \end{aligned} \tag{5.49}$$

$$\begin{aligned} B_s(k,\eta) &= (\mu_2 Cos(\theta))a_{-k} + (\nu_2 Sin\theta e^{-2i\xi})b_k^\dagger \\ &\quad +(\mu_2 Sin\theta e^{-2i\xi})b_{-k} + (\nu_2 Cos(\theta))a_k^\dagger. \end{aligned} \tag{5.50}$$

The number of particles (baryons) for each mode

$$N_{kB}(\eta) = \langle B_s^\dagger(k,\eta)B_s(k,\eta)\rangle = |\nu_{k2}|^2, \tag{5.51}$$

and the number of anti-particles (anti-baryons) for each mode

$$N_{k\overline{B}}(\eta) = \langle A_s^\dagger(k,\eta)A_s(k,\eta)\rangle = |\nu_{k1}|^2. \tag{5.52}$$

Therefore the baryon asymmetry is

$$\triangle N_k = N_B(\eta) - N_{\overline{B}}(\eta) \;\; = \;\; \frac{\Omega^6}{\omega^2(4\Omega^4 - \omega^4)}, \tag{5.53}$$

where $\Omega^2 = \lambda\sqrt{(\rho^2 + \delta^2)}$ and $\rho = 3(\chi_{10}^2 - \chi_{20}^2)$ and $\delta = 3\chi_{10}\chi_{20}$. We find that $N_B(\eta) - N_{\overline{B}}(\eta)$ is dependent on the vacuum expectation values of real and imaginary parts of scalar field and the coupling constant λ.

The total asymmetry is given by,

$$\begin{aligned} \int_0^\infty \triangle N_k d^3k &= \int_0^\infty k^2 dk \frac{\Omega^6}{\omega^2(4\Omega^4 - \omega^4)} \\ &= \Omega^2(\sqrt{m_\eta^2 + 2\Omega^2} - \sqrt{m_\eta^2 - 2\Omega^2}). \end{aligned} \tag{5.54}$$

It is interesting to see that when $\lambda \ll 1$ (perturbative limit) and $\chi_{10} = \chi_{20} = \phi_0$, the asymmetry reduces to classical value,

$$(N_B(\eta) - N_{\overline{B}}(\eta)) \;\; = \;\; (\frac{3\lambda\phi_0^2}{4m_\eta^2}) \simeq r. \tag{5.55}$$

5.2 Evolution Of BAU in Various Reheating Scenarios:

In order to get some exact results and numerical values for the parameter r after expansion, we consider some realistic expansion scenarios such that the Bogoliubov coefficients can be evaluated exactly. The time evolution of the wave function under the action of the Hamiltonian H_f is given by (5.47). Since this Hamiltonian bears an uncanny resemblance to the one given in chapter 4, we go over to to the coordinate representation $\Pi_{(A,B)}$ and $P_{(\Pi_{A,B})}$ defined by similar relations to (4.69), (4.70), (4.71) and (4.72) to get

$$H_f(\eta) = \int \frac{d^3k}{(2\pi)^3}\frac{1}{2}[[(\Omega_+)^2\Pi_A^2(k,\eta) + P_{\Pi_A}^2(k,\eta)] + [(\Omega_-)^2\Pi_B^2(k,\eta) + P_{\Pi_B}^2(k,\eta)]] \tag{5.56}$$

where $\Omega_+ = \sqrt{m_1^2 - n_1^2} = \sqrt{\omega^2 - 2\Omega^2}$ and $\Omega_- = \sqrt{m_2^2 - n_2^2} = \sqrt{\omega^2 + 2\Omega^2}$.

The time evolution of a wave function $\psi(t)$ under the action of a Hamiltonian H(t) is again similar to (4.76), with the evolution equations for $\psi_A(t)$ and $\psi_B(t)$ are given by

$$\psi_A''(k,\eta) + \Omega_-^2 \psi_A(k,\eta) = 0, \quad (5.57)$$

$$\psi_B''(k,\eta) + \Omega_+^2 \psi_B(k,\eta) = 0. \quad (5.58)$$

where $\Omega_- = k^2 + m_\phi^2 a^2 - \frac{a''}{a} - 2\lambda\sqrt{\rho^2+\delta^2}$ and $\Omega_+ = k^2 + m_\phi^2 a^2 - \frac{a''}{a} + 2\lambda\sqrt{\rho^2+\delta^2}$.

These equations can be written as *Schrodinger like* equations in η

$$\psi_A'' + (E + V_1(a))\psi_A = 0, \quad (5.59)$$

and

$$\psi_B'' + (E + V_2(a))\psi_B = 0, \quad (5.60)$$

where

$$E = k^2 + m_\eta^2, \quad m_\eta^2 = m_\phi^2 a^2 - \frac{a''}{a}, \quad V_1(a) = -2\lambda\sqrt{\rho^2+\delta^2}, \quad V_2(a) = 2\lambda\sqrt{\rho^2+\delta^2}, \quad (5.61)$$

and $\sqrt{\rho^2+\delta^2} = 3\sqrt{\chi_{10}^4 + \chi_{20}^4 - \chi_{10}^2\chi_{20}^2}$. Again, we use the machinery of potential barrier reflection and transmission problems in quantum mechanics to calculate N(k) $=|\nu|^2$.

There are two factors that contribute to the time evolution of the particle and anti-particle modes, the time dependence of the background classical solution and the time dependence of the expansion factor '$a(\eta)$'.

5.2.1 Slow Expansion

As a first approximation, consider the case when $\frac{a''}{a} = 0$, i.e, a radiation dominated universe. In this case, as seen in the chapter 2, the classical solutions for χ_{10} and χ_{20} are given by

$$\chi_{10} = \frac{\lambda\phi_0}{(m_\eta)^3} sin(m_\eta \eta + \epsilon) \quad (5.62)$$

$$\chi_{20} = \frac{\phi_0}{m_\eta} sin(m_\eta \eta). \quad (5.63)$$

To first order in λ, we have $\lambda\sqrt{\rho^2+\delta^2} = \overline{\varphi}^2 sin^2(m_\eta \eta)$, where $\overline{\varphi}$ is a slowly decreasing amplitude given by $\overline{\varphi}^2 = 3\lambda(\frac{\phi_0}{m_\eta})^2$.

The equations (5.59) and (5.60) become

$$\psi_A'' + (k^2 + m_\eta^2 - \overline{\varphi}^2 sin^2(m_\eta \eta))\psi_A = 0, \tag{5.64}$$

and

$$\psi_B'' + (k^2 + m_\eta^2 + \overline{\varphi}^2 sin^2(m_\eta \eta))\psi_B = 0. \tag{5.65}$$

These are the Mathieu equations, which are familiar from the parametric amplification problem in reheating scenarios after inflation [66] .

$$\psi_A'' + \omega_{1k}^2(1 + g_1 cos(\gamma\eta) - \frac{a''}{\omega_{1k}^2 a})\psi_A = 0, \tag{5.66}$$

and

$$\psi_B'' + \omega_{2k}^2(1 - g_2 cos(\gamma\eta) + \frac{a''}{\omega_{2k}^2 a})\psi_B = 0, \tag{5.67}$$

where, $g_1 = \frac{\overline{\varphi}^2}{\omega_{1k}^2}$, $g_2 = \frac{\overline{\varphi}^2}{\omega_{1k}^2}$, $\omega_{1k}^2 = k^2 + m_\phi^2 - \overline{\varphi}^2$, $\omega_{2k}^2 = k^2 + m_\phi^2 + \overline{\varphi}^2$ and $\gamma = 2m_\eta$.

To solve these equations, the method given in [67] and [68] is followed. From the theory of parametric resonance, the resonance is strongest if the frequency is twice ω_{ik}, hence we put $\gamma = 2\omega_{ik} + \varepsilon$ with $\varepsilon \ll \omega_{ik}$. The resonance condition will be satisfied if $\overline{\varphi}^2 - \varepsilon^2 > 0$ or $|\varepsilon| < \overline{\varphi}$.

We introduce a trial wave function

$$\psi_A = c(\eta) f[a(\eta)] cos[(2\omega_{ik} + \varepsilon)\eta] + d(\eta) f[a(\eta)] sin[(2\omega_{ik} + \varepsilon)\eta]. \tag{5.68}$$

The function $f(a)$ is introduced to include the effects of expansion. The functions $c(\eta)$ and $d(\eta)$ are slowly varying with time compared to the frequency γ.

Then it is self-consistent to put $c', d' = O(\varepsilon)$, and to neglect higher derivatives. In general, resonant solutions can be found for frequencies $n\gamma/2 \backsimeq \omega_k$, but each integer $n > 1$ corresponds to keeping $O(g_1^n)$ in the perturbation expansion. As we are only considering upto $O(g_1)$ we make the following approximation.

$$cos(\gamma\eta/2)cos(\gamma\eta) \backsimeq \frac{1}{2}cos(\gamma\eta/2), sin(\gamma\eta/2)cos(\gamma\eta) \backsimeq \frac{1}{2}sin(\gamma\eta/2). \tag{5.69}$$

Then substituting the trial function in the differential equation then we get

$$\omega_k A(\eta)cos(\gamma\eta/2) + \omega_k B(\eta)sin(\gamma\eta/2) + O(\varepsilon^2, g_1^2) = 0, \tag{5.70}$$

where

$$A(\eta) = -2c'f - cf'(2 + \frac{\varepsilon}{\omega_k}) + \frac{1}{\omega_k}(2d'f' + df'') - df(\frac{1}{2}g\omega_k a^2 + \varepsilon + \frac{1}{\omega_k}\frac{a''}{a}), \tag{5.71}$$

$$B(\eta) = -2d'f - cf'(2 + \frac{\varepsilon}{\omega_k}) + \frac{1}{\omega_k}(2c'f' + cf'') - cf(\frac{1}{2}g\omega_k a^2 - \varepsilon + \frac{1}{\omega_k}\frac{a''}{a}). \tag{5.72}$$

Equation (5.70) requires that both the coefficients A and B terms should vanish identically. This gives a pair of coupled differential equations involving the coefficients $c(\eta), d(\eta)$, and f(a(η)). To solve these equations we set $c(\eta) = Cexp[s\eta]$, $d(\eta) = Dexp[s\eta]$, to get

$$sC + \frac{1}{2}(\frac{1}{2}g_1\omega_k + \varepsilon)D = 0, \tag{5.73}$$

$$sD + \frac{1}{2}(\frac{1}{2}g_1\omega_k - \varepsilon)C = 0. \tag{5.74}$$

Solving these equations we get

$$s = \frac{1}{2}\sqrt{(\frac{1}{2}(g_1\omega_k)^2 - \varepsilon^2)}. \tag{5.75}$$

Thus, the solution is

$$\psi_A = \frac{exp[\pm s\eta]}{\gamma y_k}[y_k cos(\gamma\eta/2) \pm sin(\gamma\eta/2)], \tag{5.76}$$

where $y_k = \pm\sqrt{\frac{\frac{1}{2}g_1\omega_k - \varepsilon}{\frac{1}{2}g_1\omega_k + \varepsilon}}$. Similarly we can get ψ_B. A new variable $l = \frac{\varepsilon}{\varphi}$ is defined so that resonance occurs $-1 < l < 1$. Then by using [68], the number of particles produced is given by

$$N_{1k} = \frac{1}{1 - l^2}Sinh^2(\sqrt{\overline{\varphi}^2 - \varepsilon^2}\eta). \tag{5.77}$$

In the similar fashion we get the number of anti-particles is

$$N_{2k} = \frac{1}{1 - l^2}Sin^2(\sqrt{\overline{\varphi}^2 - \varepsilon^2}\eta). \tag{5.78}$$

The BAU is given by

$$N_{1k} - N_{2k} = \int_0^\infty dkk^2 n_{1k} - \int_0^\infty dkk^2 n_{2k}. \quad (5.79)$$

This gives us the dependence of BAU as a function of time η.

Now we calculate the BAU as a function of the comoving wave number k. For this, as in chapter 4, a broad resonance is assumed such that the Mathieu equation has instability bands within which parametric resonance occurs. In this region, the oscillating potential is replaced with an asymptotically flat potential near its zeros

$$|\overline{\varphi}|^2 sin^2(m_\phi \eta) \simeq 2|\overline{\varphi}|^2 tanh^2(m_\phi \frac{(\eta - \eta_i)}{\sqrt{2}}). \quad (5.80)$$

Then (5.59) and (5.60) become

$$\frac{d^2\psi_A}{dy^2} + \left[\kappa_1^2 + \overline{\varphi}^2 sech^2(y)\right]\psi_A = 0, \quad (5.81)$$

$$\frac{d^2\psi_B}{dy^2} + \left[\kappa_2^2 + \overline{\varphi}^2 tanh^2(y)\right]\psi_B = 0, \quad (5.82)$$

where,

$$\begin{aligned} \kappa_1^2 &= \frac{k^2 - \overline{\varphi}^2}{m_\phi^2} + 1, \\ \kappa_2^2 &= \frac{k^2}{m_\eta^2} + 1 \\ y &= m_\phi(\eta - \eta_i). \end{aligned} \quad (5.83)$$

These resemble the equation (4.109) and (4.118) in the chapter 4. The number of particles is

$$n_{1k} = |\nu_{1k}|^2 = \frac{\left(cos^2(\pi\sqrt{\overline{\varphi}^2 + \frac{1}{4}})\right)^2}{(sinh^2(\pi\kappa_1))^2}. \quad (5.84)$$

The number of antiparticles is

$$n_{2k} = |\nu_{2k}|^2 = \frac{\left(cosh(\pi\sqrt{\overline{\varphi}^2 - \frac{1}{4}})\right)^2}{sinh^2\left(\pi\sqrt{(\overline{\varphi}^2 + \kappa_2^2)}\right)}. \quad (5.85)$$

The BAU is given by

$$N_{1k} - N_{2k} = \int_0^\infty dkk^2 n_{1k} - \int_0^\infty dkk^2 n_{2k}. \tag{5.86}$$

5.2.2 Rapid Expansion

Now consider the effect of rapid expansion on the asymmetry parameter. In this case, the rate of expansion dominates over the oscillation period of the classical background solution, so that ρ and δ can be considered as time independent. We consider an expansion where there is transition from de Sitter epoch (inflationary) to the FRW epoch.

$$a(\eta) = (a_0\eta)^{\frac{p}{1-p}} \ \eta < \eta_0, \tag{5.87}$$

$$a(\eta) = C(\eta - \eta_0) \ \eta > \eta_0, \tag{5.88}$$

where $\eta_0 = \eta_* - (a_0^2\eta_*)^{-1}$. It is convenient to set $a(\eta_0) = 1$ which sets $\eta_* = a_0^{-1}$, and thus $\eta_0 = 0$ and $C = a_0^{-1}$ where $a_0 = -H_0$ for de Sitter spacetime. $p = \frac{1}{2}$ corresponds to radiation dominated universe and $p \to \infty$ corresponds to de Sitter epoch. The change in expansion parameter gives rise to non equilibrium particle production.

Thye evolution equations (5.59) and (5.60) are given by

$$\frac{d^2\psi_A}{d\eta^2} + \left[k^2 - 2\lambda\sqrt{\rho^2 + \delta^2} - \frac{p(2p-1)}{(p-1)^2\eta^2} + \frac{m^2}{H^2\eta^2}\right]\psi_A = 0 \quad \eta < \eta_0, \tag{5.89}$$

$$\frac{d^2\psi_A}{d\eta^2} + g_1\psi_A = 0 \ \ \eta > \eta_0 \tag{5.90}$$

and

$$\frac{d^2\psi_B}{d\eta^2} + \left[k^2 + 2\lambda\sqrt{\rho^2 + \delta^2} - \frac{p(2p-1)}{(p-1)^2\eta^2} + \frac{m^2}{H^2\eta^2}\right]\psi_B = 0 \quad \eta < \eta_0, \tag{5.91}$$

$$\frac{d^2\psi_B}{d\eta^2} + g_1\psi_B = 0 \ \ \eta > \eta_0. \tag{5.92}$$

The equation (5.89) can be written as

$$\frac{d^2\psi_A}{d\tau_1^2} + \left[\frac{\frac{1}{4} - q^2}{\tau_1^2} + 1\right]\psi_A = 0, \tag{5.93}$$

where $q^2 = \frac{(3p-1)^2}{4(p-1)^2} - \frac{m_\phi^2}{H}$, $\tau_1 = \sqrt{g_1}\eta$ and $g_1 = k^2 - 2\lambda\sqrt{\rho^2 + \delta^2}$. The solution is given by

$$\psi_A = (\sqrt{g_1}\eta)^{\frac{1}{2}}[A_k H_q^{(1)}(\sqrt{g_1}\eta) + B_k H_q^{(2)}(\sqrt{g_1}\eta)], \tag{5.94}$$

where $H_q^{(1)}$ and $H_q^{(2)}$ are Hankel functions.

The solution for 5.90 is given by

$$\psi_A(\eta > \eta_0) = \frac{1}{\sqrt{2k}}[\mu e^{-i\sqrt{g_1}\eta} + \nu e^{i\sqrt{g_1}\eta}]. \tag{5.95}$$

The Bogoliubov coefficients are obtained [68] by matching the wave functions and its first derivative at $\eta = \eta_0$.

The number of particles is given by.

$$N_1(k) = |\nu_1|^2 = 4^{q-2}(\frac{\sqrt{g_1}}{a_0})^{-2q-1}(q - \frac{1}{2})^2\Gamma^2(q). \tag{5.96}$$

Using similar methods for (5.91) the number of anti particles produced is given by

$$N_2(k) = |\nu_2|^2 = 4^{q-2}(\frac{\sqrt{g_2}}{a_0})^{-2q-1}(q - \frac{1}{2})^2\Gamma^2(q). \tag{5.97}$$

where $g_2 = k^2 + 2\lambda\sqrt{\rho^2 + \delta^2}$, a_0 is the reference scale of H_0, H_0 is constant for de Sitter expansion.

5.3 Results

In this section, we will discuss the results of the each of the cases we studied.

In the slow expansion scenario, we consider BAU as a function of time η and comoving wave number.

For parametric resonance, it is important that the inflaton stays in resonance band and this is possible as long as its amplitude is slowly varying function of time. The time dependence of the number of particles and antiparticles comes from the slow time variation of the decaying amplitude, which we phenomenologically approximate with $\overline{\varphi}^2 \simeq \overline{\varphi}^2 e^{-\eta/\tau}$ where τ is damping scale.

In figure (a) and (b) we have plotted $N_1(k) - N_2(k)$ as a function of the conformal time for values of $\overline{\varphi} = 10^{-1}e^{-\eta/3}$ and $\overline{\varphi} = 10^{-2}e^{-\eta/5}$, respectively. It can be seen that the value of asymmetry due to the parametric amplification of the quantum fluctuations reduces the asymmetry from the classical value of 10^{-2} to an acceptable lower value 10^{-8} with time.

Now we consider BAU as a function of comoving wave number k, we have plotted $N_1(k)$ and $N_2(k)$ as a function of of the comoving number k, for various values of $\overline{\varphi}$. In the figure (c)and (d) the evolution of particles and antiparticles for values of $\overline{\varphi} = .27$ and $\overline{\varphi} = .28$ are plotted respectively. It can be seen clearly that the number of particles increases and number of antiparticles decreases due to differential amplification of particle and antiparticle modes.

In the case of rapid expansion, we have plot $(N_1(k) - N_2(k))$ as a function of comoving number k. In the rapid expansion there are two case one $p \longrightarrow \infty$ de Sitter expansion and another $p \longrightarrow \frac{1}{2}$ radiation dominated era.

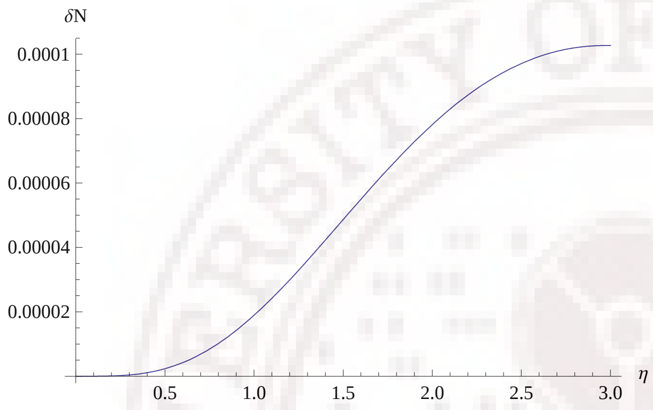
First, consider the case $p \rightarrow \infty$. The vacuum fluctuations of massive fields on an exact de Sitter background leads to density perturbations only for $\frac{m^2_{\phi_0}}{H} < 2$ and $2 < m^2 < \frac{9H_0^2}{4}$. The corresponding characteristic values are $q = 0$, and $\frac{m^2}{H^2} = \frac{9}{4}$ for de Sitter case. In the figure (e) $(N_1(k) - N_2(k))$ is plotted, with $q = .6$ $q = .7$ and $q = .8$, which corresponds to $\frac{m^2_{\phi_0}}{H} < 2$. For de Sitter epoch, at this values, the condensate starts oscillating and gives rise to fluctuations. In the figure (f) $N_1(k) - N_2(k)$ is plotted for the different values of ϕ_0 for a fixed $q = .6$. From the figure, we can see that as the ϕ_0 value decreases the value of asymmetry reduces. In the figure (g) $N_1(k) - N_2(k)$ is plotted for the different values of λ for a fixed $q = .6$ and ϕ_0. From the figure we can see that as the λ value decreases the value of asymmetry reduces.

Large occupation number in a given mode means that quasi-particles formed a condensate. Therefore, from the figure we can see that once the quantum fluctuations are switched on the value of asymmetry reduces but does not goes to zero.

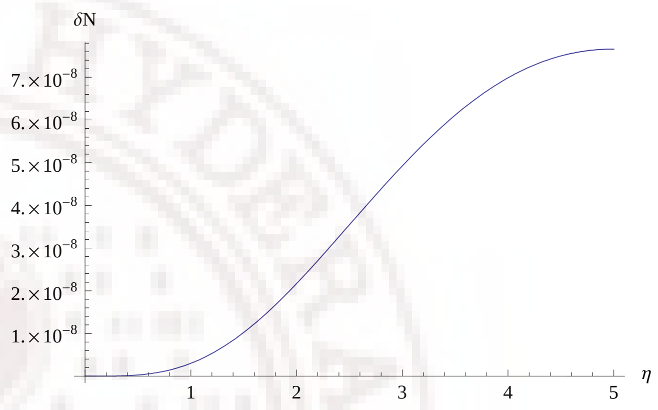
Now we consider the second case $p \to \frac{1}{2}$. In the radiation dominated universe, when $\frac{m^2_{\phi_0}}{H} < .25$, then only the vacuum fluctuations of massive fields will be relevent, and the characteristic value for $q = 0$ is $\frac{m^2}{H^2} = \frac{1}{4}$.

In the figure (h) $N_1(k) - N_2(k)$ is plotted for the different values of ϕ_0 for a fixed $q = 0.4$. which corresponds to $\frac{m^2_{\phi_0}}{H} = 0.16$. From the figure we can see that, again, once the quantum fluctuations are switched on the value of asymmetry reduces. In the figure (i) $N_1(k) - N_2(k)$ is plotted for the different values of ϕ_0 for a fixed $q = .4$, in this case the asymmetry goes to $10^{-3}\frac{\lambda\phi_0^2}{m_\phi^2}$.

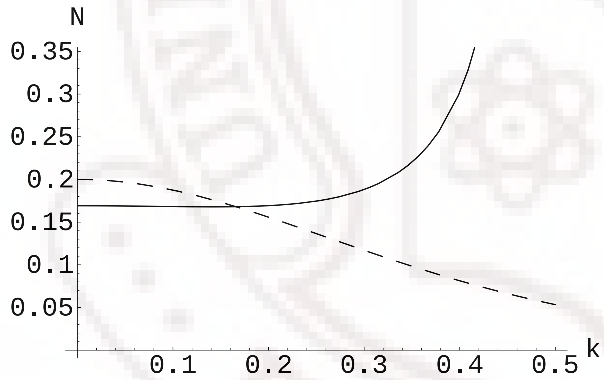
Since, the classical value of the asymmetry is given by $r = \frac{\lambda\phi_0^2}{m_\phi^2}$, we see from the above, the detailed analysis of amplifacation of quantum fluctuations combined with expansion parametrer from de Sitter phase to a FRW phase reduces the baryon asymmetry of universe to a much closer value to the observed one . This encouraging result suggests that quantum fluctuations and expansion effects play a significant role in the calculation of the asymmetry parameter and motivates the inclusion of these in other realistic models of BAU.



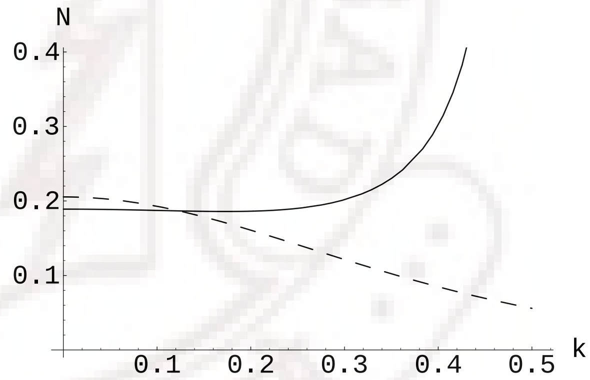
(a) $((\delta N = N_1(k) - N_2(k))$ vs η and $\overline{\varphi} = 10^{-1}e^{-\eta/3}$.



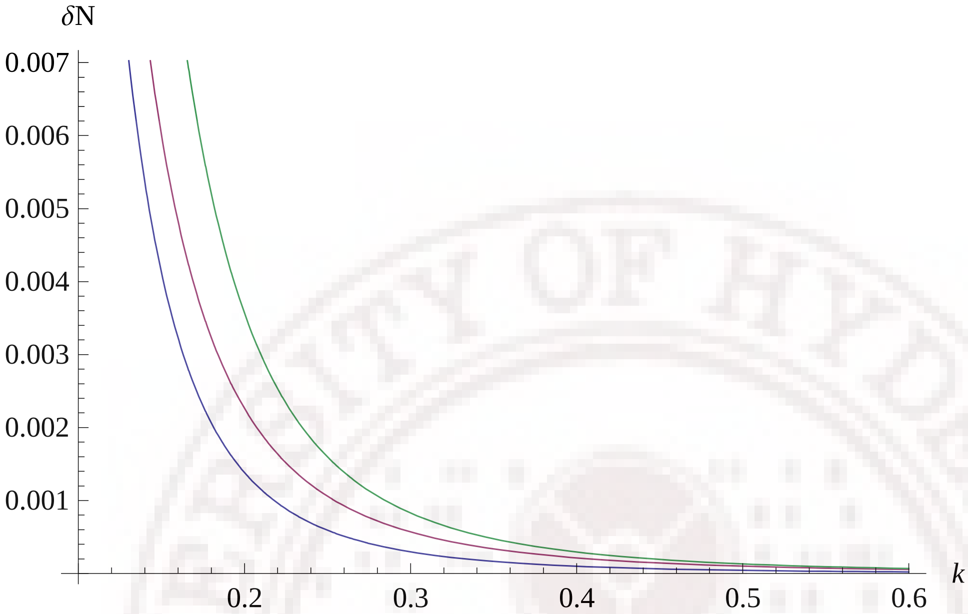
(b) $(\delta N = N_1(k) - N_2(k))$ vs η and $\overline{\varphi} = 10^{-2}e^{-\eta/5}$.



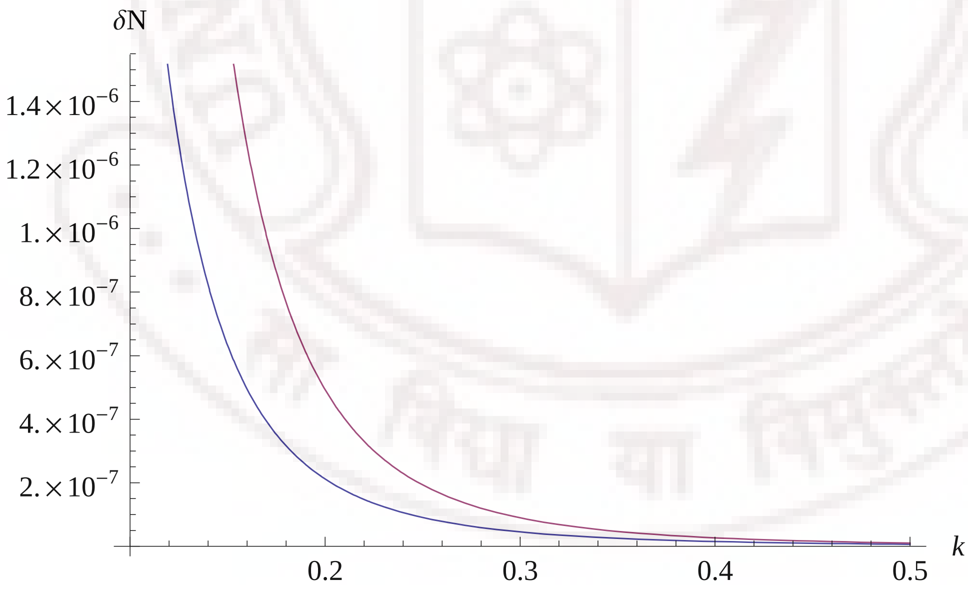
(c) $N_1(k)$ particles(solid line) and $N_2(k)$ and antiparticles(dash line) Vs k and $\overline{\varphi} = .27$.



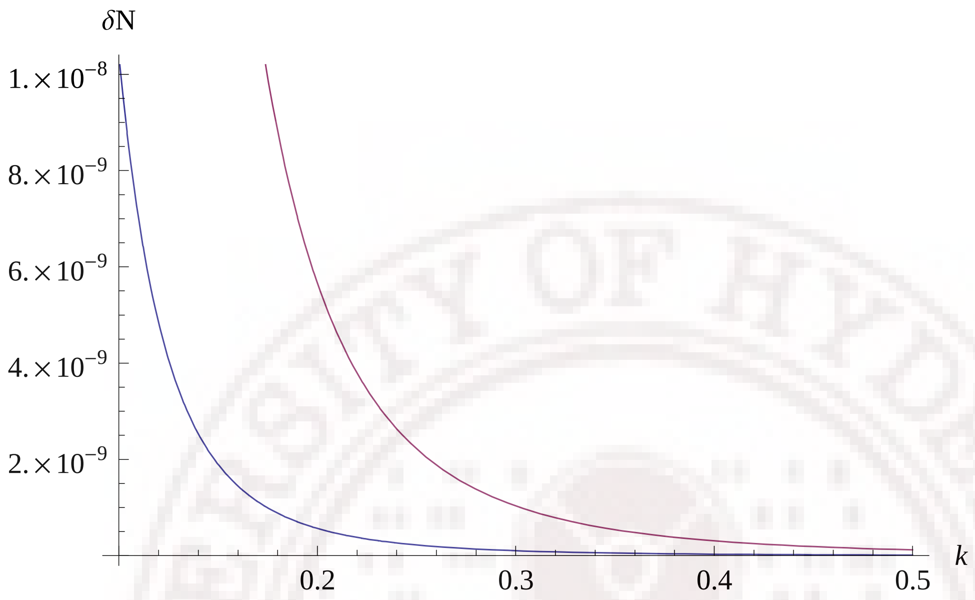
(d) $N_1(k)$ particles(solid line) and $N_2(k)$ and antiparticles(dash line) Vs k and $\overline{\varphi} = .28$.



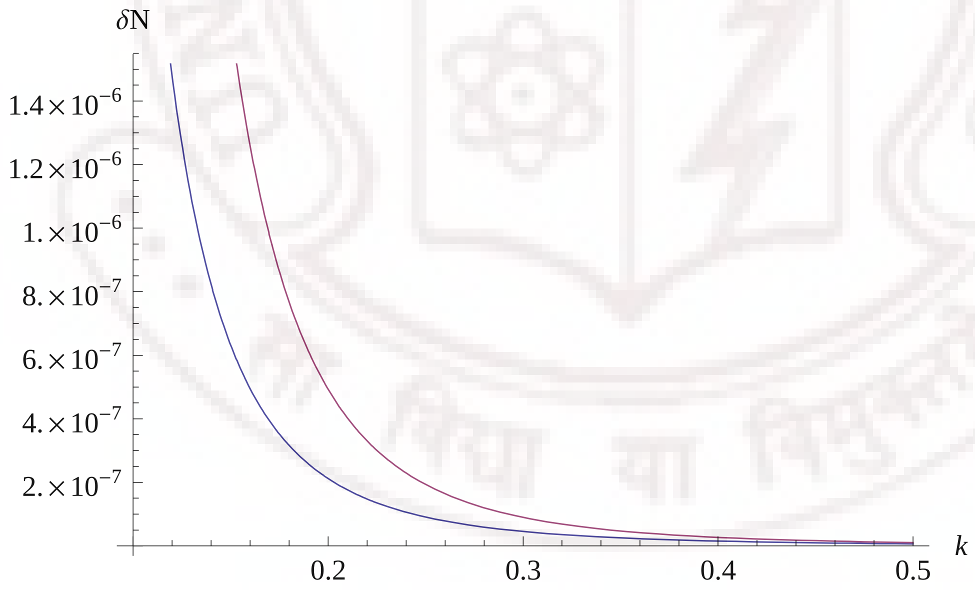
(e) ($\delta N = N_1(k) - N_2(k)$)vs k, for various values of q, $q = .6$ (blue),$q = .7$ (purple), $q = .8$ (green) here $\eta = 1$ $\lambda = 10^{-2}$ and $\phi_0 = 10^{-2}$.



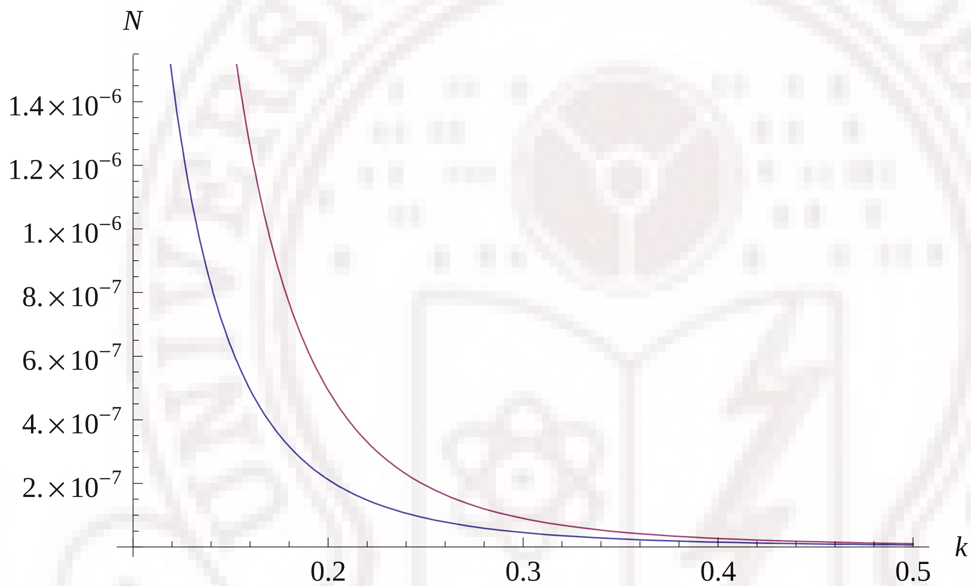
(f) ($\delta N = N_1(k) - N_2(k)$)vs k, for various values of ϕ_0 here $q = .6$, $\eta = 1$ $\lambda = 10^{-3}$ $\phi_0 = 10^{-3}$(purple) and $\phi_0 = 10^{-4}$ (blue).



(g) ($\delta N = N_1(k) - N_2(k)$)vs k, for various values of λ, $q = .6$, $\eta = 1$, $\overline{\varphi} = 10^{-2}$ and $\lambda = 10^{-6}$(purple), $\lambda = 10^{-7}$(blue).



(h) ($\delta N = N_1(k) - N_2(k)$)vs k, for various values of ϕ_0 here $q = .4$, $\eta = 1$ $\lambda = 10^{-3}$ $\phi_0 = 10^{-3}$ (purple) and $\phi_0 = 10^{-4}$ (blue).



(i) ($\delta N = N_1(k) - N_2(k)$)vs k, for various values of λ, $q = .4$, $\eta = 1$, $\overline{\varphi} = 10^{-2}$ and $\lambda = 10^{-6}$(purple), $\lambda = 10^{-7}$(blue).

CHAPTER 6

Non-Equilibrium Pion Production from Disoriented Chiral Condensates.

We have applied our formalism to a particular non equilibrium process in the early universe, the baryon asymmetry of universe. The question arises, are there any non equilibrium processes, with testable signals, available in laboratory conditions to which this formalism can be applied? In particular, the goals of the present and the next generation collider (RHIC and LHC) experiments are to create the conditions to understand the early evolution of our universe in the lab [69, 70, 71, 72]. This may seem a difficult goal, but, it is important to understand the physics for future observations of the universe. Very high energy hadronic and nuclear collisions are one of the experiments whose aim is to recreate the conditions of early universe in the lab. In such experiments, there has already been indication that highly excited state of quarks and gluons are known as quark gluon plasma (QGP) has been created [73]. The conditions of these experiments involve heating of matter upto $10^{12^0}K$, which amounts to 175 MeV per particle. Lead and gold nuclei are accelerated to ultra-relativistic speeds, pass through each other and create a hot fireball in the collision, this fire ball expands rapidly creating a non equilibrium situation and particles (mostly pions) emerge carring information of their hot and dense formation region. Thus, this evolution of QGP is a breeding ground for many non equilibrium phenomena. One such phenomenon is the interesting possibility of formation of metastable collective state called disoriented chiral condensate (DCC)[74, 75, 76, 77, 78, 79].

QCD has an approximate $SU(2)_L \times SU(2)_R$ chiral symmetry. For this symmetry, there exists a continuum of nearly degenerate pseudo vacuum states. This symmetry is spontaneously broken at low energies and pions acquire masses. If the symmetry is broken exactly, pions would be massless Nambu-Goldstone bosons. A low energy effective sigma [80, 81] model is used to study the QCD chiral phase transition as it respects $SU(2)_L \times SU(2)_R$ symmetry. This contains a scalar field (Σ) that has the same chiral properties as the quark condensate. The vacuum is represented by chiral four-vector of fields (Σ, $\vec{\pi}$), where Σ is associated with the order parameter for the chiral phase transition and $\vec{\pi}$ is the vector representing the pion fields. In the physical vacuum, the order parameter points in the Σ direction. However under appropriate non equilibrium conditions in high energy collisions, there could exist a region of space, where, for some period of time a metastable state is formed where order parameter has a non trivial pionic component, this is called the DCC.

A plausible mechanism for DCC formation has been identified as the following: due to the large energy deposit in the collision zone, QGP in a hot chirally symmetric state is formed, because of the rapid expansion, the system is suddenly quenched to a low temperature phase, where chiral symmetry is broken. This far from equilibrium evolution results in the formation of metastable states such as the DCC and is characterised by an amplification of low momentum pion modes and an asymmetry in pion multiplicity distributions. This a classic example of a non equilibrium high energy physics situation and has connection with non-equilibrium processes in early time cosmology. In this chapter, we will apply the formalism used to study the BAU, to study the formation and decay of the DCC.

6.1 The Model

For the most general treatment of DCC formation [54], the field theoretic $O(4)$ symmetric σ model is used with the following action

$$S = \int d^4x[\frac{1}{2}\partial^\mu \phi_i \partial_\mu \phi_i - \frac{\lambda}{4}(\phi_i^2 - v^2)^2 + H\Sigma], \tag{6.1}$$

where, the chiral field is $\phi = \Sigma + i\overrightarrow{\tau} \cdot \overrightarrow{\pi}$, $\overrightarrow{\pi}$ is the pion vector field and Σ is a scalar field (which is a quark condensate), $\overrightarrow{\tau}$ are the isospin Pauli matrices and $H \sim m_q$ gives the small explicit chiral symmetry breaking term that arises due to quark mass. The pion masses in this model are given by $m_\pi^2 = \frac{H}{f_\pi} = \lambda(f_\pi^2 - v^2)$, where f_π is the pion structure function. The sigma mass is given by $m_\Sigma = 2\lambda f_\pi^2$. However, in the strong coupling limit (λ large) there is a exists a sphere of vacua $\overrightarrow{\pi}^2 + \Sigma^2 = f_\pi^2$. Thus, in this limit, the normal vacuum structure is given by $<\pi>= 0$ and $<\Sigma>= f_\pi$. However, there can exist non-equilibrium situations, such as a quench, such that the system goes through metastable disordered vacua given by $<\pi>= f_\pi \overrightarrow{n} sin(\theta)$ and $<\Sigma>= f_\pi cos(\theta)$, here $\overrightarrow{n}$ is unit vector and θ measures the degree of disorientation of the condensate in isospin space. When the system reaches equilibrium, it falls to its natural ground state $<\pi>= 0$ and $<\Sigma>= f_\pi$, emitting pions. This situation is analogous to the formation of ferromagnetic domains in a Heisenberg ferromagnet.

In our model, we consider the case where H is small so that the σ model has full $SU(2)_L \times SU(2)_R$ symmetry. Then the action becomes upto a constant is

$$S = \int d^4x[\frac{1}{2}\partial^\mu \phi_i \partial_\mu \phi_i - \frac{1}{2}m^2\phi_i^2 - \frac{\lambda}{4}\phi_i^4], \tag{6.2}$$

here we identify $m^2 = \lambda v^2$, thus this action reduces to eq(6.1) in the limit $H = 0$. The minimum of the potential is given by

$$< \phi >^2 = < \Sigma^2 + \vec{\pi}^2 > = < v^2 > . \tag{6.3}$$

At high temperatures, the symmetry is restored and the potential is shown in fig(6.1), and at low temperature the symmetry is broken and shown in fig(6.2).

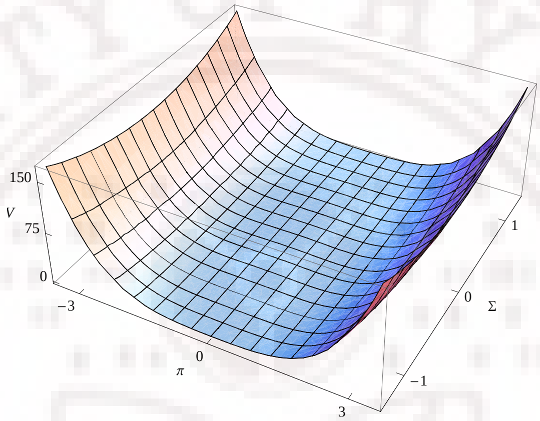


Figure 6.1: The potential with unbroken symmetry.

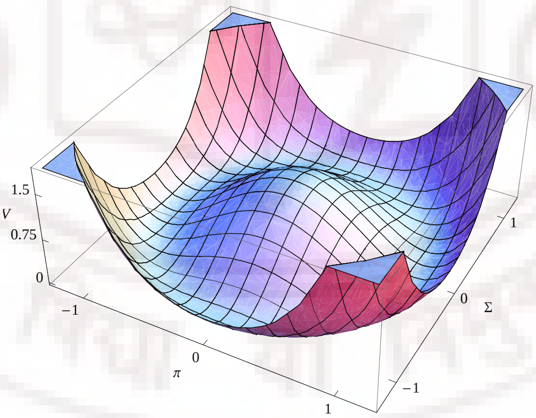


Figure 6.2: The potential with broken symmetry.

We mimic the expansion of the plasma, by using an expanding metric

$$ds^2 = dt^2 - a(t)^2 dx^2. \tag{6.4}$$

with an expansion rate $a(t)$. We notice that this is similar to the FRW metric used in cosmology.

The action of the $O(4)$ sigma model in this expanding metric becomes:

$$S = \int d^3x dt a(t)^3 (\frac{1}{2}\dot{\phi}_i^2 - \frac{1}{2a^2}(\nabla\phi_i)^2 - \frac{1}{2}m^2\phi_i^2 - \frac{\lambda}{4}(\phi_i^2)^2). \tag{6.5}$$

The background field analysis is employed to study the quantum effects [28, 29, 30]. Assuming ϕ_i has a background classical component ϕ_{icl} and the quantum fluctuations $\hat{\phi}_i$ and following the procedure in chapter 5, restricting the analysis to quadratic fluctuations only, the action reduces to

$$S_2 = \int d^3\underline{x} dt \frac{a^3}{2}(\dot{\hat{\phi}}_i^2 - \frac{1}{a^2}(\nabla\hat{\phi}_i)^2 - m^2\hat{\phi}_i^{\,2} - \hat{\phi}_i \frac{\delta^2 V}{\delta\hat{\phi}_i\delta\hat{\phi}_j}|_\phi \hat{\phi}_j). \tag{6.6}$$

By carrying out a Legendre transformation, the Hamiltonian density is

$$\mathcal{H} = \frac{1}{2a^3}\hat{p}_i^{\,2} + \frac{a}{2}(\nabla\hat{\phi}_i)^2 + \frac{a^3 m^2}{2}\hat{\phi}_i^{\,2} + \frac{a^3}{2}(\hat{\phi}_i \frac{\partial^2 V}{\partial\phi_i\partial\phi_j}|_\phi \hat{\phi}_j). \tag{6.7}$$

The classical background fields are the vacuum configuration given by

$$< \phi >= \begin{pmatrix} v_1 \\ v_2 \\ v_3 = v \\ \sigma \end{pmatrix}. \tag{6.8}$$

We parametrize the background field by three angles

$$\phi_i = \begin{pmatrix} vCos(\rho)Sin(\theta)Sin(\alpha) \\ vCos(\rho)Sin(\theta)Cos(\alpha) \\ vSin(\rho)Sin(\theta) \\ vCos(\theta) \end{pmatrix}. \tag{6.9}$$

We consider a parametrization of the background field which is simplified to two angles, θ and ρ. We impose charge symmetry by setting $\alpha = \frac{\pi}{4}$, then, $v\pm = \frac{v}{\sqrt{2}}Cos(\rho)Sin(\theta)$, $v_3 = vSin(\rho)Sin(\theta)$, and $\sigma = vCos(\theta)$. Other interesting scenarios with charge asymmetry would involve arbitrary α, which we will not consider here.

The Hamiltonian is quantised using mode expansion of the fields

$$\begin{aligned}
\hat{\pi}_0(x,t) &= \int\sqrt{\frac{1}{2\omega_\pi}}\frac{d^3k}{(2\pi)^3}(a_k e^{ik\cdot x} + a_k^\dagger e^{-ik\cdot x}), \\
\hat{\pi}_-(x,t) &= \int\sqrt{\frac{1}{2\omega_\pi}}\frac{d^3k}{(2\pi)^3}(b_k e^{ik\cdot x} + c_k^\dagger e^{-ik\cdot x}), \\
\hat{\pi}_+(x,t) &= \int\sqrt{\frac{1}{2\omega_\pi}}\frac{d^3k}{(2\pi)^3}(c_k e^{ik\cdot x} + b_k^\dagger e^{-ik\cdot x}), \\
\hat{\Sigma}(\underline{x},t) &= \int\sqrt{\frac{1}{2\omega_\Sigma}}\frac{d^3k}{(2\pi)^3}(d_k e^{ik\cdot x} + d_k^\dagger e^{-ik\cdot x}).
\end{aligned}$$

The mode Hamiltonian is

$$H = H_{neutral} + H_{charged} + H_{mixed}, \tag{6.10}$$

where

$$\begin{aligned}
H_{neutral} &= \int \frac{d^3\underline{k}}{(2\pi)^3}\frac{1}{2}(\frac{\omega_\pi}{a^3}(a_k^\dagger a_k + a_k a_k^\dagger) + \frac{\omega_\pi}{2a^3}(\frac{\Omega_\pi^2}{\omega_\pi^2} - 1)(a_k^\dagger a_k + a_k a_k^\dagger + a_{-k}a_k + a_{-k}^\dagger a_k^\dagger) \\
&+ \frac{\omega_\Sigma}{a^3}(d_k^\dagger d_k + d_k d_k^\dagger) + \frac{\omega_\Sigma}{2a^3}(\frac{\Omega_\Sigma^2}{\omega_\Sigma^2} - 1)(d_k^\dagger d_k + d_k d_k^\dagger + d_{-k}d_k + d_{-k}^\dagger d_k^\dagger)),
\end{aligned} \tag{6.11}$$

$$\begin{aligned}
H_{charged} &= \int \frac{d^3\underline{k}}{(2\pi)^3} \\
&+ \frac{\omega_\pi}{a^3}(b_k^\dagger b_k + c_k c_k^\dagger) + \frac{\omega_\pi}{2a^3}(\frac{\Omega_{\pi\pm}^2}{\omega_\pi^2} - 1)(b_k^\dagger b_k + c_k c_k^\dagger + b_{-k}c_k + c_{-k}^\dagger b_k^\dagger) \\
&+ \frac{\lambda a^3 v^2 cos^2(\rho) sin^2(\theta)}{4\omega_\pi}(b_k b_{-k} + b_k c_k^\dagger + c_k^\dagger b_k + c_k c_{-k} + c_k^\dagger c_{-k}^\dagger + c_k b_k^\dagger + b_k^\dagger c_k + b_k^\dagger b_{-k}^\dagger)),
\end{aligned} \tag{6.12}$$

and

$$
\begin{aligned}
H_{mixed} &= \int \frac{d^3\underline{k}}{(2\pi)^3} \\
& \Big(\frac{\lambda a^3 v^2 cos(\rho) sin(\rho) sin^2(\theta)}{2\omega_\pi}(b_k a_{-k} + b_k a_k^\dagger + c_k^\dagger a_k + c_k a_{-k} + c_k^\dagger a_{-k}^\dagger + c_k a_k^\dagger + b_k^\dagger a_k + b_k^\dagger a_{-k}^\dagger) \\
&+ \frac{\lambda a^3 v^2 sin(\rho) sin(\theta) cos(\theta)}{\sqrt{\omega_\pi \omega_\Sigma}} d_k a_{-k} + d_k a_k^\dagger + d_k^\dagger a_k + d_k^\dagger a_{-k}^\dagger \\
&+ \frac{\lambda a^3 v^2 cos(\rho) sin(\theta) cos(\theta)}{\sqrt{\omega_\pi \omega_\Sigma}}(b_k d_{-k} + b_k d_k^\dagger + c_k^\dagger d_k + c_k d_{-k} + c_k^\dagger d_{-k}^\dagger + c_k d_k^\dagger + b_k^\dagger d_k + b_k^\dagger d_{-k}^\dagger)\Big).
\end{aligned}
\tag{6.13}
$$

Here

$$
\frac{\omega_\pi^2(k)}{a^6} = \frac{\omega_{\pi_0}^2(k)}{a^6} = \frac{\omega_{\pi\pm}^2(k)}{a^6} = (m_\pi^2 + \frac{\underline{k}^2}{a^2}),\ \frac{\omega_\Sigma^2(k)}{a^6} = (m_\Sigma^2 + \frac{\underline{k}^2}{a^2}), \tag{6.14}
$$

and

$$
\begin{aligned}
\frac{\Omega_{\pi_0}^2 - \omega_{\pi_0}^2}{a^6} &= \lambda[v^2 + 2v_3^2], \\
\frac{\Omega_{\pi\pm}^2 - \omega_{\pi\pm}^2}{a^6} &= \lambda[v^2 + 2v_+ v_-], \\
\frac{\Omega_\Sigma^2 - \omega_\Sigma^2}{a^6} &= \lambda[v^2 + 2\sigma^2],
\end{aligned}
\tag{6.15}
$$

with

$$
2v_+ v_- + v_3^2 + \sigma^2 = v^2. \tag{6.16}
$$

The dynamical evolution of a system governed by this Hamiltonian is considered from a non equilibrium state of restored symmetry $<\phi>=0$ at high temperatures to the equilibrium state of broken symmetry $<\phi>=v$, due to the subsequent expansion and cooling of the QGP. The nonequilibrium situation arises when the expansion $a(t)$ is very rapid, the system goes from a state of restored to broken symmetry very fast compared to the relaxation time of the field which gives rise to a sudden quench i.e. $<\phi>$ suddenly jumps from $<\phi>=0$ to $<\phi>=v$. Another non-equilibrium situation will arise, when the system goes through a metastable disordered vacuum given by $<\pi>= v\vec{n} sin(\theta)$ and $<\Sigma>= vcos(\theta)$, then relaxes by quantum fluctuations to an equilibrium configuration. Each of these situations gives rise to DCC domains which decay into pions. The effect of the disorientation shows up in the pion distributions, which we shall calculate below. The time evolution can be modeled through a time dependence of the parameter $<\phi>(\tau)$, which depends on the relaxation rate τ and $a(t)$ which depends on the rate of the expansion of the plasma. The following scenarios are possible:

- (a) $\theta = 0$: This gives a sudden quench from a state of restored symmetry $<\phi>= 0$ to the ground $<\phi>= v$.
- (b) $\rho = \frac{\pi}{2}$: In this case the system goes from $<\phi>= 0$, through a metastable state $<\phi>= \begin{pmatrix} 0 \\ 0 \\ vSin(\theta) \\ vCos(\theta) \end{pmatrix}$ and then relaxes to value $<\phi>= v$.
- (c) $\theta = \frac{\pi}{2}$:In this case the system goes from $<\phi>= 0$, through a metastable state $<\phi>= \begin{pmatrix} \frac{1}{\sqrt{2}}vCos(\rho) \\ \frac{1}{\sqrt{2}}vCos(\rho) \\ vSin(\rho) \\ 0 \end{pmatrix}$ and then relaxes to value $<\phi>= v$.

We will consider each of the above cases a, b, and c.

Case a. In this case, the Hamiltonian is :

$$\begin{aligned} H &= \int \frac{d^3\underline{k}}{(2\pi)^3}[\frac{\omega_\pi}{2a^3}(N) + \frac{\omega_\pi}{4a^3}(\frac{\Omega_\pi^2}{\omega_\pi^2} - 1)(D + D^\dagger) \\ &+ \frac{\omega_\Sigma}{2a^3}(N_1) + \frac{\omega_\Sigma}{4a^3}(\frac{\Omega_\Sigma^2}{\omega_\Sigma^2} - 1)(D_1 + D_1^\dagger], \end{aligned} \tag{6.17}$$

where the operators D and D_1 are defined as

$$D = a_k a_{-k} + b_k c_{-k} + c_k b_{-k}, \quad D_1 = d_k d_{-k}, \tag{6.18}$$

$$D^\dagger = a^\dagger_{-k} a^\dagger_k + c^\dagger_{-k} b^\dagger_k + b^\dagger_{-k} c^\dagger_k, \quad D_1^\dagger = d^\dagger_k d^\dagger_{-k}, \tag{6.19}$$

$$N = \frac{1}{2}(a^\dagger_k a_k + a_{-k} a^\dagger_{-k} + b^\dagger_k b_k + b_{-k} b^\dagger_{-k} + c^\dagger_k c_k + c_{-k} c^\dagger_{-k} + 3),$$

$$N_1 = \frac{1}{2}(d^\dagger_k d_k + d_{-k} d^\dagger_{-k} + 1). \tag{6.20}$$

The $D, D^\dagger$ and N satisfy an su(1,1) algebra and independently $D_1, D_1^\dagger$ and N_1 satisfy another su(1,1) algebra. The Hamiltonian factors into two parts, one involving pion fields and one involving the sigma field. By the method followed in previous chapter, this Hamiltonian is diagonalised by a squeezing transformation,

$$U(r,t) = e^{\int \frac{d^3k}{(2\pi)^3}([r_k D^\dagger - r_k^* D] + [r_{k1} D_1^\dagger - r_{k1}^* D_1])}, \tag{6.21}$$

where r is the squeezing parameter related to the physical variables $\Omega_\pi(k,t)$ and $\omega_\pi(k)$ through

$$Tanh(2r_k) = \frac{(\frac{\Omega_\pi(k,t)}{\omega_\pi})^2 - 1}{(\frac{\Omega_\pi(k,t)}{\omega_\pi})^2 + 1}. \tag{6.22}$$

Similarly we can define r_{k1} by replacing π by Σ.

The diagonalized Hamiltonian is

$$H = \int \frac{d^3\underline{k}}{(2\pi)^3} \frac{1}{2a^3} [\Omega_\pi((A^\dagger_k A_k + \frac{1}{2}) + (C^\dagger_k C_k + B^\dagger_k B_k + 1)) + \Omega_\Sigma(E^\dagger_k E_k + \frac{1}{2})]. \tag{6.23}$$

The operators, A and $A^\dagger$ are related to the a_k and a_k by a unitary transformation given by

$$A(k,t) = U(r,t,)a_k U^\dagger(r,t), \tag{6.24}$$

It can be shown, using su(1,1) algebras that this reduces to squeezing transformation given by $A(k,t) = \nu a^\dagger_k + \mu a_{-k}$, $B(k,t) = \nu b^\dagger_k + \mu c_{-k}$, $C(k,t) = \nu c^\dagger_k + \mu b_{-k}$ and $E(k,t) = \sigma d^\dagger_k + \rho d_{-k}$ and their complex conjugates. Here $\nu = cosh(r)$ and $\mu = sinh(r)$, $\sigma = cosh(r_1)$ and $\rho = sinh(r_1)$ are the Bogoliubov coefficients. In this case, we see that the sigma mode, $E^\dagger_k E_k$, is decoupled from the Hamiltonian. As we are calculating the multiplicity of pions, from now on we will only consider the pion modes (i.e. A_k, B_k and C_k modes).

The initial vacuum $|0, 0 >$ defined by $a_k|0, 0 >= 0$ (similarly for other modes) and the final vacuum $|0(t), 0(t) >$ defined by $A_k|0(t), 0(t) >= 0$ (similarly for other modes), are related as follows

$$|0(t), 0(t) >= e^{\int \frac{d^3\underline{k}}{(2\pi)^3}[r_k(k,t)(D^\dagger - D)]}|0, 0 > . \tag{6.25}$$

Following the method from the previous chapters and going over to the coordinate representation, we get the Hamiltonian

$$H = \int \frac{d^3\underline{k}}{(2\pi)^3} \sum_{i=A,B,C} \frac{1}{2}[(\frac{\Omega_\pi}{a^3})^2 \Pi_i^2(k,t) + P_{\pi_i}^2(k,t)]]. \tag{6.26}$$

The time evolution of a wave function $\psi(t)$ is

$$H(t)\psi(t) = i\frac{d}{dt}\psi(t). \tag{6.27}$$

Again following from the previous chapter, we get the evolution equation for the wave function ψ with a time dependent potential $V(t) = \lambda(< \phi^2 > (t) - v^2)$ as

$$-\psi'' + V(t)\psi = \omega^2(k)\psi. \tag{6.28}$$

The quench is modeled by the taking

$$< \phi > (t) = f_\pi[tanh((t - \tau)/a) - tanh(-\tau/a)]. \tag{6.29}$$

Then taking the limit $a \to 0$ gives the quench and the $a \to \infty$ is the adiabatic limit. Here τ represents the relaxation time for the field to evolve from the metastable vacuum to the normal vacuum and a is given in the units of τ. We plot $< \phi > (t)$ in figure (6.3) and see that in the quench limit $< \phi > (t)$ becomes a step function representing a sudden transition from $< \phi >= 0$ to $< \phi >= v$.

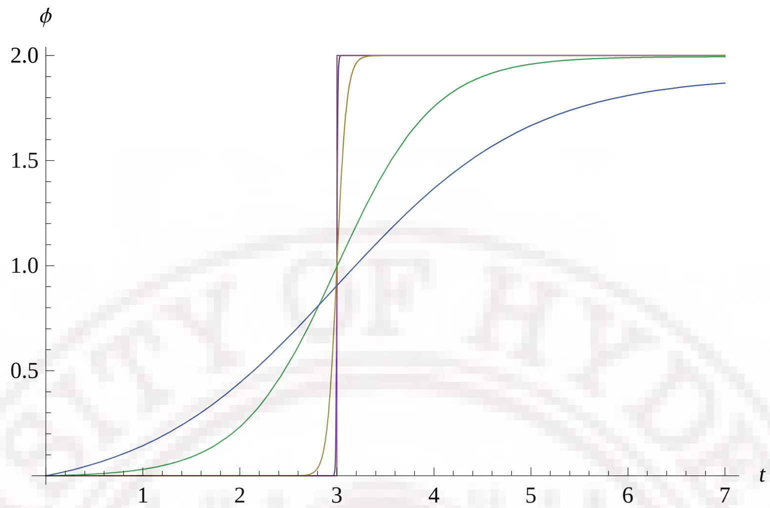


Figure 6.3: Shows the variation of ϕ with t, for the values $a = 10^{-4}(violet), a = .1(lightgreen), a = 1(green), a = 2(blue)$

This problem reduces to a potential barrier problem similar to the one given in chapter 3. The production of pions can be calculated by

$$< n_k >= sinh^2(r_k) = \frac{|1 - T|}{|T|}, \tag{6.30}$$

where T is the transmission coefficient. We calculate $< n_k >$ and plot it as a function of k. It can be seen from the figure (6.4) that the long wavelength modes are clearly amplified when the squeezing is very pronounced which is the squeezing limit. In the adiabatic limit there is a little squeezing present. This clearly gives the connection between the quenching and squeezing. A quench implies large amount of squeezing and the adiabaticity implies low squeezing. Thus the quenching process is responsible for squeezing and for the amplification of the low momentum modes that characterize a DCC.

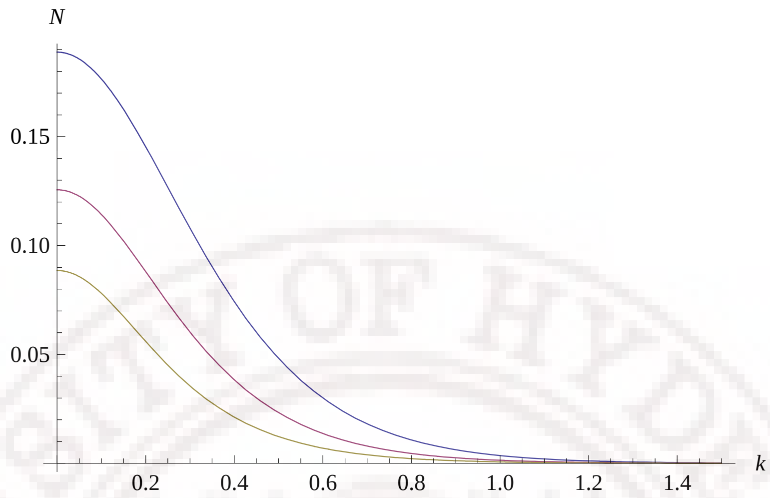


Figure 6.4: Shows the variation of N with k, for the values a=.2 (blue), a = .15(purple) a = .1 (light green)

The low momentum modes are amplified and high momentum modes are suppressed. Hence, we calculate the pions distributions in the limit $k \rightarrow 0$. In this limit, the pion states near zero momentum factors into a squeezed state for the neutral pions and a Caves-Schumaker state for the charged pions.

The k=0 wave function is

$$|\psi >= e^{r_0(D^\dagger - D)}|\psi_0(0) >= e^{r_0(a_0^{\dagger^2} + c_0^\dagger b_0^\dagger + b_0^\dagger c_0^\dagger - a_0^2 + b_0 c_0 + c_0 b_0)}|\psi_0(0) > . \tag{6.31}$$

The pion multiplicity distribution is given by

$$P_{n_0,n_+,n_-} = | < n_0, n_+, n_-|\psi > |^2 =< n_0|e^{r_0(D^\dagger - D)}|\psi_0(0) >= e^{r_0(a_0^{\dagger^2} - a_0^2)}|0 > \\ < n_+, n_-|e^{2r_0(b_0^\dagger c_0^\dagger - b_0 c_0)}|0 > |^2, \tag{6.32}$$

defining $S(r_0)$ as the two mode squeezing operator

$$\begin{aligned} S(r_0) &= < n_0|e^{r_0(a_0^{\dagger^2} - a_0^2)}|0 > \\ &= S_{n_0,0}. \end{aligned} \tag{6.33}$$

$S^{tm}(r_0)$ is then the two mode squeezing operator

$$< n_+, n_-|e^{(r_0(b_0^\dagger c_0^\dagger - b_0 c_0))}|0 >$$

$$= S^{tm}_{n_+,n_-,0}. \tag{6.34}$$

The neutral and charged pion distribution is:

$$P_{n_0,n_c} =< S_{n_0,0} >^2 < S^{tm}_{n_+,n_-,0,0} >^2, \tag{6.35}$$

which is just the product of squeezed distributions for charged and neutral pions and only even number of pions emerge. Writing $n_+ = n_- = n_c$, we get the distribution of charged particles to be

The pion multiplicity distribution is given by

$$P_{n_c} = \sum_{n_0} P_{n_0,n_c} = \frac{(tanh(r_0))^{2n_c}}{(cosh(r_0))^2}, \tag{6.36}$$

and

$$P_{n_0} = \sum_{n_c} P_{n_0,n_c} = \frac{n_0!(tanh(r_0))^{n_0}}{((\frac{n_0}{2})!)^2 cosh(r_0) 2^{n_0}}. \tag{6.37}$$

The generalized squeezed eigenstate leads to products of two types of squeezed states of pions at zero momentum, the neutral pions being in a one mode squeezed state and the charged pions being in an SU(1,1) coherent or two-mode squeezed state. Thus the neutral and charged pion distributions are significantly different as the two types of states have different properties. We now illustrate the effect of squeezing in these two distributions. Figure (6.6) show the difference in the charged and neutral pion distributions as we vary from the squeezing parameter from a low value to a high value.

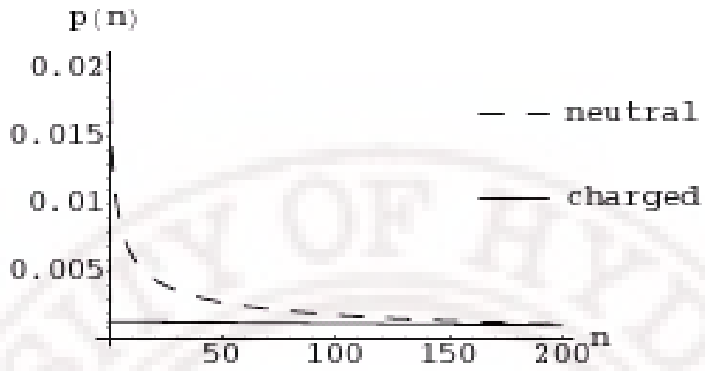


Figure 6.5: Shows the variation of P_{n0} (dashed line) and P_{nc} (solid line) with n for the quenched limit ($r_0 = 4$)

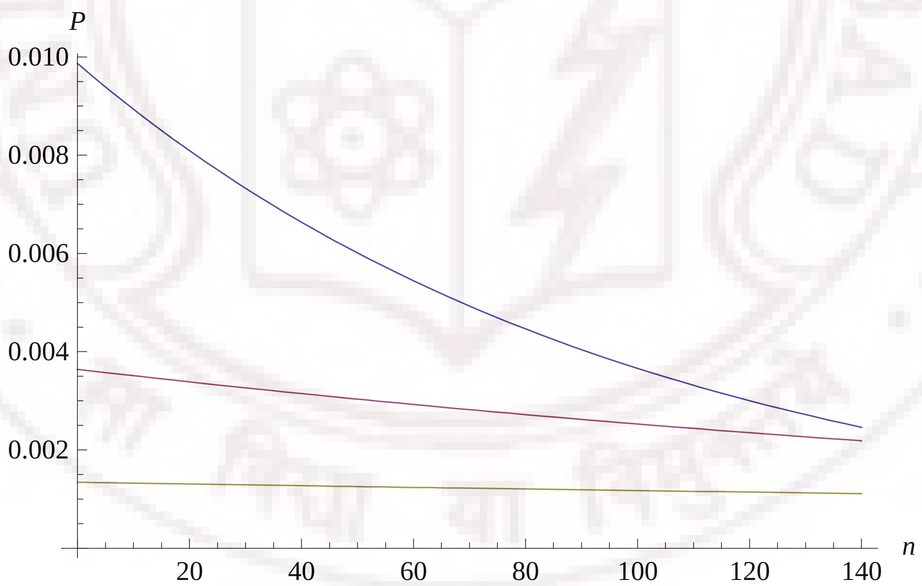


Figure 6.6: Shows the variation of P_{n0} with n, for the values r=3 (light green), $r = 3.5$ (purple),$r = 4$(blue)

Case b: In this case the Hamiltonian is

$$\begin{aligned}
H &= \int \frac{d^3\underline{k}}{(2\pi)^3}\frac{1}{2}(\frac{\omega_\pi}{a^3}(a_k^\dagger a_k + a_k a_k^\dagger) + \frac{\omega_\pi}{2a^3}(\frac{\Omega_\pi^2}{\omega_\pi^2} - 1)(a_k^\dagger a_k + a_k a_k^\dagger + a_{-k}a_k + a_{-k}^\dagger a_k^\dagger) \\
&+ \frac{\omega_\Sigma}{a^3}(d_k^\dagger d_k + d_k d_k^\dagger) + \frac{\omega_\Sigma}{2a^3}(\frac{\Omega_\Sigma^2}{\omega_\Sigma^2} - 1)(d_k^\dagger d_k + d_k d_k^\dagger + d_{-k}d_k + d_{-k}^\dagger d_k^\dagger)) \\
&+ \frac{\omega_\pi}{a^3}(b_k^\dagger b_k + c_k c_k^\dagger) + \frac{\omega_\pi}{2a^3}(\frac{\Omega_{\pi\pm}^2}{\omega_\pi^2} - 1)(b_k^\dagger b_k + c_k c_k^\dagger + b_{-k}c_k + c_{-k}^\dagger b_k^\dagger) \\
&+ \frac{\lambda a^3 v^2 sin(\theta) cos(\theta)}{\sqrt{\omega_\pi\omega_\Sigma}}[d_k a_{-k} + d_k a_k^\dagger + d_k^\dagger a_k + d_k^\dagger a_{-k}^\dagger]. \qquad (6.38)
\end{aligned}$$

The charged sector is decoupled from the neutral sector and the diagonalization procedure is same for $H_{charged}$ as in the case (a). Again we diagonalize the neutral sector Hamiltonian using the su(2) and the su(1,1) symmetries. The su(2) unitary transform is

$$U(r,t) = e^{\int \frac{d^3k}{(2\pi)^3}[\theta(J_1 e^{2i\xi} + J_2 e^{2i\xi})]}, \qquad (6.39)$$

where the mixing is given by

$$\begin{pmatrix} A_{\theta(k)} \\ E_{\theta(k)} \end{pmatrix} = \begin{pmatrix} Cos(\theta) & Sin(\theta) \\ -Sin(\theta) & Cos(\theta) \end{pmatrix} \begin{pmatrix} a_k \\ d_k \end{pmatrix}, \qquad (6.40)$$

The resultant Hamiltonian reduces to

$$H'_{neutral} = \int \frac{d^3\underline{k}}{(2\pi)^3}[m_1(N_2) + n_1(D_2 + D_2^\dagger) + m_2(N_3) + n_2(D_3 + D_3^\dagger) \qquad (6.41)$$

where the operators D_2 and D_3 are defined by

$$D_2 = A_{\theta(k)}A_{\theta(-k)}, \;\; D_3 = E_{\theta(k)}E_{\theta(-k)}, \qquad (6.42)$$

$$D_2^\dagger = A_{\theta(k)}^\dagger A_{\theta(-k)}^\dagger, \;\; D_3^\dagger = E_{\theta(k)}^\dagger E_{\theta(-k)}^\dagger, \qquad (6.43)$$

$$N_2 = A_{\theta(k)}^\dagger A_{\theta(k)} + 1/2, \;\; N_3 = E_{\theta(k)}^\dagger E_{\theta(k)} + 1/2, \qquad (6.44)$$

and m_1, m_2, n_1 and n_2 are given by

$$2m_2 = (\frac{\Omega_\pi^2 + \omega_\pi^2}{\omega_\pi} + \frac{\Omega_\Sigma^2 + \omega_\Sigma^2}{\omega_\Sigma} + \frac{\lambda v^2}{2\sqrt{\omega_\Sigma\omega_\pi}}), \qquad (6.45)$$

$$2m_1 = (\frac{\Omega_\pi^2 + \omega_\pi^2}{\omega_\pi} + \frac{\Omega_\Sigma^2 + \omega_\Sigma^2}{\omega_\Sigma} - \frac{\lambda v^2}{2\sqrt{\omega_\Sigma \omega_\pi}}). \quad (6.46)$$

While,

$$2n_2 = \frac{\sqrt{\Omega_\pi \Omega_\Sigma}}{4a^3} \left(\sqrt{\frac{\Omega_\pi}{\Omega_\Sigma}} \left(\frac{\Omega_\pi}{\omega_\pi} - \frac{\omega_\pi}{\Omega_\pi} \right) \left(1 + \frac{1}{2}\sqrt{\frac{\omega_\pi}{\omega_\Sigma}} \right) + \sqrt{\frac{\Omega_\Sigma}{\Omega_\pi}} \left(\frac{\Omega_\Sigma}{\omega_\Sigma} - \frac{\omega_\Sigma}{\Omega_\Sigma} \right) \left(1 + \frac{1}{2}\sqrt{\frac{\omega_\Sigma}{\omega_\pi}} \right) \right) \quad (6.47)$$

and

$$2n_1 = \frac{\sqrt{\Omega_\pi \Omega_\Sigma}}{4a^3} \left(\sqrt{\frac{\Omega_\pi}{\Omega_\Sigma}} \left(\frac{\Omega_\pi}{\omega_\pi} - \frac{\omega_\pi}{\Omega_\pi} \right) \left(1 - \frac{1}{2}\sqrt{\frac{\omega_\pi}{\omega_\Sigma}} \right) + \sqrt{\frac{\Omega_\Sigma}{\Omega_\pi}} \left(\frac{\Omega_\Sigma}{\omega_\Sigma} - \frac{\omega_\Sigma}{\Omega_\Sigma} \right) \left(1 - \frac{1}{2}\sqrt{\frac{\omega_\Sigma}{\omega_\pi}} \right) \right) \quad (6.48)$$

This rotated Hamiltonian is diagonalized by the squeezing transformations given by

$$U(r,t) = e^{\int \frac{d^3k}{(2\pi)^3}[r_2 D_2^\dagger - r_2^* D_2] + [r_3 D_3^\dagger - r_3^* D_3]}, \quad (6.49)$$

where r_2 is defined by the relations $sinh^2(r_2) + cosh^2(r_2) = m_1$ and $sinh(r_2)cosh(r_2) = n_1$, similarly for r_3 is defined by $sinh^2(r_3) + cosh^2(r_3) = m_2$ and $sinh(r_3)cosh(r_3) = n_2$.

Thus the final diagonalised neutral sector Hamiltonian is

$$H_{mixed} = \int \frac{d^3\underline{k}}{(2\pi)^3} (\frac{\sqrt{\Omega_\pi \Omega_\Sigma}}{4a^3} \left((F_k^\dagger F_k + \frac{1}{2}) + (G_k^\dagger G_k + \frac{1}{2}) \right). \quad (6.50)$$

The total Hamiltonian with the charged sector is given by

$$\begin{aligned} H_{\rho/2} &= \int \frac{d^3\underline{k}}{(2\pi)^3} (\frac{1}{2a^3}[\Omega_{\pi\pm}(C_k^\dagger C_k + B_k^\dagger B_k + 1)) \\ &+ (\frac{\sqrt{\Omega_\pi \Omega_\Sigma}}{4a^3} \left((F_k^\dagger F_k + \frac{1}{2}) + (G_k^\dagger G_k + \frac{1}{2}) \right). \end{aligned} \quad (6.51)$$

Following the procedure to calculate the the pion multiplicity distribution for case a.

$$P_{n_0} = \frac{(n_0!)^2 (tanh(r_2 cos^2(\theta))(tanh(r_3 sin^2(\theta))))^{2n_0}}{((\frac{n_0}{2})!)^2 (cosh(r_2 cos^2(\theta)) cosh(r_3 sin^2(\theta)))2^{n_0}} \frac{(tanh(r_2 sin(2\theta) tanh(r_3 sin(2\theta))^{n_0}}{((cosh((r_2 - r_3) sin(2\theta)))} \quad (6.52)$$

The distribution for the charged pions is still that given in eq (6.36).

In the figure (6.7), P_{n_0} is plotted as a function of θ for different values of r_2 and r_3. We see that that there are oscillations in the probability distributions, and regions where the number of neutral pions vanished. This depends on the orientation of the intermediate

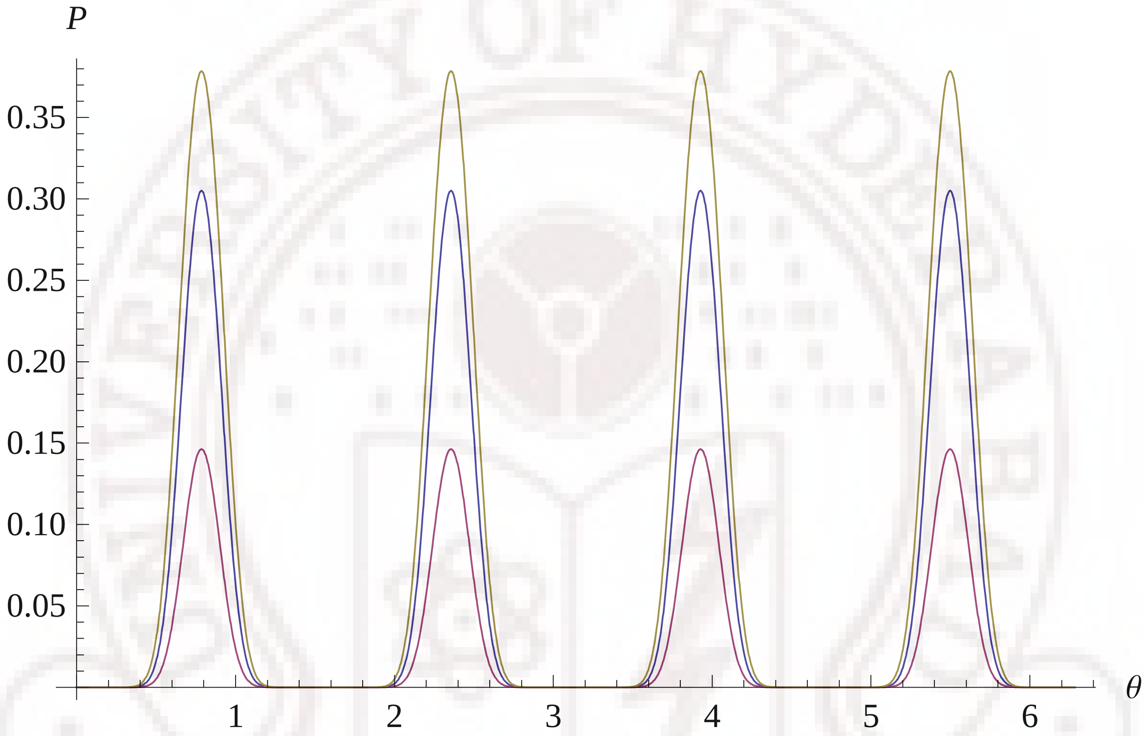


Figure 6.7: Shows the oscillations in the number of neutral pions as a function of the disorientation θ for the value of the squeezing parameters $r = 3.5$ (light green), $r = 3$ (blue) and $r = 2.5$ (purple)..

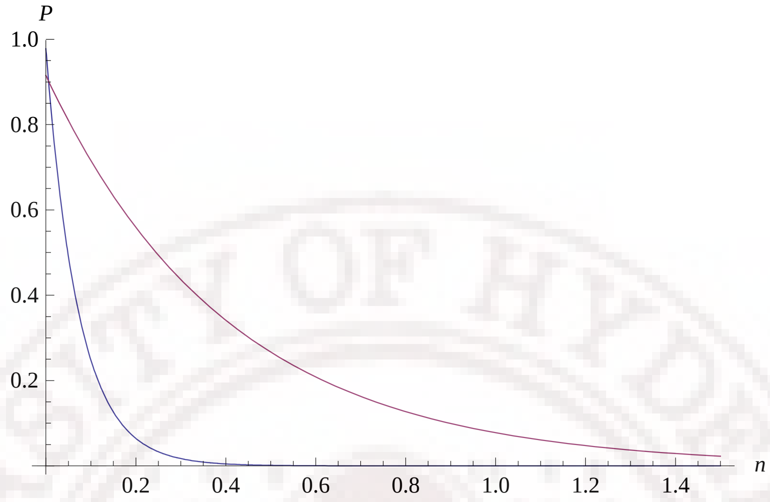


Figure 6.8: Shows the the distribution of number of neutral pions(purple) and charged pion (blue) as a function of n, for the same value of the squeezing parameters $r = .2$.

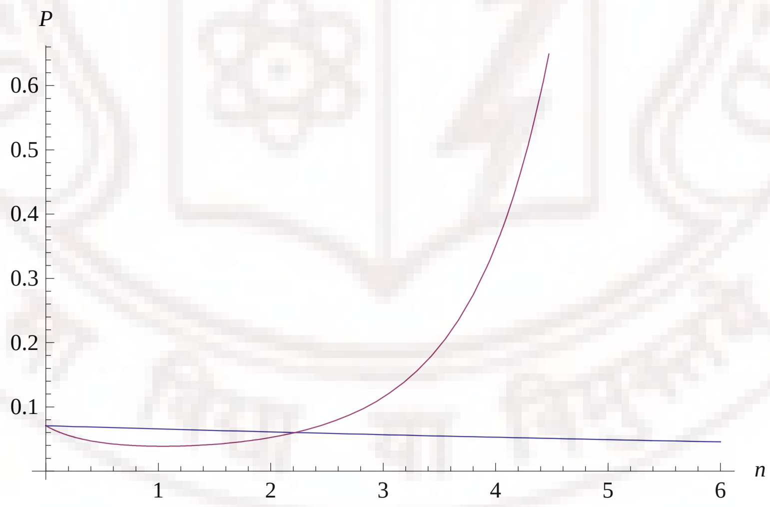


Figure 6.9: Shows the the distribution of number of neutral pions(purple) and charged pion (blue) as a function of n, for the same value of the squeezing parameters $r = 4$.

metastable state. If the orientation corresponds to the angle at which the distribution of neutral pions goes to 0, then there will be only charged pions produced.

Taking the value $\theta = \pi/4$, when there are maximum number of neutral pions , in the figures (6.8) and (6.9) we compare the multiplicity of neutral pions with the charged pion multiplicity distribution and see that for a small squeezing parameter (adiabatic limit) there is no substantial difference between the charged and neutral pions, but at large squeezing the neutral pions are enhanced compared to charged pions.

Case c: In this case the Hamiltonian is

$$\begin{aligned} H &= \int \frac{d^3\underline{k}}{(2\pi)^3}\frac{1}{2}(\frac{\omega_\pi}{a^3}(a_k^\dagger a_k + a_k a_k^\dagger) + \frac{\omega_\pi}{2a^3}(\frac{\Omega_\pi^2}{\omega_\pi^2} - 1)(a_k^\dagger a_k + a_k a_k^\dagger + a_{-k}a_k + a_{-k}^\dagger a_k^\dagger) \\ &+ \frac{\omega_\Sigma}{a^3}(d_k^\dagger d_k + d_k d_k^\dagger) + \frac{\omega_\Sigma}{2a^3}(\frac{\Omega_\Sigma^2}{\omega_\Sigma^2} - 1)(d_k^\dagger d_k + d_k d_k^\dagger + d_{-k}d_k + d_{-k}^\dagger d_k^\dagger)), \\ &+ \frac{\omega_\pi}{a^3}(b_k^\dagger b_k + c_k c_k^\dagger) + \frac{\omega_\pi}{2a^3}(\frac{\Omega_{\pi\pm}^2}{\omega_\pi^2} - 1)(b_k^\dagger b_k + c_k c_k^\dagger + b_{-k}c_k + c_{-k}^\dagger b_k^\dagger) \qquad (6.53) \\ &+ \frac{\lambda a^3 v^2 cos^2(\rho)}{4\omega_\pi}(b_k b_{-k} + b_k c_k^\dagger + c_k^\dagger b_k + c_k c_{-k} + c_k^\dagger c_{-k}^\dagger + c_k b_k^\dagger + b_k^\dagger c_k + b_k^\dagger b_{-k}^\dagger)), \\ &+ \frac{\lambda a^3 v^2 cos(\rho) sin(\rho)}{2\omega_\pi}(b_k a_{-k} + b_k a_k^\dagger + c_k^\dagger a_k + c_k a_{-k} + c_k^\dagger a_{-k}^\dagger + c_k a_k^\dagger + b_k^\dagger a_k + b_k^\dagger a_{-k}^\dagger)). \end{aligned} \qquad (6.54)$$

The sigma modes are decoupled form the neutral and the charged sector. Again using the su(2) and su(1,1) symmetries, the Hamiltonian is diagonalized. The su(2) unitary transformations is

$$U(r,t) = e^{\int \frac{d^3k}{(2\pi)^3}[\theta(J_a e^{2i\xi} + J_b e^{2i\xi})]} e^{\int \frac{d^3k}{(2\pi)^3}[\theta(J_{a1} e^{2i\xi} + J_{a2} e^{2i\xi})]} e^{\int \frac{d^3k}{(2\pi)^3}[\theta(J_{a3} e^{2i\xi} + J_{a3} e^{2i\xi})]}, \qquad (6.55)$$

The mixing is given by

$$\begin{pmatrix} A_{\rho(k)} \\ E_{\rho(k)} \end{pmatrix} = \begin{pmatrix} Cos(\rho) & Sin(\rho) \\ -Sin(\rho) & Cos(\rho) \end{pmatrix} \begin{pmatrix} a_k \\ b_k \end{pmatrix}, \qquad (6.56)$$

$$\begin{pmatrix} F_{\rho(k)} \\ G_{\rho(k)} \end{pmatrix} = \begin{pmatrix} Cos(\rho) & Sin(\rho) \\ -Sin(\rho) & Cos(\rho) \end{pmatrix} \begin{pmatrix} a_k \\ c_k \end{pmatrix}, \qquad (6.57)$$

and

$$B_{\rho(k)} = b_k + c_k, \quad C_{\rho(k)} = b_k - c_k. \tag{6.58}$$

Then the Hamiltonian reduces to

$$\begin{aligned} H_{neutral} &= \int \frac{d^3\underline{k}}{(2\pi)^3}[h_1(N_{\rho 1} + N_{\rho 2}) + h_2(D_{\rho 1} + D_{\rho 2} + D^\dagger_{\rho 1} + D^\dagger_{\rho 2}) \\ &+ h_3(N_{\rho 3} + N_{\rho 4}) + h_4(D_{\rho 3} + D_{\rho 4} + D^\dagger_{\rho 3} + D^\dagger_{\rho 4}) \\ &+ h_5(N_{\rho 5} + N_{\rho 6}) + h_6(D_{\rho 5} + D_{\rho 6} + D^\dagger_{\rho 5} + D^\dagger_{\rho 6}). \end{aligned} \tag{6.59}$$

The operators $D_{\rho i}$ are defined by

$$\begin{aligned} D_{\rho 1} &= A_{\rho(k)}A_{\rho(-k)}, \quad D_{\rho 2} = E_{\rho(k)}E_{\rho(-k)}, \quad D_{\rho 3} = F_{\rho(k)}F_{\rho(-k)}, \\ D_{\rho 4} &= G_{\rho(k)}G_{\rho(-k)}, D_{\rho 5} = B_{\rho(k)}B_{\rho(-k)}, \quad D_{\rho 6} = C_{\rho(k)}C_{\rho(-k)}, \quad (6.60) \\ D^\dagger_{\rho 1} &= A^\dagger_{\rho(k)}A^\dagger_{\rho(-k)}, \quad D^\dagger_{\rho 2} = E^\dagger_{\rho(k)}E^\dagger_{\rho(-k)}, \quad D^\dagger_{\rho 3} = F^\dagger_{\rho(k)}F^\dagger_{\rho(-k)}, \\ D^\dagger_{\rho 4} &= G^\dagger_{\rho(k)}G^\dagger_{\rho(-k)}, D^\dagger_{\rho 5} = B^\dagger_{\rho(k)}B^\dagger_{\rho(-k)}, \quad D^\dagger_{\rho 6} = C^\dagger_{\rho(k)}C^\dagger_{\rho(-k)}, \quad (6.61) \\ N_{\rho 1} &= A^\dagger_{\rho(k)}A_{\rho(-k)} + 1/2, \quad N_{\rho 2} = E^\dagger_{\rho(k)}E_{\rho(-k)} + 1/2. \\ N_{\rho 3} &= F^\dagger_{\rho(k)}F_{\rho(-k)} + 1/2, \quad N_{\rho 4} = G^\dagger_{\rho(k)}G_{\rho(-k)} + 1/2. \\ N_{\rho 5} &= B^\dagger_{\rho(k)}B_{\rho(-k)} + 1/2, \quad N_{\rho 6} = C^\dagger_{\rho(k)}C_{\rho(-k)} + 1/2. \quad (6.62) \end{aligned}$$

and

$$\begin{aligned} h_1 &= \frac{\omega_\pi}{2a^3} + \frac{\lambda a^3 v^2}{4\omega_\pi}, \quad h_2 = \frac{\omega_\pi}{2a^3} + \frac{3\lambda a^3 v^2}{4\omega_\pi}, \quad h_3 \frac{\lambda a^3 v^2}{4\omega_\pi}, \quad h_4 = \frac{3\lambda a^3 v^2}{4\omega_\pi}, \\ h_5 &= \frac{\omega_\pi}{2a^3} + \frac{\lambda a^3 v^2}{4\omega_\pi}(1 + cos^2(\rho)), \quad h_6 = \frac{\omega_\pi}{2a^3} + \frac{\lambda a^3 v^2}{4\omega_\pi}(1 + 3cos^2(\rho)). \end{aligned} \tag{6.63}$$

The squeezing transformations are given by

$$U(r,t) = e^{\int \frac{d^3k}{(2\pi)^3}[r_{\rho 1}D^\dagger_{\rho 1} - r^*_{\rho 1}D_{\rho 1}] + [r_{\rho 2}D^\dagger_{\rho 2} - r^*_{\rho 2}D_{\rho 2}] + [r_{\rho 3}D^\dagger_{\rho 3} - r^*_{\rho 3}D_{\rho 3}] + [r_{\rho 4}D^\dagger_{\rho 4} - r^*_{\rho 4}D_{\rho 4}] + [r_{\rho 5}D^\dagger_{\rho 5} - r^*_{\rho 5}D_{\rho 5}] + [r_{\rho 6}D^\dagger_{\rho 6} - r^*_{\rho 6}D_{\rho 6}]}, \tag{6.64}$$

here $r_{\rho i}$ are defined by $sinh^2(r_{\rho 1,2}) + Cosh^2(r_{\rho 1,2}) = h_1 \; sinh(r_{\rho 1,2})Cosh(r_{\rho 1,2}) = h_2$,

$sinh^2(r_{\rho 3,4}) + Cosh^2(r_{\rho 3,4}) = h_3$, $sinh(r_{\rho 3,4})Cosh(r_{\rho 3,4}) = h_4$,
$sinh^2(r_{\rho 5,6}) + Cosh^2(r_{\rho 5,6}) = h_5$ and $sinh(r_{\rho 5,6})Cosh^2(r_{\rho 5,6}) = h_6$.

The final diagonalised Hamiltonian is given by

$$H_{\rho/2} = \int \frac{d^3\underline{k}}{(2\pi)^3}(\frac{\Omega_{\pi\pm}}{a^3}(C_k^\dagger C_k + B_k^\dagger B_k + 1) + \frac{\sqrt{\Omega_\pi \Omega_\Sigma}}{4a^3}((I_k^\dagger I_k + \frac{1}{2}) + (H_k^\dagger H_k + \frac{1}{2})$$
$$+(L_k^\dagger L_k + \frac{1}{2}) + (M_k^\dagger M_k + \frac{1}{2})). \tag{6.65}$$

Similarly following the method for case a,the pion multiplicity distribution of the neutral pions is given by

$$P_{n_0} = \frac{(n_0!)^2((tanh(r_{\rho_1})(tanh(r_{\rho_3})))^{2n_0}}{((\frac{n_0}{2})!)^2(cosh(r_{\rho_1})cosh(r_{\rho_3}))2^{n_0}}. \tag{6.66}$$

The multiplicity distribution for the charged pionsP_{n_c} is given by

$$P_{n_c} = \frac{2cosh(r_{\rho_1}sin(2\rho)cosh(r_{\rho_3}sin(2\rho)(tanh(r_{\rho_1}sin(2\rho)tanh(r_{\rho_3}sin(2\rho))^{2n_c}}{((cosh(r_{\rho_1}sin(2\rho) + (cosh(r_{\rho_3}sin(2\rho)))}\frac{((tanh(r_{\rho 5})))^{n_c}}{((cosh(r_{\rho 5})} \tag{6.67}$$

From the figure (6.10) we see oscillations in the number of charged pions as the disorientation ρ changes. Unlike case b, here, there are regions depending on the orientation ρ of the intermediary metastable state,where the charged pions vanish. In these regions there will be only neutral pions observed.

Taking the value $\rho = \pi/4$, when there are maximum number of charged pions, we plot the charged and neutral pion distributions in figures (6.11) and (6.12). Again for a small squeezing parameter(adiabatic limit) there is no substantial difference between charge and neutral pions, but for large squeezing there is a difference.

Our detailed analysis shows that the multiplicity distributions can be a good mirror for the disorientation of the DCC and can be supplemented by other measures such as pion correlations, to get a good picture of the formation process of the DCC, when data becomes available. It also shows that the dramatic signal of only charged or neutral pions is not the only signal for DCC formation, but more detailed analysis of the distributions of pions has to be done.

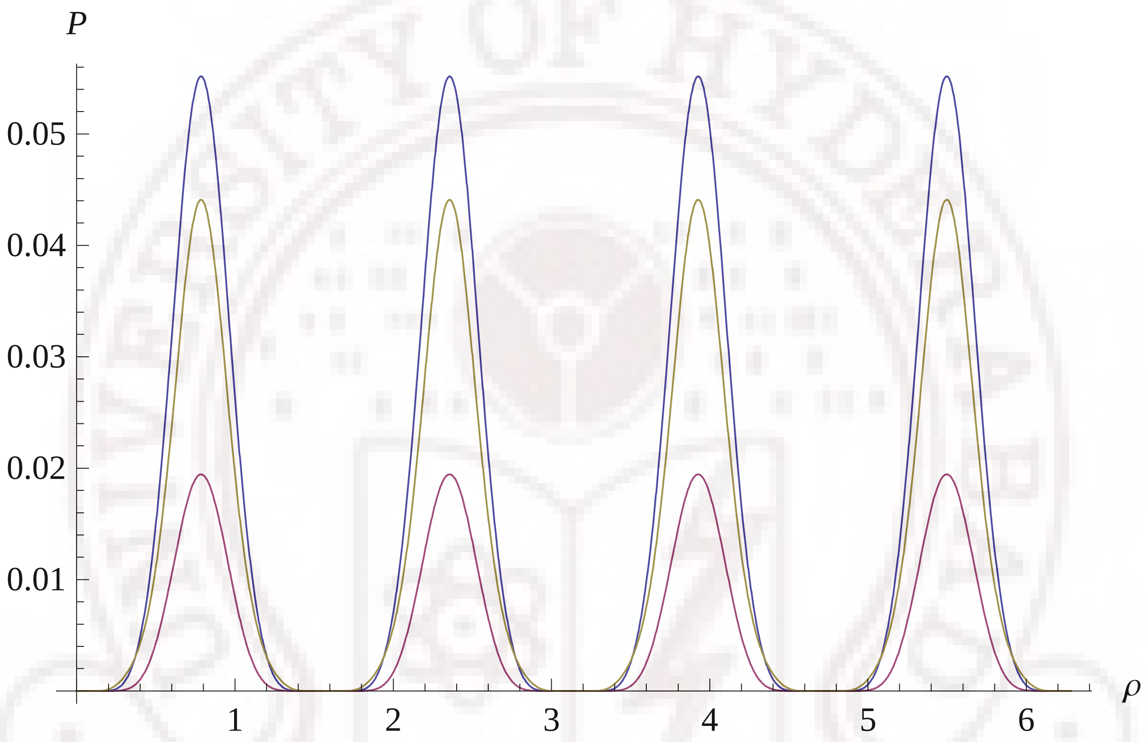


Figure 6.10: Shows the oscillations in the number of charged pions as a function of the disorientation ρ for the value of the squeezing parameters $r = 4$ (blue), $r = 3.5$ (light green) and $r = 4$ (purple).

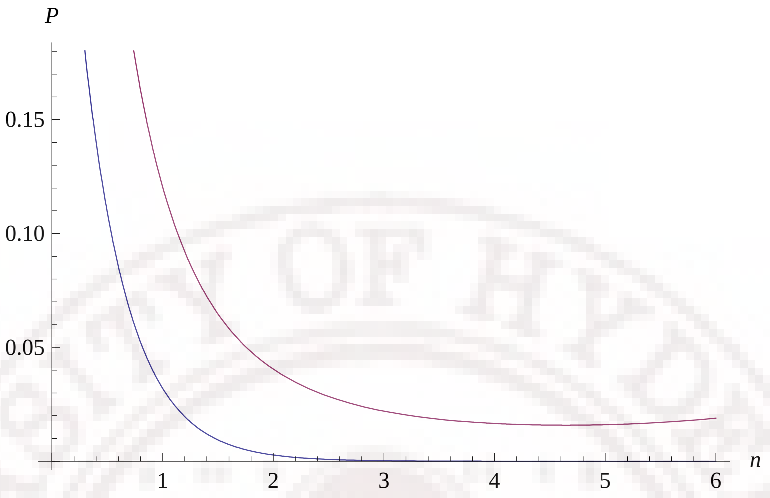


Figure 6.11: Shows the the distribution of number of neutral pions(purple) and charged pion (blue) as a function of n, for the same value of the squeezing parameters $r = .5$.

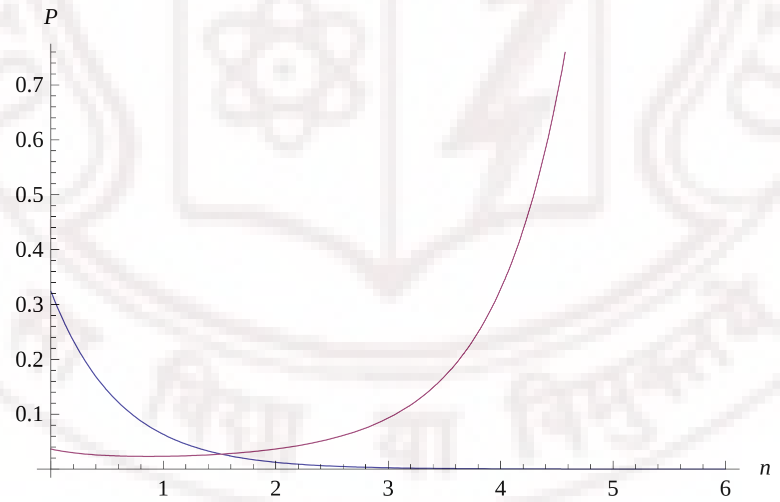


Figure 6.12: Shows the the distribution of number of neutral pions(purple) and charged pion (blue) as a function of n, for the same value of the squeezing parameters $r = 4$.

To summarize, we have used a quantum field theoretical model and the squeezed states formalism to calculate the multiplicities of pions which would be observed if the QGP undergoes a rapid quench. We have studied specific signatures, such as anomalous charged-to-neutral ratio of produced pions, which result in the process. In particular, the mixing between various erstwhile pion states in the collision region which can produce differential multiplicities of the charged and neutral final state pions has been found.

CHAPTER 7

Conclusion

7.1 Conclusion

A definitive solution to the mystery of baryon asymmetry awaits future experiments, both particle physics ones and cosmological ones, that will shed light not only on electroweak symmetry breaking, but also the existence of supersymmetry. As these facilities yield information on the conditions of the history of the universe nearer to the energy scales at which baryogenesis took place, we might find an unambiguous answer to the question as to why the universe is made of more matter than antimatter. Although we have seen that GUT baryogenesis is inadequate to account for BAU, the basic philosophy of these pioneering models, that the processes responsible for baryon number generation is the decay of super heavy bosons (complex scalar fields) carrying nonzero baryon number, survives in many subsequent models. Since it is generally believed that matter is produced in the reheating process after inflation, it is natural to consider inflationary reheating models that will produce such bosons, which would subsequently decay into baryons, in this epoch. Moreover, these must carry baryon number and the CP violation required by Sakharov conditions must be inbuilt into the inflaton potential responsible for this reheating process in the out of equilibrium expansion of the early universe. It is with this philosophy that we have considered baryon asymmetry in two scenarios of the post inflationary reheating process.

In this thesis, we have developed a general formalism based on squeezed rotated states to calculate the BAU in the wake of inflation, by considering the production of particles and anti-particles in various reheating scenarios. We have also used the formalism developed to study non-equilibrium processes during the chiral symmetry breaking phase transition of the QGP, which may produce a disoriented chiral condensate that decays to particles (pions).

In the first model, we have studied the generation of BAU in a model originally used for gravitational baryogenesis [25]. Sakharov's requirements are fulfilled through an effective two particle interaction with the graviton that violates baryon number conservation as well as CP invariance. This model was reincarnated after the inflationary scenario was proposed and the graviton was replaced by the inflaton [26]. In earlier work, [26] Green's function techniques were used to solve the equations of motion for the mode functions of the quantum fields. The estimation of baryon asymmetry was shown to vanish to first order in the perturbation parameter and appeared at the second order. The creation and annihilation operators of complex scalars with non-zero baryon numbers at the end of reheating are related to those at an earlier time by using the results of [25] and are therefore perturbative, and does not yield a sufficient BAU.

We have re-examined this model from a different point of view, in the hope that an alternative look at the problem, employing new techniques, will yield better information. On second quantizing this model, we get a Hamiltonian that resembles quantum optical Hamiltonians generated by beam splitting and parametric amplification, which suggest the use of su(2) and su(1,1) coherent states. We have used these tools of quantum optics, through the squeezed rotated states formalism, to diagonalize this Hamiltonian exactly. In particular, we have found that at any time, this Hamiltonian has $su(2)$ and $su(1,1)$ symmetries, which we have efficiently exploited. The diagonalization procedure further allowed us to relate the rotated and squeezed operators to the traditional Bogoliubov transformations between the mode functions at an earlier time to those at later times and provides us with a fully non-perturbative calculation for the asymmetry to all orders in the coupling parameter λ. This methodology has allowed us to derive the exact evolution equations for the particle and anti-particle modes leading to an asymmetry between the heavy bosons of opposite baryon number, which, when they decay to fermions, produce the required BAU. The approximations that we have utilized at the end to find explicit variation of the number of particles/anti-particles created as a function of the co-moving wave number are only illustrative. It is entirely possible to carry out the calculations without these approximations. Of course, we will then not have the luxury of the vast literature on the solutions of the Mathieu equations and instability regions allowing for parametric resonant solutions will have to be looked for. On the plus side, the full equations we derived allows the possibility of including the contributions to the baryon asymmetry coming from the expansion of the Universe. The centerpiece of our effort are the equations we derived, which are exact and show the role played by the expansion factor and the inflaton field on the evolution of the particle and antiparticle modes. These can be solved for any type of inflationary scenario. .

In the second model, we have studied the non-equilibrium quantum effects on Affleck Dine (AD) baryogenesis [11] in the post inflationary scenario. The reason for this is that supersymmetry has a number of flat directions, which are lifted when supersymmetry is broken. The Affleck Dine mechanism uses the flat directions in supersymmetric models to generate a Lagrangian which naturally satisfies Sakharov's conditions. Therefore, this can provide a very promising mechanism for baryon asymmetry generation. In the initial model, Affleck and Dine have used classical arguments to generate BAU and found that this gives a very large value. Means to reduce the rather over efficient generation of baryon asymmetry in Affleck Dine mechanism requires a through study. In this thesis, we have done a systematic study of the effects of inflation and parametric amplification of quantum fluctuations on the baryon asymmetry generated in post-inflationary Affleck Dine baryogenesis. We have taken into account the amplification of the quantum fluctuations in the Affleck Dine scenario using the background field method, and investigated whether these quantum effects coupled with reheating after inflation has any effect on the calculated BAU. The model used by Affleck and Dine to illustrate the generation of BAU lends itself very well to the quantum optical formalism we have developed in examining the first model. Again we get a Hamiltonian having su(2) and su(1,1) the symmetries, which we can diagonalize exactly. Using Bogoliubov transformations we have been able to derive the connection between the initial vacuum and the final vacuum. This has allowed us to calculate the effect of quantum fluctuations on the BAU exactly and also get the classical value in the appropriate limit.

This procedure has further allowed us to derive the evolution equations for the particle (fields with baryon number 1) and anti-particle modes(fields with baryon number -1). The effective potential encountered by these particles and anti-particles in their evolution is dependent upon the expansion parameter $a(\eta)$ and oscillation of the inflaton field (leading to parametric resonance). Thus, we have considered two scenarios: the slow expansion and the rapid expansion of the universe. In the slow expansion scenario, the predominant effect on the evolution is the oscillation of the inflaton field, which gives rise to the parametric resonance. This equation resembles the Mathieu equation, and the solutions of this equations has instability bands within the resonance band. Thus, the number of particles are exponentially enhanced and the number of anti-particles are damped. We have studied one more case in this scenario where the broad resonant band is considered by replacing the oscillating function with a asymptotically flat potential near it zeros. Here the BAU is calculated as a function of comoving wave number k. Again we find that the differential amplification of particle to anti-particle modes gives rise to BAU. In the rapid expansion scenario, the rapid transition of expansion parameter from the de Sitter metric (inflationary) to the FRW metric (normal expansion) gives gives rise to non-equilibrium particle and antiparticle production. The amount of particle production in the both the case is calculated as the tunneling through a barrier of potential $V(a)$. We found that by considering different inflationary scenarios coupled with parametric resonance we can control the asymmetry parameter "r" to a much lower value than that obtained in the classical Affleck Dine model. Since a variant of the Affleck Dine mechanism is also used to account for dark matter [82], as well as in most exotic scenarios of baryogenesis, our method should be useful in this context also. Such a study will be the subject of future investigations.

To probe the fundamental constituents of matter, the present the collider experiments are going to reach the order of the energy scales of the early universe. At the present one such experiment, which can probe scales of the first few seconds of the early universe is RHIC, where an evidence for the creation of an excited states of quarks and gluons known as quark gluon plasma (QGP) has been found. These results will be further consolidated by the second phase of the Large Hadron Collider (LHC). Since baryon asymmetry cannot be probed at these machines, we have to look at other non equilibrium situations which give rise to the production of other types of asymmetry such as charge asymmetry to test our techniques. On such situation has been hypothesised in the evolution of the QGP. This is the non equilibrium situation in which a metastable collective state called disoriented chiral condensate (DCC) is formed. We have used the techniques developed in the study of BAU to examine the formation and decay of the DCC. In the last chapter, we have studied the the evolution of the disoriented chiral condensate through both a sudden quench and with a transition through a metastable state with arbitrary disorientation. Using the O(4) sigma model and a background field analysis we constructed an effective Hamiltonian which has su(1,1) and su(2) symmetries, allowing us to use the formalism we have developed in this thesis. We have modelled three different scenarios of metastable intermedeary states that can give rise to DCC's. In all the cases the mode Hamiltonians have su(2) and su(1,1) symmetries allowing us to diagonalise them exactly using our formalism. The evolution of the wave function in the expanding metric, similar to that of the expansion of the universe, is considered thoroughly. The competing effects of expansion and the relaxation of the system from the state of restored symmetry to the broken symmetry have been examined. We have shown that there is an asymmetry in isospin space of the total multiplicity distributions of charged and neutral pions, which carry information about the non-equilibrium formation process of the DCC. These are dramatic characteristic signals for the DCC and are related directly to the way in which the DCC forms. These are unambiguous, therefore they must be examined thoroughly in future searches for the DCC. We are encouraged by the first preliminary data from the analysis of the WA98 experiment at the CERN SPS in which some events show an excess of photons (neutral pion excess) within the overlap region of charged and photon multiplicity detectors. One of the aims of LHC (phase 2) will be a search for the DCC and we hope that our results will be useful in that context.

This last application may seem out of place with the preceding chapters, but it is illustrative of the flexibility of the methods that we have developed in this thesis. Encouraged by this, we intend, in the future to apply it to other models of asymmetry generation. One example of this is the problem of particle production in the presence of warped extra dimensions [48] . One possible future problem is to construct models incorporating the Sakharov conditions to calculate the BAU using such models as the Randall-Sundrum [48] and DGP (G. Dvali, G. Gabadadze, and M. Porrati) models[83]. Since these are inspired from string models, that naturally have a plethora of scalar fields, they can give complex inflaton field candidates. It would be interesting to investigate how the BAU is affected by the nature of particle production on the brane, the nature of warping,cosmic evolution and reheating.

In conclusion, we end with the optimistic hope that the LHC will shed light on the viability of any of the conjectures on the origin of BAU. Vital evidence on supersymmetry, large extra dimensions, disoriented chiral condensates and other exotic scenarios is awaited from the results it will provide.

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