An Analysis of Stock Prices in India: Wavelets and Spectral Applications

A Thesis Submitted for the Degree of
Doctor of Philosophy
in Economics

By

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To My Parents

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This is to certify that I, Pratap Chandra Biswal have carried the research embodied in the present thesis entitled "An Analysis of Stock Prices in India: Wavelets and Spectral Application" within the full period prescribed under Ph.D. ordinances of the University of Hyderabad.

I declare to the best of my knowledge that no part of the thesis was earlier submitted for the award of research degree of any University or Institute.

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(Pratap

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An Approach to Spectral and Wavelet Analysis of Stock Prices

1.0 Introduction

The rich stock of theoretical and empirical literature in the area of stock price behaviour reveals that stock price is not merely a number as we observe; rather it is a manifestation of all available information in the market at a certain point of time. This notion receives formal and systematic treatment in the framework of Efficient Market Hypothesis (EMH) wherein, an efficient market is expected to fully reflect all the available information and hence no arbitrage is possible in such a state. For practitioners, studying pattern in stock price behaviour is quite important in the sense that it is to be used to predict future movements in prices, which will guide the trading rule. The policy makers, on the other hand, view stock prices as indicators of policy changes in monetary and fiscal front, i.e. any change in the economy gets reflected in stock prices and hence studying stock price behaviour assumes immense importance from policy point of view. And any kind of extraordinary or random movement in prices exhibiting gross deviation from that implied by economic fundamentals raises concern both for practitioners in market place as well as policy makers. Therefore, understanding and analyzing stock price behaviour have been of direct interest to academics in general and model builders in particular.

Over the past few decades, there has been a dramatic reversal in opinion concerning the degree of information contained in the data, leading to a new approach towards the analysis of economic and financial data. The early postwar period until the eighties was characterized by the overriding belief that macroeconomic data contained a lot of structural information that could be captured in the estimation of systems of simultaneous equation models with a limited number of external forcing terms driving the economic system. The models were based on concepts of static equilibria that were perturbed to new equilibria by external forces; the role of the transients in the adjustment process tended to be ignored.

Consequently, the economists' job was to estimate the appropriate parameters relating the steady state of the system to potentially controllable forces to ensure an economic dynamical path that was optimal, at least approximately. In contrast, financial data were regarded as unforecastable; current price reflects all current information on values so that no gains from trade could be made with publicly available information, this was the famous "efficient markets hypothesis (EMH)".

Both these ideas were modified during the eighties. While the EMH still prevails in more sophisticated form, there is some disputed, but growing evidence that the stock market is not entirely efficient and that very small, but noticeable / profitable deviations exist at least for some time. The chaos revolution of the last fifteen years has had a dramatic impact on the modelling and understanding of many scientific disciplines. Unfortunately, this has not been of much use with economic and financial data (Jaditz and Sayers, 1995; Scheinkman and LeBaron, 1987). The Grasberger-Procaccia approach to dimension calculation seems to throw little evidence for an attractor in the financial data; this is especially true with returns in the stock market. The conventional wisdom that stock market prices are examples of geometric Brownian motion is supported by these procedures. However, there are other views in the financial literature that indicate that at various scales, there are aspects of the data that indicate observable differences with the geometric Brownian motion hypothesis (Jegadeesh, 1990; Mankiew, Romer, and Shapiro, 1991). Early use of spectral techniques in the analysis of economic and financial data revealed the "typical spectral shape": high power at very low frequencies and rapidly declining thereafter, a shape that is consonant with geometric Brownian motion. A fundamental advantage of spectral analysis, unlike most of the previous analysis, is that it provides a mathematical tool, which can deal specifically with the regularity and randomness typically found in economic and financial data. Moreover, spectral analysis involves no loss of data unlike traditional smoothing techniques. It is important to emphasize, however, that spectral analysis follows directly from an assumption of stationarity and does not require the specification of a model.

A major difficulty with most of the previous analyses is that staionarity of time series has been assumed and possible presence of highly time localized patterns in the structure of the data have been ignored. Consequently, there is a need for alternative ways to examine and evaluate the data that do not condition upon the assumption of sationarity that can detect singularities, and do not require very large numbers of observations. Wavelet analysis can provide a particular answer.

1.1 Spectral Analysis: An Overview

There are two major approaches to analyze the behaviour of a time series, *viz.* time domain and frequency domain. In fact, the analysis of time series got started in frequency domain, which had its seed in physical sciences and astronomy. The fact that light passing through a prism gets decomposed into its various colour components as per corresponding frequencies, led to the idea that a time series could also be similarly broken into components with different frequencies. This was technically termed as spectral analysis of time series, which developed as a distinct and parallel discipline in contrast with the analysis of time series in time domain.

If the concept of frequency is used, the phenomena which possess a wave-like structure can be discussed, i.e. can be represented as a composition of sine and cosine waves with different amplitudes and frequencies. All the so called cycles in economics, in no sense, are not necessarily regular, the cycles being no more than fluctuations. One of the advantages of using the spectral methods is that they provide a mathematical approach to this mixture of regularity and non-regularity. It might be noted that fluctuations similar to the business cycle such as movements in stock prices could be described as the sum of a number of sine series with suitably chosen amplitudes and phases and all having nearly equal periods. The interactions between the terms would produce changes in both the direction and amplitude of the fluctuation, but nevertheless the average length of the fluctuation would remain constant. The spectral approach essentially describes fluctuations of the business cycle type, moving from a sum of sine terms to an integral of a sine function over a band of periods.

As discussed above, spectral analysis decomposes a stationary time series into a set of frequency bands in terms of its contribution to the overall variance (termed power) of the series. The power spectrum is looked upon as the graph of variance against frequency. If a band of frequencies is important, the spectrum will exhibit a relative peak in this frequency band. On the other hand, if the spectrum is flat (i.e. a horizontal straight line parallel to X-axis) indicating that every component is present in equal amount, the interpretation is that the series is merely a sequence of uncorrelated readings which is purely random or technically a white noise series.

The relationship between a pair of series is analyzed by use of the cross-spectrum. There are two basic diagrams of importance, viz. the coherence and phase diagrams. The former records the estimate of the square of the correlation coefficients between corresponding frequency components. A noteworthy feature is that the coherence between two series may attain a peak or be high at a certain cyclical frequency even though the spectrum of the neither series exhibits peak, or is high at this frequency. The phase diagram provides evidence of time lags between components but is generally more difficult to interpret. This is partly because the significance of this diagram varies with the corresponding values of the coherence diagram. It is clearly not worth discussing the time lag between two unrelated or hardly related components. In simple terms, if the phase diagram appears to lie about a straight line over a region where coherence is reasonably high, this can be considered to indicate a simple time lag between the two series for those components, the extent of the lag- lead depending upon the slope of the line.

The two approaches, "time domain" and "frequency domain" are mathematically related in that the autocovariance function of the time domain approach is the Fourier transform of the spectrum of the frequency domain formulation and vice versa. Very often it is not clear as to why one approach should be preferred over other. The choice between the two procedures depends on both technical and substantive use.

Any stationary time series without a purely deterministic component can be expressed as an expansion in terms of a causal moving average of an uncorrelated white noise process (Wald's general linear representation theorem). Alternatively, a stationary time series can be represented as a Fourier transformation of orthogonal random measures (Cramer's spectral representation theorem). However, there are differences between these two approaches that are of importance to us. An initial reason for using the frequency domain approach is when the time series is presumed to be the sum of 'harmonic components', that is, the sum of sinusoidal terms. If financial time series are generated, at least in part, by harmonic components, then the use of spectral analysis is an obvious tool. More relevantly, if economic or financial time series contain frequency bands of high power, the use of spectral analysis is a natural choice.

An important reason for choosing spectral technique is that the frequency domain approach does not assume a specific model, whereas the time domain approach does: that is, the frequency domain approach "lets the data tell the story", when the time domain formulation is believed to be the more applicable in physical model, there may be some difficulty in determining what the corresponding time domain model is. The advantage of such a representation lies in the ease with which various operations can be handled in the transformed frequency domain as opposed to the time domain. While the analogy should not be pressed, that the transformation from reals to the complex domain simplifies many problems and provides solutions to the problems that may be much more difficult to solve in the domain of the reals.

The benefit of the spectral approach is to be discovered in the properties of the relationships between functions defined on frequencies relative to the corresponding relationship between functions defined on the time domain. For example, the fact that compression in the time domain is equivalent to expansion in the frequency domain, that a time shift yields a phase shift in the frequency domain and vice versa. The resurgence of interest in dynamical analysis during the last few years argues for the use of spectral techniques as they provide a natural analytical link to the solution of the differential equations describing dynamical systems. The ARIMA approach is tied to the relation formulation of difference equations, whereas the spectral approach can be used to explore both difference and differential equation formulations of dynamical systems and is not constrained to linearity in the difference or differential equations.

Finally, an important case for choosing the frequency domain approach stems from the function analytic aspects of spectral analysis. The frequency domain expansion of a time series is an expansion in terms of eigen vectors in the discrete case and in terms of eigen functions in the continuous case. The Fourier expansions have advantage of being orthogonal complete and convenient.

1.2 Wavelets: An Overview

By design, the usefulness of wavelets is its ability to localize data in time-scale space. At high scales (shorter time intervals), the wavelet has a small time support and is thus, better able to focus on short lived, strong transients like discontinuities, ruptures and singularities. At low scales (longer time intervals), the wavelet's time support is large, making it suited for identifying long periodic features. Wavelets have a intuitive way of characterizing the physical properties of the data. At low scales, the wavelet characterizes the data's coarse structure i.e. its long-run trend and pattern. By gradually increasing the scale, the wavelet begins to reveal more and more of the data details, zooming in on its behavior at a point in time.

Wavelet analysis is the analysis of change. A wavelet coefficient measures the amount of information that is gained by increasing the frequency at which the data are sampled, or what needs to be added to the data in order that it looks like as if it were measured more frequently. For instance, if a stock price does not change during the course of a week, the wavelet coefficients from the daily scale are all zero during that week. Wavelet coefficients that are non-zero at high scales typically characterize the noise inherent in the data. Only those wavelets at very fine scales will try to follow the noise, whereas those wavelets at coarser scales are unable to pick up the high frequency nature of the noise. If both the low and high scaled wavelet coefficients are non-zero then something structural is occurring in the data. A wavelet coefficient that does not go to zero as the scale increases indicates a jump (non-differentiable) has occurred in the data. If the wavelet coefficients do go to zero then the series is smooth (differentiable) at this point.

Very little needs to be known about the relevant information or the information that one wants to extract. Because the wavelet transform captures the characteristics of the data in a few wavelet coefficients, if the wavelet coefficients having magnitudes less than some prescribed value are set to zero and the few non-zero wavelet coefficients are used to recreate the data, the resulting data set will contain only the relevant information.

Wavelet filter provides insight into the dynamics of financial time series beyond that of current methodology. A number of concepts such as non-stationarity, multi-resolution and approximate decorrelations emerge from wavelet filters. Wavelet analysis provides a natural platform to deal with the time varying characteristics found in most real world time series and thus the assumption of stationarity may not be invoked. Wavelets provide an easy way to the study the multiresoution properties of a process. It is important to realize that financial time series may not need to follow the same relationship as a function of time horizon (scale). Hence, a transform that decomposes a process into different time horizons is appealing as it reveals structural breaks and identifies local and global dynamic properties of a process at these time scales. Further, wavelets provide a convenient way of dissolving the correlation structure of a process across time scales. This would indicate that the wavelet coefficients at one level are not associated with coefficients at different scales or within their scale.

1.2.1 Spectral Versus Wavelet Analysis

Spectral tools based on Fourier transformation are a linear combination of sines and cosines. Fouries basis functions (sines and cosines) are very appealing when working with time series. However, restricting the research in finance to stationary time series is not very appealing since most financial time series exhibit quite complicated patterns over time (e.g. trends, abrupt changes, transient events etc.). The spectral tools cannot efficiently capture these events. In fact, if the frequency components are not stationary such that they may appear, disappear and then reappear over time, spectral tools may miss such frequency components.

The spectral density does not provide any information on the time localization of different frequency components. That is, on the basis of spectral density, it is not possible to identify the exact time period when the frequency components are active. By considering the frequency representation of time series, one may know which frequency components are active but not when they were active. The reverse is true in time domain - one knows when things happened but has no information about corresponding frequency.

To overcome the problem of simultaneous analysis of time and frequency, a new set of basis functions are needed. The wavelet transform uses a basis function (called wavelets) that is stretched and shifted to capture features that are local in time and local in frequency. The wavelet filter is long in time when capturing low-frequency events and hence has good frequency resolution. Conversely, the wavelet is short in time when capturing high-frequency events and therefore has good time resolution for these events. By combining several combinations of shifting and stretching of the wavelets, the wavelet transform is able to capture all the information in a time series and associate it with specific time horizon and locations in time.

The wavelet transform adapts itself to capture features across a wide range of frequencies and has the ability to capture events that are local in time. This makes the wavelet transform an ideal tool for studying nonstationary or transient time series. The following points demonstrate the convenient usage of wavelet based methods.

1.2.2 Time-Scale Decompositions

One of the most useful properties of the wavelet approach is the ability to decompose any signal into its time-scale components. It is well known that like physical and biological processes, in economics, participants show different behaviour across time-scales. Consider the participants of securities markets who are made of traders with different trading horizons. There are traders who take a very long view (years in fact) and consequently concentrate on what are termed 'market fundamentals'; these traders ignore ephemeral phenomena. In contrast, other traders are trading on a much shorter time-scale and as such are interested in temporary deviations of the market from its long term growth path, their decisions have a time horizon of a few months to year. And yet other traders are in the market for whom a day is a long time. Each of these classes of traders may have their own trading tool sets consistent with their trading horizon may possesses a homogenous appearance within their own class. Overall, it is the sum of the activities of all traders for all horizons that generates the market prices. Therefore, market activity is heterogeneous with each trading horizon (trader class) dynamically providing feedback across all trader classes.

In such a heterogeneous market, a low frequency shock to the system penetrates through all layers. The high frequency shocks, however, would be short lived and may have no impact outside their boundaries. This apparent aggregate heterogeneity requires econometric methods that can simultaneously study and forecast the underlying structure at different time-scales (horizons). This involves the separation of the local dynamics from the global one, and the transitory from the permanent. Wavelet methods provide a natural platform to distinguish these effects from one another by decomposing a time series into different time-scales. Furthermore, wavelet methods are localized in time so that they can easily identify nonstationary events, such as sudden regime shifts and transient shocks to a system.

1.2.3 Non-stationarity and Structural Breaks

The development of new time series models assumes that the mean and the covariance of the process do not vary over time but for most of the financial time series this assumption is suspect and statistical testing is useful in detecting and locating deviation from stationarity at specific point. The wavelet has a small time support at high frequencies to focus on short lived strong transient like discontinuities ruptures and singularities and also it has longer time support at low scales for identifying long periodic features.

Another important point is that given the potential effects of sudden regime changes and isolated shock to the system, non-stationarity may arise due to structural break. An interesting wavelet based approach is to test the wavelet coefficients on a level-by-level basis to detect structural breaks. If the structural break refers to change in variance, then the low level of wavelet coefficients (which are associated with high frequency content of the time series) should retain this sudden shift in variability while the high level coefficients should be stationary. If the structural break is a possible change in the long range dependence of the series, then all levels of wavelet coefficients should exhibit a structural change, especially the low-frequency ones.

1.2.4 Denoising

Given the recording errors that occur during the short intense trading periods and transient shocks that are caused by news reports, the denoising technique is important to financial data like stock prices. Methods which address the irregularity of high frequency data are needed and wavelet analysis is one possible solution. Because of its localization in time and scale, wavelets are able to extract relevant information from data set while disregarding the noise. Utilizing the thresholding one may remove (hard thresholding) or shrink towards zero (soft thresholding). Wavelet coefficients at each level of decomposition is an attempt to eliminate the noise from the signal. Thresholding wavelets coefficients is appealing since they capture information at different combinations of time and frequency, and hence the wavelet based estimate is locally adaptive.

1.2.5 Multi-Scale Correlation

The decomposition of time series on a scale-by-scale basis has the ability to unveil structure at different time horizons. The wavelet regression between two time series on a scale-by-scale basis make it possible to see how the association between two time series changes as a function of time horizon. This feature, which reveals important information about relationship between two time series such as spillover effect, will not be remained hidden with the multi-scale decomposition of the wavelet transform.

1.2.6 Local Inhomogenity and Density Estimation

The estimation of density and spectral density functions in economics and finance requires the recognition of the presence of spatial inhomogeneties. Further, it is well known that a prominent characteristic of economic and financial data is the presence of spatial inhomogeneties, so that wavelet analysis is a natural choice for analyzing such data due to its localization property. By performing wavelet decomposition of time series, one makes an implicit assumption about the underlying nature of the process. Specifically, there is a horizontal structure present in the data so that information in the time series is evolving at different time horizons. When determined appropriately one may easily view the individual factors that make up the complicated process.

1.3 Need for the Present Study

The discussion thus far reveals that the use of spectral methods and wavelets in analyzing stock prices enables as to capture certain useful and important aspects, which have hitherto been neglected in the literature, largely by virtue of concentrating on the time domain analysis. Especially, the ability of detecting hidden informations has been the most important feature of the frequency domain approach. A fundamental advantage of spectral analysis is that it deals with the regularity and randomness typically found in financial data. Moreover, spectral analysis involves no loss of data and does not require the specification of a model. On the other hand, the wavelet transform intelligently adapts itself to capture features across a wide range of frequencies and thus has the ability to capture events that are local in time (non-stationarity and transient times series). The most useful applications of wavelet analysis involve segregating observed data into time-scales, identification of structural breaks, denoising, scaling and finding the relationship among different time series. Though there are a number of studies, which deal with analyzing stock price behaviour, use of spectral methods and wavelets has been sporadic and incomprehensive. Besides, in India while the analysis of stock price behaviour using spectral methods has taken a back seat barring a few studies (Sharma and Kennedy, 1977, Kulkarni, 1978; Ranganatham and Subramaniam, 1993), use of wavelets is completely absent. This clearly identifies the research gap, which the present study will make an attempt to fill up.

1.4 Objectives of the Study

In the light of the above the major objectives of the study are:

- To analyze stock price behaviour in India in spectral domain.
- To show the wavelets as an alternative approach in analyzing stock price behaviour in India.
- To carry out certain applications of wavelet analysis in Indian stock market, viz. long memory pattern and spillover effect.

1.5 Nature and Sources of Data[1]

The study has made use of daily data on four broad based market indices viz. BSE Sensitive Index, BSE National Index, NSE S&P CNX 500 index and NSE S&P CNX Nifty Index. The Sensitive Index constitutes 30 Blue Chip securities traded in Bombay Stock Exchange and represents major portion of BSE in terms of market capitalization. The National index explains the stock market movement on national level since it constitutes 100 scrips traded in five major stock exchanges in India. The last two indices represent the National Stock Exchange (NSE) and we have chosen these two indices given the method of their compilation and representation of different securities traded in NSE. The study period spans over a period of January 1991 to December 2001, thus involving around 2052 number of data points, which we believe constitute a rich data set for our analysis. It may be noted here that though trading on equities in NSE started in 1994, we have used the data since 1991 by taking the simulated data provided by NSE. The data are collected from respective web pages of BSE and NSE. For estimation purpose we have taken into account daily closing values of these indices and have converted them to daily compounded return by taking the log first difference. However, we are using the data excluding the dividends for two important reasons namely, (a) daily data on dividends are not available (b) we are concerned only with unforecastable market rate of return rather than total return on these indices.

Apart from the above, we have also considered three major developed stock markets indices, viz. Dow Jones Industrial Average (DJ1A) of the U.S., FTSE 100 index of the U.K., and Nikkei 225 of Japan spanning over a period January 2000 through December 2001. We have deliberately chosen theses three indices keeping in mind their importance in world equity market.

1.6 Organization of the Study

The remaining part of the study consists of the following four chapters. Chapter II reviews the theoretical and empirical contributions of spectral and wavelet analysis. First, we present a comprehensive review of select works on spectral analysis of

[1] The detailed discussion on **the data** sets **used is presented in the appendix.**

stock price behavior, followed by a select review of some theoretical and empirical applications of wavelets in stock price analysis.

Chapter-Ill presents, in detail, the spectral analysis, its various approaches and inference procedures for analyzing stock prices. The chapter also presents the estimation procedure of spectral methods in a brief manner. This is followed by a rigorous empirical exercise for Indian stock return data, where the prime emphasis has been on analyzing stock prices using power spectrum and cross-spectrum.

Chapter IV is dedicated to introduce wavelets and carry out the empirical estimation using the same. A thorough discussion on wavelets and its mechanical properties will be undertaken. Here an attempt is made to discuss briefly the implementation of wavelet transformation. Latter we invoke the estimation of the wavelet methods by confronting with daily stock return data in India.

Two interesting, yet important stylized facts associated with stock prices such as long memory nature and spillover effects are discussed in Chapter V with the help of above mentioned methods. An effort is made to investigate the long range dependence in stock returns series using both spectral and wavelet regression. Towards the end, we analyze the international transmission of international movements using wavelets.

Finally, we present an overview of the study followed by some economic and policy implications. Some suggestions for further research in this area are also given.

Chapter II

Spectral and Wavelet Analysis of Stock Price Behaviour: An Overview of the Studies

2.0 Introduction

The behaviour of financial markets, in particular stock markets, has always remained as an intriguing phenomenon for the general public as well as academics and policy makers. An important reason may be that these markets are characterized by complex dynamics, which cannot be captured by traditional approaches developed in various spheres of analyzing these phenomena. To be specific, the quest for retrieving the hidden information in a financial data has caught the imagination of researchers in recent years. Barring standard statistical and econometric treatments of financial data, researchers in present times have venture into quite alien areas to explain the observed dynamics in financial markets. In this regard, two prominent approaches namely spectral methods and wavelets may be mentioned. Though these methods are not of recent origin, their application in financial markets is relatively new and at its infancy. Therefore, it is useful as well as interesting to take cognizance of these methods. This chapter goes to present an overview of these two methods in the analysis of stock price behaviour.

The chapter proceeds as follows: we present a comprehensive review of select works on spectral analysis of stock price behaviour. This section goes to discuss a range of studies relating to various aspects of spectral analysis. This is followed by a select review of application of wavelets in stock market analysis. Section 2.3 concludes the chapter.

2.1 Spectral Analysis of Stock Prices: A Review of Select Works

An early application of spectral methods to economic data was in the year 1959[1]. The first report was published in 1961 (Granger, 1961) and the complete report resulted in the form of a book by Granger and Hatanaka (1964). Beginning with

[1] **This was initiated in a project at Princeton University to investigate the usefulness of spectral methods in economics.**

this, the use of spectral methods spread to different spheres of economics. Here, we present a select review of studies pertaining to stock price behaviour with a passing reference to few other macroeconomic applications.

2.1.1 Studies Based on Power Spectrum

An apparent feature of a univariate power spectrum that can be easily noted are the peaks, such as at the seasonal frequencies and any shape that is complicated compared to the simple shapes that arise from a white noise or first order autoregressive and moving average models. Economies have been seen to follow swings with alternating periods of prosperity and depression, known as the business cycle. An early application of spectral techniques was to investigate these swings. It should be emphasized that the business cycle has never been at all regular, or deterministic, and so corresponds to one, or several, frequency bands rather than to particular frequency points. The apparent problem with this topic is that the business cycle corresponds to rather low frequencies and so estimation of this component is difficult unless very long series are available. The situation is a little improved by considering a number of different series from the same economy, as this provides little extra information; most parts of the economy are inclined to move together at low frequencies. Although some evidence was found for certain low frequency components being especially important (see, for instance, Howrey, 1968 and Harkness, 1968), in general all low frequencies were usually observed to be important for the levels of major economic variables. The relative importance of low frequency components compared to all higher components was found so frequently that a spectrum that steadily declined from low to higher frequencies, except possibly at seasonal frequencies, was called the 'typical spectral shape' in Granger (1966). The resulting spectrum of the New York commercial paper rate for the period 1876-1914, is not a typical one for an economic series as the low frequencies are considerably more imposing and a peak has been found for a frequency other than that correspondingly to the annual component (Granger and Hatanaka, 1964).

Granger and Hatanaka (1964) estimated the spectrum taking the monthly mid range of Woolworth stock prices quoted on the New York stock exchange for the period January 1946 to December 1960. The spectrum was seen to be very smooth and with the low frequencies predominating a shape frequently found for the spectrum of an economic series. There were no important peaks except that centered on period 2.8 months, which is the 'alias' of a weekly cycle. In another study in the same year for the period 1879-1914, they found that the estimated power spectrum of the call money rate was similar in general appearance to that of the commercial paper rate but with less prominent peaks corresponding to the annual and 40-months components and their harmonics.

There are a number of studies relating to testing of random walk hypothesis of stock price behaviour. If the spectrum is flat *(i.e.* a horizontal straight line parallel to x-axis) indicating that every component is present in an equal amount, the interpretation is that the series is nearly a sequence of uncorrelated readings, which is purely random or technically a white noise process. If the spectrum shows any clear peak(s) and spike(s) at a particular frequency, then the conclusion that there is one frequency or frequencies which are of particular importance, resulting in a periodic cycle appearing in the series. Granger and Morgenstern (1963) applying spectral technique to New York stock market prices found that short run movements of series obey the simple random walk hypothesis but that long run components are of greater importance than suggested by this hypothesis. The seasonal variation and the business cycle components are shown to be of little or no importance and a surprisingly small connection was found between the amount of stocks sold and the stock price series.

In another attempt to test for random walk hypothesis of share prices on the New York and London stock exchanges, Godfrey *et al.* (1964) found that most of the points of the spectra were within the 95% confidence limits. However, the strong long run (two or more years) components were larger than expected. The annual component was not apparent in any of the series studied, but several series showed very faint evidence at the harmonics of the seasonal components. In a similar fashion Granger and Rees (1968) found that a random walk model with no seasonal component appeared to fit the shorter run fluctuations of the series rather well for rate of return on a bond of specific maturity. Granger and Morgenstern (1970) studied the spectrum of the Sydney ordinary share index over 160 weeks from 1961-64. They observed that the spectrum seemed flat, which was not surprising, as the series was not long enough to reveal other components if they were present.

Sharma and Kennedy (1977) made a comparative analysis of stock price behaviour on the Bombay, London, and New York stock exchanges. Their results indicated that the spectral densities estimated for the first differences series (raw and log transformed) of each index, confirmed the randomness of series, and no systematic cyclical component or periodicity was present. Based on these tests, their view was that stocks on the Bombay Stock Exchange obeyed a random walk and were equivalent in this sense to the behaviour of stock prices in the markets of advanced industrialized countries.

In contrast with the above studies, Praetz (1973) found departures from the random walk hypothesis in case of Australian share prices and share price indices and moreover there was a clearly defined seasonal pattern in share price indices. Kulkarni (1978) presented auto-spectral test of the random walk hypothesis about share price movements on the Indian stock exchanges. All the weekly series (all India, Bombay, Calcutta, Madras, Ahmedabad, and Delhi) seemed to behave alike except for Calcutta and Delhi. The presence of 4 week lags and hence auto-covariance function of 4 lags was an indication of non-random walk among weekly series. All other weekly series except these two seemed to be free from any seasonal or other cycles. Of the six monthly series analyzed, four of them show a lag structure of four months in their spectral representation. Thus, this result indicated the presence of non-random walk behaviour. All the monthly series were found to be influenced by one or two seasonal and other harmonics (cycles). In another study, Ranganatham and Subramaniam (1993) made an attempt to test weak form of efficient market hypothesis (EMH) and failing to find any support for this, concluded that there was no random walk.

Poterba and Summers (1987) and Lo and MacKinlay (1988) challenged the conventional view that stock price returns were unpredictable i.e., do not form a martingale difference sequence. They suggested that the spectral shape tests may be interpreted as searching over all frequencies of spectral density for martingale difference violations, whereas the variance bounds tests may be interpreted as examining the zero frequency in isolation. Durlauf (1991) extended this literature on using spectral shape to test various hypotheses, which has concentrated on a single statistic, to more general question of analyzing spectral distribution deviations from the straight line as a problem of weak convergence in a random function space. A general asymptotic theory for spectral distribution function permits the construction of many test statistics of the martingale hypothesis. Using the data sets of Lo - MacKinlay (1988) and Poterba and Summers (1987), he found that weekly and monthly stock returns revealed some evidence against the null hypothesis that holding returns are martingale differences. His result confirmed that stock price exhibited long run mean reversion. Violations of the random walk theory appear to be robust to a relatively diffuse formulation of a researcher's beliefs concerning the class of alternatives. Another study by Fond and Ouliaris (1995) supported the view against the martingale hypothesis for exchange rates data and they viewed this rejection is due to long memory influences.

Apart form the studies relating to random walk and martingale behaviour, a few studies have been made to analyze some other aspects of stock price behaviour. On the basis of eleven original descriptive spectral characteristics obtained from log spectrum of the return on Helsinki stock exchange, Knif and Luoma (1992) developed three main principal characteristics of the spectrum, i.e., size, shape, and variability. The empirical results indicated that the spectral approach could be used for the descriptive as well as the analytical analysis of stock market behaviour. Knif, Pynnonen and Luoma (1995) studied the differences in the spectral characteristics between the two stock markets - the Finnish and Swedish. Their results indicated differences between the return spectra of two markets and more volatile Swedish market exhibited a two-day periodicity and autoregressive dependence of about two weeks. In a recent study, Barkoulas and Baum (2000) used the spectral regression test for fractional dynamic behaviour in a number of Japanese financial time series, *viz.* spot exchange rates, forward exchange rates, stock prices, and currency forward premia. Long memory is indicated by the fact that the spectral density becomes unbounded as the frequency approaches zero; the series has power at low frequencies.

2.1.2 Studies Based on Cross-Spectrum

Potentially the most important technique available in the early period was the cross spectrum and the functions derived from it, the coherence, the phase and gain diagrams. The coherence measures the strength of relationships between corresponding frequency components in the two series. As components with different frequencies are necessarily uncorrelated for jointly stationary series, the coherence thus totally measures the strength of the relationships between the series. The gain essentially measures the regression coefficient of the frequency component of x_t on the corresponding component of y_t. In the case where one series is leading the other, the phase diagram can be used to measure this lead.

Many studies have applied cross-spectral techniques to economic data as the primary method of analysis and no attempt will be made to summarize them all. Instead, a brief account will be given of results mainly in three fields, *viz.* in stock markets, the term structure of interest rates, and evaluation of leading indicators.

The first ever application of cross spectrum started with the study of Granger and Hatanaka (1964). The cross spectrum between the Dow-Jones industrial stock prices and the industrial production index revealed quite irregular oscillations. The high coherence observed at scattered points of frequency when more lags were used really represented sampling fluctuations. Thus, the Dow-Jones industrial stock price series could not be a good indicator. The phase analysis indicated some evidence that this series was leading production, but the magnitude of the lead was only about one month.

Godfrey, *et al.* (1964) studied series of share prices, share price indices and volume series at daily, weekly and monthly intervals on the New York and London Stock Exchanges. The connection between different industry group series was not high and very often weak and there was no evidence of one series leading another. Also, the result failed to suggest any evidence to associate prices and volumes of shares sold. The relationship between stock prices and volume of sales has also been investigated by a few other studies. A study by Ying (1966) showed that most of the variations were due to the trend or cycles of long length. A high value of coherence indicated that there was a strong tendency for the two series to exhibit a lead-lag relationship over time. This was, however, criticized by Granger and Morgenstern (1970) on the ground that adjustments to data by Ying induced a series of local trends, which led to dependence. Apart from this, they also found inter-relationships of various stock exchanges around the world. In a related study by Praetz (1973), clearly defined seasonal patterns in Australian share price indices was reported and some indices exhibited lead or lag relationship with the share market as a whole. At the same time there was no support for the presence of any lead-lag relations of overseas markets.

The interrelationship among bonds of different lengths of maturity was examined by Granger and Rees (1968) who reported close correlation between bonds of nearly same term, which was not unexpected, as it had always been considered that arbitrage by holders is usually closest between adjacent maturities. One-year rate was found leading the other rates for long run fluctuations, but this was not the case for shorter fluctuations (higher frequencies).

The use of cross-spectrum in analyzing interrelationship among various stock markets has been quite popular. A study by Hilliard (1979) focussed on detecting frequency domain correlation among ten world equity markets and the conclusion was that there were significant co-movements (coherence) among European intra-continental markets, with little co-movements across continents. Fischer and Palasvirta (1990) measured coherence among 23-country equity markets during the period surrounding the 1987 Crash. They compared the average of mean coherence from their sample with mean coherence for 10 countries used in Hilliard (1974). For each of these 10 countries compared, the results indicated that coherence had increased from 1974 to 1988. This led them to conclude that markets had become more inter-dependent, both in 1974 to 1988, as well as in 1986 to 1988 period. Knif *et al.* (1995) studied lead-lag structure between market return on two stock markets, *viz.* the Finnish and Swedish. The cross spectrum of two return series showed a Swedish lead of about 10 days, which decreased to 5 days for the later part of the observations and the non-linearity of the phase, however, indicated a compound effect of several leading terms.

The studies of Granger and Morgensten (1970), Hilliard (1979), and Fischer and Palasvirta (1990) had a common objective of testing for systematic relationship among major world equity markets (Smith, 1999). Smith mentioned that overriding practical implication of this line of research was to estimate benefits to investors from international portfolio diversification. The greater the coherence between these markets, the less the benefit from the international diversification. His study aimed at testing whether six (G-6) largest world equity markets had experienced increased co-movements since the 1987 Crash period with a lengthy sample period. His results were consistent with the hypothesis that several of the markets studied (US, UK, Germany and France) had become more closely linked; particularly true at longer frequencies, with the exception of US/Canada and US/Japan. With the movement towards the European Economic and Monetary Union, the UK, German and French markets were leading the way towards greater economic inter-dependence in Europe. The phase analysis indicated that lead for the US market over the UK, French and Japanese markets had decreased in the post crash period. The UK lead over the French market appeared to have decreased as well.

In another recent study, Asimamakopoulos *et al.* (2000) examined interrelationship between daily returns generated by one US (S & P 500) and three major European (FTSE 100, DA 30, CSE 40) share price indices. They found a strong interdependence between the European return series, as well as a lead-lag relationship between the US and each of the European series. The spectra also gave some evidence of cyclical fluctuation. However, the patterns were similar among the European series for cycles of all frequencies, while similarities between the US and the European series were evident at low, but not at high frequencies.

The only study on transmission of volatility and information between markets by Diagler and Herbst (1996) reported significant coherence with very large majority of significant phases for S & P 500, MMI, and NSYE futures contracts. Three theories concerning the transmission of information were examined, *viz.* dominant market theory, pure information theory and independents market theory. The results supported only dominant market theory of information transfer. It was similar to the change in direction and strength of the stock index lead-lag volatility relationships as found by Kawaller *et al* (1990).

The co-spectral analysis by Wilson and Okunev (1998) for various series of real estate and stock markets in the USA, UK and Australia offered some support for the existence of co-cycles. The finding that phase differences on the co-cycles in the US data set were very small might be of particular interest to portfolio managers seeking to optimize risk reduction opportunities. A similar study by Oppenheimer and Grissom (1998) examined the coherence between real estate investment trust (REITs) and stock market indices and REITs and US treasury debt indices. Their results of the coherence spectra showed significant co-movement between REITs and stock market indices, while debt instruments showed very few frequencies with significant coherence. Furthermore, phase spectra provide evidence of contemporaneous movement between REITs and stock indices at all frequencies.

In contrast with above studies, Ramsey and Thomson (1998) examined the spectral properties of some indices of production and of S& P 500 stock market index by using some new procedures for estimating spectra such as multiple window estimation, slepian functions and sequences, harmonic f-tests, and outlier detection, which they claimed to be more efficient and more powerful than conventional Fast Fourier Transform (FFT). They found that real indices have high coherence between themselves and exhibited strong seasonal components at harmonic frequencies of one to five cycles per year. Also, there was evidence of coherence between the stock market index and the indices of production at business cycle frequencies.

Andersen (1982) evaluated two methods, *viz.* the cross-correlation method and the cross-spectra approach, which have been used for the purpose of establishing the presence of a causal relationship between a pair of variables. He established a relationship between a pair of variables using artificially generated data. The results obtained by him suggest that (a) neither method is successful in all circumstances, (b) the correlation approach can do a reasonable job of picking up the order a lagged relationship provided the underlying generating process is not too complex. The spectral technique, on the other hand, cannot indicate the order of the lags but it is more successful than the correlation approach in detecting the presence of a lead or lag relationship when the generating process is allowed to become more complex.

A number of techniques have been employed for the investigation of the term structure of interest rates. Spectral analytic methods are probably the most interesting techniques so far used in terms of information obtained and they provide certain advantages in studying time varying behaviour. Spectral studies of the term structure include Dobell and Sargent (1969), Fand (1966), Granger and Rees (1968), Melnik and Kraus (1969), Sargent (1968), Cargil and Meyer (1972), Smith and Marcis (1972), and Porsius (1977). These studies examined the spectral density functions of various interest rate series for differences in cyclical and seasonal movements between different maturities. Short-term securities exhibited seasonality and some evidence of cyclical behaviour while longer-term securities indicate little or no evidence of Seasonality. They also examined the cross-spectral relationship between long and short term rates of interest, which was characterized by coherence, gain and phase. Each of these statistics measured a particular aspect of the relationship between the two interest rate series at a specific frequency component. One of the most interesting results obtained from cross-spectral analysis was the presence of a high degree of association among various interest rates, which varied inversely with respect to difference in maturities.

The timings of the long swings in the macroeconomy is very irregular. Prediction of turning points, the upwards and downwards, is of considerable interest. One method of prediction is to find series that consistently lead at the turns. The National Bureau of Economic Research (NBER) has suggested many such leading indicators and also an index of these indicators. A possible way of evaluating the claims made for these indicators, in terms of their consistency and extent of leads, is by looking at the coherence and phase diagrams at low frequencies from the cross spectrum between the indicator series and a measure of the state of the economy. This was done by Hatanaka (chapter 12 of Granger and Hatanaka, 1964) and by Hymans (1973). They found that the indicators did lead, in that the phase diagrams indicated such a lead, but the coherences were often lower than might be hoped for and leads were less than those suggested by NBER.

The potentially important partial cross-spectral techniques, in which the relationship between a pair of series is considered in the frequency domain after removal of the effects of one or more other series, have been little used in economics. One application was by Hatanaka (in Granger and Hatanaka, 1964) who considered inventory cycles and the acceleration principle. A further application was in the Harkness (1968) who attempted to resolve the long-swing controversy. Evidence showed that long-swings occurred in the growth rates of the Canadian economic variables. The first application of spectral methods to long swing measurement was done by Adelman (1965). She concluded that the biases introduced by smoothing raw time series were sufficient to explain long swings found. Hatanaka and Hawrey (1969) pointed out that number of lags used by Adelman were so few that she could not distinguish a long swing from a estimated log linear trend, unless by chance she had achieved perfect trend removal. After increasing the number of lags and correcting for biases created by the elimination of trend, they found peaks in the estimated spectra at long swing frequencies for only 3 of the 22 series analyzed.

2.1.3 Studies Based on Bispectrum

As we discussed earlier in section-2, traditional spectral techniques cannot deal with non-stationary time series and unable to detect non-linearity in the data. Bispectrum is the appropriate technique for such kinds of series. For example Godfrey (1965) had estimated the bispectrum being the Fourier transform of their lagged moments, such as $E[X_t X_{t-p} X_{t-q}]$ when X_t has zero mean, for two economic series- a stock price series for a single company and the 'Federal Float'[2]. He found that a transformation of the stock prices P_t to $log\ (P_t+a)$, where a is near zero produced a series that more nearly obeyed a linear model. The Federal Float series, which was strongly seasonal, produced a bispectrum, which rejected "the hypothesis that the entire process including the seasonal frequencies is well represented as a linear process". Thus the bispectral results were found useful for detecting non-linearity.

To test the implications of CAPM, Goldberg and Vora (1978) used bi-spectrum and concluded that market index did not perfectly explain individual portfolio movements for all portfolios and that despite cyclical betas that were fairly stable, the true value of beta appeared to be different for cycles of differing durations. Joyeux (1979) discussed estimation and interpretation of the harmonizable spectrum, and applied the technique to two individual economic

[2] Which is a quantity of cash held within the Federal Reserve System and is used to measure the level of activity of the member banks in the system.

series. It was found that the high and low frequency components were inter-correlated and thus the series was nonstationary. Barnett *et al.* (1999) used polyspectral methods to investigate whether there existed any nonlinear structure in the difference between the divisia and estimated theoretical monetary aggregates and also used this as a form of residual analysis to explore the dynamic properties of the divisia index as an approximation to the theoretical aggregate.

This section has presented a review of some select works relating to the application of spectral methods in stock markets with a passing reference to other macroeconomic applications. The spectral approach considered here are to test for random walk, to trace the lead lag relationship, analysis of capital asset pricing models et cetera. However, the spectral methods are crippled by the underlying assumption of stationarity. Given the fact that most of the economic and financial time series are non-stationary, spectral approach fails to take account of this. In contrast, wavelets tend to uncover the dynamics of a financial time series in a superior manner when compared to the spectral approach. A discussion pertaining to this is presented in the following section.

2.2 Wavelets: A Selective Review of the Literature

While applications in finance have not yet extensively used the special properties of the wavelets, there have been a number of interesting applications. We might discuss the various applications under four categories. The first category covers the studies that emphasize on the role of non-stationarity and the ability to handle complex functions. The second category covers those studies that are most concerned with structural change and the role of local phenomena. The third and fourth involve the use of time-scale decompositions, density estimation respectively.

2.2.1 Non-Stationarity and Complex Functions

An early paper that warrants discussion was by Johnstone and Silverman (1997), who analyzed the statistical properties of wavelet coefficient estimators under the assumption of stationarity but correlated, data both in short run and long run. Further, Von Sachs and MacGibbon (1997) derived the bias and variance for wavelet coefficient estimators allowing for non-stationarity. An important assumption in their analysis is that while the non-stationarity is slowly changing over the entire sample period, it does so in such a manner that more observations per unit time would lead to locally asymptotic convergence of the estimators. The importance of this paper stems from the fact that the single most obvious variation in the financial data, especially after first differencing, is the presence of second order non-stationaity, which is sometimes modeled as an ARCH process.

Pan and Wang (1998) introduced a novel approach when the data generating mechanism must be regarded as evolutionary, so that the wavelet coefficients are varying over time. The model is

$$Y_t = W_t^T w_t + \varepsilon_t.$$

The authors interpret the model in terms of a Kalman filter, so that the pair of equations that define the time path of coefficients is given by

$$Y_t = W_t^T w_t + \varepsilon_t ,$$
$$w_t = w_{t-1} + V_t$$

where w_t is the wavelet coefficients and V_t is the time varying covariance for the wavelet coefficients. The model was applied to the monthly stock price index, y_t , that was regarded as some function some function $f(r_t)$, where r_t is the dividend yield. $f(r_t)$ was represented in the model by the wavelet transform $W_t^T w_t$. The empirical results were encouraging both as to the overall degree of approximation and to the extent that turning points were correctly indicated.

The paper that represents research arising from earlier concerns in the analysis of the stock market is Ramsey *et al.* (1995). In that paper, the chief topics of interest were the degree of statistical self-similarity of the daily stock return data and whether there was any evidence of quasi-periodicity. The wavelet analysis indicated that while there was little evidence of scaling in the data, there was surprisingly clear evidence for quasi-periodic sequences of shocks to the system. Thus, these results confirm what is by now a commonplace statement about this type of financial data (see Ramsey & Thomson, 1998). Financial data are very complex and are more structured than mere representations of Brownian motion.

Ramsey and Jhang (1996) examined the 16384 daily observations of the S & P 500 stock price index on the New York Stock Exchange from January 3rd 1928 to November 18th 1988. The time-frequency distributions indicated virtually no power for any frequencies; although there was evidence that some frequencies expand and disappear in strength. However, most of the power was in time-localized bursts of activity. A plot of the squared weights of the coefficients relative to those that would be obtained from random data indicated very strong approximations with relatively few coefficients.

The similar techniques have been applied by Ramsey and Jhang (1997) to tic-by-tic foreign exchange rates worldwide for a year. The three exchange rates so examined were the Deutschmark-US dollar, the Yen-US dollar and the Yen-Deutschmark. The results obtained were qualitatively similar for all three exchange rates. Both the levels and the first differenced data were examined, because there has been some controversy in economics literature about the appropriate data generating mechanism; the presence of a unit root in the data being of great concern. In analyzing the levels data using the waveform dictionary approach, some evidence of structure was discovered, but only with very low power. Another key insight provided by these data is that despite the relatively low number of atoms needed to provide a very good approximation to the data, about 100 is sufficient, there is little opportunity for improved forecasting. This is because, while relatively few structures are needed to represent the data, the bulk of the power is in chirps and that does not seem to be any way of predicting the occurrences of the chirps. In short, as most of the energy of the system is in randomly occurring local behaviour, there is little opportunity to improve one's forecast.

2.2.2 Structural Change

Given the concerns of the economists over the potential effects of sudden regime changes and isolated shocks to the system, wavelets can detect structural breaks and outliers of an economic time series due to its good local properties. Strang and Nguyen (1996) summarize how to detect discontinuities by Daubechies wavelets. For data with no noise, wavelets can detect discontinuities at the finest resolution level. As noise increases, it becomes difficult to detect discontinuities at the finest resolution. In this case we can find them by investigating wavelet coefficients for several resolutions. Wang (1995) proposed a method to detect jumps and sharp cusps for signals with noise. Using the fact that the wavelet coefficients at finer resolutions show local changes more precisely, he considered intervals where wavelet coefficients were bigger than his threshold as the location of jumps or sharp cusps given at the resolution levels. He applied his method to the US stock market monthly data from 1953 to 1991. The Daubechies - 8 wavelets with reflecting boundary condition was used to compute the empirical wavelet coefficients and the results indicated that these coefficients exceed the threshold line at two locations. The locations of these large empirical wavelet coefficients were near the observation of October 1974 and October 1987, so there were local structural changes near the corresponding times. He suggested that they were caused by the recession in 1974 and the New York stock market crash in 1987.

Further, Cheng and Ogden (1997) developed three basic wavelet methods for testing for a change point in an independent sequence along with their null distributions and powers under local alternatives. A simulation study compares power of the tests, using both Haar basis as a smoother Daubechies wavelets. Because of their good time -frequency localization, among other reasons, wavelets have been proven to be useful in many applications. In particular, they are well equipped for dealing with abrupt jumps and other irregular futures on non-parametric regressions. Under the bull hypothesis of no change, the empirical wavelet coefficients will be *i.i.d* $N(0, 1)$ random variables. The heuristic underlying the development of the methods in his study is that under the alternatives, most of the empirical coefficients will still be near zero, but that a few coefficients, localized to the area of change point will exhibit significant signals. The tests derived can easily be adapted to test for jumps or other irregularities in an otherwise smooth function.

2.2.3 Time-Scale Decomposition

For economics and finance, one of the most useful properties of the wavelet approach is the ability to decompose any series into its time scale components. In financial economics, one must allow for quite different behaviour across time scales. When traders in the security market decide to buy a security, they show different behaviour according to the market conditions at different time horizons – hour, day, month, year et cetera. An effort along these lines is illustrated in Davidson *et al* (1997), who investigated the US commodity prices.

Borck (2000) discussed the need for time-scale analysis of economic time series when he brought up work on the interface between ecology and economics. The effects of periodic market closures (Brock and Klidon, 1992) also brings up the need for decomposing observed financial time series in order to better observe the range of contributing oscillations. Time-scale analysis has been used in Ramsey and Zhang (1997) to better differentiate between short (high-frequency) and longer (low-frequency) movements in the financial time series. Genacy *et al* (2002) performed various levels of wavelet decompositions of the IBM return series (N = 368) using both Haar and Daubechies - 8 wavelets. The first and second scales of wavelet coefficients show a large group rapidly fluctuating returns between observations 250 and 300. Since the IBM return series does not exhibit low-frequency oscillations, the higher scale wavelet coefficients do not indicate large variations from zero. In a similar study, Genacy *et al* (2002) looked at the validity of the IBM stock price series a Discrete Wavelet Transform (DWT) Multi Resolution Analysis (MRA). Given the choice of wavelet filter, the MRA will mimic its aesthetic characteristics. The Daubechies - 8 wavelet is relatively smooth, when compared with the Haar wavelet filters and therefore produces a smoothly varying MRA. According to the dbg MRA, the large fluctuations in volatility occur across the first three scales across trading days 230 to 299. There is a lack of volatility activity at the fourth scale, but an increase during those days in the low-frequency content (wavelet smooth). Even though the differences across scales were not pursued fully, the authors did consider different properties of the wavelet coefficients across scales and calculated a measure of the relative importance of the coefficient between scales.

In two related papers Lee (2000 and 2001) attempted to examine price and volatility spillovers effects across international stock markets via wavelet analysis. They first proposed a new testing strategy to test for spillover effects based on discrete wavelet decomposition, which is then applied to investigate the dynamics and potential interaction in international stock markets. By examining the relationship between high-frequency fluctuations in stock returns obtained from reconstruction of the data by wavelet coefficients, they investigated international transmission of news across stock markets. Lee (2000) using the data on daily stock returns of Dow Jones Industrial Average (DJIA) and of the KOSPI (Korean Composite Stock Price Index), string evidence is found for price as well as volatility spillover effects from the US stock market to the Korean counterpart but not the vice versa. Further, Lee (2001) investigated whether and to what extent the MENA (Middle East and North American) markets were integrated globally with developed stock markets such as the US, Japan and Germany using discrete wavelet decomposition. The basic finding was that price as well as volatility spillover effects from the developed stock markets to the MENA counterparts, but not vice versa. Also discussed was on the dependence of the inter-dependence of the major MENA stock markets.

Understanding the importance of potential stock market wealth effects on consumption, Cheng (2000) tried to narrow the gap between the studies done in Canada and the US. Using wavelet decomposition technique, his study revealed a rich pattern of components among consumption, income and wealth across different time-scales. At the scale of quarter year, the stock market wealth at current and one lagged period had relatively stronger effect on the consumption than the current disposable income and lagged non-human wealth excluding equity. As the time - scale increases to 2 years, the disposable income dominated the determinants of consumption, while the consumption moved ahead of the income and wealth, displaying a feature of expectations.

In two related papers (Ramsey and Lampart, 1998 a, b), a very different approach was taken to the use of scale in wavelet analysis. In both the papers, the concept was that the relationship between two variables might well vary across scales. The objective in the two papers were to examine this issue on the context of two relationships; one was that between personal income and consumption and the other was that between money and GDP. Wavelets were used to provide both an orthogonal time-scale decomposition of the data and a non-parametric representation of each individual time series. At each scale a regression was run between consumption and income, and the results were compared across scales.

2.2.4 Density Estimation

The estimation of density functions has been a tradition in economics and finance and the debate concerning the distribution of income and wealth in particular is of great concern. Further, the income distributions frequently exhibit anomalous local behaviour that is best analyzed in the context of wavelets. Hardle *et al* (1998) illustrate the benefits of using a wavelet approach to the estimation of densities relative to the standard kernel and smoothing procedures. A recent study and important line of research in finance is the analysis of fractionally integrated models. Jensen (1999, a, b) in a recent series of the fractional differencing coefficients. Jensen had several reasons for choosing wavelets as a basis for the analysis of the long memory processes defined by the ARFIMA models. The major reasons were that the wavelet based estimations were far less intensive to calculate than the regular exact maximum likelihood estimation (MLE), the estimates were much more robust to modeling errors and the estimates were robust to not knowing the mean of the process. A key element was the computational gains achieved using wavelet coefficients, especially for the longer time-scale components of the process.

Further, in another paper Jensen (2000) estimated the time varying long memory parameter of stochastic volatility models, by introducing a semi parametric, ordinary least squares estimator based on the log-linear relationship between the local variance of the maximum overlap discrete wavelet transforms wavelet coefficients and the wavelet scales. He applied his estimator to a year's worth of tick-by-tick Deutchemark - US dollar return data measured at five minutes interval and found that the long memory parameter to be positive over a larger percentage of the sample. The circumstances behind those periods where the long memory parameter was negative were associated with prescheduled new announcements and unexpected market crashes or political upheavals. Moreover, it has been hypothesized that long memory is present in daily stock price volatility (Lobato and Savin, 1998). Genacy *et al* (2002) defined the volatility as the absolute values of the returns for the IBM series for the estimation of long memory parameter. Applying the methodology of fractional difference processes to a partial discrete wavelet transform using the db_4 wavelet filter of the volatility series, they concluded that the IBM volatility series may be adequately modeled via a stationary long memory time series.

2.3 Concluding Remarks

Keeping in mind various aspects of wavelets and spectral methods that are most likely to be useful in financial application, the literature on the application of those techniques in this filed is reviewed. The chapter has presented a review of some select works relating to the application of spectral methods and wavelets in stock markets. The spectral approach considered here are to test for random walk, to trace the lead lag relationship, analysis of capital asset pricing models et cetera. The chapter has reviewed the ability of spectral analysis to describe the hidden periodicities of data, to decompose variance into various components et cetera. Especially, studies relating to power spectrum, cross-spectrum and bispectrum receive due attention.

The robustness of wavelet analysis lies to erroneous assumptions, flexibility of regression fit, its ability to handle complex relationships, efficiency of the estimators to be able to make useful distinctions on a few data points, and simplicity of implementation. The most important property of wavelets for finance is decomposition by time-scale. Financial systems, like many other systems, contain variables that operate on a variety of time-scales simultaneously so that the relationship between variables may well differ across time-scales. Moreover, it is apparent that wavelets are particularly well adapted to the statistical analysis of economic and financial data. The potential benefits of wavelets are for greater than mere application of new techniques.

Chapter III
The Spectral Domain Approach and its Applicability to Indian Stock Market

3.0 Introduction

In spectral analysis interest is centered on the contributions made by various periodic components in the observed series. Such components do not need to be identified with regular cycles. In fact, regular (time invariant) cycles are unusual in economic time series and when talking about periodic components, what is meant is a tendency towards cyclical movements around a particular frequency. A frequency domain analysis may give important insights, which would not be apparent in an analysis in the time domain only. Furthermore, the estimation of the spectrum raises a number issues that are not encountered in the estimation of the auto covariance function in the time domain (Jacquier, 1992). The standard spectral analysis is univariate. In order to study the relationship between two series in the frequency domain, the characteristics of the cross-spectrum are analyzed. Here, in this chapter we apply the various aspects of spectral domain analysis to four stock price indices in India.

The chapter is organized as follows: sections 3.1 to 3.4 devoted to a complete and thorough discussion on spectral methods and its various approaches. The estimation procedure of spectral methods is presented in section 3.5. We have undertaken the empirical applications of spectral methods in stock prices in section 3.6 followed by concluding remarks in section 3.7.

3.1 Time Series Analysis in the Time Domain

We need to introduce some notation and definition from time series analysis in the time domain. Let X_t be a univariate stochastic process with T observations. X_t is said to be stationary (weakly or covariance stationary) if its mean, variance and autocovariances are constant over time.

$$E(X_t) = \mu$$

$$var\,(X_t) = E\,[(X_t - \mu)^2] = \gamma_0$$

$$autocovar\ (X_t - X_{t-\mu}) = E[(X_t - \mu)(X_{t-\tau} - \mu)]$$

$$= \gamma_\tau \quad \tau = 0, 1, 2, ..., T.$$

Where E is the expectations operator.

The autocorrelation between $X_t and X_{t-\tau}$ *is* defined as

$$autocorr\ (X_t, X_{t-\tau}) = \rho(\tau) = \gamma_\tau / \gamma_0 = auto\,\mathrm{cov}\ ar(X_t, X_{t-\tau}) / \ \mathrm{var}(X_t).$$

Notice that $\rho(0) = 1$ and $\rho(\tau) = p$ (-r).

3.1.1 Cycles

A stochastic process or economic time series is periodic if there exits an integer $x > 0$ such that

$$X_t = X_t + \mathrm{r}.$$

The smallest r, for which this holds, is called the period and it is the number of periods from peak to peak or from trough to trough of a periodic time series. The trigonometric function

$$X = cos\,(\omega)$$

is a periodic function as is the $sin(\omega)$ function where $\omega\ e\ [-n, \pi]$ is the angular frequency measured in radians.

Since a circle with unit radius has *2n* in circumference, one cycle corresponds to 2π measured in radians and *X* goes through its full components of values as w goes from zero to *2n.* This pattern is repeated such that for any integer *k, X* is a periodic function with period *2n.*

$$cos\,\{co)\text{-}\ cos\,(\omega + k2\pi)$$

A Second order autoregressive process with complex roots is **also a periodic** function and has the solution

$$X_t = Ar^t\,cos\,(\omega t + \theta) \ldots \qquad (3.1)$$

where Ar^t is the amplitude, θ **is** the phase which measures the shift in time **and** *t* is time. The AR(2) process has dampened oscillations and is stationary , if

$$\lim Ar^{t} = 0$$

which is the case if r < / (r is always non-negative) undamped oscillations when *r* - *1* and explosive oscillations when *r* > *1*.

The period (or wave length) of a cycle is the minimum number of periods between two peaks of the cycle and is given by

$$\text{period} = 2\pi / \omega$$

for quarterly data, if $\omega = n/4$ in (3.1) then the period of the cycle is 8 quarters or 2 years.

Notice that the highest frequency about which we have direct information is π. This is known as the Nyquist frequency. With quarterly data we cannot detect cycles with a higher frequency than two quarters, ω = *n*. This is related to the problem of time aggregation or sampling. The process of converting a continuous time signal to a discrete time sequence of numbers such as quarterly national accounts is called sampling in the signal processing literature or time aggregation. The collection of quarterly data for national accounts leads to distortions of the underlying continuous time stochastic process, of which aliasing is the most serious problem[1]. If the actual times series contains cycles at a higher frequency than quarterly, say monthly, then these will be imputed to cycles with frequency between zero and *n*. This is known as aliasing,

A single series may be said to posses a cycle if its covariogram (the plot of autocovariance against the time lag) or the correlogram (the plot of autocorrelation function) is characterized by (damped) oscillations or, in the frequency domain, if there is a clear peak in the spectrum at a particular frequency.

[1] **See Porat (1997) for detailed treatment of the problem of sampling and reconstruction.**

3.2 The Autocovariance Generating Function

The dynamic or cyclical pattern of an economic time series is summarized by the autocovariance function or autocorrelation function, which contains all the information about the time dependence of individual observations in the time series, Analysis of the autocovariances form the basis for the analysis of the cyclical properties of time series both in the time domain and in the frequency domain.

A generating function is a way of recording the information of some sequence. Consider the sequence $a_0, a_1, a_2, \ldots$ of possibly infinite length, then the generating function is defined as

$$a(Z) = \sum_{j=0}^{\infty} a_j Z^j \quad \ldots \quad (3.2)$$

The quantity Z does not necessarily have any interpretation and may be considered as the carrier of information in the sequence. The concept of a generating function is useful because it can be manipulated in simpler ways than the whole sequence a_j. It is seen that the polynomial in the lag-operator is a generating function with $Z = L$. Let X_t *be* a real valued stochastic process with variance equal to γ_0 and the τ^{th} autocovariance equal to γ_τ. Let γ_τ be the sequence of autocovariances. If the sequence of autocovariances is absolutely summable, then the autocovariance generating function is given by

$$g_x(Z) = \sum_{\tau=-\infty}^{\infty} \gamma_\tau Z^\tau \quad \ldots (3.3)$$

where the argument of the function, Z, is a complex scalar. It is seen that replacing Z with the lag operator, the sequence of autocovariances can be formulated as

$$\gamma_\tau = \sum_{\tau=-\infty}^{\infty} \gamma_0 L^\tau$$

Of particular interest as an argument for the autocovariance generating function is any value of Z that lies on the complex unit circle.

$$Z=\cos(\omega)-i\sin(\omega)=e^{-i\omega} \quad ...(3.4)$$

where $i=\sqrt{-1}$ and *co* is the radian angle that Z makes with the real axis.

3.3 Time Series Analysis in the Spectral Domain

3.3.1 The Spectral Representation

There is a general result known as the representation theorem which says that any covariance stochastic process *x*, can be given an alternative representation expressed in terms of an infinite weighted sum of (orthogonal) periodic functions of the form $cos(\omega t)$ and $sin(\omega t)$, where *co* is the angular frequency measured in radians, which may take any value in the range$[-\pi\ ,\ \pi]$. Such a representation is called the spectral representation or Cramer's representation.

$$x_t=\mu+\int_0^{n}\alpha(\omega)\cos(\omega t)d\omega+\int_0^{n}\delta(\omega).\sin(\omega t)d\omega \quad ... (3.5)$$

The goal of spectral analysis is to determine how important cycles of different frequencies are in accounting for the behavior of x_t.

3.3.2 The Spectrum

The autocovariance generating function was defined above as

$$g_x(Z)=\sum_{\tau=-\infty}\gamma_\tau Z^\tau$$

where the argument of the function, Z, is a complex scalar and γ_τ are the autocovariances. If we replace Z with the value $e^{-i\omega}$ and divide by 2π, the resulting function of ω is called the population **spectrum** or power spectrum (strictly: the power spectral density **function**)[2] of x_t.

$$f(\omega)=\frac{1}{2\pi}g_x(e^{-i\omega})=\frac{1}{2\pi}.\sum_{\tau=-\infty}^{\infty}\gamma_\tau e^{-i\omega\tau},$$

$$-\pi<\omega<\pi,$$

or it can be written as

$$\begin{aligned} f(\omega)&=\frac{1}{2\pi}.\sum_{\tau=-\infty}^{\infty}\gamma_\tau e^{-i\omega\tau} \\ &=\frac{1}{2\pi}(\gamma_0+\sum_{\tau=1}^{\infty}\gamma_\tau(e^{i\omega\tau}+e^{-i\omega\tau})) \\ &=\frac{1}{2\pi}(\gamma_0+2\sum_{\tau=1}^{\infty}\gamma_\tau\cos(\omega_\tau)) \quad ...(3.6) \end{aligned}$$

Assuming that the sequence of autocovariances γ_t is absolutely summable, the power spectrum exists. The spectrum has a number of important properties:

1. $f(\omega)$ is a non negative, continuous, real valued function of *co.*
2. If the γ_τ are autocovariances of a covariance stationary stochastic process, then the spectrum will be non-negative for all values of *co.*
3. Since *cos* $(\omega\tau)$ = *cos {-cot),* the spectrum is symmetric around *co* = 0: $f(\omega)=/(-\omega)$.
4. Since *cos* $(\omega\tau+2\pi k\tau)$ — *cos* $(\omega\tau)$ for any integer r and k, the spectrum is a periodic function of *co:* $f(\omega)=f(\omega+2\pi k)$: knowledge of the value of $f(\omega)$ for any value of *co* $\in[0,\pi]$ implies knowledge *off(co)* for any value of *co.*

The spectrum is a kind of covariance generating function and it can be viewed as a device for decomposing the variance of a series by frequency. The area under the spectrum over the interval $[-\pi,\pi]$, the integral from $-\pi$ to π, is equal to the variance of

[2] The spectrum is sometimes standardized by dividing by γ_0 or by replacing the autocovariances by the corresponding autocorrelations.

$$\int_{-n}^{n} f(\omega)d\omega = \int_{0}^{\pi} 2f(\omega)d\omega = \gamma_0 \quad ... (3.7)$$

This sheds some light on the interpretation of the variance in the time domain, since the variance is the sum of the spectra over all frequencies from $-\pi$ to n. The variance of a time series is generally distributed unevenly over frequencies where for example the growth component, the business cycle component or the seasonal components contribute differently to the variance. A more general function of (3.7) implies that

$$\int^{n} f(\omega)e^{i\omega\tau}d\omega = \int^{n} f(\omega)\cos(\omega\tau)d\omega \; \gamma_\tau \quad ... (3.8)$$

There exists a bounded non decreasing function $F(\omega)$ called the spectral distribution function such that

$$g_x(\tau) = \int e^{i\omega\tau}F(d\omega) \quad ...(3.9)$$

Where $g_x(\tau)$ is the autocovariance generating function. The spectral distribution function determines a measure *F (A)* called the spectral distribution of the time series where $f(\omega)$ - $F\,'(\omega)$ and over the mass of frequencies *A*.

$$F(A) = \int_{A} f(\omega)d\omega \quad ...(3.10)$$

The spectrum of a stochastic process contains the same information as the autocovariance generating function since it is simply a linear combination of autocovariances. Looking at the graph of the spectrum of a series, a peak can be interpreted as indicating a "cyclical" component in the series having almost constant period. It is worth considering a number of cases.

(1) If the estimated spectrum is flat without any peaks and without any clear tendency to follow a smooth curve, then the time series is close to a white noise. The white noise process $x_t = \varepsilon_t$ has variance $\gamma_0 = \sigma_\varepsilon^2$ and autocovariance $\gamma_\tau = 0$ for $T \neq 0$. The covariance generating function is $g_x(z) - \sigma_\varepsilon^2$ and the spectrum is a constant: $f(e^{-i\omega}) = \frac{1}{2n}\sigma_\varepsilon^{\,2}$. *A* plot of the spectrum of a white

noise process is flat and equals $\frac{1}{2\pi\sigma_{\varepsilon}^{2}}$ at all frequencies. This is the reason behind the term "white noise" like white light; it consists of an infinite number of frequencies each having equal weight.

(2) If the time series contains a clear cyclical component at some frequency as a deterministic second order difference equation with complex roots, the spectrum will have a tall, narrow peak, having finite area.

(3) A time series with an important trend component will have strong peak at the very low frequencies, known as the "typical spectral shape" of Granger (1966). If the time series contains an autoregressive unit root, the peak will be infinite at zero frequency.

3.3.3 Fourier Analysis

The trigonometric functions $cos(\omega)$ and $sin(\omega)$ are periodic functions with *2n* and any linear combination of these functions are also periodic with period *2n.* If we consider any function $P(\omega)$ of the form

$$P(\omega) = \sum_{t=0} (a_t \cos(\omega t) + b_t \quad \sin(\omega t)) \dots (3.11)$$

where a_t and b_t are arbitrary sequences of constants subject to the constraint that the infinite series on the right hand side of (3.11) converges for all ω, then $P(\omega)$ must always be a periodic function with period *2n.* The trigonometric functions *cos* and *sin* may be considered the fundamental building blocks of periodic functions. The fundamental idea in Fourier analysis is that any deterministic function of ω can be approximated by an infinite sum of trigonometric functions called the Fourier series representation, e.g. Priestly (1981).

Let y_t be a sequence of real or complex numbers with $\sum |y_t| < \infty$, then there exists a complex - valued function, $P(\omega)$, called the Fourier transform belonging to the interval $[-\pi, n]$ such that

$$P(\omega)= \int_{-\infty}^{\infty} y_t e^{-i\omega t} = \int_{-\infty}^{\infty} y_t \cos(\omega t) - i\sin(\omega t)$$

The power spectrum is the Fourier transform of the **covariogram and** is as such a kind of covariance generating function. The inverse Fourier transform is given by

$$y_t = \frac{1}{2\pi} \int_{\pi} P(\omega) e^{-i\omega t} d\omega$$

If the frequencies take values in a discrete state space with T **equally** spaced valued in the range *[0,* 2π], where T is the number of observations in the signal, y_t, so the sampling interval is $2n/T$, using the frequencies

$$\omega(k) = \text{—,} \qquad 0 \le k \le T-1.$$

the result is the discrete Fourier transform (DFT)

$$P(\omega) = \sum_{t=0}^{T-1} y_t e^{\frac{-i2\pi kt}{T}}$$

and the inverse discrete Fourier transform

$$y_t = \frac{1}{T} \sum_{k=0}^{T-1} P(\omega) e^{\frac{-2\pi kt}{T}}, \quad 0 \le t \le T-1.$$

3.4 Multivariate Spectral Analysis

3.4.1 Vector Autocovariance Generating Functions

Let X, be a stationary stochastic n-dimensional vector process with mean vector $E(X_t)$ μ and the τ*'th* autocovariance matrix given by

$$\Gamma(\tau) = E\,[(X_t - \mu)\,(X_{t-\tau} - \mu)]$$

Notice that, by contrast to the univariate case, $\Gamma(\tau)$ and $\Gamma(-\tau)$ are not in general identical. However, the ij^{th} element of $\Gamma(\tau)$ is identical to *the* ji^{th} element of $\Gamma(-\tau)$ SO that

$$\Gamma(\tau) = \Gamma(-\tau), \qquad \tau = 1, 2, \qquad ...(3.12)$$

If the sequence of matrix au toco variances Γ_τ is absolutely summable and if z is a complex scalar. The matrix autocovariance generating function, called the cross-covariance generating function of X_t is given by

$$F_x(Z)=\sum_{\tau=-\infty}^{\infty}\Gamma(\tau)Z^{\tau} \quad ...(3.13)$$

where $F_x(z)$ is an $(n \times n)$ dimensional matrix of complex numbers.

Let X_t be vector of covariance stationary stochastic processes. The vector processes are said to be jointly covariance stationary if the cross covariance $\Gamma(\tau)= E[(X_t - \mu)(X_{t-\tau}-\mu)]$ depends only on r and not on time t.

Consider the case, where $n = 2$, and

$$X=\begin{bmatrix} x_t \\ y_t \end{bmatrix}.$$

Define the cross covariance between the two covariance stationary stochastic process, x_t and y_t as

$$\gamma_{xy}(\tau)=E[(x_t-E(x_t))(y_{t-\tau}-E(y_t))]$$

written out fully

$$\Gamma_x(\tau)=E\begin{bmatrix} (x_t-\mu_x)(x_{t-\tau}-\mu_x)(x_t-\mu_x)(y_{t-\tau}-\mu_y) \\ (y_t-\mu_y)(x_{t-\tau}-\mu_x)(y_t-\mu_y)(y_{t-\tau}-\mu_y) \end{bmatrix}$$

$$=\begin{bmatrix} \gamma_{xx} & \gamma_{xy} \\ \gamma_{yx} & \gamma_{yy} \end{bmatrix} \quad ... \quad (3.14)$$

3.4.2 The Multivariate Spectrum

Evaluating the cross-covariance generating function at the value $Z = e^{i\omega}$ and divide by 2π, we have the multivariate spectrum (strictly cross spectral density function).

$$f_x(\omega)=\frac{1}{2\pi}\sum_{\tau=-\infty}^{\infty}\Gamma(\tau)\,e^{-i\omega\tau} \quad ... \quad (3.15)$$

where $f_x(\omega)$ is an $(n \times n)$ matrix.

The diagonal elements are the power spectrum of the individual processes, which are real valued and nonnegative for all *co.* The off diagonal elements are the cross spectrum. Unlike the power spectrum the cross spectrum is in general a complex number at each frequency, a consequence of the fact that it is in general not symmetric. The ji^{t} element of $\Gamma(\tau)$ is the complex conjugate of the ij^{th} element.

As for the univariate case, the multivariate power spectrum is integrated to the autocovariances

$$\int_{-\pi}^{\pi} f_x(\omega)\, e^{i\omega\tau}\, d\omega = \Gamma(\tau)$$

and for $\tau = 0$

$$\int_{-\pi}^{\pi} f_x(\omega) d\omega = \Gamma(0)$$

The area under multivariate power spectrum is the conditional variance-covariance matrix of x_t.

Consider the case $n = 2$, where $X = [x_t\ y_t]'$, where x_t and y_t are two jointly stationary stochastic processes with continuous power spectra f_x (co) and f_y (co), then the cross spectrum between x_t and y_t is defined as

$$f_{xy}(\omega) = \frac{1}{2\pi} \sum_{\tau=-\infty}^{\infty} \gamma_{yx}(\tau) e^{-i\omega\tau}$$

or written out fully

$$f_x(\omega) = \begin{bmatrix} f_{xx}(\omega) & f_{xy}(\omega) \\ f_{yx}(\omega) & f_{yy}(\omega) \end{bmatrix}$$

$$\frac{}{2\pi} \begin{bmatrix} \sum_{\tau=-\infty}^{\infty} \gamma_{xx}(\tau)\, e^{-i\omega\tau} & \sum_{\tau=-\infty}^{\infty} \gamma_{xy}(\tau)\, e^{-i\omega\tau} \\ \sum_{\tau=-\infty}^{\infty} \gamma_{yx}(\tau)\, e^{-i\omega\tau} & \sum_{\tau=-\infty}^{\infty} \gamma_{yy}(\tau)\, e^{-i\omega\tau} \end{bmatrix}$$

Using the trigonometric function $e^{i\omega\tau} = \cos(\omega\tau)\ -i \sin(\omega\tau)$, the spectra and cross spectra can be reformulated as

$$\sum_{\tau=-\infty}^{\infty} \gamma_{xx}(\tau)\, e^{-i\omega\tau} = \sum_{\tau=-\infty}^{\infty} \gamma_{xx}(\tau)\{\cos(\omega\tau) - i\sin(\omega\tau)\}$$

$$\sum_{\tau=-\infty}^{\infty} \gamma_{xy}(\tau)\, e^{-i\omega\tau} = \sum_{\tau=-\infty}^{\infty} \gamma_{xy}(\tau)\{\cos(\omega\tau) - i\sin(\omega\tau)\}$$

$$\sum_{\tau=-\infty}^{\infty} \gamma_{yx}(\tau)\, e^{-i\omega\tau} = \sum_{\tau=-\infty}^{\infty} \gamma_{yxx}(\tau)\{\cos(\omega\tau) - i\sin(\omega\tau)\}$$

$$\sum_{\tau=-\infty}^{\infty} \gamma_{yy}(\tau)\, e^{-i\omega\tau} = \sum_{\tau=-\infty}^{\infty} \gamma_{yy}(\tau)\{\cos(\omega\tau) - i\sin(\omega\tau)\}$$

Using (3.12), $sin\ (\omega\tau) = -sin\ (\omega\tau)$, and $sin\ (0) = 0$, we have,

$$f_{xx}(\omega) \qquad f_{xy}(\omega)$$

with

$$f_{xx}(\omega) = \frac{1}{2\pi} \sum_{\tau=-\infty}^{\infty} \gamma_{xx}(\tau)\, \cos(\omega\tau)$$

$$f_{xy}(\omega) = \frac{1}{2\pi} \sum_{\tau=-\infty}^{\infty} \gamma_{xy}(\tau)\, \{\cos(\omega\tau) - i\sin(\omega\tau)\}$$

$$f_{yx}(\omega) = \frac{1}{2\pi} \sum_{\tau=-\infty}^{\infty} \gamma_{yx}(\tau)\, \{\cos(\omega\tau) - i\sin(\omega\tau)\}$$

$$f_{yy}(\omega) = \frac{1}{2\pi} \sum_{\tau=-\infty}^{\infty} \gamma_{yy}(\tau)\, \cos(\omega\tau)$$

It is seen that the imaginary components disappear from the diagonal terms. The spectrum of x_t and of y_t, denoted by $f_{xx}(\omega)$ and $f_{yy}(\omega)$, are real valued.

3.4.3 A Polar Decomposition

The cross spectrum $f_{xy}(\omega)$ is a complex quantity and can be reformulated in terms of two real quantities, the cospectrum, $co(\omega)$ and the quadrature spectrum, $qu(\omega)$

$$f_{xy}(\omega) = c+_{xy} i.qu_{xy}(\omega) \qquad \ldots \qquad (3.16)$$

The cospectrum is given by

$$co_{xy}(\omega) = \frac{1}{2\pi} \sum_{\tau=-\infty}^{\infty} \gamma_{xy}(\tau)\, \cos(\omega\tau) \qquad \ldots (3.17)$$

where

$$co_{xy}(\omega) = co_{yx}(\omega)$$

$$co_{xy}(\omega) = co_{xy}(-\omega)$$

Using (3.12) and cos $(\omega\tau)$ = *cos* $(-\omega\tau)$, and the quadrature spectrum is given by

$$qu_{xy}(\omega) = \frac{1}{2\pi}\sum_{\tau=-\infty}^{\infty}\gamma_{xy}(\tau)\sin(\omega\tau) \qquad \dots(3.18)$$

where

$$qu_{xy}(\omega) = -qu_{yx}(\omega)$$

$$qu_{xy}(\omega) = -qu_{xy}(\omega)$$

The cospectrum between x_t and y_t at frequency ω has the interpretation of the covariance between x_t and y_t that is attributable to cycles with frequency *co*. The cospectrum can be positive over some frequencies and negative over others since the autocovariances can be both positive and negative.

The quadrature spectrum from y_t to x_t at frequency *co* is proportional to the portion of the covariance between y_t and x_t due to cycles of frequency *co*. Cycles of frequency *co* may be quite important for both x_t and y_t individually as reflected by large values for $f_x(\omega)$ and $f_y(\omega)$ yet fail to produce much contemporaneous covariance between the variables because at any given date the two series are in different phases of the cycle. The quadrature spectrum looks for the evidence of out-of-phase cycles.

3.4.4 Gain, Phase and Coherence

The cross spectrum between x_t and y_t is given by

$$f_{xy}(\omega) = \sum_{\tau=-\infty}^{\infty}\gamma_{xy}(\tau)e^{-i\omega\tau} \qquad \dots(3.19)$$

The gain is defined as

$$G(\omega) = \frac{\left|f_{xy}(\omega)\right|}{f_{yy}(\omega)} \qquad \dots(3.20)$$

and the phase is

$$ph(\omega) = \tan^{-1}\left(\frac{qu(\omega)}{co(\omega)}\right) \quad ...(3.21)$$

The gain is seem to be the ratio between the cross spectrum between x_t and y_t to the spectrum of y, which is the regression coefficient at the *co* frequency of x_t series on the *co* frequency of y_t series and so it can be interpreted as an expression of how the amplitude of the y_t series is multiplied in contributing to the amplitude in the x_t series. The gain of a time series measures - at the specified frequency *co* - the increase in amplitude of one series over the other

The phase gives the lead of x over y at frequency *co*. The phase diagram, the plot of the phase against the frequency gives information about the lag relationship between two series. If the phase is a straight line over some frequency band, the slope is equal to the time lag and thus tells which series is leading and by how many periods. For a given $ph(\omega)$, the shift in time is $ph(\omega)/\omega$. Further more, the concept of phase is closely connected to the concept of Wiener-Granjer causality

The coherence between two time series is defined as

$$coh(\omega) = \frac{|f_{xy}(\omega)|^2}{f_{yy}(\omega) f_{xx}(\omega)} \quad ...(3.22)$$

The coherence is a real valued function where $0 < coh(\omega) < 1$. The coherence between x_t and y_t are jointly influenced by cycles at frequency *co*. From (3.22) it is seen that the coherence is the ratio of the squared cross spectrum to the product of the two spectra. This is analogue to the well known squared coefficient of correlation.

The coherence between two or more time series can be used to measure the extent to which multiple time series move toghther and a graph of the coherence shows how strongly correlated two time series are at different frequencies.

3.5 Estimating the Spectrum

In empirical research, we cannot compute the spectrum analytically as above and we have to rely on econometric methods for estimating the power spectrum from observed economic time series. Given an observed time series x_t, how might $f_x(\omega)$ be estimated ? Two methods are available:

(1) Estimation of the sample periodogram. The periodogram can either be estimated directly in the time domain or by using an algorithm known as the Fast Fourier Transform (FFT). The sample periodogram is not a consistent estimator of the power spectrum but we can get a consistent estimator by smoothing the periodogram using spectral windows. These estimation methods are non-parametric in the sense that they do not assume a specific parametric model for the observed series.

(2) If we know that the true data generating process is an autoregressive process or if it can be approximated by an AR-process and any linear stochastic process can be approximated by an AR-process of finite order, then the parameters of the AR-process can be estimated by OLS and the theoretical spectrum can be computed from these parameter estimates. Spectral estimation based on this technique is parametric and is called autoregressive spectral estimation. We have used the nonparametric estimation in this study.

3.5.1 Estimating the Sample Periodogram

The sample periodogram corresponding to autocovaraince (3.3) can be expressed as

$$\hat{f}_x(\omega) = \frac{1}{2\pi}(\lambda_0 + 2\sum_{\tau=1}^{t-1}\hat{\gamma}\cos(\omega\tau)\ ),\quad 0 \le \omega \le \pi \quad \dots \quad (3.23)$$

where γ_τ is the $\tau'th$ sample autocovariance and (3.23) is the sample spectral density.

Fuller (1976) shows that when $\omega \neq 0$ and T is large, then

$$\frac{2\hat{f}_x(\omega)}{f_x(\omega)} \approx \chi^2(2)$$

where $f_x(\omega)$ is the population spectrum and

$$E(\frac{2\hat{f}_x(\omega)}{f_x(\omega)}) \cong 2 \Rightarrow E\ (\hat{f}_x(\omega)) \cong f_x(\omega)$$

and the estimate is approximately unbiased. However, the variance of the estimate does not depend on T and so it does not give a consistent estimator of the spectrum at a given frequency (Hamilton, 1994). A further disadvantage with estimating the sample periodogram is that the sample spectra density has a jagged and irregular appearance, even when estimating a white noise (Harvey, 1993). This can be remedied by the technique of spectrum averaging using window. This gives for the spectral density the consistent estimator

$$\hat{f}_x(\omega) = \sum_{\tau=-m/2}^{m/2} \lambda_k \hat{f}_x(\omega_{i+\tau}) \text{.........}(3.24)$$

where the λ_i's are spectral window weights. Two commonly used sets of weights are:

(i) Parzen weights or Parzen window

$$\begin{aligned} \lambda_k &= 1 - 6k^2 (1-|k|/m)/m^2, & 0 \le |k| \le m/2 \\ &= 2(1-|k|/m)^3, & m/2 \le |k| \le m \\ &= 0 & |k| \ge m \end{aligned}$$

(ii) the Tukey-Hanning weights or Tukey-Hanning window

$$\begin{aligned} \lambda_k &= \tfrac{1}{2}(1+\cos \pi/m), & |k| < m \\ &= 0 & |k| \ge m \end{aligned}$$

We use Parzen weights because it has a wider window and the leakage is less between estimates for non-neighboring frequency bonds.

3.5.2 Tapering

The idea of tapering a stationary time series, in addition to smoothing in the frequency domain, has become widely applied in practical spectrum estimation (Walden, 1990). Tapering has the benefit of reducing window leakage, but at the expense of an increase in the variance of the spectrum estimator. A taper is applied to the unpadded part of a series prior to taking the Fourier transform. The taper reduces the window leakage by scaling the ends of the data so they merge smoothly with the zeros on either side. The taper we used here is the cosine taper, defined as

$$b(t) = \begin{cases} 0.5[1-\cos(\pi t/m)] & 1 \le t \le m \\ 1 & m+1 \le t \le N-m \\ 0.5[1-\cos(\pi(N-t+1)/m)] & N-m+1 \le t \le N \end{cases}$$

where, m is the number of affected entries at each end.

Estimation of the spectrum corresponding to a theoretical $f(\omega)$ often uses as it is impossible to estimate over all values of ω, $0 < \omega < n$, a finite set of values denoted $\{\omega_j\}$, $j = 0, 1, ..., n$. *A* very commonly used set of values is equispaced set defined by

$$\omega_j = \pi_j / m.$$

Assuming n data points are available, x_t , $t = 0, l, ..., n$ then the choice $m < n$ is quite subjective. If m is too small, the graph of the spectrum is too coarse and so bias results. If m is too large, the graph of the spectrum has very large variance and so is blurred. Conventional wisdom suggests $m = n/5$ to $n/6$ and $< n/3$, and for n, it suggests around 150 although smaller values may give a good estimated depending on the complexity of the spectrum.

3.6 Empirical Results

As discussed in previous section, spectral analysis decomposes a stationary time series into a set of frequency bands in terms of its contribution to the overall variance (termed power) of the series. The power spectrum is looked upon as the graph of variance against frequency. If a band of frequencies is important, the spectrum will exhibit a relative peak in this frequency band. On the other hand, if the spectrum is flat (i.e. a horizontal straight line parallel to X-axis) indicating that every component is present in equal amount, the interpretation is that the series is merely a sequence of uncorrelated readings which is purely random or technically, a white noise series.

Since, spectral analysis is based upon the assumption of stationarity and there are various formal mathematical definitions of stationarity ranging from complete stationarity to weak and wide sense of stationarity, our definition and usage **here** correspond to wide sense stationarity, which implies that mean and variance are independent of time. The fact that a portion of trend remained would first negate the estimates to some extent because the stationarity assumption would be violated. But more important, due to a property of the estimating procedure known as ' leakage*, this trend portion in the spectrum would tend to swamp the low frequencies in which the long cycles reside. Thus, since the trend component will usually contribute the largest portion of the total variance of the series, the value of a spectrum at the zero frequency band will be large and the leakage effect will make it likely that the estimated spectrum will be quickly yet smoothly decreasing for the next few frequency bands. One of the methods of eliminating trend in data is to do a transformation using log first differences which has been used in present study while transforming stock prices to return series (see Appendix A). From other points of view also, this transformation is justified from stationarity test shown in table 3.1.

The relationship between a pair of series is analyzed by use of the cross-spectrum. There are two basic diagrams of importance, viz. the coherence and phase diagrams. The former records the estimate of the square of the correlation coefficients between corresponding frequency components. A noteworthy feature is that the coherence between two series may attain a peak or be high at a certain cyclical frequency even though the spectrum of the neither series peaks, or is high, at this frequency. The phase diagram provides evidence of time lags between components but is generally more difficult to interpret. This is partly because the significance of this diagram varies with the corresponding values of the coherence diagram. It is clearly worthless discussing the time-lag between two unrelated or hardly related components. In simple terms, if the phase diagram appears to lie about a straight line over a region where coherence is reasonably high, this can be taken to indicate a simple time lag between the two series for those components, the extent of the lag - lead depending upon the slope of the line.

Table 3.1 reports the summary statistics for all the four return series studied. The values of mean and standard deviation are almost similar for all cases. All the return series show similar nature of the overall variance. In all the cases, the excess kurtosis and skewness measures are indicative of evidence against normal distribution. It is important that the data and thus the Fourier transforms should be independent of time. Therefore, Augmented Dickey-Fuller (ADF) test for unit roots of the returns data are performed[3]. Absolute value of all the ADF statistics are greater than the MacKinnon critical values at conventional significance level, indicating the rejection of the null hypothesis of unit root for all of the return series.

The plots of the power spectrums for all the four indices considered are shown in figures 3.1 to 3.4. For each return series the power spectrum plots are divided into four parts (a, b, c, d), where 'a' shows the plots of simple estimated power spectrum, 'b' shows plot of the tapered power spectrums, 'c' is the log power spectrum and finally 'd' refers to log power spectra with tapering. A mere visualization of the plots of the power spectrum against frequency for all the indices shows very similar nature due to overall equal variance in the return series. The spectrum at the zero frequency fails to indicate trended return series. One important thing to note here is that the interpretation of low frequency range is difficult when compared with high frequency range. At low frequencies one is actually estimating with a less number of repetitions of a cycle than at a high frequency range. It is suggested by Granger that in economic time series, generally low frequencies are considerably more important than high frequencies, and plotting on a log scale allows all points to be shown conveniently. Granger and Hatanaka (1964) opine that data having a length of at least seven times that of the largest periodic cycle are needed for a proper determination of amplitude.

The largest periodic component reported in figures 3.1(a) to 3.1(d) for BSE Sensex return series correspond to a cycle of 244 days with a frequency of 0.0041. The amplitude at this frequency is realistic since the number of data points used in the estimation is at least 10 times the length of this periodic component. In every case the resulting spectrum is quite flatter over the whole frequency range apart from a few

[3] To ensure white noise residuals the Dickey-Fuller tests are conducted with a general model including a constant, trend and four lags of the dependent variable.

exceptional frequencies, which will be discussed below. In figure 3.1 (a) we find some peaks at frequency 0.0041, 0.041, 0.068, 0.171, 0.228, 0.281, 0.342 and 0.453 that correspond to around 244, 25, **14.5,** 5.8, 4, 3.5, 3, and 2 days respectively. The frequency 0.0041 corresponds to around **244** days on about **one** year. The peak at frequency 0.171 corresponds to 5.8 days i.e. for about **one week in the stock market.** The peaks at frequency 0.04 corresponds to 25 days, which shows **around a month** in the stock market. After smoothing the Sensex return series by tapering barring **two** frequencies, all others are almost flat. These two frequencies are 0.068 and 0.453, which are present there in spectrum without tapering. The log power spectrum of Sensex return series without tapering captures some periodic cycles present there in first spectrum. Again after smoothing the log power spectrum it is able to capture only two peaks around 14.5 and 2 days corresponding to frequencies 0.068 and 0.453. As a whole, we find for all the four cases discussed above power spectra are almost flat. Though the log spectra show power both at low frequencies and at relatively high frequencies, there is no evidence of significant periodic cycles in the movement of Sensex, which is in support of the assertion of the random walk hypothesis (RWH) or weak form of efficient market hypothesis (EMH)[4].

The power spectrum plots for National index return series are presented in figures 3.2(a) to 3.2(d). The spectra for this index show almost similar results as in case of Sensex return series. Since the National index is a broad based index of the same stock exchange, it shows similar frequency domain behaviour. After tapering **and** taking log spectral density, the power spectra plots of National index return series are almost flat.

Figures 3.3(a) to 3.3(d) show the estimated power spectra of the NSE S & P CNX Nifty index return series. The peaks are traced at frequencies 0.04, 0.11, 0.17, 0.33, 0.39, and 0.46 corresponding to 25, 9, 5.8, 3, 2.5 and 2 days respectively. After tapering and taking logarithm of the spectral density, the peaks at frequency 0.17 **and**

[4] Technically speaking, the random walk hypothesis (RWH) is more stringent and restrictive than the weak for of EMH. Stock prices can conform to weak form of EMH without meeting the conditions of RWH.

0.33 retain their contribution to overall variance in the frequency domain, but it is not significant. One interesting thing here to note is that, the peak at frequency 0.17 corresponding to 5.8 days i.e. about one week is common for all the four index returns. But the power spectra of S & P CNX 500 index return series show the significant peaks at frequencies 0.20 and 0.30, thus corresponding to 5 and 3.3 days respectively.

The power spectrum results for all the four indices indicate that even after smoothing the data series, apart from some exceptional peaks, the resulting spectra are almost flat over whole frequency range. Tapering removes the contribution of some frequencies, which acts as a leakage in the untappered power spectra. The conclusion, we find from the power spectra analysis of four indices is that the spectra of all the returns series are flat indicating random or white noise behavior on their part and thus the stock prices follow random walk model (which is synonymously with weak form of EMH).

The study uses seven indices for cross-spectrum analysis, four of which are Indian stock market indices and other three are from developed stock markets. Four Indian indices are Sensex, National, S & P CNX Nifty and S & P CNX 500 and three developed stock market indices are Dow Jones Industrial Average (DJIA) of New York stock exchange, FTSE index of London stock exchange and Nikkei 225 index of Tokyo stock exchange. The data consists of daily observations (closing price) from January 2000 to December 2001 (see Appendix A). Tables 3.2 and 3.3 report descriptive statistics and the correlations for all the indices used for cross-spectrum analysis. The mean for all the series are negative. The excess kurtosis and skewness show the non-normality for all the return series. The correlation statistics do not show any high degree of relationships between Indian stock markets and its developed counter parts.

The results of our analysis pertaining to coherence and phase spectrum are shown in figures 3.5 to 3.16. Figures 3.5(a) and 3.5(b) show the movement of the Sensex and DJIA coherences. The coherence is high at extreme frequencies. The average coherence between these indices is low (less than 0.3). This result is consistent with the low correlation coefficient between these two markets. The coherence between National index and DJIA index return series is presented in figure 3.6(a) and 3.6(b). The average coherence here is low except at certain frequencies. The evidence of coherence is mixed over all the range of frequencies i.e. it does not show any consistent pattern of relationship. Figures 3.7(a) and 3.7(b) exhibit a somewhat different picture. The coherence between S & P CNX Nifty and DJIA indices return series fluctuates within a fairly narrow range (0.08 to 0.40). The coherence is almost evenly distributed over all frequencies. Coming to the coherence diagram between S & P CNX 500 and DJIA returns series shown in figures 3.8(a) and 3.8(b) we find tapering increases the coherence at extreme frequencies. Altogether, the coherence between Indian stock market and the U.S. stock market appears to be small and does not show any consistent path. The interpretation is that both the stock markets do not have similar long run development features.

The U. K./India coherences are plotted in figures 3.9 to 3.12. The coherence between Sensex and FTSE return series are similar to coherence between Sensex and DJIA. The coherence plot is within a narrow range (0.02 to 0.50) in the middle part of the frequencies. The plot of coherence between National and FTSE return series are presented in figures 3.10(a) and 3.10(b). The coherence between NSE indices and FTSE index return series are very low and almost same over all the frequencies. The overall results reveal that coherence between India and U.K. stock markets is low at almost all frequencies. This result is consistent with the correlations between all the Indian return series and FTSE return series.

The India/Japan coherence relationships are shown in figures 3.13 to 3.16. The coherences between all the four Indian return series and Nikkei 225 return series of Japan are low for all the frequencies with the exceptions at some extreme frequencies. To conclude, results here are generally consistent with the idea that developed stock markets are not closely linked with Indian stock market. This is particularly true for all frequencies with the exceptions at certain extreme frequencies.

Figures 3.17 to 3.28 show the phase spectra plot and are organized in the same manner as coherence diagrams. As discussed in previous section, it is not worth discussing the phase relationship if there is no or hardly any coherence between two series. But for the sake of completeness we have gone for analyzing phase spectra between all pairs of series analyzed for coherences. Figures 3.17 to 3.20 graph the phase lead-lag of the Indian market (input) over the U.S. market (output). The phase diagram between Sensex and DJIA return series are presented in figures 3.17(a) and 3.17(b), without and with tapering respectively. The figures indicate that the Indian market lags U.S. market at low frequencies, but at high frequencies both markets are in phase. And mostly at middle part of the frequencies, there is no clear pattern of leads and lags, with the phase plot crossing the horizontal axis several times. This result is consistent with the coherence results that there is relatively a greater degree of frequency domain correlation between the return series both at low and high frequencies. Similar result obtained from the phase diagrams of National and DJIA index returns series presented in figures 3.18 (a) and 3.18 (b). The phase diagrams for two NSE indices and DJIA index returns series shown in figures 3.19 and 3.20 do not show any clear pattern of lead and lags over all frequencies, with the phase plot crossing the horizontal axis various times over the full range of frequencies. There is a great deal of volatility in the phase lead around zero frequency that requires some further explanation.

Figures 3.21 to 3.24 show the phase movements between India and the U. K. markets. The phase plots of Sensex and FTSE returns series, presented in figures 3.21 (a) and 3.21 (b), show similar behaviour like Sensex and DJIA returns series. We find no clear pattern of phase leads and lags in the phase movement of these two market returns series. As shown in figures 3.22 (a) and 3.22 (b), the phase diagram of National and FTSE return series gives a better plot at low frequencies after tapering. At high frequencies portion both return series are in phase. But as whole, we are not able to find the exact nature of the phase plot since it crosses the horizontal axis several times over the full range of frequencies. Coming to the phase spectra of NSE and FTSE returns series, shown in figures 3.23 and 3.24, we find non-linear behaviour in the phase spectra; with the frequent change in angles in the phase plot at various frequencies.

The phase spectra plots of India and Japan markets presented in figures 3.25 to 3.28 show very similar kind of behaviour like India and other two markets discussed earlier. These phase spectra results can generally be summarized as follows: i) since there is no high degree of frequency domain correlation between **India and other** developed markets, one should not expect a linear and consistent pattern of leads and lags; ii) the phase spectra show a highly volatile nature around the zero frequency that necessitates further explanations; iii) there is no clear pattern of leads and lags, with the phase plot crossing the horizontal axis several times over the full range of frequencies and iv) It is found that over all the full range of frequencies the angles of phase spectra are changing, which suggests that there may be non-linearities in the phase spectra. These descriptive empirical results indicating inconsistent time lags between different periods suggest non-linear relationships between India and other developed markets return series.

3.7 Concluding Remarks

In this chapter we have used spectral analysis to reveal possible common cyclical components in the stock returns series. Spectral analysis describes the properties of a time series using the decomposition of the variance into components, which are explained by cycles of all possible frequencies. In doing so, it provides a set of frequency domain descriptors (power spectrum, coherence, and phase spectra), which are complementary to their more familiar time domain counterparts (variance and autocovariance).

The univariate spectra (power spectrum) for the four returns series suggest that there are no significant cyclical patterns present in four stock price indices. This view is consistent with the findings of Sharma and Kennedy (1977) and Kulkarni (1978) and

against the findings of Ranganatham and Subramanian (1993) in case of Indian stock market. The coherence analysis suggests that there are no similar long term development features between India and developed stock markets studied here. The analysis of phase spectra shows no clear pattern of leads and lags between India and developed counterparts, with the phase plot crossing the horizontal axis several times. In addition, it is found that over all the full range of frequencies the angles of phase spectra are changing, which suggests non-linear relationships between the India and developed markets return series.

Table 3.1: Summary Statistics of Returns Series Used for Power Spectrum Analysis

Series	Mean	SD	Sk	Kurtosis	ADF
Sensex	0.020	0.829	0.104	6.510	-21.933"*
National	0.018	0.794	0.106	8.652	-21.187*"
CNX Nifty	0.020	0.803	0.022	7.936	-21.307•••
CNX 500	0.012	0.775	-0.219	9.031	-20.396""

Note: The superscripts *, **, • indicate statistical significance for the t-statistics at the 1, 5, and 10 per cent levels respectively. The Mackinnon critical values for unit root tests are -3.9671, -3.4141, and -3.1289 at the 1, 5, and 10 per cent levels respectively.**

Table 3.2: Summary Statistics of Returns Series Used for Cross-Spectrum Analysis

Series	Mean	SD	Sk	Kurtosis	ADF
Sensex	-0.052	1.092	-0.230	4.498	-10.371""
National	-0.061	0.946	-0.442	3.964	-9.302•••
CNX Nifty	-0.023	0.608	0.013	4.373	-9.933"*
CNX 500	-0.042	0.788	-0.435	4.727	-9.654"*
DJIA	-0.012	0.579	-0.331	5.344	-10.056"*
FTSE	-0.019	0.588	-0.053	3.214	-9.859•••
Nikkei	-0.54	0.734	0.007	4.861	-10.089"*

Note: The superscripts *, **, * indicate statistical significance for the t-statistics at the 1, 5, and 10 per cent levels respectively. The Mackinnon critical values for unit root tests are -3.9820, -3.4214, and -3.1331 at the 1, 5, and 10 per cent levels respectively.**

Table 3.3: Correlation Coefficients of Returns Series Used for Cross-Spectrum Analysis

Series	Sensex	National	CNX Nifty	CNX 500	DJIA	FTSE	Nikkei
Sensex	1.000						
National	0.165	1.000					
CNX Nifty	0.012	0.065	1.000				
CNX 500	0.354	0.105	0.015	1.000			
DJIA	0.044	-0.004	0.077	-0.0005	1.000		
FTSE	0.082	0.063	0.016	0.086	0.153	1.000	
Nikkei	0.237	0.096	0.272	-0.049	0.090	-0.026	1.000

Figure 3.1 (a): Estimated Power Spectrum of BSE Sensex Return Series

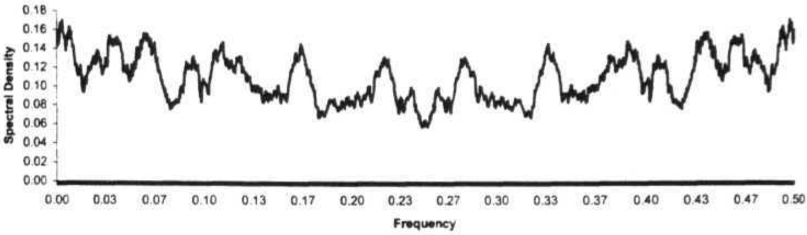


Figure 3.1 (b): Estimated Power Spectrum of BSE Sensex Return Series (Tapered)

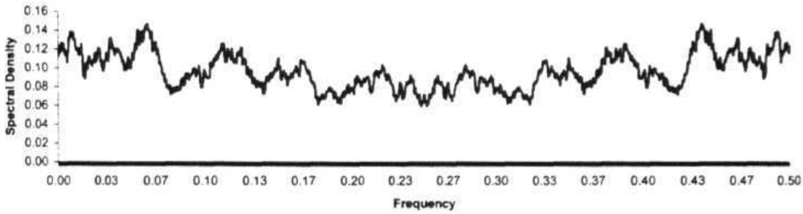


Figure 3.1 (c): Estimated Log Power Spectrum of BSE Sensex Return Series

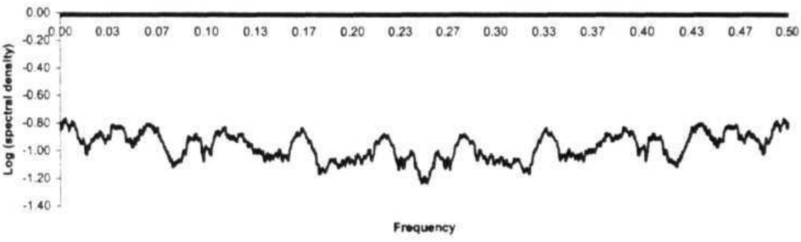


Figure 3.1 (d): Estimated Log Spectrum of BSE Sensex Return Series (Tapered)

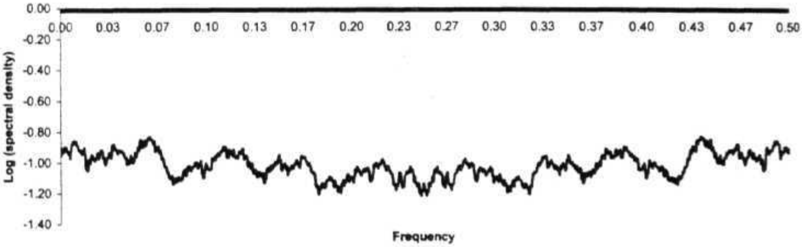


Figure 3.2 (a): Estimated Power Spectrum of BSE National Index Return Series

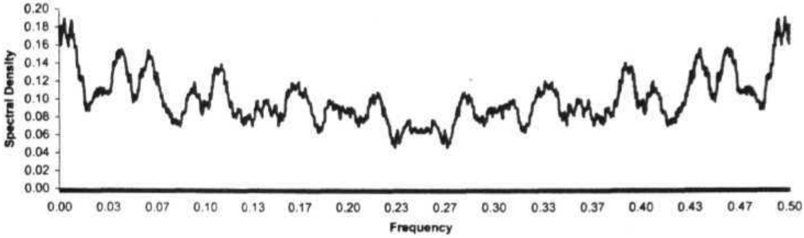


Figure 3.2 (b): Estimated Power Spectrum of BSE National Index Return Series (Tapered)

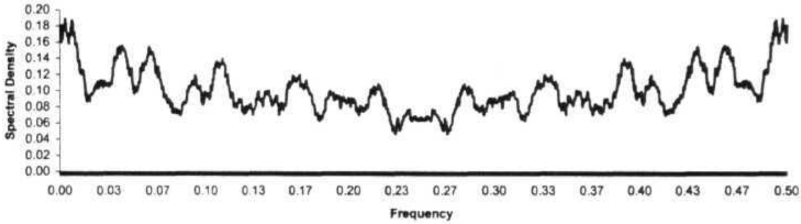


Figure 3.2 (c): Estimated Log Spectrum of BSE National Index Return Series

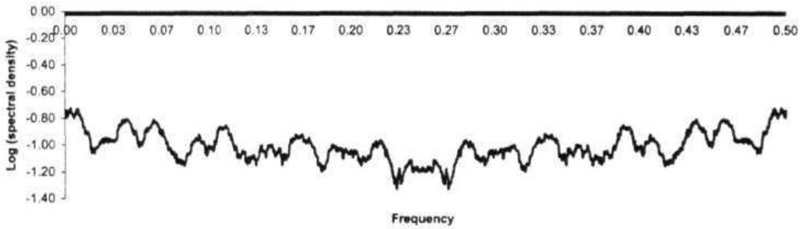


Figure 3.2 (d): Estimated Log Spectrum of BSE National Index Return Series (Tapered)

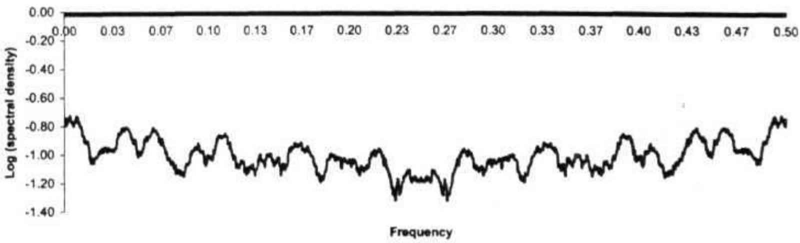


Figure 3.3 **(a): Estimated Power** Spectrum of NSE S & **P CNX Nifty Index Return** Series

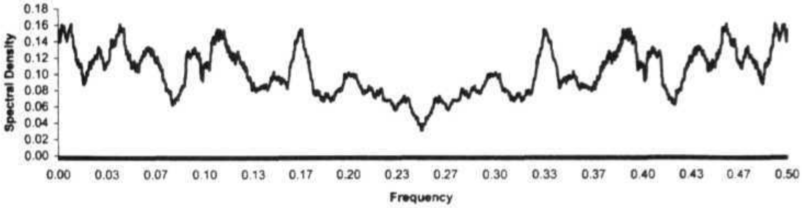


Figure 3.3 (b): Estimated Power Spectrum of NSE S & P CNX Nifty Index Return Series (Tapered)

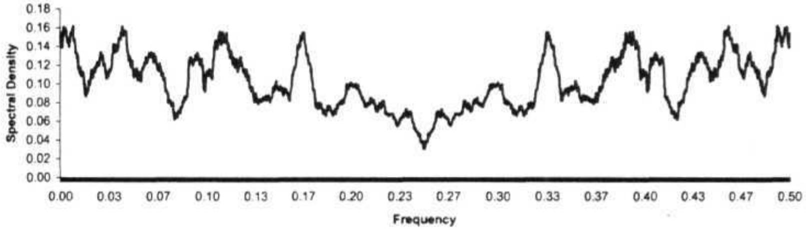


Figure 3.3 (c): Estimated Log Spectrum of NSE S & P CNX Nifty Index Return Series

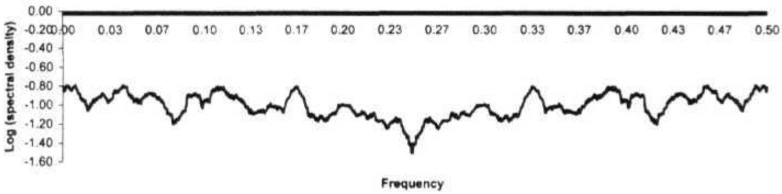


Figure 3.3 (d): Estimated Log Spectrum of NSE S & P CNX Nifty Index Return Series (Tapered)

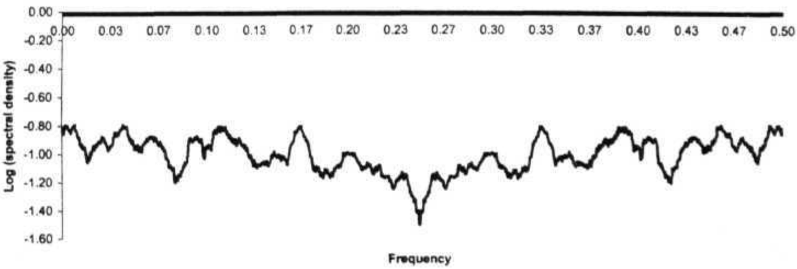


Figure 3.4 (a): Estimated Power Spectrum of NSE S & P CNX 500 Index Return Series

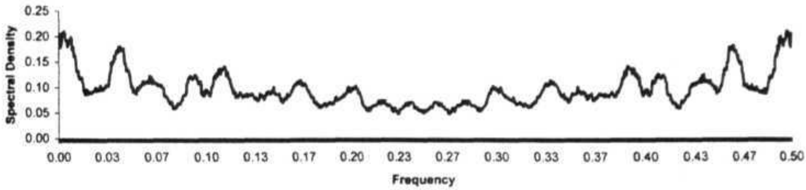


Figure 3.4 (b): Estimated Power Spectrum of NSE S & P CNX 500 Index Return Series (Tapered)

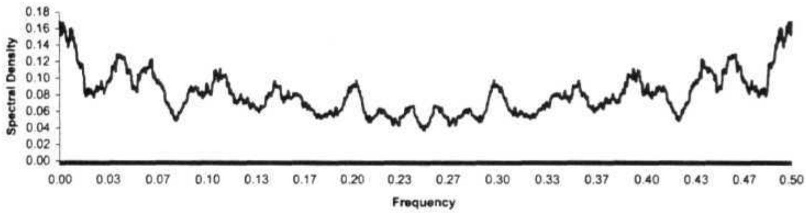


Figure 3.4 (c): Estimated Log Spectrum of NSE S & P CNX 500 Index Return Series

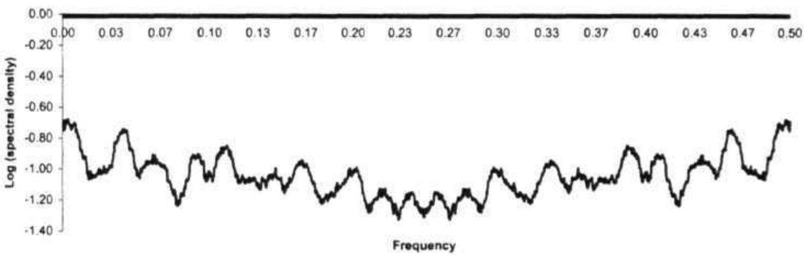


Figure 3.4 (d): Estimated Log Spectrum of NSE S & P CNX 500 Index Return Series (Tapered)

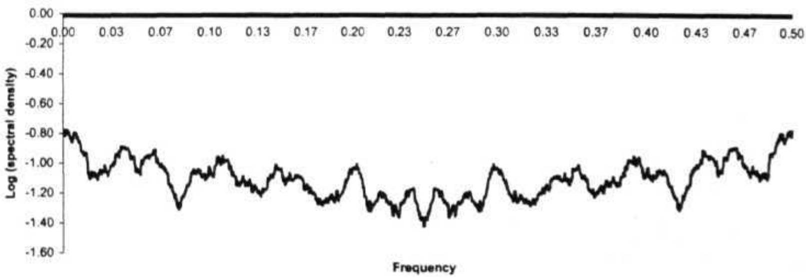


Figure 3.5 (a): Coherence Between BSE SENSEX Return and DJIA Index Return of New York Stock Exchange

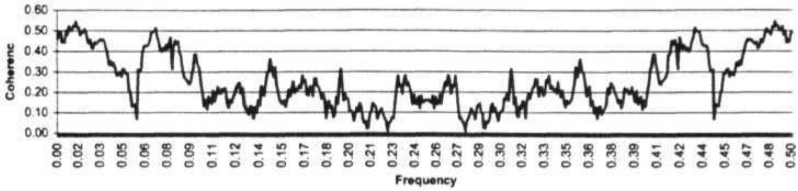


Figure 3.5 (b): Coherence Between BSE Sensex Return and DJIA Index Return of New York Stock Exchange (Tapered)

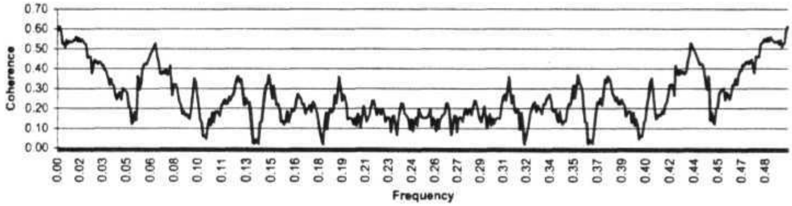


Figure 3.6 (a): Coherence Between BSE Sensex Return and FTSE Index Return of London Stock Exchange

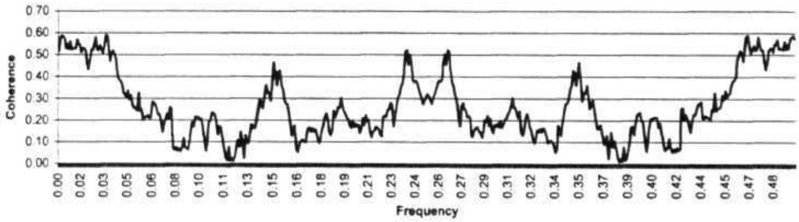


Figure 3.6 (b): Coherence Between BSE Sensex Return and FTSE Index Return of London Stock Exchange (Tapered)

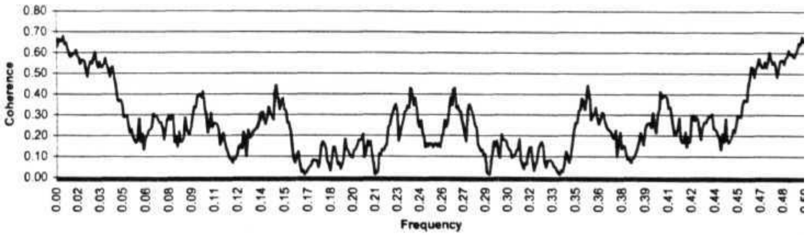


Figure 3.7 (a): Coherence Between BSE Sensex index Return and Nikkei Index Return of Japan

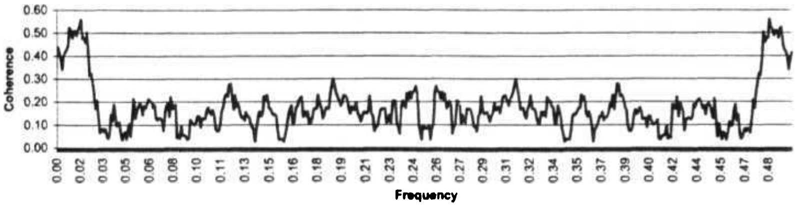


Figure 3.7 (b): Coherence Between BSE Sensex Index Return and Nikkei Index Return of Japan (Tapered)

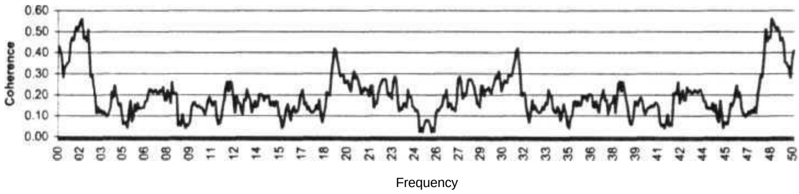


Figure 3.8 (a): Coherence Between BSE National Index Return and DJiA Index Return of New York Stock Exchange

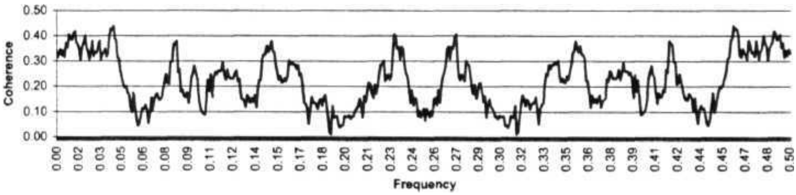


Figure 3.8 (b): Coherence Between BSE National Index Return and DJiA Index Return of New York Stock Exchange (Tapered)

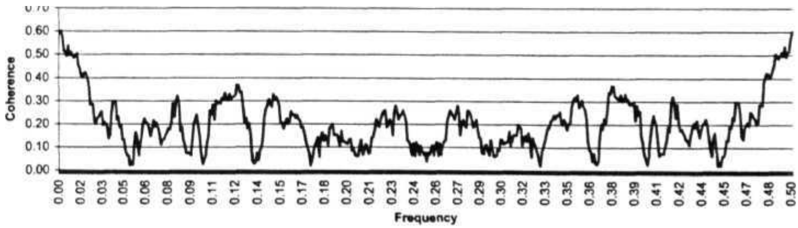


Figure 3.9 (a): Coherence Between BSE National Index Return and FTSE Index Return of London Stock Exchange

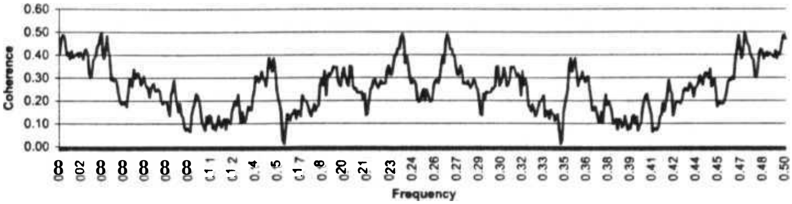


Figure 3.9 (b): Coherence Between BSE National Index Return and FTSE Index Return of London Stock Exchange (Tapered)

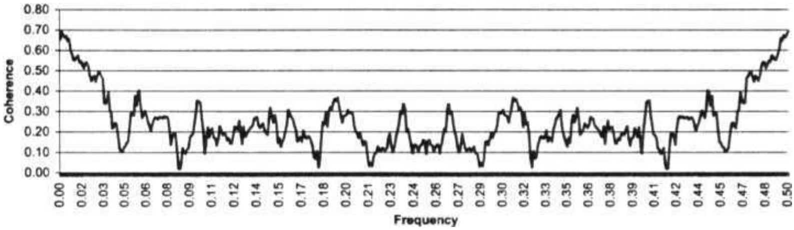


Figure 3.10 (a): Coherence Between BSE National Index Return and Nikkei Index Return of Japan

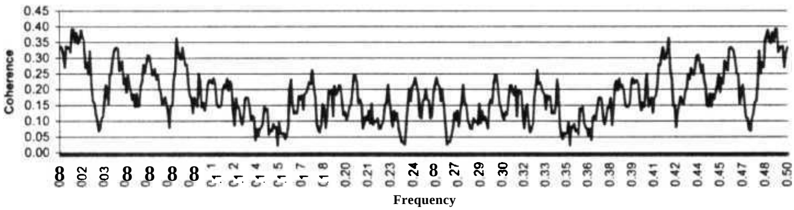


Figure 3.10 (b): Coherence Between BSE National Index Return and Nikkei Index Return of Japan (Tapered)

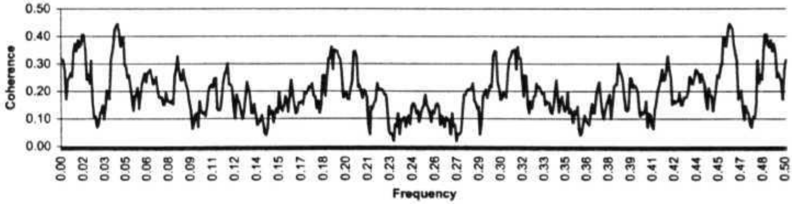


Figure 3.11 (a): Coherence Between NSE Nifty **Index** Return **and DJIA index** Return of **New** York Stock Exchange

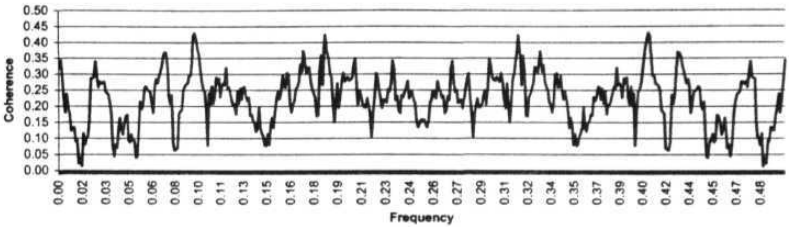


Figure 3.11 (b): Coherence Between NSE Nifty **Index** Return and DJIA Index Return of **New** York Stock Exchange (Tapered)

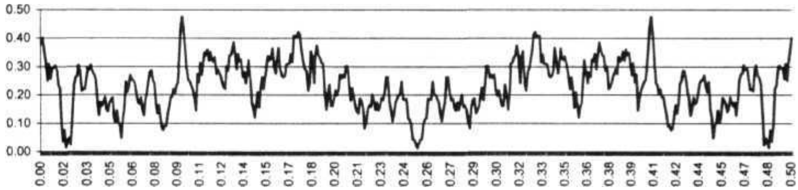


Figure 3.12 (a): Coherence Between CNX Nifty index Return and FTSE Index Return of London Stock Exchange

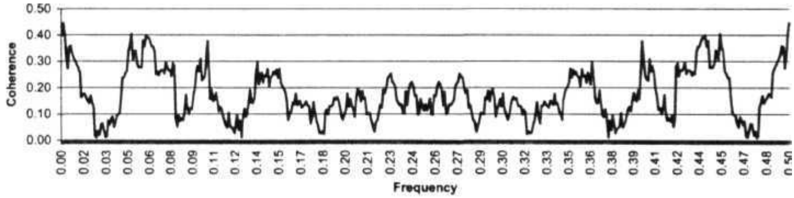


Figure 3.12 (b): Coherence Between CNX Nifty Index Return and FTSE Index Return of London Stock Exchange (Tapered)

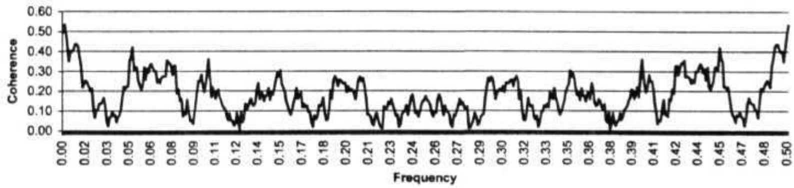


Figure 3.13 (a): Coherence Between NSE S & P CNX Nifty Index Return and Nikkei Index Return of Japan

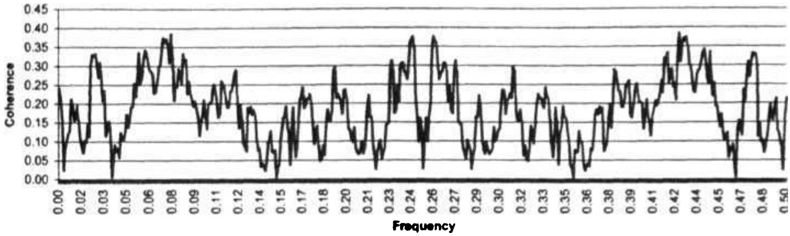


Figure 3.13 (b): Coherence Between NSE S & P CNX Nifty Index Return and Nikkei Index Return of Japan (Tapered)

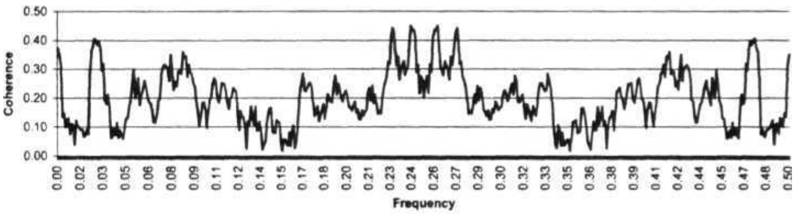


Figure 3.14 (a): Coherence Between S & P CNX Nifty Index Return and DJiA Index Return of New York Stock Exchange

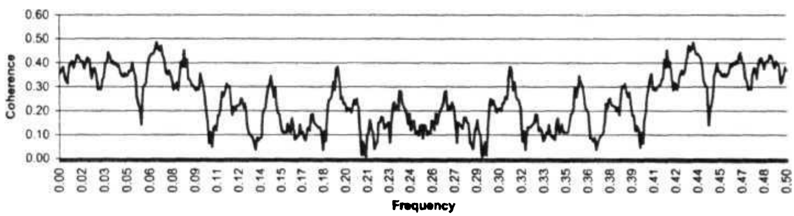


Figure 3.14 (b): Coherence Between S & P CNX Nifty Index Return and DJIA Index Return of New York Stock Exchange (Tapered)

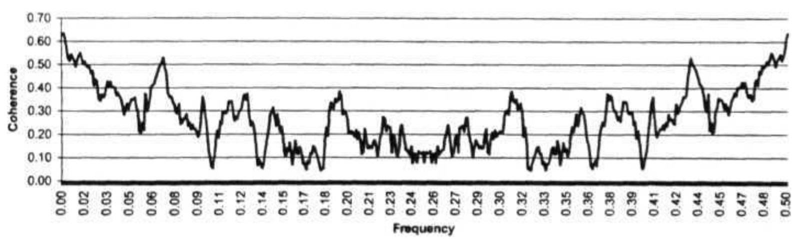


Figure 3.15 (a): Coherence Between S & P CNX Nifty Index Return and FTSE Index Return of London Stock Exchange

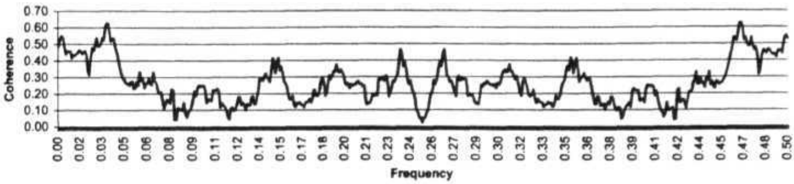


Figure 3.15 (b): Coherence Between S & P CNX Nifty Index Return and FTSE Index Return of London Stock Exchange (Tapered)

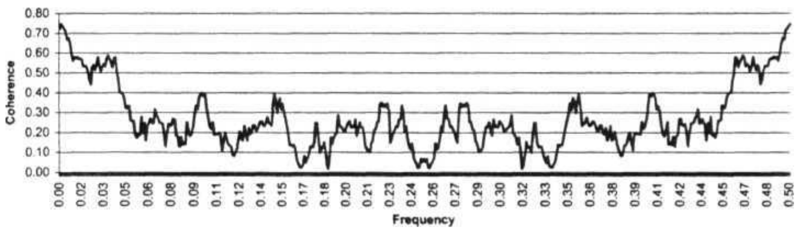


Figure 3.16 (a): Coherence Between NSE S & P CNX 500 Index Return and Nikkei Index Return of Japan

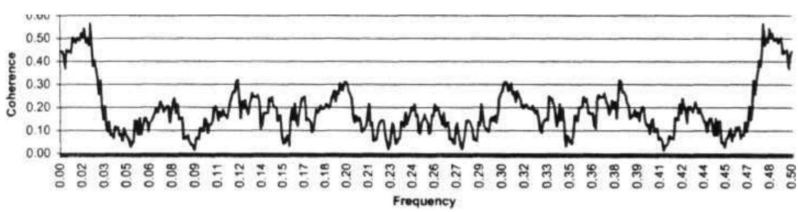


Figure 3.16 (b): Coherence Between NSE S & P CNX 500 Index Return and Nikkei Index Return of Japan (Tapered)

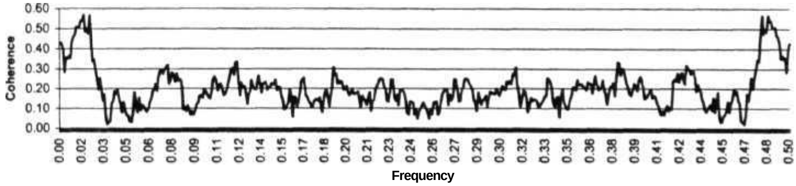


Figure 3.17 (a): Phase Between BSE SENSEX Return and DJIA Index Return of New York Stock Exchange

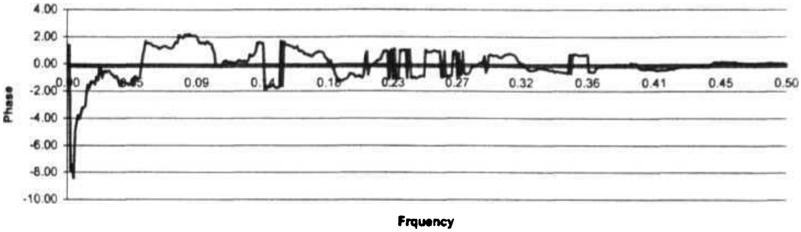


Figure 3.17 (b): Phase Between BSE Sensex Return and DJIA Index Return of New York Stock Exchange (Tapered)

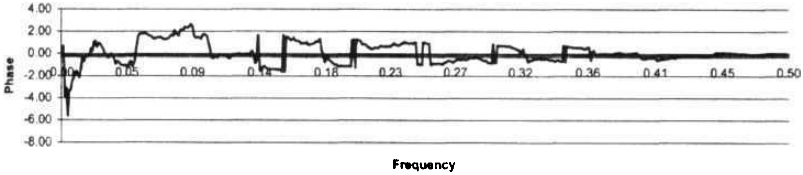


Figure 3.18 (a): Phase Between BSE Sensex Return and FTSE Index Return of London Stock Exchange

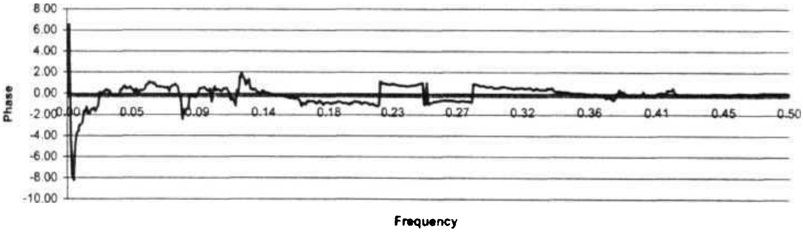


Figure 3.18 (b): Phase Between BSE Sensex Return and FTSE Index Return of London Stock Exchange (Tapered)

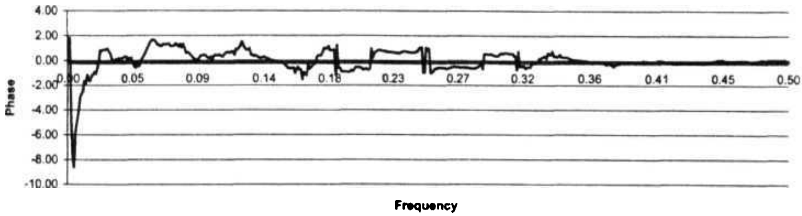


Figure 3.19 (a): Phase Between BSE Sensex Index Return and Nikkei Index Return of Japan

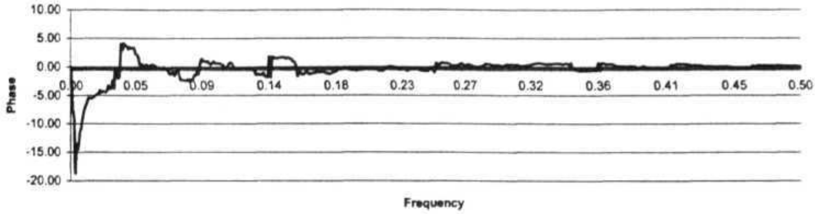


Figure 3.19 (b): Phase Between BSE Sensex Index Return and Nikkei Index Return of Japan (Tapered)

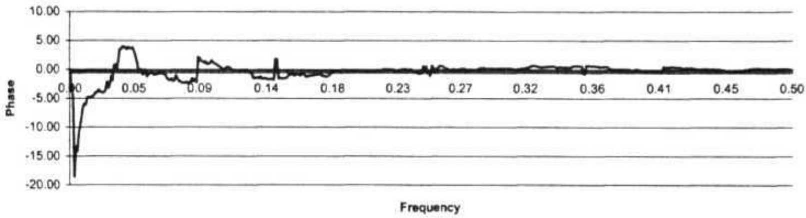


Figure 3.20 (a): Phase Between BSE National Index Return and DJIA Index Return of New York Stock Exchange

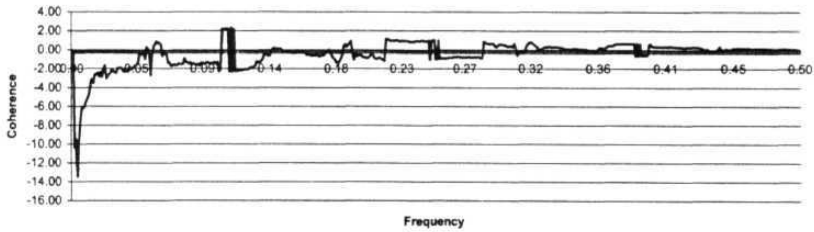


Figure 3.20 (b): Phase Between BSE National Index Return and DJIA Index Return of New York Stock Exchange (Tapered)

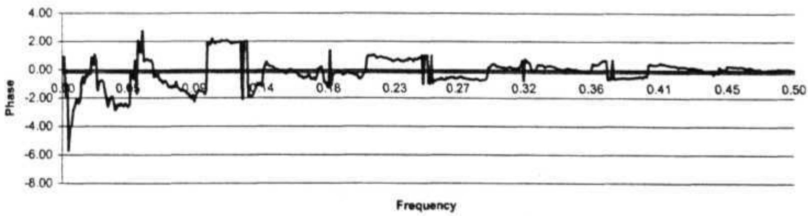


Figure 3.21 (a): Phase Between BSE National Index Return and FTSE Index Return of London Stock Exchange

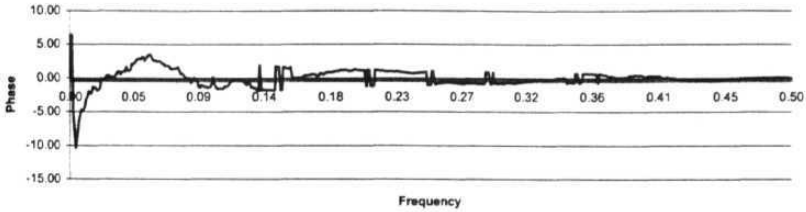


Figure 3.21 (b): Phase Between BSE National Index Return and FTSE Index Return of London Stock Exchange (Tapered)

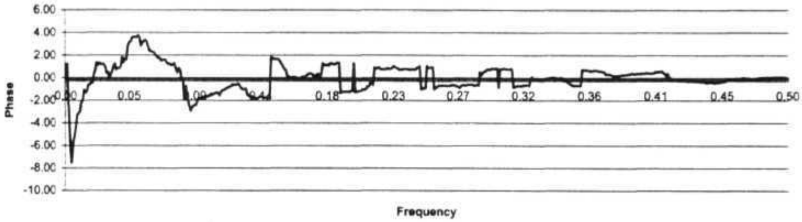


Figure 3.22 (a): Phase Between BSE National Index Return and Nikkei Index Return of Japan

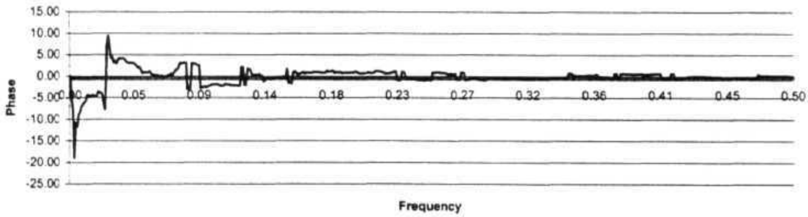


Figure 3.22 (b): Phase Between BSE National Index Return and Nikkei Index Return of Japan (Tapered)

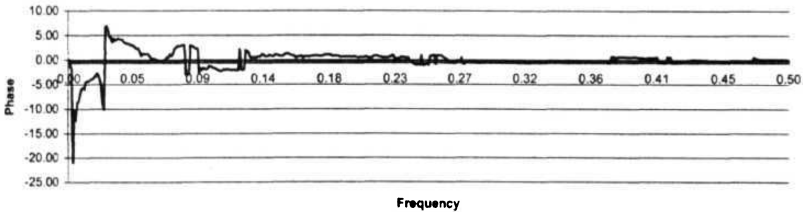


Figure 3.23 (a): Phase Between NSE Nifty Index Return and DJIA Index Return of New York Stock Exchange

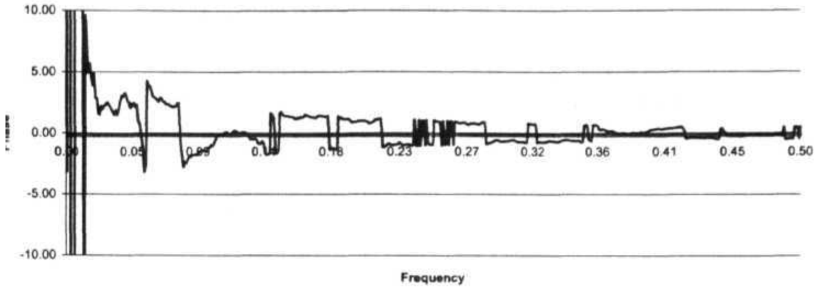


Figure 3.23 (b): Phase Between NSE Nifty Index Return and DJIA Index Return of New York Stock Exchange (Tapered)

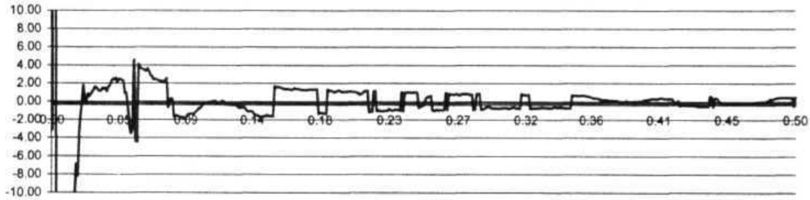


Figure 3.24 (a): Phase Between S & P CNX Nifty Index Return and FTSE Index Return of London Stock Exchange

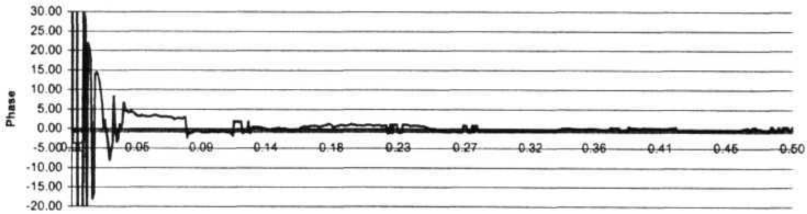


Figure 3.24 (b): Phase Between S & P CNX Nifty Index Return and FTSE Index Return of London Stock Exchange (Tapered)

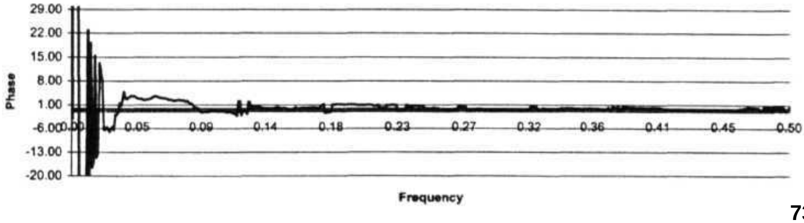


Figure 3.25 (a): Phase Between NSE S & P CNX Nifty Index Return and Nikkei Index Return of Japan

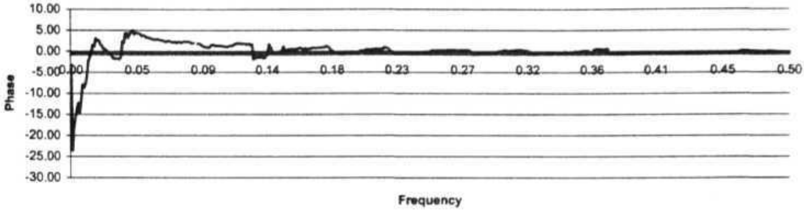


Figure 3.25 (b): Phase Between NSE S & P CNX Nifty Index Return and Nikkei Index Return of Japan (Tapered)

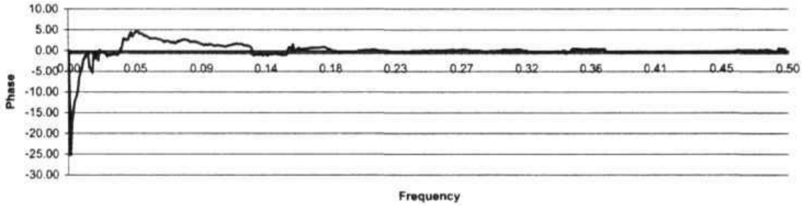


Figure 3.26 (a): Phase Between S & P CNX Nifty Index Return and DJIA Index Return of New York Stock Exchange

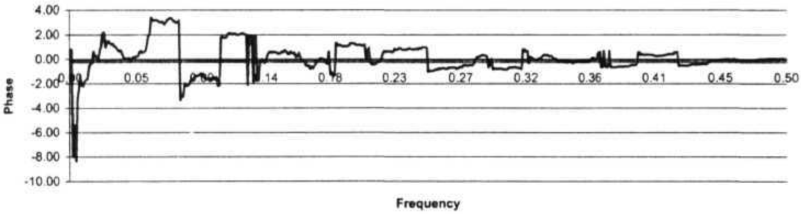


Figure 3.26 (b): Phase Between S & P CNX Nifty Index Return and DJIA Index Return of New York Stock Exchange (Tapered)

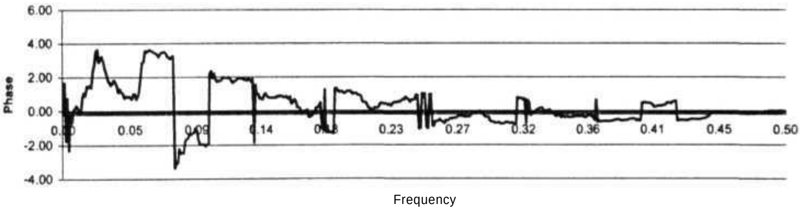


Figure 3.27 (a): Phase Between S & P CNX Nifty index Return and FTSE Index Return of London Stock Exchange

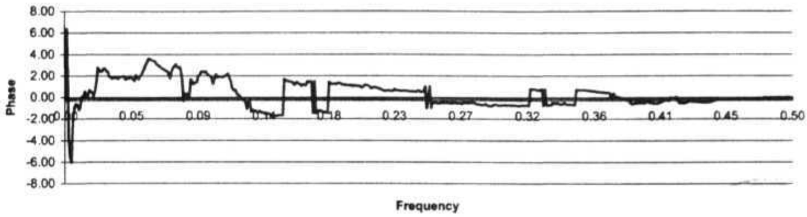


Figure 3.27 (b): Phase Between S & P CNX Nifty Index Return and FTSE Index Return of London Stock Exchange (Tapered)

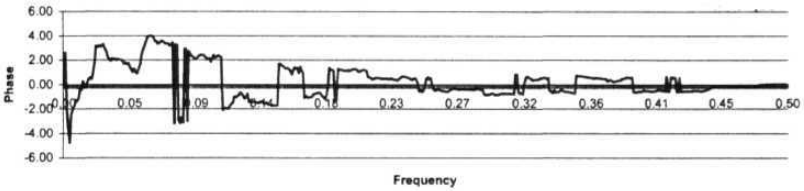


Figure 3.28 (a): Phase Between NSE S & P CNX 500 Index Return and Nikkei Index Return of Japan

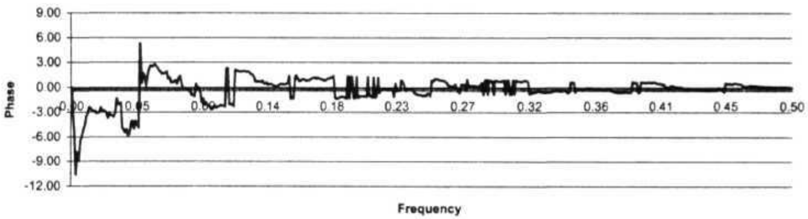
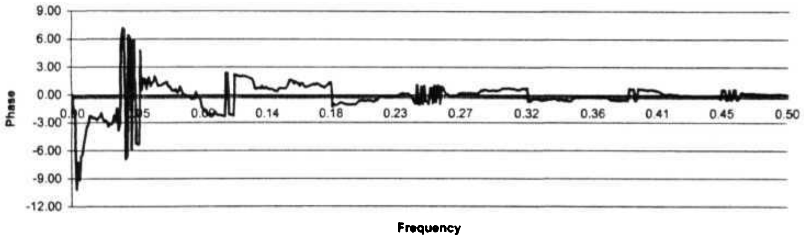


Figure 3.28 (b): Phase Between NSE S & P CNX 500 Index Return and Nikkei Index Return of Japan (Tapered)



Chapter IV

Wavelet Analysis of Stock Prices

4.0 Introduction

In the previous chapter, we analyzed stock price behaviour using various spectral methods such as power spectrum and cross-spectrum. Here we make an attempt to carry out a more exploratory analysis of the same using the sophisticated wavelet techniques. The major emphasis is on the investigating the time-scale decompositions, time frequency analysis and denoising. Besides, we also present a discussion on various dimensions of wavelet analysis that will lead to empirical analysis in subsequent sections.

The rest of the chapter is organized as follows: sections 4.1 to 4.7 are devoted to discussion on various technical aspects of wavelets. Section 4.8 documents the implementation procedure of the discrete wavelet system. We undertake the empirical analysis in section 4.6 followed by concluding remarks in section 4.10.

4.1 Wavelet Analysis

Wavelet analysis is characterized by a wavelet. A wavelet is a small wave, which has its energy concentrated in time to give a tool for the analysis of transient, non-stationary or time varying phenomenon. It still has the oscillating wave like characteristic (as Fourier analysis) but also has the ability to allow simultaneous time and frequency analysis with a flexible mathematical foundation.

There are two types of wavelets defined on different normalization and orthogonalization rules, namely, father wavelets *(p* (scaling function) and mother wavelets ψ (wavelets). The father wavelet integrates to a constant and the mother wavelets integrates to zero: that is,

$$\int \varphi(t)dt = a\ constant$$
$$\int \psi(t)dt = 0\ and$$
$$\int \varphi(t)\psi(t)dt = 0 \quad (orthogonalization\ condition)$$

4.1.1 Wavelet System

There are many different wavelets systems (Harr, Daubechies, Symmlet etc) that can be used effectively, but all seem to have the following three general characteristics

i. A wavelet system is a set of building blocks to construct or represent a signal or function. It is a two dimensional expansion set (usually a basis) for some class of one or higher dimensional signals. In other words, if the wavelet set is given by $\psi_{j,k}(t)$ for indices of $j, k = 1, 2....$, a linear expansion would be $f(t) = \sum_k \sum_j a_{j,k}\psi_{j,k}(t)$ for some set of coefficients $a_{j,k}$.

ii. The wavelet expansion gives a time-frequency localization of the signal. This means most of the energy of the signal is well represented by a few expansion coefficients $a_{j,k}$

iii. The calculation of the coefficients from the signal can be done efficiently. It turns out that many wavelet transforms (the set of expansion coefficients) can be calculated with order of N [i.e. $O(N)$] operations. This means the number of floating point multiplications and additions increase linearly with the length of the signal. More general wavelet transforms require $O(N\log(N))$ operations, the same as for the fast Fourier transform (FFT).

Virtually all wavelet systems have these characteristics. Where the Fourier series maps a one dimensional function of a continuous variable into a one dimensional sequence of coefficients, the wavelet expansion maps it into a two dimensional array of coefficients. We will see that it is the two dimensional representation that allows the localizing the signal in both time and frequency. A

Fourier series expansion localizes in frequency in that if a Fourier series expansion of a signal has only large coefficient, then the signal is essentially a single sinusoid at the frequency determined by the index of the coefficients. The simple time domain representation of the signal itself gives the localization in time. If the signal is a simple pulse, the location of that pulse is the localization in time. A wavelet representation will give the location in both time and frequency simultaneously.

4.1.2 Specific Characteristics of the Wavelet System

There are two additional characteristics that are more specific to wavelet expansion.

1. All the so called first generation wavelet systems are generated from a single scaling function or wavelet by simple scaling and translation. The two dimensional parameterization is achieved from the function (mother wavelet) $\psi(t)$ by

$$\psi_{j,k}(t) = 2^{j/2}\psi\left(2^j t - k\right) \qquad j,k \in Z \qquad (4.1)$$

   where Z is the set of all integers and the factor $2^{j/2}$ maintains a constant norm independent of scale *j*. This parameterization of the time or space location by *k* and the frequency or scale *by j* turns out to be extraordinarily effective.

2. All most all wavelet systems also satisfy the multiresolution conditions. This means that if a set of signals can be represented by a weighted sum of $\psi(t - k)$, then a larger set can be represented by weighted sum of $\psi(2t - k)$. In other words, if the basic expansion signals are made half as wide and translated in steps half as wide, they will represent a larger class of signals exactly or give a better approximation of any signal.

The operations of the translation of scaling seem to be basic to many practical signals and signal generating processes and their use is one of the reasons that wavelets are efficient expansion function. If the index *k* changes, the location of the wavelet moves along the horizontal axis, which allows the expansion to explicitly represent the location of events in time or space. If the index *j* changes, the shape of the wavelet changes in scale, which allows a representation of detail or resolution. For the Fourier series and transform, the expansion function (bases) are chosen, then the properties of resulting transform are **derived** and analyzed. For the wavelet system, these properties or characteristics are mathematically required, and then the resulting basis functions are derived. Wavelet analysis is well suited to transient signals. Fourier analysis is appropriate for periodic signals or for signals whose statistical characteristics do not change over time (stationary). It is the localizing property of the wavelets that allow a wavelet expansion of a transient event to be modeled with a small number of coefficients. This turns out to be a very useful in applications.

4.2 The Discrete Wavelet Transform

Any function $f(t) \in L^2(R)$ could be written as

$$f(t) = \sum_{k=-\infty}^{\infty} c_k \varphi_k(t) + \sum_{j=0}^{\infty} \sum_{k=-\infty}^{\infty} d(j,k) \psi_{j,k}(t) \qquad ...(4.2)$$

Using equation (4.1) a more general statement of expansion (4.2) can be given by

$$f(t) = \sum_k c_{j0}(k) 2^{j0/2} \varphi\left(2^{j0/2} t - k\right) + \sum_k \sum_{j=j_0}^{\infty} d_j(k) 2^{j/2} \psi\left(2^j t - k\right) \qquad ...(4.3)$$

or

$$f(t) = \sum_k c_{j0}(k) \varphi_{j_0,k}(t) + \sum_k \sum_{j=j_0}^{\infty} d_j(k) \psi_{j,k}(t) \qquad ...(4.4)$$

The coefficients in this wavelet expansion are called the discrete wavelet transform (DWT) of the signal / *(t).* Theses wavelet coefficients completely describe the original signal and can be used in a way similar to Fourier series coefficients for analysis, description, approximation and filtering. These coefficients can be calculated by inner products

$$c_j(k) = \langle f(t), \varphi_{j,k}(t) \rangle = \int f(t) \varphi_{j,k}(t) dt \qquad ...(4.5)$$

and

$$d_j(k) = \langle f(t), \psi_{j,k}(t) \rangle = \int f(t) \psi_{j,k}(t) dt \qquad ...(4.6)$$

The DWT is similar to a Fourier series but, in many ways, is much more flexible and informative. It can be made periodic like a Fourier series to represent periodic

signals efficiently. However, unlike Fourier series, it can be used directly on non-periodic transient signals with excellent results.

4.2.1 Analysis –From Fine Scale to Coarse Scale

In order to work directly with the wavelet coefficients, we will derive the relationship between the expansion coefficients at a lower scale level in terms of those at a higher scale level.

The basic recursive equations are:

$$\varphi(t)=\sum_n h(n)\sqrt{2}\varphi(2t-n) \qquad ...(4.7)$$

$$\psi(t)=\sum_n \hat{h}(n)\sqrt{2}\varphi(2t-n) \qquad ...(4.8)$$

and assuming a unique solution exists, we scale and translate the time variable to give

$$\varphi(2^j t-k)=\sum_n h(n)\sqrt{2}\varphi(2(2^j t-k)-n)=\sum_n h(n)\sqrt{2}\varphi(2^{j+1}t-2k-n) \qquad ...(4.9)$$

which after changing variables $m=2k+n$, becomes

$$\varphi(2^j t-k)=\sum_m h(m-2k)\sqrt{2}\varphi(2^{j+1}t-m) \qquad ...(4.10).$$

If we denote v_j as

$$\upsilon_j=\underset{k}{span}\{2^{j/2}\varphi(2^j t-k)\} \qquad ...(4.11)$$

then

$$f(t)\in\upsilon_{j+1}\Rightarrow f(t)=\sum_k c_{j+1(k)}(k)2^{(j+1)/2}\varphi(2^{j+1}t-k) \qquad ...(4.12)$$

is expressible at a scale of/+7 with scaling functions only and no wavelets. At one scale lower resolution, wavelets are necessary for the detail not available at scale *j*. we have

$$f(t)=\sum_k c_j(k)2^{j/2}\varphi(2^j t-k)+\sum_k d_j(k)2^{j/2}\psi(2^j t-k) \qquad ...(4.13)$$

where the $2^{j/2}$ terms maintain the unity norm of the basis functions at various scales. If $\varphi_{j,k}(t)$ and $\psi_{j,k}(t)$ are orthonormal, the j level scaling coefficients are found by taking the inner product

$$c_j(k) = \langle f(t), \varphi_{j,k}(t) \rangle = \int f(t) 2^{j/2} \varphi(2^j t - k) dt \qquad \text{...(4.14)}$$

which, by using (4.10) and interchanging the sum and integral, can be written as

$$c_j(k) = \sum_m h(m-2k) \int f(t) 2^{(j+1)/2} \varphi(2^{j+1} t - m) dt \qquad \text{...(4.15)}$$

but the integral is the inner product with the scaling function at a scale of $j+1$ giving

$$c_j(k) = \sum_m h(m-2k) c_{j+1}(m) \qquad \text{...(4.16)}$$

The corresponding relationship for wavelet coefficient is

$$d_j(k) = \sum_m \hat{h}(m-2k) c_{j+1}(m) \qquad \text{...(4.17)}$$

where $h(n)$ and $\hat{h}(n)$ are filter coefficients for scaling and wavelets respectively.

4.2.2 Synthesis – From Coarse Scale to Fine Scale

As one would expect, the reconstruction of the original fine scale coefficients of the signal can be made from a combination of the scaling function and wavelet coefficients at a coarse scale resolution. This is derived by considering a signal in the $j+1$ scaling function space $f(t) \in v_{j+1}$. This function can be written in terms of the scaling function as

$$f(t) = \sum_k c_{j+1}(k) 2^{(j+1)/2} \varphi(2^{j+1} t - k) \qquad (4.18)$$

or in terms of the next scale (which also requires wavelets) as

$$f(t) = \sum_k c_j(k) 2^{j/2} \varphi(2^j t - k) + \sum_k d_j(k) 2^{j/2} \psi(2^j t - k) \qquad \text{...(4.19).}$$

Substituting (4.7) and (4.8) into (4.19) gives

$$f(t)=\sum_k c_j(k)\sum_n h(n)2^{(j+1)/2}\varphi\left(2^{j+1}t-2k-n\right)+\sum_k d_j(k)\sum_n \hat{h}(n)2^{(j+1)/2}\varphi\left(2^{j+1}t-2k-n\right) \quad ...(4.20)$$

Because all of these functions are orthonormal, multiplying **(4.18)** and (4.20) by $\varphi(2^{j+1}t - k)$ and integrating evaluates the coefficient as

$$c_{j+1}(k)=\sum_m c_j(m)h(k-2m)+\sum_m d_j(m)\hat{h}(k-2m) \qquad ...(4.21)$$

Therefore, for the use of any wavelet, it is not **always** necessary to know the specific form of the function. If we know the filter coefficients for the scaling function *h(n)* of a particular wavelet, we can get the filter coefficients of the wavelet, $h(n)$ by using $h(n) = (-1)^n h(N-1-n)$.With the use of *h(n)* and $h(n)$ we can solve for wavelet coefficients c_j and $d_{j,k}$'s. Except in some special cases, there is no analytical formula for computing a wavelet function. Instead, wavelets are derived using a special two-scale dilation equation. For father wavelet <p *(t)*, **the** dilation equation is defined by

$$\varphi(t)=\sum_n h(n)\sqrt{2}\varphi(2t-n) \qquad ...(4.22)$$

The mother wavelet ψ *(t)* can similarly be obtained from the father wavelet by the relationship

$$\psi(t)=\sum_n \hat{h}(n)\sqrt{2}\varphi(2t-n) \qquad ...(4.23)$$

The coefficients *h(n)* and $h(n)$ are the low-pas and high-pass filter coefficients defined as:

$$h(n)=\frac{1}{\sqrt{2}}\int\varphi(t)\varphi(2t-n)dt \qquad ...(4.24)$$

$$\hat{h}(n)=\frac{1}{\sqrt{2}}\int\psi(t)\varphi(2t-n)dt \qquad ...(4.25)$$

4.3 Wavelet Approximation

Any function $f(t)$ in $L^2(R)$ to be represented by a wavelet analysis can be built up as a sequence of projections onto father and mother wavelets generated from through scaling and translation as follows:

$$\varphi_{j,k}(t) = 2^{-j/2}\varphi(2^{-j}t - n) = 2^{-j/2}\varphi\left(\frac{t - 2^j n}{2^j}\right) \qquad ...(4.26)$$

$$\psi_{j,k}(t) = 2^{-j/2}\psi(2^{-j}t - n) = 2^{-j/2}\psi\left(\frac{t - 2^j n}{2^j}\right) \qquad ...(4.27)$$

The wavelet representation of the signal or function $f(t)$ in $L^2(R)$ can be given as:

$$f(t) = \sum_k c_{J,k}\varphi_{J,k}(t) + \sum_k d_{J,k}\psi_{J,k}(t) + \sum_k d_{J-1,k}\psi_{J-1,k}(t) + ... + \sum_k d_{1,k}\psi_{1,k}(t) \qquad ...(4.28)$$

where J is the number of multiresolution components, and k ranges from 1 to the number of coefficients in the specified component. The coefficients $c_{J,k}$, $d_{J,k}$,....,$d_{1,k}$ are the wavelet transform coefficients given by the projections

$$c_{J,k} \approx \int \varphi_{J,k}(t) f(t) dt \qquad ...(4.29)$$

$$d_{j,k} \approx \int \psi_{j,k}(t) f(t) dt, \quad for \quad j = 1,2,...,J \qquad ...(4.30).$$

The magnitude of these coefficients reflects a measure of the contribution of the corresponding wavelet function to the total signal. The basic functions $\varphi_{j,k}(t)$ and $\psi_{j,k}(t)$ are the approximating wavelet functions generated as scaled and translated versions of φ and ψ, with scale factor 2^j and translation parameter $2^j k$, respectively. The scale factor 2^j is also called the dilation factor and the translation parameter $2k$ refers to the location. Here 2 is a measure of the scale or width of the functions $\varphi_{j,k}(t)$ and $\psi_{j,k}(t)$. That is, the larger the index j, the larger the scale factor 2^j, and hence the function get shorter and more spread out. The translation parameter $2k$ is matched to the scale parameter 2 in that as the functions $\varphi_{j,k}(t)$ and $\psi_{j,k}(t)$ get wider, their translation steps are correspondingly larger.

4.4 Multiresolution Analysis

The discrete wavelet transform (DWT) calculates the coefficients of the wavelet representation (4.28) for a discrete signals $f_1, \ldots, f_n$ of finite extent. The DWT maps the vector f = $(f_1, f_2, \ldots, f_n)$ to a vector of n wavelet coefficients w = $(w_1, w_2, \ldots, w_n)$. The vector w contains the coefficients $c_{J,k}$, $d_{j,k}, \ldots$, $d_{1,k}$ of the wavelet series representation (4.30). The coefficients $c_{J,k}$ are called the smooth coefficients, representing the underlying smooth behaviour of the signal at the coarse scale 2^J. On the other hand, $d_{j,k}$ are called the detailed coefficients, representing deviations from the smooth behaviour, where $d_{j,k}$ describe the coarse scale deviations and $d_{J-1,k}, \ldots, d_{1,k}$ provide progressively finer scale deviations.

In cases when n is divisible by 2^J, there are $n/2$ coefficients $d_{1,k}$ at the finest scale $2^1 = 2$. At the next finest scale $2^2 = 4$, there are $n/4$ coefficients $d_{2,k}$. Likewise, at the coarsest scale, there are $n/2^J$ coefficients each for $d_{j,k}$ and $c_{j,k}$. Summing up, we have a total of n coefficients:

$$n = n/2 + n/4 + \ldots + n/2^{J-1} + n/2^J + n/2^J.$$

The number of coefficients at a scale is related to the width of the wavelet function. At scale 2, the translation steps are $2k$, and so $n/2$ terms are required in order for the functions $\psi_{1,k}(t)$ to cover the interval $1 < t < n$. By similar reasoning, a summation involving $\psi_{j,k}(t)$ requires just $n/2!$ terms, and the summation involving $\varphi_{J,k}(t)$ requires only $n/2!$ terms. The string of coefficients can be ordered from coarse scales as:

$$\mathrm{w} = \begin{pmatrix} s_J \\ d_J \\ d_{J-1} \\ \cdot \\ \cdot \\ \cdot \\ d_1 \end{pmatrix} \qquad \ldots(4.31)$$

Each of the sets of coefficients in 'w' is called a **'crystal'**, and the wavelet associated with each coefficient is referred to as an **'atom'**.

The multiresolution decomposition of a signal can now be defined by using the product of the crystals and the corresponding wavelet atoms, namely:

$$C_J(t) = \sum_k c_{J,k}\varphi_{J,k}(t) \qquad ...(4.32)$$

$$D_j(t) = \sum_k d_{j,k}\psi_{j,k}(t) \quad for \quad j = 1,2,...,J \quad ...(4.33)$$

The functions (4.32) and (4.33) are called the smooth signal and the detail signals, respectively, which constitute a decomposition of a signal into orthogonal components at different scales. Similarly to the wavelet representation (4.28) of a signal in $L^2(R)$, a signal/ft/ can now be expressed in terms of these signals:

$$f(t) = C_J(t) + D_J(t) + D_{J-1}(t) + ... + D_1(t) \qquad (4.34)$$

As each terms in (4.34) represent components of the signal *f(t)* at different resolutions, it is called a multiresolution decomposition (MRD).

The coarsest scale signal $C_J(t)$ represents a coarse scale smooth approximation to the signal. Adding the detail signal $D_J(t)$ gives a scale 2^{J-1} approximation to the signal, $C_{J-1}(t)$, which is a refinement of the coarsest approximation $C_J(t)$.Further refinement can sequentially be obtained as:

$$C_{j-1}(t) = C_j(t) + D_{j-1}(t) = C_J(t) + D_J(t) + D_{J-1} + ... + D_j(t) \qquad (4.35)$$

The collection $\{C_J, C_{J-1}, C_{J-2}, ..., C_1\}$ provides a set of multiresolution approximations of the signal *f(t).*

4.5 Vanishing Moments of Wavelets

We now define the k^{th} moments of <p (t) and ψ *(t)* as

$$m(k) = \int t^k \varphi(t)dt \qquad ...(4.36)$$

and

$$m_1(k) = \int t^k \psi(t)dt \qquad ...(4.37)$$

and the discrete k^{th} moments of $h(n)$ and $\hat{h}(n)$ as

$$\mu(k) = \sum_n n^k h(n) \qquad ...(4.38)$$

and

$$\mu_1(k) = \sum_n n^k \hat{h}(n) \qquad ...(4.39)$$

The partial moments of $h(n)$ are defined as

$$v(k,l) = \sum_n (2n+l)^k h(2n+l) \qquad ...(4.40)$$

Note that $\mu(k) = v(k,0) + v(k,1)$.

From these equations, we obtain

$$m(k) = \frac{1}{(2^k - 1)\sqrt{2}} \sum_{l=1}^{k} \binom{k}{l} \mu(l) m(k-l) \qquad ...(4.41)$$

which can be derived by substituting recursive equation into (4.36), changing variables, and using (4.38). Similarly, we obtain

$$m_1(k) = \frac{1}{2^k \sqrt{2}} \sum_{l=0}^{k} \binom{k}{l} \mu_1(l) m(k-l) \qquad ...(4.42)$$

These equations exactly calculate the moments defined by the integrals in (4.36) and (4.37) with simple finite convolutions of the discrete moments with the lower order continuous moments.

If $\psi(t)$ is K-times differentiable and decays fast enough, then the first K-1 wavelet moments vanish *i.e.*

$$\left| \frac{d^k}{dt^k} \psi(t) \right| < \infty, \quad 0 \le k \le k \quad (4.43)$$

implies

$$m_1(k) = 0, \quad 0 \le k \le K \qquad ...(4.44)$$

Unfortunately, the converse of this theorem is not true. However, we can relate the differentiability of $\psi(t)$ to vanishing moments. There exists a finite positive integer L such that if $m_l(k) = 0$ for $0 < k < K-1$ then

$$\left|\frac{d^P}{dt^P}\psi(t)\right| < \infty \quad for\ L\ P > K.$$

For example, a three times differentiable $\psi(t)$ must have three vanishing moments, but three vanishing moments results in only one dimensional differentiability.

4.6 Types of Wavelets

Haar Wavelets

The first wavelet filter, Haar wavelet (Haar, 1910) remained in relative obscurity until the consequence of several disciplines to form what we now know in broad sense as wavelet methodology. It is the simplest wavelet with the scaling and wavelet functions which is as follows:

$$\int \phi(t)dt = 1,\ \int \psi(t)dt = 0,\ and\ \int \phi(t)\psi(t)dt = 0$$

It is a filter of length 2 that can be succinctly defined by its scaling (low-pass) filter coefficients $h_0 = h_1 = 1/\sqrt{2}$ or equivalently by its wavelet (high-pass) filter coefficients

$\hat{h}_0 = -\hat{h}_1 = 1/\sqrt{2}$.

The Haar wavelet has good properties such as simplicity, orthonormality and compact support. Although, the Haar wavelet is easy to visualize and implement, it is inadequate for most real world applications in that it is a poor approximation to an ideal band-pass filter. Also, it is discontinuous and so we have difficulty in approximately a smooth function by Haar wavelets with more regularity.

Daubechies Wavelets

Daubechies wavelets form an orthogonal basis with compact support. The Daubechies wavelet filters represent a collection of wavelets that improve upon the frequency domain characteristics of the Haar wavelet (Daubechies, 1992). Daubechies derived these wavelets from the criterion of a compactly supported function with the maximum number of vanishing moments. In general, there are no explicit time-domain formulae for this class of wavelet filters; the simplest way to define the class of Daubechies wavelets is through the filtered coefficients. As the number of filter length increases, the Daubechies wavelets length increases. If the length of the filter is 2, then the wavelet is Daubechies - 2 (db_2), which is identical to Haar wavelet. If the filter length is 4 and 6, then wavelets are Daubechies -4 (db_4) and Daubechies - $6(db_6)$ respectively. The (db_4) wavelets have following filtered coefficients:

$$h_0 = -\hat{h}_3 = \frac{1+\sqrt{3}}{4\sqrt{2}},\ h_1 = \hat{h}_2 = \frac{3+\sqrt{3}}{4\sqrt{2}},\ h_2 = -\hat{h}_1 = \frac{3-\sqrt{3}}{4\sqrt{2}}, \text{ and } h_3 = -\hat{h}_0 = \frac{1-\sqrt{3}}{4\sqrt{2}},$$

and the (db_6) wavelets filter coefficients are as follows:

$$h_0 = -\hat{h}_5 = 0.3327,\ h_1 = -\hat{h}_4 = 0.8069,\ h_2 = \hat{h}_3 = 0.4599, -h_3 = \hat{h}_2 = 0.1350,$$
$$-h_4 = -\hat{h}_1 = 0.0854,\ h_5 = -\hat{h}_0 = 0.0352.$$

The (db_4) wavelet has one vanishing moment and the (db_6) has two vanishing moments. One implication due to this property is that longer wavelet filters may produce stationary wavelet coefficient vectors from higher degree of non-stationary stochastic processes. We have mentioned in the previous section that the Haar wavelet is a poor approximation to an ideal band-pass filter but the level of approximation improves as filter length increases.

4.7 Wavelet Shrinkage

Separating noise from the signal is denoising. Wavelet shrinkage refers to thresholding, which is shrinkage of wavelet coefficients. The choice of a threshold in wavelet analysis is as important as the choice of a bandwidth in kernel smoothing. There are several thresholding rules. The key parameters in all thresholding rules is value of the threshold.

4.7.1 Value of the Threshold

This is an important step, which affects the quality of the compressed signal. The basic idea is to truncate the insignificant coefficients since less amount of information is contained. Then optimal thresholding occurs when the thresholding parameter is set to the noise level. Setting thresholding parameter less that noise level would allow unwanted noise to enter the estimate while setting thresholding parameter greater than noise level would destroy information that really belongs to the underlying signal. Donoho and Johnstone (1994) suggested a universal thresholding by setting

$$t = \sigma\sqrt{2\log N}$$

where σ is the standard deviation and N refers to total number of data points. The value of the threshold (t) should be calculated for each level of decomposition and only for the high-pass coefficients (low-pass coefficients are kept untouched so as to facilitate further decomposition).

Hard Tresholding

Literally interpreting the statement "keep or kill", hard thresholding is a straightforward technique for implementing wavelet denoising. The hard thresholding function is easy to use and gives better reconstruction of discontinuities. The threshold value t is given by

$$H(t,d) = \begin{cases} d & if\ |d| > t \\ 0, & otherwise \end{cases}$$

where d refers to the wavelet coefficients. This observation is not a continuous mapping and only affects input coefficients that are less than or equal to the threshold.

Soft Tresholding

The other standard technique for wavelet denoising is soft thresholding of the wavelet coefficient via

$$S(t,d)=\begin{cases}0 & if \quad |d|<t \\ Sign \quad (d)(|d|-t), & otherwise\end{cases}$$

$$\text{where } Sign(d)=\begin{cases}+1 & if\ d>0 \\ 0 & if\ d=0 \\ -1 & if\ d<0\end{cases}$$

Instead of forcing wavelet coefficients to zero or leaving it untouched, soft thresholding pushes all coefficients towards zero. If the wavelet coefficient happens to be smaller in magnitude than the threshold, then it is set to zero as in hard thresholding. Thus, the operation of soft thresholding is a continuous mapping. The choice between these two thresholding rules depends upon what characteristics are desirable in the resulting estimate. For instance, if large spikes are present in the observed series, then hard thresholding will preserve the magnitude of these spikes while soft thresholding, because it affects all wavelet coefficients, will suppress them. On the other hand, soft thresholding will, in general produce a smoother estimate because all wavelet coefficients are being pushed towards zero. It is up to the practitioners to weigh these differences and apply the most appropriate thresholding rule.

4.8 Implementation of the Discrete Wavelet Transform (DWT)

Let x be a dyadic length vector $(N = 2^J)$ of observations. The length N vector of discrete wavelet coefficients $\boldsymbol{w}$ is obtained via

where *to* is NxN **orthonormal** matrix defining the DWT. The vector of wavelet coefficients may be organized **into** $J+1$ vectors,

$$w=\left[d_1,d_2,\ldots,d_j,c_J\right]^T \qquad \ldots (1)$$

where w_j is a length $N/2^j$ vector of wavelet coefficients with changes on a scale of length[1] $\lambda_j = 2^{j-1}$ and c^J is a length $N/2^J$ vector of scaling coefficients on a scale of length $2 \;\; = 2\lambda_J$.

4.8.1 Pyramid Algorithm

In practice, the DWT is implemented via pyramid algorithm (Mallat, 1989) that starting with the data x_t, filters a series using h and h , sub samples both the filter outputs to half of their original lengths, keeps sub sampled output from the h as wavelet coefficients, and then repeats the above filtering operations on the sub sampled output from the h filter. Figure 4.1 gives a flow diagram for the first stage of the pyramid algorithm. The symbol $\downarrow 2$ means that every other value of the input vector is removed (down sampling by 2).

For each iteration of the pyramid algorithm, we require three objects: the vector x, the wavelet filter h and the scaling filter h. The first operation of the pyramid algorithm begins by filtering (convolving) the data with each filter to obtain the wavelet (d_1) and scaling coefficients (c_1). The N length vector of observations has been high and low-pass filtered to obtain $N/2$ coefficients. The second step of the pyramid algorithm starts by defining the data to be the scaling coefficients c_1 from the first iteration and apply the filtering operations as above to obtain the second level of wavelet (d_2) and scaling (c_2) coefficients. Now the length of the filtered coefficients is $N/4$. Keeping all vectors of wavelet coefficients and the final level of scaling coefficients, we have the following length decomposition

$$w = [d_1 \; d_2 \; c_2]^T$$

After the third iteration of the pyramid algorithm, where we apply filtering operations to c_2, the decomposition now looks like

[1] Wavelet coefficients are obtained, by projecting the wavelet filter onto a vector of observations. Since Daubechies wavelets may be considered as generalized differences, we prefer to characterize the wavelet coefficients this way. For example, a unit scale Daubechies wavelet filter is a generalized difference of length one-that is, the wavelet filter is essentially taking the difference between two adjacent observations. We call this a wavelet scale of length $\lambda_1=2^0=1$. A scale two Daubechies wavelet filter is a generalized difference of length two-that is, the wavelet filter first averages adjacent pairs of observations and then takes the differences of these averages. We call this a wavelet scale of length $\lambda_2=2^1=2$. The scale length increases by powers of two as a function of scale.

$$w = [d_1 \; d_2 \; d_3 \; c_3]^T$$

This procedure may be repeated up to J times where $J = \log_2(N)$ and gives the vector of wavelet coefficients. Inverting, the DWT is achieved through up-sampling the final level of wavelet and scaling coefficients, convolving them with their respective filters (wavelet for wavelet and scaling for scaling) and adding up the two filtered vectors. Figure 4.2 gives a flow diagram for the reconstruction of x from the first levels wavelet and scaling coefficient vectors. The symbol t 2 means that a zero is inserted before each observation in d_l and c_l (up sampling by 2). Starting with the final level of the DWT, up sampling the vector d_J and c_J will result in two new vectors:

$$d_J^0 = [0 \; d_J]^T \text{ and } c_J^0 = [0 \; c_J]^T .$$

The next step of reconstruction involves up sampling to produce d_{J-1}^0 and c_{J-1}^0. This procedure may be repeated until the first level of wavelet and scaling coefficients have been up sampled and combined to produce the original vector of observations.

4.8.2 Partial Discrete Wavelet Transform

If the data are of dyadic length, it is not necessary to implement the DWT down to level $J = \log_2(N)$. A partial DWT may be performed instead, that terminates at a level $J_P < J$. The resulting vector of wavelet coefficient will now contain $N - N/2^J_p$ wavelet coefficients and $N/2^J_p$ scaling coefficients. When we are provided with a non-dyadic length time series, e.g., 368, which is divisible by 2^4 = 16 and therefore we may perform an order J_p = 4 partial DWT on it.

4.9 Empirical Analysis

By design, the wavelet's usefulness is its ability to localize data in time-scale space. At high scales (shorter time intervals) the wavelet has a small time support and is thus better able to focus on short lived, strong transients like discontinuities, ruptures and singularities. At low scales (longer time intervals), the wavelet's time support is large making it suited for identifying long periodic features. Wavelets have a intuitive way of characterizing the physical properties of the data. At low scales, the wavelet characterizes the data's coarse structure; its long-run trend and pattern. By gradually increasing the scale, the wavelet begins to reveal more and more of the data's details, zooming in on its behavior at a point in time.

Wavelet analysis is the analysis of change. A wavelet coefficient measures the amount of information that is gained by increasing the frequency at which the data is sampled, or what needs to be added to the data in order for it to look like it had been measured more frequently. For instance, if a stock price does not change during the course of a week, the wavelet coefficients from the daily scale are all zero during that week.

Wavelet coefficients that are non-zero at high scales typically characterize the noise inherent in the data. Only wavelets at very fine scales will try to follow the noise, whereas the same at coarser scales are unable to pick up the high frequency nature of the noise. If both the low and high scaled wavelet coefficients are non-zero then something structural is occurring in the data. A wavelet coefficient that does not go to zero as the scale increases indicates a jump (non-differentiable) has occurred in the data. If the wavelet coefficients do go to zero then the series is smooth (differentiable) at this point. Because of its localization in time and scale, wavelets are able to extract relevant information from a data set while disregarding the noise that may be present. Given the recording errors that occur during short, intense trading periods, and transient shocks that are caused by news reports, this de-noising technique is important to financial data.

Very little needs to be known about the relevant information or the information that one wants to extract. Because the wavelet transform captures the characteristics of the data in a few wavelet coefficients, if the wavelet coefficients whose magnitude is less than some prescribed value are set to zero and the few non-zero wavelet coefficients are used to recreate the data, the resulting data set will contain only the relevant information.

The data considered for wavelet analysis are daily closing values of Sensex, National Index, S&P CNX Nifty and S&P CNX 500 (for detail see appendix A).

Though our sample **period is from 2nd** January 1991 to 31st December 2001 constituting of 2552 data points, for wavelet decomposition analysis, we have used two different sample periods. First one is the dyadic length sample period from 2nd January 1991 to 21st December 1999 with 2048 (2^{11}) data points. Dyadic length data series has one potential advantage of performing up to the last level of wavelet decomposition (here 10). Second type of sample period is from 2nd January 1991 to 27th September 2000 with 2240 (divisible by $2^6 = 64$) data points. In this case, we apply partial discrete wavelet decomposition and perform an order $J_p = 6$ partial DWT on it. Since our original sample period consists of 2552 data points (divisible by $2^3 = 8$), we cannot perform an order of more than three partial DWT on it. Therefore, we conveniently take 2240 (divisible by $2^6 = 64$) data points of the return series for the purpose of DWT up to level 6.

The movements of the original data series are shown in figures 4.3 and 4.6 respectively. For the interest of the reader and also for better comparison with the wavelet decomposition, some key dates are highlighted in figure 4.1. It is easily observed from the data series that there was a significant variation in stock prices during April - May 1992, possibly due to Harshad Meheta led rally and scam unravels thereafter. It has crossed the 5000 mark in December 1999 and also reached its highest mark so far in February 2000. From the return series it is evident that there are increases in variance in the first and the latter part of the series.

All the return series are now subjected to discrete wavelet transform by using Haar (equivalent to db_2), Daubechies - 4 (db_4) and Daubechies - 6 db_6) wavelets. Haar DWT results for all the four indices are reported in figures 4.7 to 4.11. The figure 4.7 reports the results of Haar DWT of a dyadic length return series (here $2048 = 2^{11}$). As we mentioned earlier in the technical section of the wavelet analysis, if the data series is of dyadic length 2^n, then we perform up to the last level of decomposition. In this case, we may perform up to 10th level of wavelet decomposition where we will have only with two coefficients for both high and low-pass filter. Moreover in this case of dyadic length return series, we may analyze time -scale decompositions from finest scale (first level) to coarsest scale (here 10th level) and it will give clear picture of both high and low-frequency fluctuations.

The DWT results of Sensex return series in figure 4.5 are arranged in to level - 1, level - 2, ... up to level - 10 coefficients, where the last level represents the low pass coefficients. The return series are plotted on the upper row of figure 4.5. The wavelet coefficient vectors $\mathbf{d}_1$, ... $\mathbf{d}_{10}$, using Haar wavelet are shown in the lower part of the figure 4.5. The first scale of the wavelet coefficient $\mathbf{d}_1$ are filtering out the high frequency fluctuations by essentially looking at adjacent differences in the data. There is a large group of rapidly fluctuating returns between observations 220 and 270. A small increase in the magnitude $\mathbf{d}_2$ is also observed between observations 250 and 300, but smaller than the unit scale coefficients. This vector of wavelet coefficients is associated with changes of scale *fa.* The higher scale (low-frequency) vectors of wavelet coefficients $\mathbf{d}_3$ to $\mathbf{d}_6$ indicate variations from zero, which implies that the Sensex return series exhibits low frequency oscillations. The next level of wavelet coefficients $\mathbf{d}_7$ and $\mathbf{d}_8$, after differencing the 64 and 128 trading days averages of returns, show a quasi-periodic behaviour. The coarsest scale wavelet coefficient $\mathbf{d}_9$ and $\mathbf{d}_{10}$ show only the four and two data points respectively, whose interpretation is not of much use.

The same Haar wavelet decomposition is performed in non-dyadic length of Sensex return series and are provided in figure 4.8. The length of the return series is N = 2240, which is divisible by $2^6 = 64$ and therefore we perform an order $J_p = 6$ partial DWT on it. The DWT results are arranged in to level–1, level–2, up to level–6. Wavelet coefficients, where the last level represents the low-pass coefficients. The level-1 coefficients are the differences of the nearest neighborhood observations of the return series multiplied by Haar wavelet filter coefficient. The next scale coefficients are the differences of the nearest neighborhood averages. To be precise, finest scale (level-1) coefficients capture day-by-day fluctuations; the level-2 coefficients represent the differences of the two day averages and analogously level–6 wavelet coefficients captures the differences of 32-day averages of the return series. Hence, it is clear that the first scale wavelet coefficients filter out high frequency fluctuations by looking at the adjustment differences in data. Our results show that there is a large group of rapidly fluctuating returns between observations 220 and 270 and further there is also a group of fluctuations in the later part of the return series. Analogous results are obtained in second and third levels of wavelet decompositions but detected fluctuations are smaller than the first level coefficients. A number of notable features appear at the 4^{th} and 5^{th} levels of decomposition, which shows that there are substantial differences between eight and sixteen days averages of the returns over the whole series. Higher scale wavelet coefficients indicate slow variations from zero, which implies low frequency fluctuations in return series. Interestingly, the 6^{th} level coefficients, after differencing the 32 trading days averaging in returns, show a quasi-periodic behaviour. As we go for higher level of wavelet decomposition, both high and low-pass coefficients become smooth which is quite obvious from the very nature of averaging and differencing the wavelet of each scale.

This analysis clearly indicates the usefulness of the time-scale decompositions and multi-scale nature of the wavelets. In the stock market, there are traders who take a very long term view and consequently concentrate on what are termed 'market fundamentals'; these traders ignore short-term phenomena. For them, the high-level wavelet coefficients are very useful and they are more concerned about the same. In contrast, other traders are trading on a much shorter time-scale and as such are interested in temporary deviations of the market from its long-term path. Their decisions have a time horizon of a few months to a year; so they are interested in middle level wavelet decompositions of the return series. And yet other traders are in the market for whom a day is a long time and consequently concentrate on day-by-day fluctuations. Therefore, low level of wavelet coefficients of return series are more useful for them in the stock market. As we have discussed earlier, wavelet coefficients that are non-zero at high scales typically characterize the noise inherent in the data series. Only wavelets at very fine scales will try to follow the noise, whereas those wavelets at coarser scales are unable to pick up the high frequency nature of the noise. If both the low and high scaled wavelet coefficients are non zero then something structural is occurring in the data series. In this case, the large group of fluctuating returns between observations 220 and 270 indicated at all levels of wavelet decompositions provides some insight in to the Indian stock market. These large swings in stock returns were probably due to Harshad Meheta led rally and scam unraveled subsequently during March-May 1992. Again, the large group of rapid fluctuations in Sensex return series in the later part indicated at various scales of wavelet decomposition may be due to Kargil war followed by lifting up of the US economic sanctions and upswing in the Indian stock market. This bullish trend in the Indian stock market during the December 1999 to February 2000 was due to favourable economic condition. The large peaks detected by some levels of wavelet decomposition analysis between observations 1950 to 2000 match with the Sensex reaching its all-time high of above 6000 mark and subsequent downslide due to union budget 2000-01.

The same Haar wavelets decomposition has been performed for other three Indian stock market indices such as National Index, S&P CNX Nifty and S&P CNX 500 and is provided from figures 4.9 to 4.11. The interpretations for each of the vectors wavelet coefficients is the same as in case of Sensex return series. In all the above cases of wavelet decompositions, each level low-pass coefficients gives a smoothed replication of the original return series. As shown in above analyzed figures, we plot the low-pass (scaling) coefficients vector of only large level (c_6) and it also reflects most of the fluctuating features of the original return series in a smooth manner. The most interesting findings from the various levels of wavelet decompositions is that with least number of data points, we are able to analyze the original return series. For example, in the first scale of wavelet decomposition the vector of wavelet coefficients consist of data points half of the return series, but still it gives a clear picture about the various useful features of the original series. In similar fashion, the sixth level wavelet coefficient vector with 35 points able to give a overall idea about the original return series.

In the next step we present the reconstruction results of the decomposed return series from figures 4.12 to 4.15. The idea behind the reconstruction of the wavelet return series is to show how wavelet coefficients are abele to capture the various properties of the return series and then reconstruct them. Figure 4.12 reports the reconstructed wavelet series of the Sensex return series. For comparison, the return series is plotted in he upper row of the figure 4.12. The reconstructed series from wavelet coefficient vectors are $Rd_1, Rd_2, \ldots, Rd_6$, using the Haar wavelets are shown in the lower part of the figure 4.12. The first scale wavelet reconstructed series Rd_1 shows exactly similar behaviour as return series by capturing all the high frequency fluctuations. The most interesting finding is that if we use the wavelet reconstructed series instead of original return series, it gives better results in economics and finance, which will be discussed in chapter V. The most promising advantages of the wavelet reconstructed series is that it removes the statistical anomalies by recognizing the potential for relationship between variables to be at scale level, not at the aggregate level. That is, one should recognize that the relationship between two variable series depending on the time scale that is involved. Another advantage is that if we are interested in fluctuations and try to find out the relationship between two variables by essentially looking at fluctuations, e.g., fluctuations due to news or other factors, wavelet reconstructed series is very much useful. The most important example in the stock market is to test for spillover effects across stock markets, where we are interested in relating fluctuations of two series, which will be discussed in chapter V. Similarly, second scale of wavelet series is reconstructed with wavelet coefficients which is $1/4^{th}$ of the original series and still it gives a clear idea about the original return series. In a similar fashion all the levels of wavelet coefficients vectors reconstruct their corresponding return series. Interestingly, the 6^{th} scale wavelet coefficient vector even with 35 observations is able to pinpoint all the low-frequency fluctuations of the original Sensex return series. The same reconstruction is performed using stock indices such as National Index, S&P CNX Nifty and S&P CNX 500 and are provided in figures 4.13 to 4.15 respectively. The interpretations for each index reconstructed return series is the same as in case of Sensex return series.

Figures 4.16 to 4.19 display the wavelet decompositions using the db_4 wavelet filter for the original return series. As we have discussed earlier in technical session, db_4 wavelet filter has one vanishing moment, i.e., it does not see straight-line part of a series, whereas Haar does not see a constant part of a series. If some part of the series is smooth then Haar wavelet coefficients go to zero and a wavelet coefficient that does not go to zero indicates a jump has occurred in the data. In addition to that if some portion of the return series is straight then db_4 wavelet coefficients go to zero. One implication of this property is that if we are interested in high fluctuations only, then db_4 wavelet coefficients are very much useful. Figure 4.16 shows the db_4 wavelet decomposition coefficient vectors for Sensex return series. The analysis for each of the vectors of coefficients is the same as the case of the Haar wavelet filter. The wavelet coefficients will be different given the length of the filter is now four versus two, should isolate features in a specific frequency interval better since the db_4 is a better approximation to an ideal band-pass filter over the Haar wavelet (Genacy et al. 2001). The wavelet coefficients in the db_4 basis capture the fluctuations very well after removing the straightline part of the return series.

To provide an analysis of the db_6 wavelet basis, we consider four return series used in this study and results are displayed in figures 4.20 to 4.23. The db_6 wavelets satisfy the criterion of a compactly supported basis set with two vanishing moments. This shows that db_6 wavelet removes the patchiness of the data and pinpoints zig-zag fluctuations in the data series. Similarly, as we go on applying longer wavelet filters, we get higher order vanishing moments wavelet coefficients. One implication of this property is that longer wavelet filters may produce stationary wavelet coefficient vectors from higher degree non-stationary stochastic processes. The longer wavelet filters induce significant amounts correlation between adjustment coefficients, thus produce even smoother vectors of wavelet and scaling coefficients.

The most useful property of higher order vanishing moments wavelets are that they remove patchiness from the data and pinpoints higher order fluctuations. The longer wavelet filters separate out order out of chaos and fractals. As it is clear if we cannot describe some part of the curve by polynomials the there may be evidences for chaotic behavior and fractal nature in the curve. Thus, the longer wavelet filters with the property of higher order vanishing moments separate out ordered part like regular and random changes from chaos and fractals. In this study we have not gone for finding out chaos and fractal nature of a curve, which may be taken as a scope for further research.

A critical innovation in estimation that is introduced by wavelets, although by no means necessarily restricted to wavelets is the idea of shrinkage. Traditionally in economic analysis the assumption has universally been made that the signal, $f(t)$, is smooth and the innovations, *eft)* are irregular. Consequently, it is a natural first step to consider extracting the signal $f(t)$ from the observed signal $y(t) = f(t) + \varepsilon(t)$. By locally smoothing $y(t)$. However, when the signal is as, or even more irregular than the noise such a procedure no longer provides a useful approximation to the signal. The process of smoothing to remove the contamination of noise distorts the appearance of the signal itself When the noise is below a threshold and the signal variation is well above the threshold. One can isolate the signal from the noise component by selectively shrinking the wavelet coefficient estimates (Donoho and Johnstone, 1995; Donoho et al, 1995). If we want the probability of any noise appearing in our signal return series to be as small as possible then applying the wavelet transform and thresholding the wavelet coefficients is a good strategy. Utilizing the threshold we may then remove (hard thresholding) or shrink toward zero (soft thresholding) wavelet coefficients at each level of the decomposition in an attempt to eliminate the noise from the signal. Inverting the wavelet transform yields an estimate of the underlying denoised signal *f(t)*. Thresholding wavelet coefficients are appealing since they capture information at different combination of time and frequency. Thus the wavelet based estimate is locally adaptive.

Figures 4.24 to 4.27 display the result of universal thresholding applied to wavelet coefficients of the four return series. In each figure the upper row is the plot of the respective return series and next two rows represent the estimates of wavelet coefficients after applying hard and soft thresholding respectively. Here we have used universal threshold of Donoho and Johnostone (1994) by setting

$$t = \sigma\sqrt{2 \log N} .$$

As we know hard thresholding affects wavelet input coefficients that are less than and equal to the threshold (t) i.e., make them as zero. Whereas the soft thresholding, instead of forcing wavelet coefficient to zero or leaving it untouched, pushes all coefficients toward zero. If the wavelet coefficient happens to be smaller in magnitude than the threshold, then it is set to zero.

Figure 4.24 shows the results of application of both hard and soft thresholding of the Sensex return series. The universal thresholding value in this case is fond to be 1.9540. Therefore large spikes greater than 1.9540 are left untouched and values smaller than and equal to 1.9540 are set as zeros in case of first scale Haar wavelet coefficients by hard thresholding. Whereas soft thresholding suppresses the larger spikes towards zero, that is why it gives relatively small spikes after applying the same. As a whole, after applying both the thresholding rules the Sensex return series retains all its spikes with a smooth plot of it. The universal threshold values for National Index, S&P CNX Nifty and S&P CNX 500 returns series are 1.74499, 1.8825, and 1.6879 respectively. The interpretations of the figures from 4.25 to 4.27 are same as Sensex return series.

Towards the end we have gone for estimating the summary statistics for original returns series and reconstructed returns series by wavelet coefficients for comparison, which are reported in table 4.1. The most interesting findings are that mean and skewness of reconstructed returns are zero. Since all the return series are used on the assumption of the zero mean, the wavelet reconstructed return series are particular answer to that. Another important thing is that standard deviations of the reconstructed series are less than original return series and skewness is zero for all the reconstructed series. Thus, they are relatively more approximated towards the normality assumption of the return series. These informations are very much useful while using return series for further statistical estimation or application.

4.10 Concluding Remarks

In this Chapter, the four returns series are subjected to discrete wavelet transform (DWT) by using Haar, Daubechies - 4 (D_4) and Daubechies - 6 (D_6). The results in this chapter have been an exploratory investigation into the applicability and usefulness of the wavelet analysis to detect element of fluctuations at various scales, and recover signals from noisy observations (also known as wavelet de-noising or wavelet shrinkage). It is apparent that wavelet analysis of return empirically explores the fluctuations, removes patchiness and looks for patterns possibly at certain levels. It also recovers signals from the noisy data by applying universal thresholding rule. In general, wavelet coefficients are very much useful for the statistical analysis of stock return series.

Table 4.1: Summary Statistics of Returns Series

	Sensex		National		CNX Nifty		CNX 500	
	RET	RET_d_1	RET	RET_d_1	RET	RET_d_1	RET	RET_d_1
Mean	0.027	0.000	0.027	0.000	0.028	0.000	0.018	0.000
SD	0.842	0.559	0.791	0.499	0.814	0.539	0.774	0.483
Sk	0.146	0.000	0.209	0.000	0.064	0.000	-0.162	0.000
Kurt	6.617	7.029	9.247	7.282	8.162	6.946	9.654	7.329
J-B	1229.3	1515.3	365807	1712.0	2488.6	1453.7	4143.1	1749.7
p-value	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Note: RET represents original return series and RET_d| represents return series reconstructed from wavelet coefficients. Jarque-Bera (J-B) statistics is approximately distributed as central Chi-square (2) under the null hypothesis of normality

Figure 4.1: Pyramid Algorithm (Down Sampling)

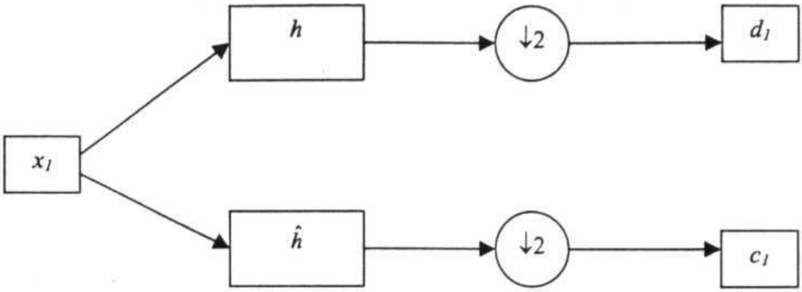
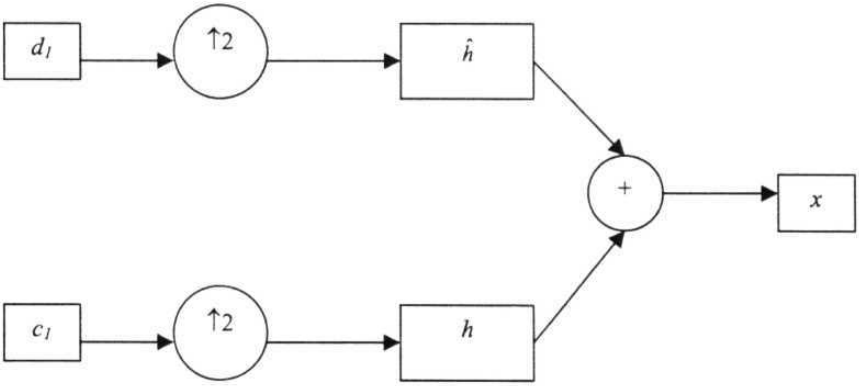


Figure 4.2: Pyramid Algorithm (Up Sampling)



Figures 4.3-4.6: Plots of Daily Closing Values of the Indices

Figure 4.3: Sensex

Figure 4.4: National **Index**

Figure 4.5: S&P CNX Nifty

Figure 4.6: S&P CNX 500

Figure 4.7: Wavelet Decompositions of a Dyadic Length Sensex Return Series (Haar)

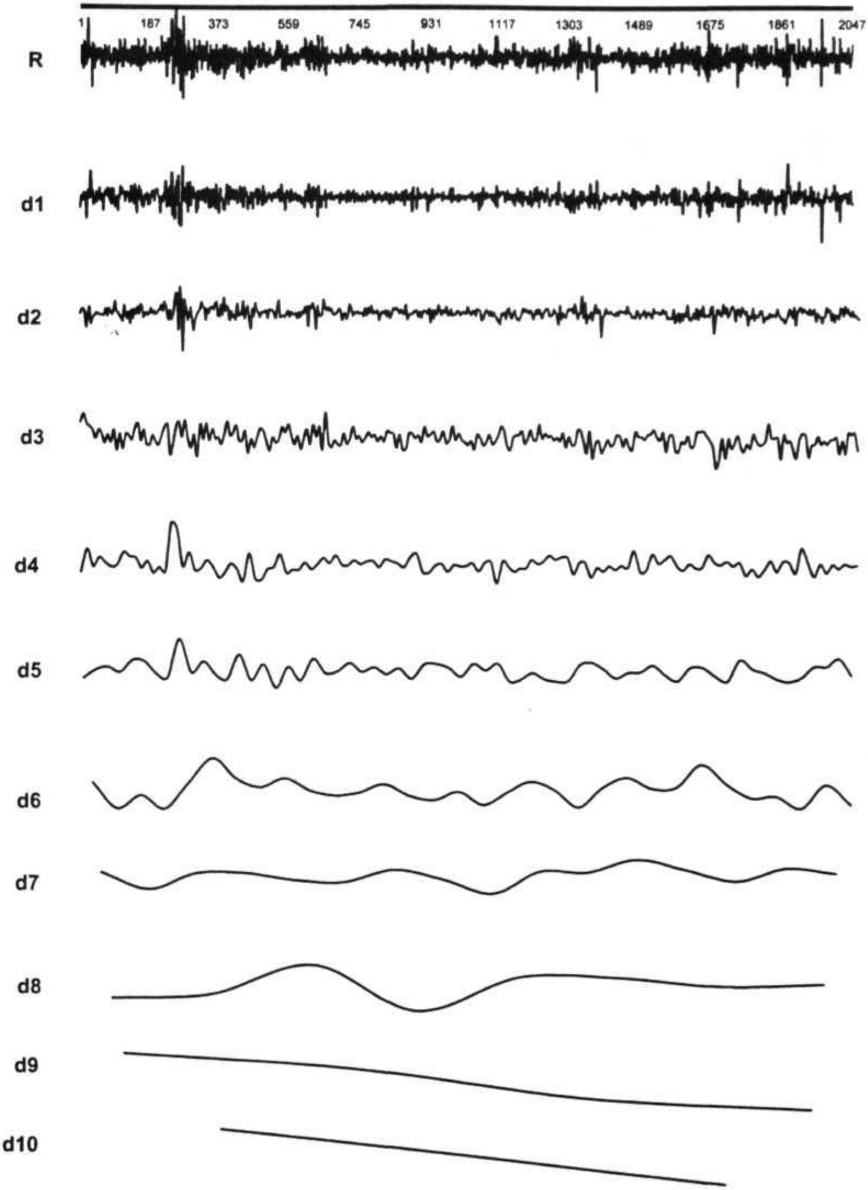


Figure 4.8: Wavelet Decompositions of Sensex Return Series (Haar)

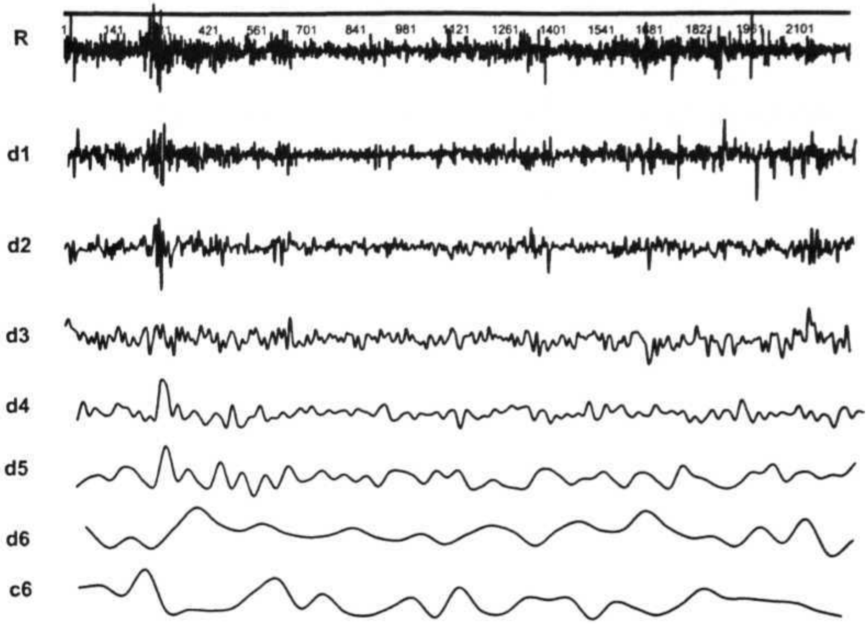


Figure 4.9: Wavelet Decompositions of National Index Return Series (Haar)

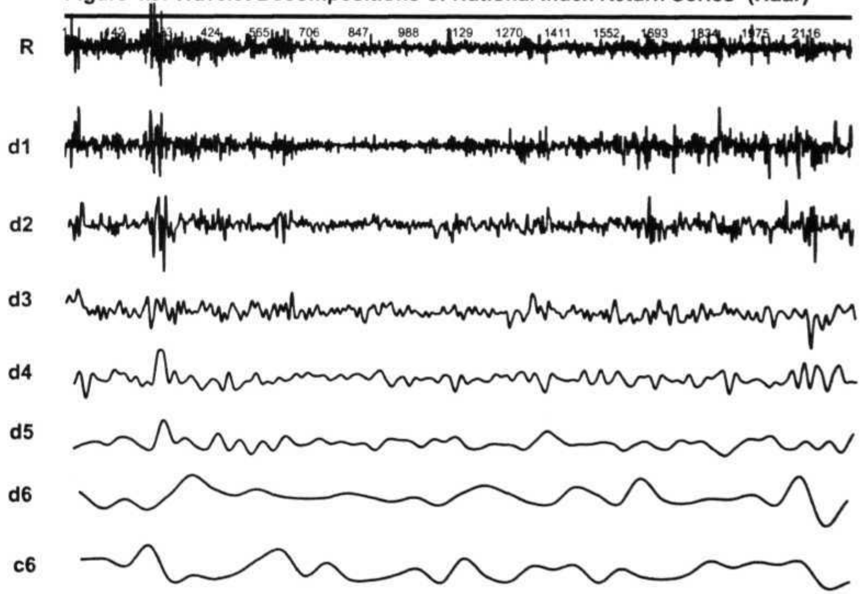


Figure **4.10:** Wavelet Decompositions of S & P CNX Nifty Return Series **(Haar)**

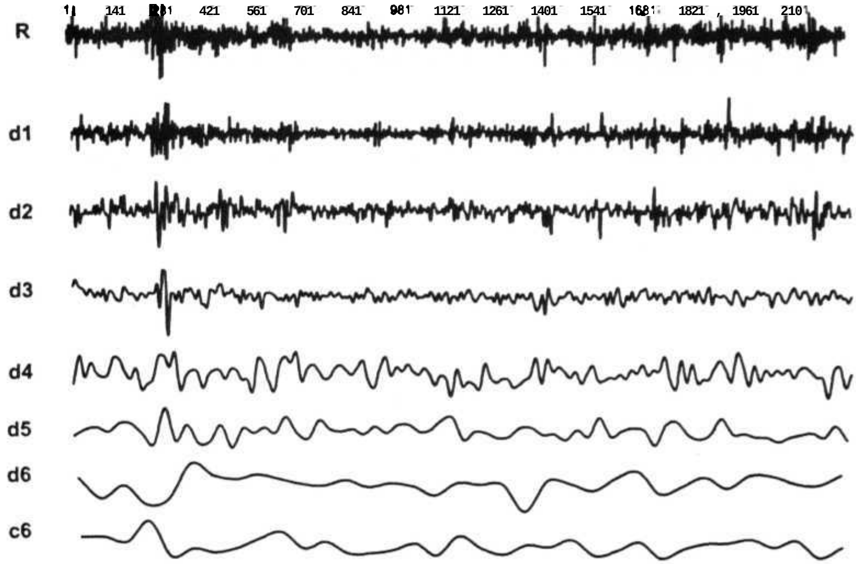


Figure 4.11: Wavelet Decompositions of S & P CNX 500 Return Series (Haar)

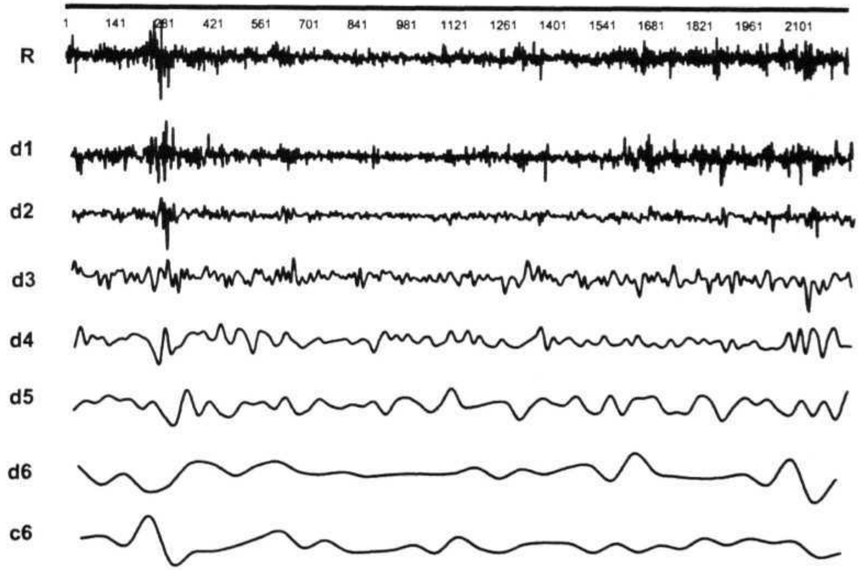


Figure 4.12: Reconstruction of Sensex Return Series from Haar Wavelet Coefficients

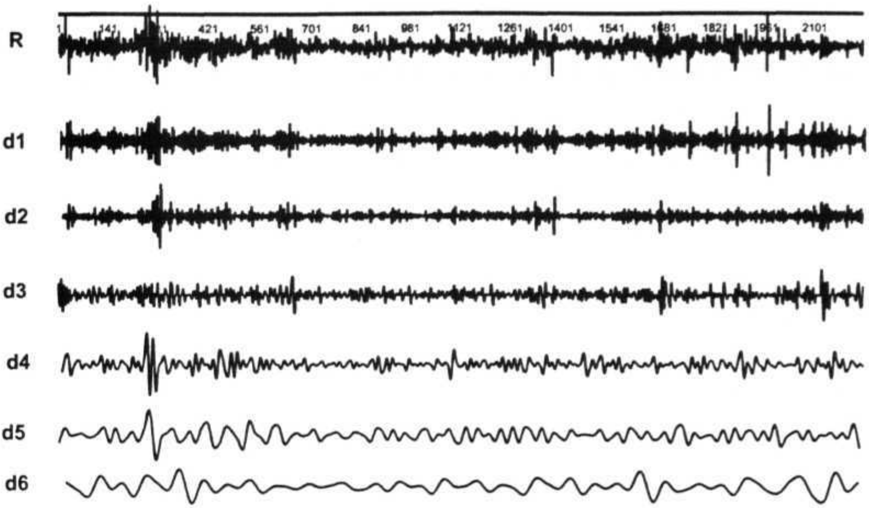


Figure 4.13: Reconstruction of National Index Return Series from Haar Wavelet Coefficients

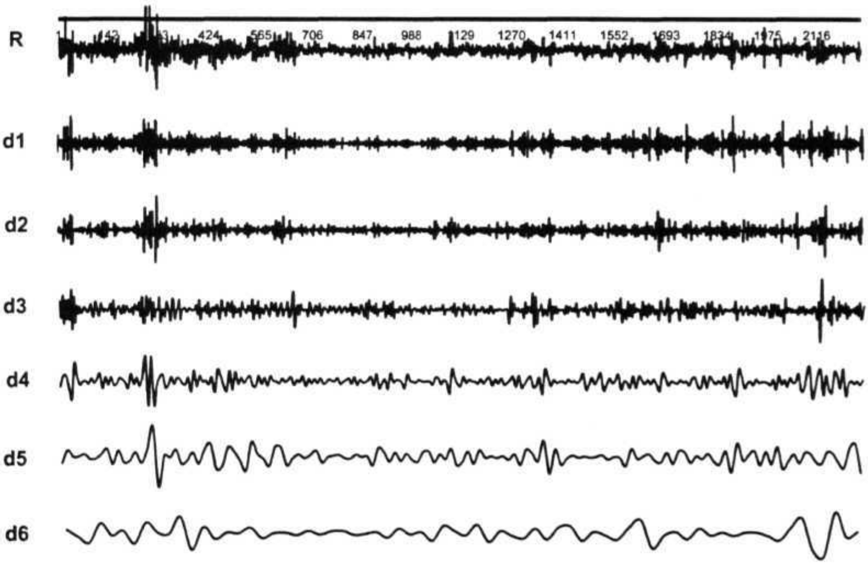


Figure 4.14: Reconstruction of S & P CNX Nifty Return Series from Haar Wavelet Coefficients

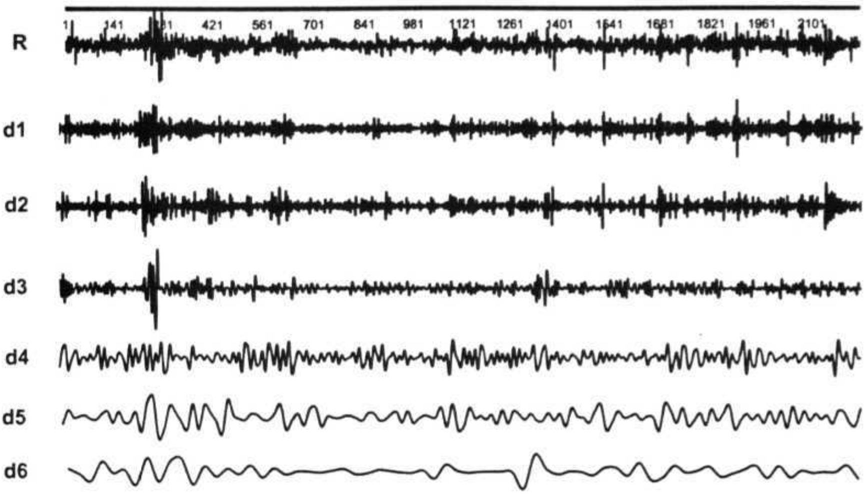


Figure 4.15: Reconstruction of S & P CNX 500 Return Series from Haar Wavelet Coefficients

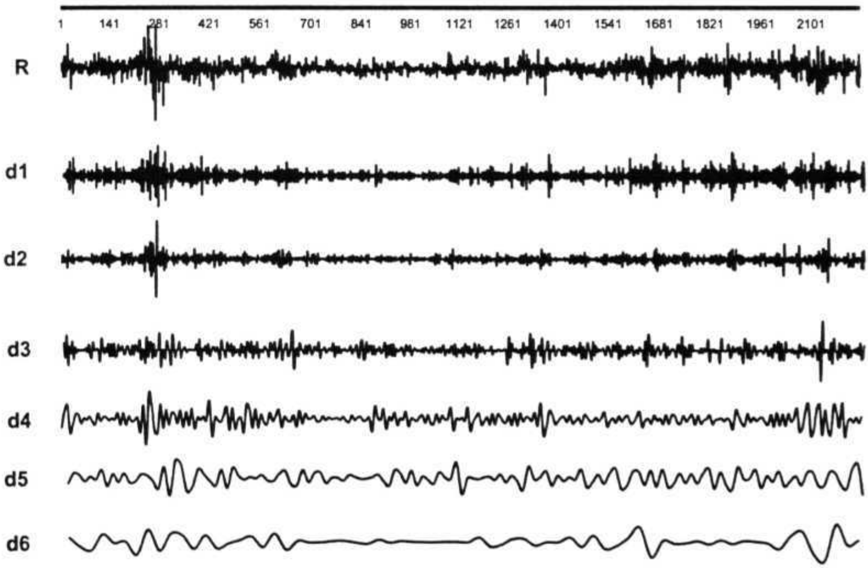


Figure 4.16: Wavelet Decompositions of Sensex Return Series (db4)

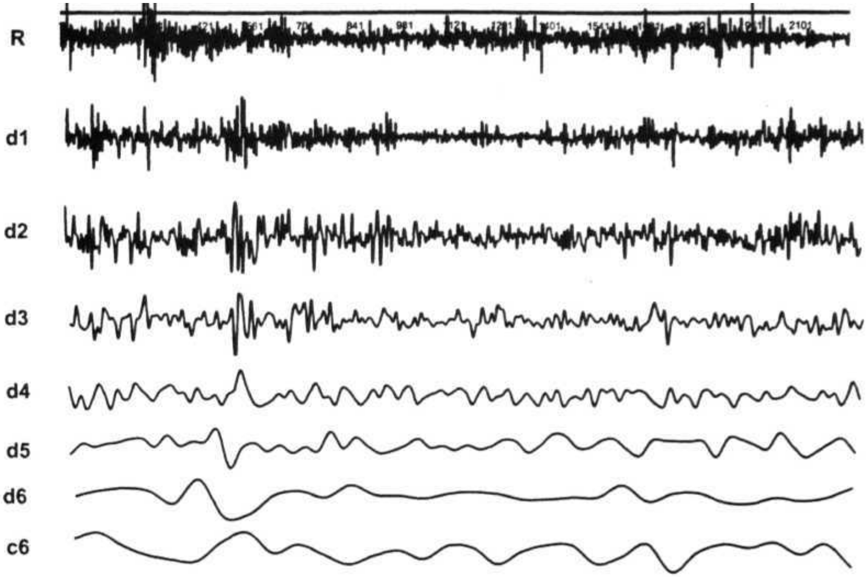


Figure 4.17: Wavelet Decompositions of National Index Return Series (db4)

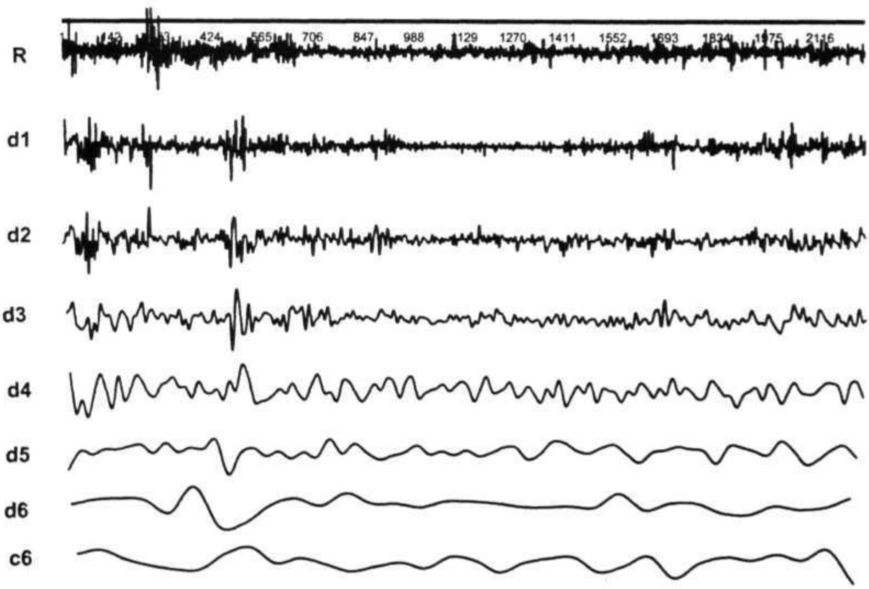


Figure 4.18: Waveiet Decompositions of S & P CNX Nifty Return Series (db4)

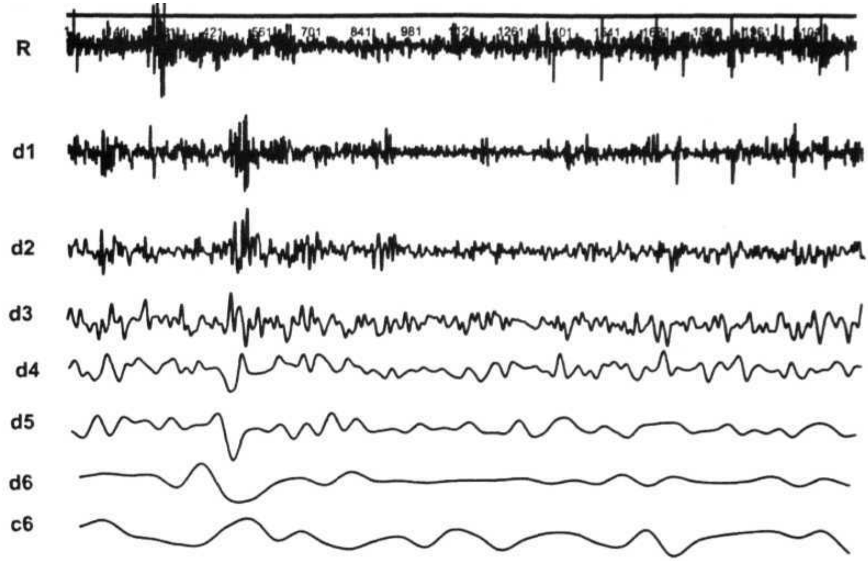


Figure 4.19: Wavelet Decompositions of S & P CNX 500 Return Series (db4)

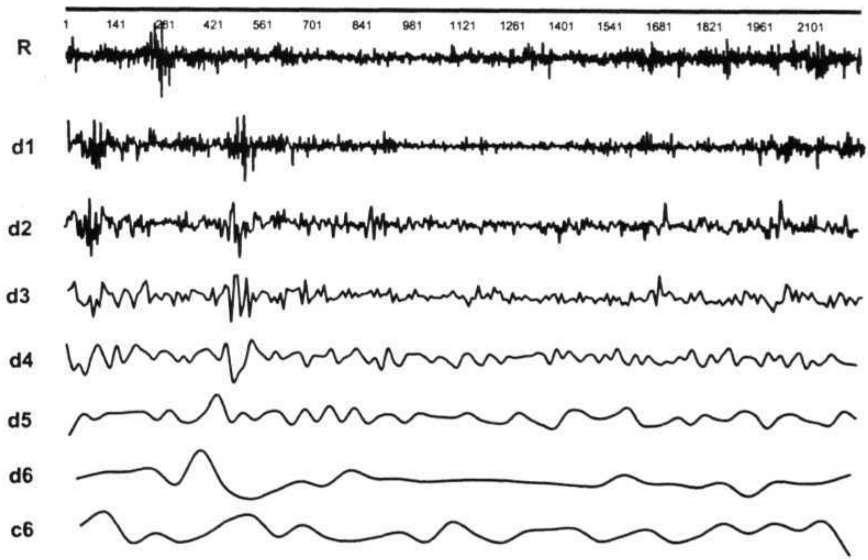


Figure 4.20: Wavelet Decompositions of Sensex Return Series (db6)

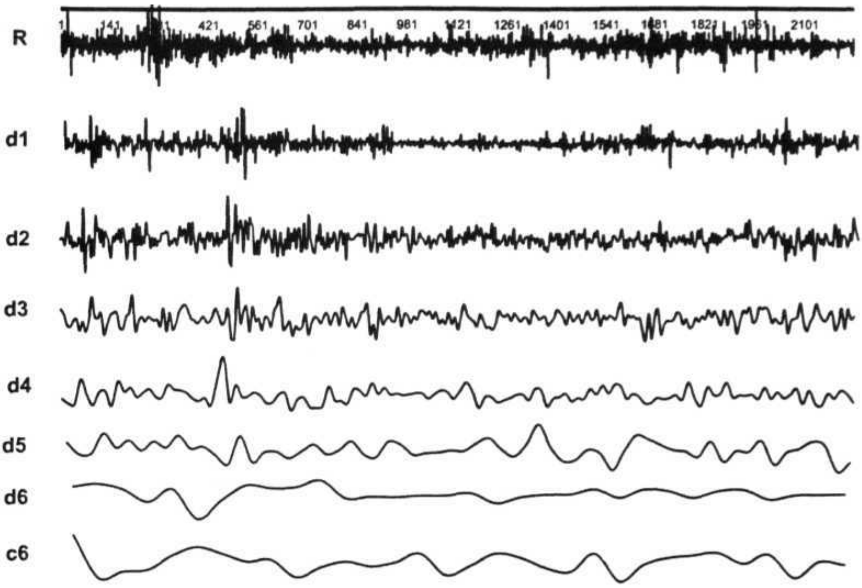


Figure 4.21: Wavelet Decompositions of National Index Return Series (db6)

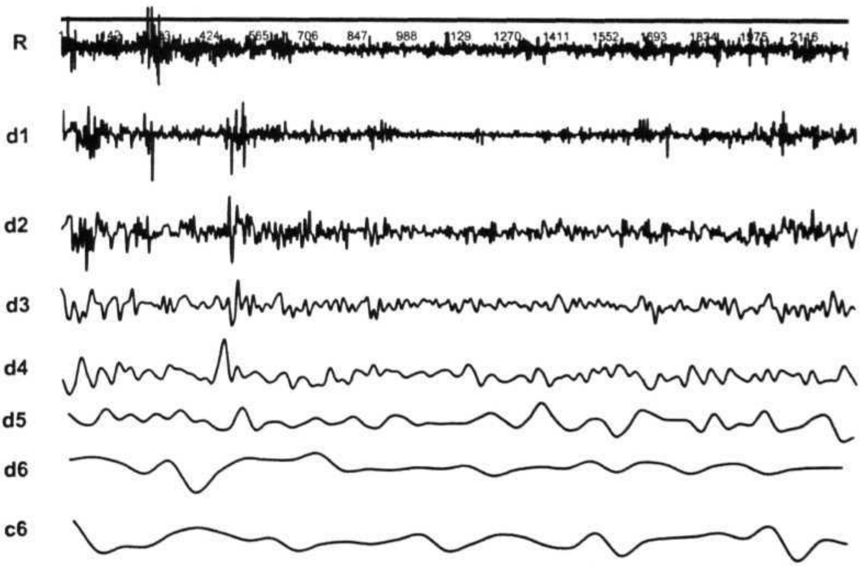


Figure 4.22: Wavelet Decompositions of S & P CNX Nifty Return Series (db6)

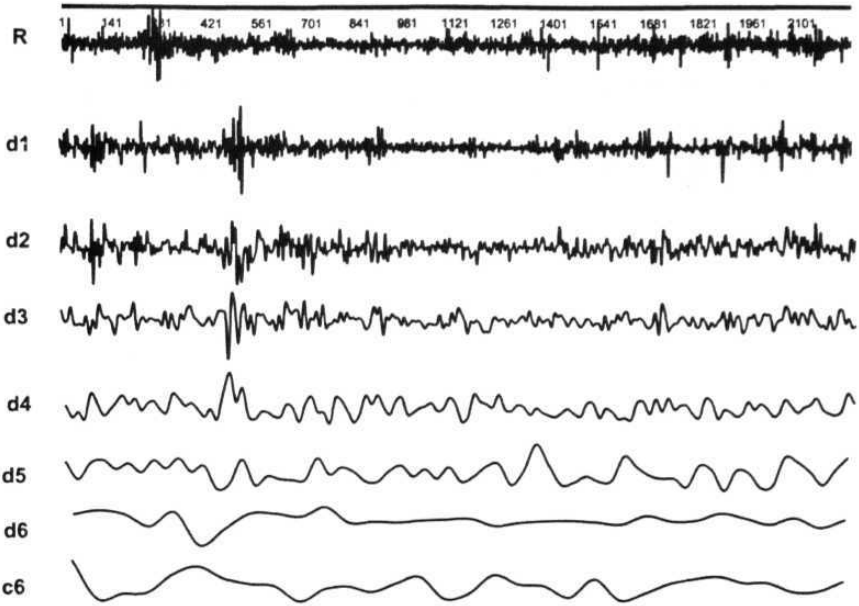


Figure 4.23: Wavelet Decompositions of S & P CNX 500 Return Series (db6)

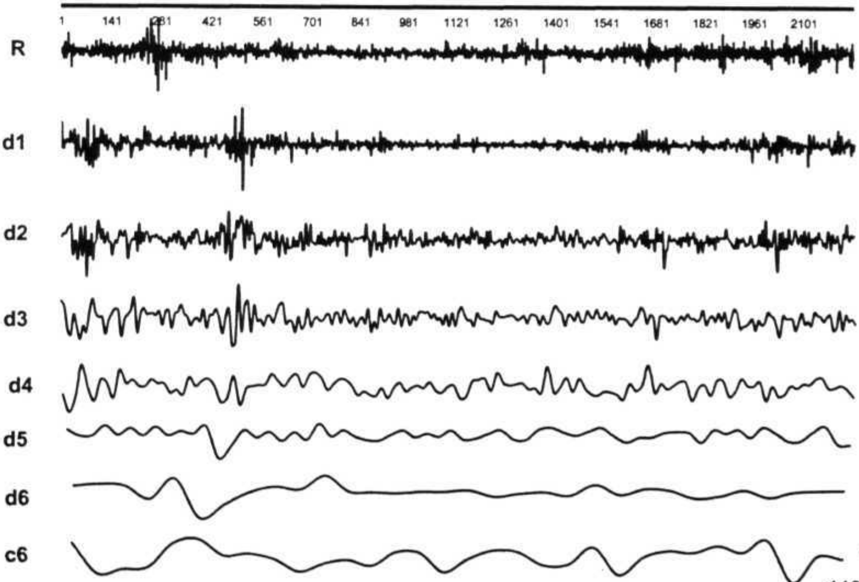


Figure 4.24: Universal Thresholding to Wavelet Coefficients of Sensex Return Series

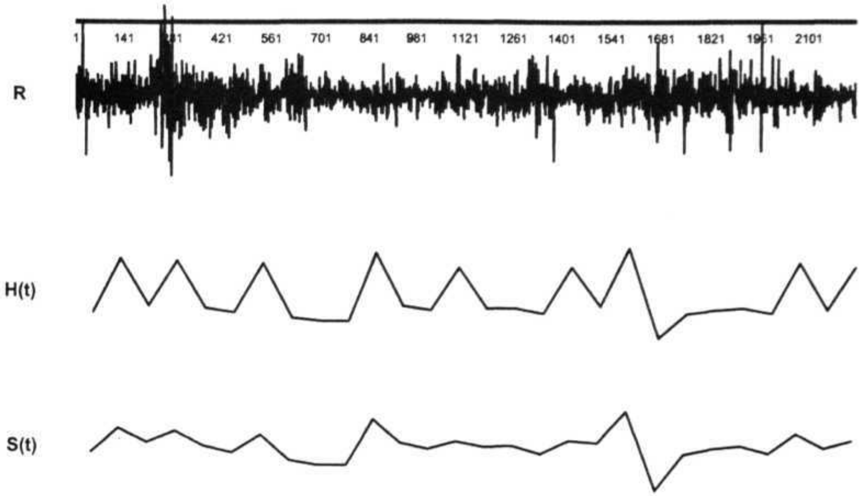


Figure 4.25: Universal Thresholding to Wavelet Coefficients of National Index Return Series

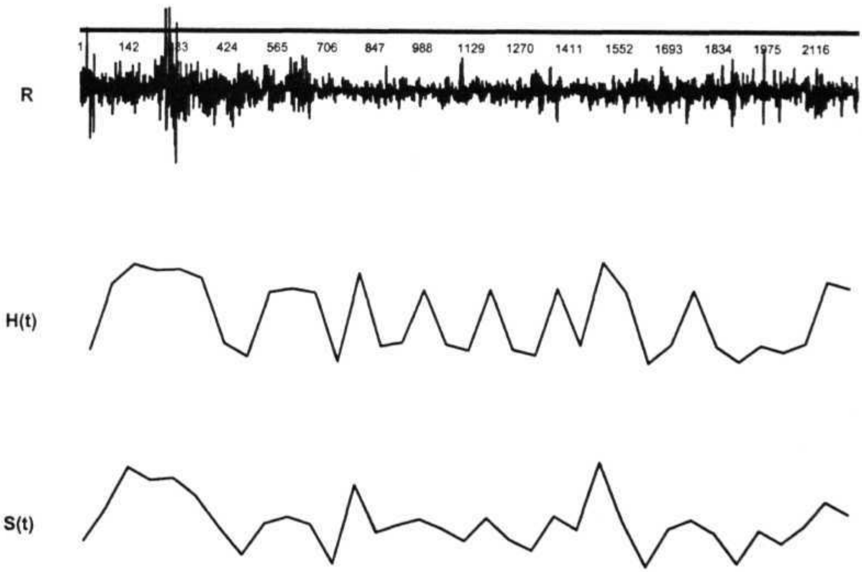


Figure 4.26: Universal Thresholding to Wavelet Coefficients of S & P CNX Nifty Return Series

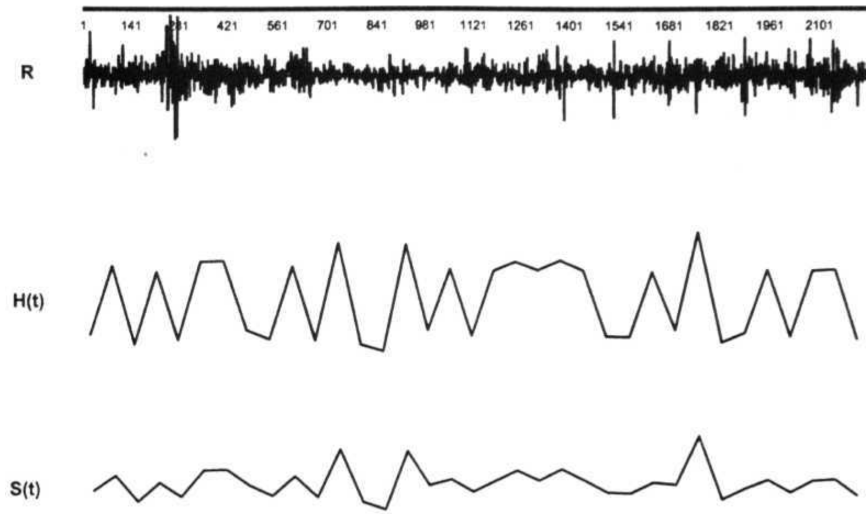
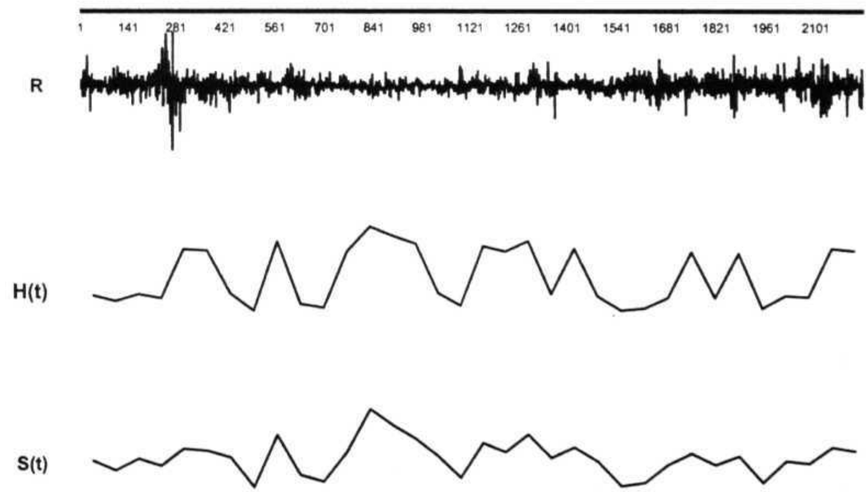


Figure 4.27: Universal Thresholding to Wavelet Coefficients of S & P CNX 500 Return Series



Chapter V

Long Memory Pattern and Spillover Effect in Stock Market

5.0 Introduction

In the preceding chapters the discussions and analysis pertained to different aspects of spectral and wavelet methods as well as their applicability in case of stock price behaviour in India. In this chapter, here we make an attempt to address two important stylized facts associated with stock prices, viz. long memory pattern and spillover effect using discrete wavelet transforms. Analysis of long memory pattern assumes immense importance in light of the fact that many financial time series including stock prices exhibit slowly decaying autocorrelation functions. This notion has important applications, in particular for futures and option pricing. We have also carried out the analysis relating to international transmission of stock returns. This is particularly interesting with the increased liberalization of capital movements across the economies and increasing dependence among international stock markets.

The organization of the chapter is as follows: section 5.1 carries out a discussion on long memory process and presents empirical findings for Indian case. A discussion of international movements of stock markets supported by empirical analysis is presented in section 5.2. Section 5.3 concludes the chapter.

5.1 Long Memory Process

The presence of long memory components in asset returns has important implications for many of the paradigms used in modern financial economics. The long memory or long term dependence property describes the higher order correlation structure of a series. If a series exhibits long memory, there is persistent temporal dependence between observations widely separated in time. Such series exhibits hyperbolically decaying autocorrelations and low frequency distributions. Fractionally integrated process can give rise to long memory (Mandelbrot, 1977; Granger and Joyeux, 1980; Hosking, 1981). On the other hand, the short memory or short-term dependence properly describes the low order correlation structure of a series. Short memory series are typified by quickly declining autocorrelations and high frequency spectral distributions. Standard autoregressive moving average process cannot exhibit long term (low frequency) dependencies they can only describe the short run (high frequency) behavior of a time.

The presence of fractional structure in asset returns raises a number of theoretical and empirical issues. First, as long memory represents a special form of non-linear dynamics, it calls into question linear modeling and invites the development of non-linear pricing models at the theoretical level to account for long memory behavior. Mandelbrot (1971) observes that in the presence of long memory, the arrival of new market information cannot be fully arbitraged away and martingales models cannot be obtained from arbitrage. Second, Pricing derivative securities with martingales methods may not be appropriate if the underlying continuous stochastic process exhibit long memory. Third, statistical inferences concerning asset pricing models based on standard testing procedures may not be appropriate in the presence of long memory series (Yajima, 1985). Finally, as long memory creates nonlinear dependence in the first moment of the distribution and generates a potentially predictable component in the series dynamics, its presence caste on the weak form of market efficiency hypothesis. The price of an asset determined in an efficient market should follow a martingale process in which each price change is unaffected by its predecessor and has no memory. The presence of long memory in asset returns implies significant autocorrelations between distant observations. Consequently, past return can help predict future returns, and the possibility of consistent speculative profits arises.

A number of studies have tested the long memory hypothesis for stock market returns. Using the rescaled-range (R/S) method of Hurst(1951), Greene and Fielitz (1977) report evidence of persistence in daily US stock returns series. A problem with the classical R/S method is that the distribution of its test statistic is not well defined and is sensitive to short term dependence and heterogeneity of the underlying data generating process. These dependencies bias the classical R/S test toward finding long memory too frequently. Lo (1991) developed a modified R/S method, which addresses these drawbacks of the classical R/S method. Using this variant of R/S analysis, Lo (1991) finds no evidence to support the presence of long memory in US stock returns. Using both the modified R/S method and the spectral regression method (described below), Cheung and Lai (1995) find no evidence of persistence in several international stock returns series. Crato (1994) reports similar evidence for the stock returns series of G-7 countries using exact maximum likelihood estimation. Lobato and Savin (1997) find no evidence of long memory in daily standard and poor 500 returns over the period July 1962-December 1994. Interestingly, Labato and Savin (1997) find some evidence of long-memory in the squared return data, which supports the conclusions of Ding et.al. (1993). Barkoulas and Baum (1998) using the spectral regression and Gaussiansemi parametric (described below) of estimating the long memory parameter for various Japanese financial time series find no evidence of long memory in stock returns. Again Barkoulas and Baum (1999) test for long term dependence in US stock returns, analyzing composite and sectoral stock indices and firm's return series to evaluate aggregation effects. They find no evidence of fractional dynamics in US stock returns, but detected presence of intermediate memory in some firms return series. Using the spectral regression method, Barkoulas et.al. (2000) find significant and robust evidence of positive long-term persistence in the Greek stock market. Henry (2000) investigated long range dependence using various parametric and non-parametric estimators in a sample of 9 international stock index returns. The results provide evidence of long memory in the German, Japanese, South Korean and Taiwanese markets and no evidence of the same in stock returns for the UK, USA, Hong Kong, Singapore and Australia. Caporale and Gil-Alana (2001) using the tests of Robinson (1994) find that there is very little evidence of fractional integration in S&P 500 daily return series, despite the length of the series.

In the early stages, Fourier based methods dominated the literature in terms of identifying and fitting models to long memory processes. But now, wavelets have shown great promise in handling long memory and combination of short memory and long memory processes. Both least squares and maximum likelihood procedures have been established for estimating the model parameters in the case of long range dependence. The empirical presence of long memory is found in the persistence of the autocorrelations. This slow decay by the autocorrelations is not consistent with either the stationary, short memory, ARMA models nor the non-stationary, unit root models. Instead, long memory falls nicely in between these two knife-edge approaches. The drawback is the dense covariance matrix it creates i.e., a large matrix with few zero elements. This dense matrix makes calculation of the exact maximum likelihood function (MLE) impossible for large data sets since inversion of the long memory's covariance matrix is an exhaustive task, requiring on the order of cubed numerical operations.

Using the logarithmic decay of long memory process's autocovariance function Jensen (1999) shows that a log-linear relationship exists between **the** variance of the wavelet coefficients from the long memory process and its scale equal to the long memory parameter. This log-linear relationship lends itself nicely to the estimation of the long memory parameter of a fractional integrated process known as the fractional differencing parameter. He shows that the wavelet OLS estimator yields a consistent estimator of the fractional differencing parameter.

In a heuristic manner, McCoy and walden (1996) have shown the existence of this log-linear relationship between the wavelet coefficients' variance and its scale, but they show it graphically with a plot of $\log_2$ of the sample variance of the wavelet coefficients from a long memory process against the $\log_2$ of the frequency and compare to the $\log_2$ of the process's power spectrum. McCoy and Welden use this log-linear relationship to calculate the maximum likelihood estimator of the fractional differencing parameter (MW estimator). By using only the wavelet coefficients' variance and ignoring their correlation, McCoy and Walden implicitly assume that the wavelet coefficients' covariance between scale and time are insignificantly different from zero, i.e. the wavelet coefficients are independent over time and scale. Hence, the MW estimator amounts to an approximate maximum likelihood estimator, the precision of which is dependent on how rapidly the wavelet coefficients' autocovariance function decays as the difference in scale and time increases.

5.1.1 Fractional Integration Tests

Let $y(t)$ be the fractionally integrated process, $I(d)$, is defined by

$$(1-L)^d y(t) = \varepsilon(t) \qquad (5.1)$$

and the model of an autoregressive fractionally integrated moving average process of order (p,d,q), denoted by ARFIMA (p,d,q) may be written using operator notation as

$$(1-L)^d \phi(L) y(t) = \theta(l)\varepsilon_t \quad ...(5.2)$$

Where $\varepsilon(t) \approx i.i..d(0, \sigma_\varepsilon{}^2), L$ is the backward shift operator and $(1\text{-}L)^d$ is the fractional differencing operator defined by

$$(1-L)^d = \sum_{j=0}^{\infty} \binom{d}{j} (-1)^j L^j$$
$$= \sum_{j=0}^{\infty} \frac{\Gamma(j-d)L^j}{\Gamma(-d)\Gamma(j+1)}$$

with Γ denoting the gamma operator. The parameter d is allowed to assume any real value. The arbitrary restriction of d to integer values gives rise to the standard autoregressive integrated moving average (ARIMA) model. The stochastic process y_t is both stationary and invertible if all roots of $\phi(L)$ and $\theta(L)$ lie outside the unit circle and $|d| < 0.5$. The process is non-stationary for $d \geq 0.5$, as it posses infinite variance (see Granger and Joyeux, 1980). Assuming that $d \in (0, 0.5)$ and $d \neq 0$, Hosking (1981) showed that the correlation function, $\rho(.)$, of an ARFIMA process decay hyperbolically to zero as $j \to \infty$ which is contrary to the faster, geometric decay of a stationary ARMA process. For $d \in (0, 0.5)$, $\sum^{n} |e(k)|$ diverges as $n \to \infty$ and the ARFIMA process is said to exhibit long memory, or long-range positive dependence. The process is said to exhibit intermediate memory (anti-persistence) or long range negative dependence, for $d \in (-0.5, 0)$. The process exhibits short memory for $d=0$, corresponding to stationary and invertible ARMA modeling. For $d \in (0.5, 1)$ the process is mean reverting, even though it is not covariance stationary, as there is no long-run impact of an innovation on future values of the process. Now let y_t be a zero mean fractional difference process (d) with $-0.5 < d < 0.5$. It is well known (Hosking, 1981) that the autocovariance function of y_t is defined to be

$$R_y(t,s) = E[y(t)\,y(s)]$$
$$= \frac{\sigma_\varepsilon^2 \Gamma(1-2d)\Gamma(|t-s|+d)}{\Gamma(d)\Gamma(1-d)\Gamma(|t-s|+1-d)} \quad ..(5.3)$$
$$\approx |t-s|^{2d-1} \qquad as\ |t-s| \to \infty$$

The slow hyperbolic decay of $R_y(t, s)$ satisfies the long memory definition of Resnick(1987).

The implications of the long memory evidence can be seen in time and frequency domains. In the time domain, long memory is indicated by the fact that the series eventually exhibit positive dependence between different observations. In the frequency domain, long memory is indicated by the fact that the spectral density becomes unbounded as the frequency approaches zero; the series has power at low frequencies.

We estimate the long memory parameter using the spectral regression and Gaussian semiparametric methods.

The Spectral Regression Method

Geweke and Porter-Hudak (1983) suggest a semi parametric procedure to obtain an estimate of the fractional differencing parameter d based on the slope of the spectral density function around the angular frequency $\omega=0$. More specifically, let $I(\omega)$ be the periodogram of y at frequency ω defined by

$$I(\omega) = \frac{1}{2\pi T}\left|\sum_{t=1}^{T} e^{it\omega}\ (y_t - \bar{y})\right|^2 \quad ...(5.4)$$

Then the spectral regression is defined by

$$\ln\{I(\omega)\} = \beta_0 + \beta_1 \ln\{4\sin^2(\frac{\omega_\lambda}{2})\} + \eta_\lambda, \quad \lambda = 1,...,\nu \quad ...(5.5)$$

Where ω_λ ———— $(\lambda = 0,....,T-1)$ denotes the Fourier frequencies of the sample, T is the number of observations, and $v = g(T) < T$ is the number of Fourier frequencies included in the spectral regression.

Assuming that $\lim_{T\to\infty} g(T) = \infty$, $\lim_{T\to\infty}\left\{\frac{g(T)}{T}\right\} = 0$, *and* $\lim_{T\to\infty} \frac{\ln(T)^2}{g(T)} = 0$, the negative of the OLS estimate of the slope coefficient in (5.5) provides an estimate of d. The properties of the regression method depend on the asymptotic distribution of the normalized **periodogram,** the derivation of which is not straightforward. Geweke and Porter-Hudak **(1983)** prove consistency and asymptotic normality for $d < 0$, while Robinson (1995a) proves consistency and asymptotic normality for $d \in (0, 0.5)$ in the case of Gaussian ARMA innovations in (5.2).

The Gaussian Semiparametric Method

Robinson (1995b) proposes a Gaussian semiparametric estimate referred to as the GS estimate hereafter, of the self-similarity parameter H, which is not defined in closed form. It is assumed that the spectral density of the time series, denoted by/f.,), behaves as

$$F(w) \sim G\, w^{1-2H} \text{ as } w \to 0^+ \quad \dots (5.6).$$

For $G \in (0, \infty)$ and $H \in (0, 1)$. The self similarity parameter H relates to the long memory parameter d by $H = d + ½$. The estimate for H, denoted by H, is obtained through minimization of the **function**

$$R(H) = \ln \hat{G}(H) - (2H-1)\frac{1}{v}\sum_{\lambda=1}^{v} \ln \omega_\lambda \qquad \dots (5.7)$$

with respect to H, where $\hat{G}(H) = \frac{1}{v}\sum_{\lambda=1}^{v} \omega_\lambda{}^{2H-1} I(\omega)$

The discrete averaging is carried out over the neighborhood of zero frequency and, in asymptotic theory, v is assumed to tend to infinity much more slowly than does T. The GS estimator has several advantages over the spectral regression estimator and its variants. It is consistent under mild conditions and under somewhat stronger conditions; it is asymptotically normal and more efficient. Gaussianity is nowhere assumed in the asymptotic theory. The GS estimator is $v^{1/2}$ consistent and the variance of the limiting distribution **is** free of nuisance parameters and equals *l/4v.*

The GPH utilizes a non parametric approach which regresses the log values of the periodogram on the log Fourier frequencies to estimate the fractional differencing parameter. However, due to the inconsistency of the periodogram as an estimator of the spectrum (Priestly, 1981), and the normalized periodogram being neither asymptotically independent nor identically distributed (Robinson, 1995), the GPH estimator has no sufficiently asymptotic properties.

Beside the GPH, the other estimates of the functional differencing parameter that exists calculate either the exact or approximate maximum likelihood estimator of the fractional differencing parameter. Although the statistical properties of the MLE are well known, their calculation is computationally intensive, suffering from the burden of inverting a dense covariance matrix at each iteration of the numerical optimization algorithm (Deriche and Tewfik, 1993), or are approximations of the likelihood function in frequency space (McCoy and Walden, 1996). An additional problem associated with the maximum likelihood approaches is their sensitivity to misidentified short memory parameters (Schmidt and Tschernig, 1995). Unlike the MLEs, the GPH and wavelet estimator do not require the inversion of the covariance matrix, nor the parameterization of the short memory parameters. Hence, they are easier to implement and take fewer cycles to compute.

5.1.2 Wavelet OLS Estimator of Fractional Differencing Parameter

Let $y(t)$ be a zero *Ifd)* process with $|d| < 0.5$. Using the autocovariance function of the *Ifd)* process found in (3), Jensen (1999) arrive at the following theorem.

Theorem: As $j \rightarrow 0$, *the wavelet coefficients,* $\omega_{.\,k}$, *associated with a mean zero Ifd) process with* $|d| < 0.5$ *are distributed* $N(0, \sigma^2 2^{-2jd})$ *Where* σ^2 *is a finite constant.*

Proof: see Appendix B.

From the above theorem, the wavelet coefficients form a *Ifd)* process have a variance that is a function of the scaling parameter, j, but is independent of the translation parameter, k. Hence, define *Rfj)* to be the wavelet coefficients' variance at scale j, i.e. $R(j) = \sigma^2 2^{-2jd}$. Taking the logarithmic transformation of $R(j)$, we obtain the relationship

$$\ln R(j) = \ln\sigma^2 - d\ln 2^{2j} \quad \ldots. \quad (5.8)$$

Where $\ln R(j)$ is linearly related to $\ln 2^{-2j}$ by the fractional differencing parameter, d. Hence, the unknown d of a fractionally integrated series can be estimated by the ordinary least squares estimator, d .

To perform this OLS regression, a estimate of the wavelet coefficients population variance, $R(j)$ is required. At scale j, define the sample variance of the wavelet coefficients as

$$\overline{R}(j) = \frac{1}{2^j}\sum_{k=0}^{2^j-1}\omega_{j,k}^2 \quad \ldots (5.9)$$

If a large number of wavelet coefficients are available for scale j, the wavelet coefficients' sample variance provides a consistent estimate of the true variance, $R(j)$ (Jensen, 1999).

5.1.3 Empirical Analysis of Long Memory Pattern

The series studied in this analysis include four aggregate stock indices at daily frequencies in India (see appendix-A). Using the spectral regression, Gaussian semiparametric and Wavelet regression methods of estimating the long memory parameter we test for fractional dynamics behaviour in four Indian stock indices. We have also tried to establish whether the degrees of dependence given by the fractional parameter (d) are constant over time. Therefore, we extend our sample period from April 1984 to December 2001 for this analysis only. Again, we split the full sample, comprising 3956 observations in two sub samples. First sub sample is from April 1984 to December 1990 and the second sub sample covers our original study period from January 1991 to December 2001.Since we deliberately extend our sample period to check whether the degree of dependence given by the fractional parameter (d) is constant over time. We have not carried out any other statistical tests for this extended sample period. The summary statistics for our original sample are shown in table 3.1 (chapter III). The data appear extremely non-normal. All of the return distributions are highly skewed. The data also display a high degree excess kurtosis. Such skewness and kurtosis are common features in stock return distributions, which are repeatedly found to be leptokurtic. The results of the ADF unit root test indicate that all of the returns series are stationary. However, stationarity does not preclude the possibility of long memory in the returns data.

Table 5.1 reports the empirical estimates for the fractional differencing parameter **'d'** as well as the test results regarding its statistical significance based on the spectral regression and Gaussian **semiparametric** methods. In estimating the fractional exponent, a choice has to be made with respect to the number of low frequency **periodogram** ordinates used. Improper inclusion of the median of the high frequency periodogram ordinates will bias the estimate of d; at the same time too small a regression sample will increase sampling variability of the estimates. We report the estimates for $T^{0.50}$ and $T^{0.55}$.[1]

As table 5.1 indicates that, there does not appear to be any consistent and convincing evidence in support of the long memory hypothesis for the returns series of any of the indices. When we consider the return series of the first sub-sample, there is evidence of fractional structure **in** two of the BSE indices in spectral regression analysis. The results indicate that degree of dependence remains relatively constant over time, with the order of integration of stock returns fluctuating slightly in spectral regression method. Gaussian-semiparametric methods of Robbinson (1995) dose not support any evidence of long memory for the return series of any of the indices.

With the exception of the first sub sample in case of spectral regression method, all of the stock returns we have examined are of short memory. The implications of the long memory evidence can be seen in both the time and frequency domains. In the time domain, long memory is indicated by the fact that the series eventually exhibit, positive dependence between distant observations. In the frequency domain long memory is indicated by the fact that spectral density becomes unbounded as the frequency approaches zero; the series has power at low

[1] We check the sensitivity of our results to the choice of estimation sample size by producing *d* estimates for plausible choices for v. The results to follow are robust to the choice of v.

frequencies. Further, the evidence of intermediate memory in time domain exhibits long range negative dependence between distant observations while, in the frequency domain the spectral density approaches zero as the frequency approaches zero

To obtain a consistent estimator of the fractional differencing parameter from a simple OLS regression is a substantial improvement over the popular GPH estimator. The wavelet coefficients' variance is a regularization of the spectrum (Percival, 1995; McCoy and Walden, 1996). Like the spectrum, which decomposes the variance of a series across different frequencies, the wavelet coefficients' variance decomposes the variance of the scales across different scales. Those scales, which contribute the most to the series variance, are associated with those wavelet coefficients with largest variance. Hence the wavelet coefficients' sample variance provides a more intuitive parametric estimates of its population variance than the nonparametric periodogram does of the power spectrum. More importantly, where the periodogram is an inconsistent estimator of the spectrum, the wavelet coefficients' sample variance is a consistent estimator of the population variance that enables the wavelet OLS estimator to be a consistent estimator of the fractional differencing parameter. Table 5.2 displays the estimates of fractional differencing parameters (d) for all the stock returns series calculated using the wavelet regressions method. There is no evidence of the long memory in any of the returns series examined. The estimate of 'd' for S&P CNX Nifty is in the stationary region.

Using the spectral regression, Gaussian-semiparametric, and wavelet regression method, we tested for stochastic long memory in four Indian stock returns series. From the analysis, we do not find any significant evidence of fractional dynamics with long memory features in the stock returns series of Indian market.

5.2 Spillover Effect

Over the past two decades, the globalization of the international stock markets have generated a large number of studies on international stock market relationship, especially, since the stock crash of 1987. Because with the development in the liberalization of the capital movements and the securitization of the stock markets, international financial markets have become increasingly interdependent. Advanced computer technology and improved worldwide network processing of news have improved the possibilities for domestic stock markets to react promptly to new information from international markets.

Using international stock return data, previous studies generally found evidence for spillover effects across international stock markets. Eun and Shim (1989) found a substantial multi-lateral interaction among the nine largest stock markets in the world. In particular, they documented that news originating in the U.S. market brings the most influential responses from other national markets. Hamao *et al* (1990) provided some evidence for spillover effects from New York to Tokyo and London and from London to Tokyo, but not from Tokyo to either to New York or London.

Studies concerning the international transmissions of stock returns and volatility include, among others, Ng *et al* (1991), Lin *et al* (1994), Karolyi (1995), Kim and Rogers (1995) and Booth *et al* (1997), where new evidence on spillover effects are discussed around the globe. For example, Ng *et al* (1991) and Ng (2000) found significant spillovers among the Pacific Rim countries, and Booth *et al* (1997) provided evidence for price and volatility spillovers among the Scandinavian countries

Among the related literature, King and Wadhwani (1990), Koch and Koch (1991), and Chowdhury (1994) used the traditional statistical methodologies to investigate the interrelationship among stock index of various countries for the data from the 1980s. They consistently found short-run interrelationships among national equity prices. Park and Fatemi (1993), however, found that the Pacific-Basin markets exhibit a weak linkage to the U.S., U.K., and Japanese equity markets. Moreover, the ambiguous results are found in papers applying the VAR technique to examine the linkages and dynamic interactions among stock markets of different economic regions. (e.g., Shachmurove (1995) found that none of the South American markets is completely independent; Elyasiani *et.al.* (1998) found no discernible interdependence between the Sri Lankan market and the equity markets of the US and the Asian markets considered; whereas, Friedman and

Shachmurove (1996) studied the transmission of innovations among European stock markets and found differential effects for the small and large markets.

The impact of the turmoil on the dynamic linkages among equity markets has drawn some attentions of economists within this decade. The elaborate work on the issue of the influence of the October 1987 stock market crash can be found in Malliaris and Urrutia (1992), Arshanapalli and Doukas (1993), Arshanapalli *et.al.* (1995), Hassan and Naka (1996), Choudhry (1996), and Masih and Masih (1997). They most found an significant influential power since the dynamic relationships among stock markets had some change from pre to post October 1987 crash.

Other evidence for the dynamic linkage among stock markets can also be found in, based on different economic regions, Chan *et.al.* (1997) for the global markets, Chung and Liu (1994), Corhay *et.al.* (1995), and Ghosh *et.al.* (1999) for the Pacific-Basin markets, and Choudhry (1996) and Gerrits and Yuce (1999) for the European markets. Most of these papers used the US market as a core to examine the long run and short-run dynamic relationships among stock markets.

As spillover effects are expected to be completed within a short period of time (Eun and Shim, 1989) much of the previous studies are concerned with the influence of any unanticipated shocks of innovations to one stock market on other markets. In order to extract new information in stock markets and hence to examine the relationship between short -term fluctuations in stock prices, the VAR methodology uses forecast errors from the regression models, and the GARCH methodology uses the estimated ARCH error terms. However, such approaches are subject to being sensitive to models specifications.

New testing strategies based on Wavelet analysis have been developed to investigate international stock marker spillover effects. The discrete Wavelet transform is very much useful in decomposing time series data into an orthogonal set of components with different frequencies. By examining the relationship between high frequency fluctuations in stock returns, obtained from reconstruction of the data by Wavelet coefficients, Lee (2000, 2001) investigated the dynamics and the potential interaction among international stock markets. Using the data on daily stock returns of the US Dow Jones Index and of the KOSPI (Korea, composite stock price index.) Lee (2000) found strong evidence for price as well as volatility spillover effects from the US stock markets to the Korean counterparts but not vice versa. Lee (2001) using the same discrete wavelet decomposition discussed international spill over effects between the developed markets (the US, Japan and Germany) and the emerging markets in the MENA (Middle East and North American) region (Turkey and Egypt). His finding was that price as well as volatility spillover effects exist from the developed stock markets to MENA counterparts but no vice versa.

While new evidence on the international spillover effects has widely been discussed around the globe India has received no attention. In view of the recent development of information network that is capable disseminating new instantaneously around the world, however, a shock in one stock market can be transmitted to another market within a very short period of time. It is thus essential to use high frequency data (See Appendix B) such as daily prices to examine spillover effects. The main purpose of this study is to apply, a new mathematical tool, 'Wavelets' to investigate whether Indian markets are integrated globally with major developed markets such as the US, the UK and Japan.

5.2.1 Empirical Results of Spillover Effect

Much of the previous studies on spillover are concerned with international transmission mechanism of any unanticipated shocks and innovations originating from one stock market to other markets. In order to examine short-term fluctuations in stock prices, a few common approaches comprises of VAR methodology that uses forecast errors, and the GARCH methodology which estimate ARCH error terms. However, such approaches are subject to being sensitive to model specifications.

In this study, we apply the tools of wavelet analysis, discussed in chapter III, to investigate whether the Indian stock market is integrated with major developed stock markets, viz, U.S.A., U.K. and Japan. We first examine the time scale properties of stock index returns based on the discrete Wavelet decomposition. As discussed in chapter III, the discrete wavelet transforms is very useful in decomposing time series data into an orthogonal set of components with different frequencies. By examining the relationship between high-frequency fluctuations in stock returns obtained from the reconstruction of data by wavelet coefficients. We can investigate the international transmission of news in stock markets. The Haar wavelet is used as the basis function in this study of the international transmission of stock markets movements.

The summary Statistics of daily returns are presented in the table 3.2 in chapter - III. In all the cases the excess kurtosis and skewness measures are indicative of the evidence against normal distribution. The results of the unit root show that all the returns series are stationary on levels. Table 5.3 provides the summary statistics for the Wavelet reconstructed data of the stock returns. The most interesting result we find here is that the mean and skewness for all the cases are zero. The test of the unit root shows evidence against the null hypothesis of non-stationarity for all cases. The correlation coefficients indicate very low levels of relationship between Indian stock market and developed stock markets except Japan.

If the price movements of one stock market affect subsequent price movements in other, then the innovations of the influential market should lead to subsequent changes in other markets. Given earlier empirical results that the U.S market is the most influential in the World (see, for example, Eun and Shim, 1989) we first examine whether innovations in the U.S. stock market are transmitted to Indian market, then between U.K.- India and Japan-India stock markets.

In order to investigate international stock market spillovers, we need to figure out the innovations in stock markets. While the GARCH type models have mainly been used to capture such innovations, the wavelet decomposition approach is applied here to derive high-frequency fluctuations in stock market data. Based on the reconstruction of the stock returns, we can investigate the relationship between various pairs of rescaled data to discuss about the spillover effects from the developed stock markets returns to Indian counterparts. We have used two Indian stock indices such as sensex and S&P CNX Nifty for this analysis. In order to investigate whether the U.S. stock market movements are transmitted to Indian market, we start with a simple regression of the sensex returns on the DJIA returns of the previous trading day[2]. Table 5.4 reports the coefficient estimates from a sequence of least squared regressions obtained via wavelet decompositions. As for the raw data on stock returns, the slope coefficient of the regression between India (Sensex) and U.S. is significant. However, such a result may not be interpreted as direct evidence on the international transmission mechanism of stock market movements. We do not find any significant coefficient between S & P CNX Nifty (India) and U.S. markets. If one stock market is causally prior to other markets, the price movements of the influential market should affect subsequent price changes in other markets but are not affected by price movements of other markets in earlier period. In order to see whether the Indian market movements may also explain the U.S. stock prices, we estimate a reverse regression where the U.S. stock return of the same calendar day now becomes the dependent variable. In this case, the estimated coefficient turns out to be significant (at 10% significance level). Such result is in contrast with earlier findings that no single foreign market can significantly affect the U.S. market.

In fact spillover effects are concerned with the effects of any unexpected developments in one stock market. In order to figure out the international transmission mechanism of 'news' in stock markets, we need to focus on the relationship between high frequency fluctuations in stock returns. Based on the reconstructions of the returns data, we next examine the relationship between the finest components (d_1) in stock returns, reported in table 5.5. The estimates of the slope coefficients are affected in case of Sensex and DJIA and are insignificant. These results are not in agreement with earlier empirical findings that innovations in the US stock markets are rapidly transmitted to other markets. Again the reverse regression slope coefficients from regressions of d_1 scale data turns out to be far from being significant. Thus, unlike the results from the raw data consistent observations are obtained when compared with previous empirical findings, when we focus on high frequency data fluctuations.

The relationship of Indian stock market with the UK stock market is quite poor as shown in table 5.6. The regression results using return series show evidence

[2] On a calendar day, the Indian market opens earlier than the U.S. market. Thus if the U.S. market is causally prior to the Indian market, 'news' in the U.S. market should be followed by a response in the Indian market on the next trading day.

against any relationship between these two stock markets barring the case of S&P CNX Nifty and FTSE. So far as the spillovers are concerned similar findings are obtained in case of relationship between the Indian market and the UK market. Further, reverse regressions result in insignificant estimates.

For the relationship between Indian stock market and Japanese market, a somewhat different scenario emerges. Due to overlapping trading hours we have used close-to-close returns of the same trading day in case of India and Japan regression analysis. First, the slope coefficient estimated from a simple regression of the Indian returns on the Japanese return of the same trading day is significant. Further, as presented in table 5.6 reverse regressions of the Japanese returns on the Indian returns lead to significant estimates. The regressions of the Indian reconstructed returns on the Japanese reconstructed returns lead to significant estimates, which show evidence for price spillover effects from Japanese stock market to its Indian counterpart. An analysis of the reverse regression results for spillover effects also leads to significant estimates. Thus, the Japanese stock market also appears to be influenced by the Indian stock market.

To recapitulate, price spillover effects are found from Japanese market to Indian counterpart and vice versa. But we do not find any evidence of price spillover effects from either the US or the UK stock markets to Indian markets and vice versa. To conclude, little evidence is found for price spillover effects from the developed stock markets to the emerging Indian counterpart, though we cannot draw too strong conclusions from such mixed results.

5.3 Concluding Remarks

Applications of discrete wavelet transforms across scales have received considerable attention by the economists for their special time-scale properties. In this chapter we use discrete wavelet transform to comment upon two important phenomena associated with the nature of stock price behavoiur, viz., long memory pattern and spillover effect. Using the spectral regression, Gaussian-semiparametric, and wavelet regression method, we tested for stochastic long memory in four of the Indian stock returns series. From the analysis, we do not find

any significant evidence of fractional dynamics with long memory features in the stock returns series of Indian market.

Again, we try to investigate international stock market spillovers by using the wavelet decompositions approach to derive high frequency fluctuations in stock market data. Price spillover effects are found from Japanese market to Indian counterpart and vice versa. But in case of the U.K. and the U.S. we do not find any evidence of price spillover effects from either side. To conclude, little evidence is found for price spillover effects from the developed stock markets to the emerging Indian counterpart, though we cannot draw too strong conclusions from such mixed results.

Table 5.1: Estimates of the Fractional Differencing Parameter d for Stock Return Series

Series	Spectral Regression Estimates		Gaussian Semiparametric Estimates	
	d (0.50)	d (0.55)	d (0.50)	d (0.55)
Sensex	0.027 (0.078)	0.102 (0.072)	-0.041 (-0.064)	-0.042 (-0.065)
National	-0.009 (.087)	0.062 (0.073)	-0.062 (-0.094)	-0.067 (-0.101)
CNX Nifty	0.070 (0.105)	0.025 (0.080)	-0.027 (-0.042)	-0.067 (-0.101)
CNX Nifty	0.070 (0.105)	0.025 (0.080)	-0.027 (-0.042)	-0.067 (-0.101)
Sensex (first)	0.184 (0.127)	0.189 (0.107)	0.077 (0.133)	0.063 (0.106)
National (first)	0.175 (0.127)	0.143 (0.102)	0.025 (0.041)	0.048 (0.081)
Sensex (second)	0.066 (0.109)	0.0001 (0.079)	-0.018 (-0.029)	-0.053 (-0.082)
National (second)	0.007 (0.089)	-0.015 (0.069)	-0.026 (-0.041)	-0.071 (-0.107)

Notes: The data for the BSE Sensex and National indices covers the period 03/04/84 to 31/12/2001 daily observations. The sample period spans 01/01/91 to 31/12/2001 for the daily NSE S & P CNX 500 and S & P CNX Nifty indices. The sample period is divided to two parts (first and second) in case of BSE indices. First sub-sample covers the period 03/04/88 to 31/12/90 and second sub-sample spans 01/01/91 to 31/12/2001. d (0.50) and d (0.55) give the d estimates corresponding to estimation sample size $v = T^{0.50}$ and $v = T^{0.55}$, respectively. The figures in parentheses are standard error and t - statistics in spectral regression estimates and Gaussian semiparametric estimates respectively. The null hypothesis is $d = 0$ against the alternative of $d \neq 0$.

Table 5.2: Wavelet Estimates of the Fractional Differencing Parameter d for Stock Return Series

Series	**Parameter - d**	**t- statistics**
Sensex	-0.014	-0.285
National	-0.037	-0.818
CNX Nifty	-0.0006	-0.010
CNX 500	-0.098	-1.925*
Sensex (first)	-0.076	-2.238*
National (first)	-0.099	-2.935"
Sensex (second)	0.002	0.443
National (second)	-0.024	-0.472

Notes: The data for the BSE Sensex and National indices covers the period 17/02/86 to 05/10/95 for a total of 2048 daily observations. The sample period spans 01/01/91 to 21/12/99 for a total of 2048 daily observations of NSE S & P CNX 500 and S & P CNX Nifty indices. The sample period is divided to two parts (first and second) in case of BSE indices. First sub-sample covers the period 17/12/86 to 31/12/90 (1024 observations) and second sub-sample spans 01/01/91 to 21/12/99 (2048 observations). The sample periods have been selected using dyadic length return series for optimum level of wavelet decompositions.

Table 5.3: Summary Statistics of Reconstructed Returns Series

Series	**Mean**	SD	**Sk**	**Kurtosis**	**ADF**
d_1_Sensex	**0.000**	**0.835**	**0.000**	**5.582**	-17.056***
D_1_CNX Nifty	**0.000**	**0.531**	**0.000**	**6.702**	-17.250***
d_1_DJIA	**0.000**	**0.399**	**0.000**	**3.771**	-17.849***
d_1_FTSE	**0.000**	**0.409**	**0.000**	**3.804**	-17.202***
d_1_Nikkei	**0.000**	**0.549**	**0.000**	**3.787**	-16.270***

Note: The superscripts *, **, • indicate statistical significance for the t-statistics at the 1, 5, and 10 per cent levels respectively.**

Table 5.4: Correlation Coefficients of Reconstructed Returns Series

Series	d_1_Sensex	d_1_CNX Nifty	d_1_DJIA	d_1_FTSE	d_1_Nikkei
d_1_Sensex	1.000				
d_1_CNX Nifty	0.083	1.000			
d_1_DJIA	-0.003	0.052	1.000		
d_1_FTSE	0.010	-0.063	0.045	1.000	
d_1_Nikkei	0.140	0.238	0.010	-0.155	1.000

Table 5.5: Regressions of Stock Returns for Testing Spillover Effect

INDIA (Sensex and S & P CNX Nifty) and U. S. A. (DJIA)

Regression		R_t^{IND} on R_{t-1}^{US}			R_t^{US} on R_t^{IND}		
		Intercept	Slope	R^2	Intercept	Slope	R^2
Sensex	Rt	-0.0488 (-0.9392)	0.1460 (1.6137)*	0.0054	-0.0084 (-0.3196)	0.0425 (1.8594)*	0.0078
	d_1	-0.0049 (-0.1292)	-0.1085 (-1.1141)	0.0026	0.0000 (0.0000)	0.0154 (0.7216)	0.0011
Nifty	Rt	-0.0420 (-1.0923)	0.0838 (1.2516)	0.0062	-0.0100 (-0.3816)	0.0060 (0.1935)	0.0007
	d_1	0.0018 (0.0671)	0.0443 (0.6501)	0.0009	0.0000 (0.0000)	0.0350 (1.1458)	0.0027

Notes: 1) R_t^{IND} on R_t^{US} denote the Sensex and S & P CNX Nifty of India and DJIA index of U.S.A returns at calendar day t, respectively. 2) The figures in the parentheses denote t-statistics of the coefficients. 3) The superscripts *, **, * indicate statistical significance for the t-statistics at the 1, 5, and 10 per cent levels respectively.**

Table 5.6: Regressions of Stock Returns for Testing Spillover Effect

INDIA (Sensex and S & P CNX Nifty) and U. K. **(FTSE)**

Regression		R_t^{ITO} on R_{t-1}^{UK}			R_t^{UK} on R_t^{IND}		
		Intercept	**Slope**	R^2	**Intercept**	**Slope**	R^2
Sensex	**Rt**	-0.0399 (-0.8073)	0.1352 (1.5182)	0.005	-0.0205 (-0.8157)	0.0324 (1.3991)	0.004
	d_1	0.0008 (0.0217)	-0.0503 (-0.5452)	0.001	0.0000 (0.0000)	0.0166 (0.7334)	0.001
Nifty	**Rt**	-0.0330 (-0.9377)	0.2089 (3.2935)***	0.022	-0.0203 (-0.8048)	0.0435 (1.3464)	0.004
	d_1	0.0005 (0.0243)	0.0492 (0.8662)	0.002	0.0000 (0.0000)	-0.0395 (-1.0721)	0.002

Notes: 1) R_t^{IND} on R_t^{UK} denote the Sensex and S & P CNX Nifty of India and DJIA index of U.K returns at calendar day t, respectively. 2) The figures in the parentheses denote t-statistics of the coefficients. 3) The superscripts ***, **, * indicate statistical significance for the t-statistics at the 1, 5, and 10 per cent levels respectively.

Table 5.7: Regressions of Stock Returns for Testing Spillover Effect

INDIA (Sensex and S & P CNX Nifty) and Japan (NIKEI)

Regression		R_t^{IND} on R_t^{Japan}			R_t^{Japan} on R_t^{INt}		
		Intercept	**Slope**	R^2	**Intercept**	**Slope**	R^2
Sensex	**Rt**	-0.0333 (-0.6792)	0.3534 (5.2921)	0.056	-0.0460 (-1.3965)	0.1597 (5.2921)	0.056
	d_1	0.0000 (0.0000)	0.2133 (3.0624)	0.019	0.0000 (0.0000)	0.0921 (3.0624)	0.019
Nifty	**Rt**	-0.0266 (-0.7574)	0.2931 (6.1374)	0.074	-0.0436 (-1.3358)	0.2540 (6.1375)	0.074
	d_1	0.0000 (0.0000)	0.2297 (5.2920)	0.056	0.0000 (0.0000)	0.2457 (5.2920)	0.056

Notes: 1) R_t on R_t^{pan} denote the Sensex and S & P CNX Nifty of India and DJIA index of Japan returns at calendar day t, respectively. 2) The figures in the parentheses denote t-statistics of the coefficients. 3) The superscripts ***, **, * indicate statistical significance for the t-statistics at the 1, 5, and 10 per cent levels respectively.

Table 5.8: Regressions of Stock Returns for Testing Spillover Effect

BSE Sensex and NSE S & P CNX Nifty

Regression		Intercept	Slope	R^2
Nifty	**Rt**	-0.0296 (-0.8099)	0.2380 (7.4607)***	0.1043
	d_1	0.0000 (0.0000)	-0.1066 (-3.3949)***	0.0235
Sensex	**Rt**	-0.0272 (-0.5486)	0.4382 (7.4607)***	0.1043
	d_1	0.0000 (0.0000)	-0.2207 (-3.3949)***	0.0235

Summary of the Findings and Scope for Further Research

Stock market is a dynamic system characterized by the decentralized interactions among a large number of participants. These participants while attempting to sell and buy securities in the market place tend to take their decisions on the basis of their individual judgments as well as are influenced by the publicly available information or in other words by the prevailing prices in the market. Thus, in contrast with a perfectly competitive market as we view in economics, a stock market embodies agents who influence and in turn are influenced by the prices. Hence, this institution has evolved over time and need to be evaluated in relations to the external factors on the one hand and of fundamental market forces on the other. Academic research on financial markets suggests that the traders operate on a wide range of time-scales, indicating that not only current events but also the past as well as the future. This has always attracted the researchers to extract the underlying information in stock prices which are of direct interest to public in general and policy makers and researchers in particular. But, academics in this connection is always featured by mutual incomprehension and contradictions. The quest for appropriate methods is always there. Taking a cue from this, the present study makes a sincere attempt in the realm of stock price analysis by invoking two sophisticated methods, viz. spectral methods and wavelets, which have not been excessively tried out in the literature.

The study, as such, does not undertake a comparative exercise for these methods vis-a-vis the traditional ones; rather it highlights the facets of these approaches and their utility to uncover the dynamics and numerous facts associated with stock prices. Taking note of this, at the outset of the study we present a brief and non-technical view of both spectral and wavelet analysis. Here, we make an attempt to discuss various aspects of spectral analysis and their usefulness. Furthermore, the present study highlights the important properties of wavelets and their applications.

Any sincere attempt to understand and analyze the stock price behaviour necessitates a careful review of the existing literature. Keeping this in mind, we present the theoretical and empirical contributions of the spectral and wavelets analysis in the second chapter. First, we present a comprehensive review of select works applying spectral methods. Then, we proceed to review some theoretical and empirical applications of wavelets in stock price analysis.

Stock prices are portrayed by a mixture of regular and non-regular components. While explaining the non-regular components has always remained as a trivial task, the quest for retrieving the hidden periodicities has caught the imaginations of researchers in recent years. In this connection, the spectral analysis provides a way out. This particular notion gets treatment in chapter III. Broadly, the results here refer to power spectrum and cross spectrum analysis of stock prices in India. The power spectrum analysis for all the four indices does not indicate the presence of significant periodic cycles. Even after frequency domain smoothing (tapering), there are no significant peaks, which contribute significantly to overall variance of the return series. Tapering, a technique for reducing *window leakage', removes the contribution of some insignificant frequencies to the overall variance. The conclusion that follows from the power spectrum analysis of four indices is that the stock prices in India follow a random walk model (which is synonymously with weak form of efficient market hypothesis). This view is consistent with the findings of Sharma and Kennedy (1977) and Kulkarni (1978) and against the findings of Ranganatham and Subramanin (1993) in case of Indian stock market.

In light of the ever-expanding globalization of the economies and consequently increased dependence among international stock markets have always led the researchers to explore the dynamic and interactive relationships among the markets. The coherence analysis makes an attempt towards this, the results of which suggest that there are no similar long-term development features between India and developed stock markets. Following this, the phase spectra results can generally be summarized as follows: i) since there is no high degree of frequency domain correlation between India and other developed markets, one should not expect a linear and consistent pattern of leads and lags; ii) the phase spectra show a highly volatile nature around the zero frequency that necessitates further explanations; iii) there is no clear pattern of leads and lags, with the phase plot crossing the horizontal axis several times over the full range of frequencies; iv) in addition, it is found that over all the full range of frequencies the angles of phase spectra are changing, which suggest that there may be non-linearities in the phase spectra. These descriptive empirical results indicating inconsistent time lags between different periods might suggest non-linear relationships between India and other developed markets returns.

Notwithstanding the fact that spectral methods facilitate to uncover the underlying periodicities, at the same time theses are crippled by the inherent assumption of stationarity. With the established fact that most of the economic and financial time series are non-stationary, spectral methods fail to take account of this. In contrast, wavelet filter provides better insight into the dynamics of the time series when compared to the spectral approach. The principal features that characterize wavelet analysis include non-stationarity, structural changes, multi-resolution and denoising. Taking note of these features, chapter IV extends the application of wavelets to the four indices considered for analysis in this study. Specifically, the returns on these indices are subjected to discrete wavelet transform (DWT) by using Haar, Daubechies - 4 (db_4) and Daubechies - 6 (db_6). It is clear that the Haar basis is special, since it is the only wavelet, which is symmetric and compactly supported. For db_4 and db_6 wavelets, the number of vanishing moments is one and two respectively. One implication of this property is that longer wavelets may produce stationary wavelet coefficient vectors from higher degree non-stationary stochastic processes. This chapter has been an exploratory investigation into the applicability and usefulness of the wavelet analysis to detect elements of fluctuations at various scales, and to recover signals from noisy observations (also known as wavelet de-noising or wavelet shrinkage). It is apparent that wavelet analysis of returns empirically discovers the fluctuations, removes patchiness and looks for patterns possibly at certain levels. It also recovers signals from the noisy data by applying universal thresholding rule. In general, wavelet coefficients are very much useful for the statistical analysis of stock return series.

Stock prices are evidenced to exhibit a high degree of persistence, yet a highly turbulent period in the market is followed by a tranquil period, though the process may be quite slow. Technically, this notion refers to the long memory pattern and this has tremendous implications for pricing derivative securities. In this regard, we make an attempt to analyze this via spectral and wavelet regressions in chapter V. At the outset, we tested for stochastic long memory via the spectral regression, Gaussian semiparametric method and ordinary least squares using wavelets for four stock return series in India. Consistent with the findings for major capital markets, significant evidence of long term persistence is not found in Indian stock market.

The fact that share price movement might be correlated should not be considered surprising. The major world equity markets are related through trade and investment, so that news about economic fundamentals and other changes in one market can be expected to have implications for stock prices in other markets as well. Typically, this refers to international spillover. In the latter part of chapter V, we look for international transmission of news between India and developed stock markets, by examining the relationships between high frequency fluctuations in stock returns obtained from reconstructions of the data by wavelet coefficients. Considering three developed stock markets and two Indian indices (Sensex and S&P CNX Nifty), no evidence is found for price spillover effects from the developed markets to their Indian counterparts and vice versa.

The stock market is an institution of considerable interest to the public at large and of real importance to researchers. The variables, which make up a stock market may not directly affect the mechanism of the economy but they certainly influence the psychological climate within which the economy works. In this regard, the study on stock prices assumes immense importance. So far as the policy implications of the study are concerned, it is pertinent to note that we are not going to prescribe any specific policy implications. But the study assumes enormous importance in light of the ever-increasing importance of stock return volatility. A measure of the asset's total risk refers to asset return fluctuations or volatility. Investors carefully assess the total risk, as this is one of the factors determining investor's optimal portfolio of stocks. Also, in the absence of proper measure of risk, inefficient capital allocation could arise which has costs for the entire economy and this is the reason as to why governments are also too concerned for asset price behaviour. In addition, understanding and appropriately modelling the characteristics of stock returns will aid investors to build their accurate and precise portfolio and risk management. Not only can the analyzing and modelling of stock prices assist the investor in the choice of individual assets to include in the portfolio, but also investors could use the same methodology to analyze and measure the risk on the entire portfolio held. Another importance of modelling and analyzing stock prices is that investors and traders can predict future prices with the help of resultant patterns in data.

The time-scale decompositions or multi-scale nature clearly indicate the usefulness of the wavelets. In the stock market, there are traders who take a very long-term view and consequently concentrate on what are termed ' market fundamentals'; these traders ignore short-term phenomena. For them, high-level wavelet coefficients may be useful. In contrast, other traders are trading on a much shorter time-scale and as such are interested in temporary deviations of the market from its long-term path. Their decisions have a time horizon of a few months to a year; so they may be interested in middle level wavelet decompositions of the return series. And yet other traders are in the market for whom a day is a long time and consequently concentrate on day-by-day fluctuations. Given these observations, our study makes an important contribution to the existing literature, which would certainly provide a broad based framework to study and analyze stock price behaviour.

Suggestions for Further Research

The goal of this study has been to analyze the stock price behaviour in India in both spectral and wavelet domain. The study makes an attempt to touch many important aspects in both the domains and finds very interesting and useful results for Indian stock markets, However, there still exist some aspects, which may be considered for further research. First, in the present study, no attempt has been made to address filtering methods in spectral domain. Besides, spectral estimation based on the actual underlying data generating process may be considered for further study. Second, research and application of higher order vanishing moments wavelets are still at a preliminary stage. Third, maximum likelihood procedure of fractional difference processes may be established for estimating the model parameters in case of long range dependence using wavelets. Fourth, the wavelet approach can be used to investigate whether innovations in one market may lead to asymmetric impact on other markets.

Appendix-A

The data set for the present study constitutes four broad based stock market indices of India, viz., Sensex, National Index, S&P CNX Nifty Index and S&P CNX 500 Index. The study spans over a period 2nd January 1991 through 31st December 2001. The description of these indices, in brief, is as follows.

Sensex: This comprises of 30 companies from specified group in Bombay Stock Exchange constituting 150 shares. The selection is made on the basis of liquidity, depth, floating stock adjusted depth and industry representation. The compilation of the index is based on the weighted aggregate method.

National Index: It is a broad based index that represents movement of stock prices on a national scale. The equity shares of 100 companies from both specified and non-specified groups of five major stock exchanges, viz., Bombay, Calcutta, Delhi, Ahemadbad and Madras have been selected for the purpose.

S&P CNX 500 Index: This comprises of a select group of 500 stocks traded in National Stock Exchange (NSE) and is a market capitalization weighted index. Stocks are selected based on their market capitalization, industry representation,, trading interest, and financial performance. The index contains 71 industry groups representing over 73% of total market capitalization and 97% of total turnover at NSE, thus making it an optimal benchmark.

S&P CNX Nifty: This index also belongs to NSE and is a market capitalization based index comprising of 50 stocks. Stocks are selected on **the** basis of their market capitalization and liquidity. The index represents about 46% of total market capitalization of the stocks listed in the Indian bourses.

Note: It may be noted that, though NSE started security trading only in 1994, for our purpose we are using the simulated data for the last two indices provided by NSE since 1991.

Besides four Indian indices, the additional data used in the study for the analysis of cross-spectrum and spillover effect consists of three major developed stock markets (U.S., U.K., and Japan). From among developed stock markets, much of earlier studies found that the U.S. stock market is, by far, the most influential in the world. The U.K. and Japanese markets are also examined to discuss the global linkage of emerging capital markets to movements in other matured markets.

The data for three developed markets are obtained from the website at http://finance.yahoo.com. These stock price indices include Dow Jones Industrial Average for the U.S. (denoted as DJIA); FTSE 100 for U.K. (denoted as FTSE); and Nikkei 225 for Japan (denoted as Nikkei). All of theses stock indices are measured at closing times, in terms of local currency. The data set for these analysis ranges from January 2000 to December 2001. Since Indian stock market and these developed stock markets are operating in different time zones with different holiday and trading day schedules as well as different opening and closing times, some daily observations are deleted. Since there is no overlapping trading hours between Indian stock markets in one hand and New York and London on the other hand, current day data set for India is juxtaposed with the data set for these two countries with one-day lag. However, for Japan the data for the same day are considered, as there is the presence of overlapping trading hours. After matching the daily observations, we have about 472 through 496 observations depending on which pairs of countries are under investigation.

Appendix – B

Proof of the Theorem:

Let *y(t)* be mean zero *I(d)* process with $|d| < 0.5$ and σ_ε^2. The expected value of $\omega_{j,k}$ can easily be shown to equal zero, since

$$E(\omega_{j,k}) = 2^{j/2} \int E[x(t)]\, \psi(2^j t - k) dt$$

The variance of the wavelet coefficients equals

$$\begin{aligned} \mathrm{var}[\omega_{j,k}] &= E[\omega_j, k^2] \\ &= 2^j \int dt \int ds E[x(t)x(s)]\, \psi(2^j t - k)\, \psi(2^j s - k) \end{aligned}$$

Using the fractionally integrated processes' autocovariance function found in (3) and by a change of variables

$$\mathrm{var}[\omega_{j,k}] = k2^{-j} \int dt \int ds \frac{\Gamma(2^{-j}|t-s| + d)}{\Gamma(2^{-j}|t-s| + 1 - d)}\, \psi(t)\, \psi(s) \quad \ldots(-1)$$

Because $\Gamma(k+a)/\Gamma(k+b)$ is approximated well by k^{a-b} for large *k*, and for normalization purposes $j \in \Im = \{0,1,2,\ldots\ldots,p-1\}$

$$\mathrm{var}[\omega_{j,k}] = k2^{-2jd} \int dt \int ds |t-s|^{2d-1}\, \psi(t)\, \psi(s) \quad as\ j \to 0$$

By another change of variables

$$\mathrm{var}[\omega_{j,k}] = k2^{-2jd} \int dt\, |t|^{2d-1}\, \Lambda(1, t)$$

Where $\Lambda(1.t) = \int \psi(s)\, \psi\,(s-t)\, ds$ is the wavelet transformation of the mother wavelet. Collecting terms we find

$$\mathrm{var}[\omega_{j,k}] = \sigma^2 2^{-2jd}$$

Where $\sigma^2 = k^1 \int dt\, |t|^{2d-1}\, \Lambda(1,t) < \infty$, since $\Lambda\,(1, t)$ is finite. Thus, $\omega_{jk} \sim N\,(0, \sigma^2\, 2^{-2jd})\ as\ j \to 0$.

Bibliography

Adelman, I (1965), "Long Cycles - Fact or Artifact?", *American Economic Review,* 55, 444-466.

Anderson, J S (1982), "Methods of Investigating Causal Relationship Between **Time Series", *Reserve Bank of Australia Research Discussion Paper, 8205.***

Arshanapalli, B., and Doukas. J. (1993), "International Stock Market Linkages: Evidence from the Pre- and Post-October 1987 Period," *Journal of Banking **and Finance,*** **193-208.**

Arshanapalli, B., Doukas, J., and Lang, Larry H.P. (1995), "Pre and post-October 1987 stock market linkages between U.S. and Asian markets," Pacific-Basin Finance Journal, 3, 57-73.

Asimakopoulos, I, J Goddard, and C Siriopoulos (2000), "Interdependence Between the US and Major European Equity Markets: Evidence From Spectral Analysis", ***Applied Financial Economics,*** **10,41 -47.**

Baillie, R. (1996), "Long Memory Processes and Fractional Integration in Econometrics", *Journal of Econometrics,* 73, 5-59.

Barkoulas, J T and C F Baum (1998), "Fractional Dynamics in Japanese Financial Time Series", *Department of Economics,* Boston College.

Barkoulas, J T and C F Baum (1999), "Long Term Dependence in Stock returns", ***Department of Economics,*** **Boston College.**

Barkoulas, J T and C F Baum (2000), "Fractional Dynamics In Japanese Financial **Time Series", *From Internet.***

Barkoulas, J T, C F Baum and N Travlos (2000), "Long Memory in the Greek Stock Market", *Department of Economics,* Boston College.

Barnett, W A, M J Hinich, and P Yue (1999), "The Exact Theoretical Rational Expectations Monetary Aggregate", *From Internet.*

Blackman, R B and J W Tukey (1958), "The Measurement of Power Spectra from the View Point of Communications Engineering", *Bell System Technological **Journal,*** **37.**

Booth, G.G., T. Martikainen and Y. Tse (1997), "Price and Volatility Spillovers in Scandinavian Stock Markets." *Journal of Banking & Finance,* 21: 811-823.

Brock, W A (2000), "Whither Nonlinear?", *Journal of Economic Dynamics and Control,* 24, 663-78.

Brock, W A and A W Kleidon (1992), "Periodic Market Closure and Trading Volume: A Model of Intra day Bids and Asks", *Journal of Economic Dynamics and Control,* 16, 451-89.

Caporale, G M and L A G Alana (2001), "Long Range Dependence in Daily Stock Returns", *Center for Monetary and Financial Economics*, **South Bank** University.

Chan, K.C., Gup, B. E., and Pan, M. S. (1997), "International Stock Market Efficiency and Integration: A Study of Eighteen Nation," *Journal of Business and Financial Accounting*, 1-18.

Cheng, C and R T Ogden (1997), "Change, Detection on an Independent Sequence Using Wavelets", *Technical Report,* Department of Statistics, University of South Carolina.

Cheng, Y (2000), "Stock Market Wealth Effect", *Department of Economics, University of Toronto.*

Cheung, Y. and K. Lai (1995), "A Search for Long Memory in International Stock Market Returns," *Journal of International Money and Finance*, 14, 597-615.

Choudhry, Taufiq. (1996), "Interdependence of Stock Markets: Evidence from Europe During the 1920s and 1930s," *Applied Financial Economics,* 6, 243-149.

Chowdhury, Abdur R. (1994), "Stock Market Interdependencies: Evidence from the Asian NIEs", *Journal of Macroeconomics*, 16, 4, 629-651.

Chui, C. (1992), <u>An introduction to wavelets</u>, New York, Academic Press.

Chung, Ping J. and Liu, Donald J. (1994), "Common Stochastic Trends in Pacific Rim Stock Markets," *The Quarterly Review of Economics and Finance,* 34, 3, 241-259.

Clark, P. K. (1973) "A Subordinated Stochastic Process Model with Finite Variance for Speculative Prices", *Econometrica*, 41, 135-256.

Corhay , A., Rad, A. Tourani and Urbain, J. P (1995), "Long-Run Behavior of Pacific-Basin Stock Prices," *Applied Financial Economics,* 5, 11-18.

Daigler, R T and A F Herbst (1996), "Information Dispersal: A Microstructure Analysis of Stock Index Futures Volatility Patterns", *From Internet.*

Daubechies, I. (1992) Ten Lectures on Wavelets. Volume 64 of CBMS-NSF Regional Conference Series in Applied Mathematics, Society for Industrial and Applied Mathematics, Philadelphia.

Davidson, R, W C Labys and J B Lesourd (1997), "Wavelet Analysis of Commodity Price Behaviour", *Computational Economics,* **11,** 103-28.

Deriche, M and A H Tewfik (1993), "Maximum Likelihood Estimation of the Parameters of Discrete Fractionally Differenced Gaussian Noise Process", ***IEEE Transaction on Signal Processing,* 41, 2977-89.**

Dickey, D and W Fuller (1981), "Likelihood Ratio Statistics for Autoregressive Time Series with a Unit Root", *Econometrica,* 49, 1057-72.

Ding, Z, C W J Granger and R F Engle (1993), "A Long Memory Property of Stock Returns and a New Model", *Journal of Empirical Finance,* 1, 83-106.

Dobell, R and T J Sargent (1969), "The Term Structure of Interest Rates in **Canada", *Canadian Journal of Economics,* 2, 65-77.**

Donoho, D and I M Johnstone (1984), "Ideal Spatial Adaptation by Wavelet Shrinkage", *Biometrica,* 81, 425-55.

Donoho, D and I M Johnstone (1995), "Adapting to Unknown Smoothness by **Wavelet Shrinkage", *Journal of the American Statistical Association,* 90,** 1200-24.

Donoho, D, I Johnstone, G Kerkyacharian, and D Picard (1995), "Wavelet **Shrinkage: Asymptopia?", *Journal of the Royal Statistical Society, Series B,*** 57.

Durlauf, S N (1991), "Spectral Based Testing of the Martingale Hypothesis", ***Journal of Econometrics,* 50, 355 - 376.**

Elyasiani, Elyas., Perera, Pruyal., and Puri, Tribhuvan N. (1998), "Interdependence and Dynamic Linkages Between Stock Markets of Sri Lanka and its Trading **Partners," *Journal of Multinational Financial Management*, 8,** 89-101.

Eun, Cheol S and Shim, Sangdal (1989), "International Transmission of Stock **Market Movements", *Journal of Financial and Quantitative Analysis,* 24, 2,** 241-256.

Fama, E F (1970), "Efficient Capital Markets: A Review of Theory and Empirical **Work", *Journal of Finance,*** 283 - 417.

Fand, D (1966), "A Time Series analysis of "Bills Only" Theory of Interest Rates", ***Review of Economics and Statistics,* 48, 361 - 371.**

Fischer, K P and A P Palasvirta (1990), "High Road to a Global Market Place: The International Transmission of Stock Market Fluctuations", *Financial Review,* 25,371-393.

Gencay, R., F. Selcuk, and B. Whitcher (2002), An Introduction to Wavelets and Other Filtering Methods in Finance and Economics. Academic Press.

Gerrits, Robert-Jan., and Yuce, Ayse (1999), "Short- and Long-term Links among Euorpean and US Stock Markets," *Applied Financial Economics,* 9, 1-9.

Geweke J. and S. Porter-Hudak, 1983, The Estimation and Application of Long Memory Dependence", *Annals of Statistics* 13, 1048-1072.

Ghosh, Asim, Saidi, Reza and Johnson, Keith H. (1999), "Who Moves the Asia-Pacific Stock Markets -US or JapanDEmpirical Evidence Based on the Theory of Cointegration," *The Financial Review*, 34, 159-170.

Godfrey, M D (1965), "An Exploratory Study of BiSpectrum of Economic Time **Series", *Applied Statistics,* 14, 48 - 69.**

Godfrey, M, C Granger, and O Morgenstern (1964), "The Random Walk Hypothesis of Stock Market Behaviour", *Kyklos,* 17, 1 - 30.

Goldberg, M A and A Vora (1978), "Bivariate Spectral Analysis of the Capital **Asset Pricing Model", *Journal of Financial and Quantitative Analysis,* 435 -** 455.

Gordon, R A (1961), Business Fluctuations, New York.

Granger, C W J (1961), "First Report of the Princeton Economic Time Series Project", *L'Industria,* 34.

Granger, C W J (1966), "The Typical Spectral Shape of an Economic Variable", ***Econometrica,* 34, 150-161.**

Granger, C W J (1969), "Investigating Causal Relations by Econometric Models and Cross-Spectral Methods", *Econometrica,* 37, 424-438.

Granger, C W J and H G B Rees (1968),"Spectral Analysis of the Term Structure of Interest Rates", *Review of Economic Studies,* 35, 67-76.

Granger, C W J and M Hatanaka (1964), *Spectral Analysis of Economic Time Series,* Princeton University Press, Princeton.

Granger, C W J and O Morgenstern (1963), "Spectral Analysis of New York Stock Market Prices", *Kyklos,* 16, 1-27.

Granger, C W J and O Morgenstern (1970), ***Predictability of Stock Market Prices,*** D C Heath & Co, Lexington, Mass.

Granger, C W J and R Engle (1983), "Applications of Spectral Analysis in Econometrics", in D R Brillinger and P R Krishnaiah, ed., *Handbook of Statistics,* Vol. III, Elsevier Science Publishers Amsterdam, 93 - 109.

Granger, C. W. J. and R. Joyeux (1980), "An Introduction to Long Memory Time Series Models and Fractional Differencing", *Journal of Time Series Analysis,* ***1,15-39.***

Grassberger, P. and I. Procaccia (1983) "Characterization of Strange Attractors", ***Physical Review Letter,*** **50, 346-49.**

Grassberger, P. and I. Procaccia (1983) "Measuring Strangeness of Strange Attractors", *Physica D9,* 189-208.

Greene, M. T. and B. D. Fielitz (1977), "Long Term Dependence in Common **Stock Returns",** ***Journal of Financial Economics,*** **4, 339-49.**

Grenander, U and M Rosenblatt (1957), ***Statistical Analysis of Stationary Time*** *Series,* Wiley, New York.

Haar, A (1910), "Zur Theorie Der Orthogonalen Funktionensysteme", *Mathematische Annalen,* **69, 331-71.**

Hamao, Y, R.W. Masulis and V. Ng. (1990), "Correlations in Price Changes and Volatility across International Stock Markets", *Review of Financial Studies,* 3, 281-307.

Hamilton, J D (1994), Time Series Analysis. Princeton University Press.

Hardle, W, G Kerkyacharian, D Picard and A Tsybakov (1998), Wavelets. Approximation and Statistical Applications, Springer-Verlag.

Harkness, J P (1968), "A Spectral Analytic Test of the Long Swing Hypothesis in **Canada",** ***Review of Economics and Statistics,*** **50.**

Harvey, A C (1993), Time Series Models. 2nd Edition, London.

Hassan, M. Kabir. and Naka, Atsuyuki (1996), "Short-Run and Long-Run Dynamic Linkages Among International Stock Markets," *International Review of* ***Economics and Finance, 5,*** **4, 387-405.**

Hatanaka, M and E P Howrey (1966), "On the Long-Swing Hypothesis", *Research Paper No. 15, Princeton, N.J.,* **Mimeo.**

Henry, O T (2000), "Long Memory in Stock Returns: Some International Evidence", *Department of Economics, University of Melbourne.*

Hilliard, J E (1979), "The Relationship Between Equity Indices on World Exchanges", *Journal of Finance,* 34, 103 - 114.

Hosking, J. R. M. (1981), "Fractional Differencing", *Biometrika,* 68, 165-176.

Howrey, E P (1968), "A Spectral Analysis of Long Swing Hypothesis", *International Economic Review,* 9, 228-252.

Hubbard, B. B. (1996) The World According to Wavelets: the Story of Mathematical Technique in the Making, A K Peters, Wellesley, Massachusetts.

Hurst, H E (1951), "Long Term Storage Capacity of Reservoirs", *Transactions of the American Society of Civil Engineers*, 116, 770-99.

Hymans, S (1973), "On the Use of Leading Indicators to Predict Cyclical Turning Points", *Brookings Papers on Economic Activity,* 339 - 384.

Jacquier, E (1992), "Business Cycles Components in Stock Market Return Series: Empirical Evidence", Paper Presented at *The European Finance Association Meeting,* Lisbon.

Jaditz, T and C Sayers, "Is Chaos Generating in Economic Data?", *International Journal of Bifurcation Chaos.*

Jegadeesh, J (1990), "Evidence of Predictable Behaviour of Security Returns", *Journal of Finance,* 45, 881-98.

Jensen, M (1999a), "Using Wavelets to Obtain a Consistent Ordinary Least Squares Estimator of the Long Memory Parameter", *Journal of Forecasting,* 18, 17-32.

Jensen, M (1990b), "An Approximate Wavelet MLE of Short and Long Memory Parameters", *Studies in Nonlinear Dynamics and Economics,* 3, 239-53.

Johnstone, I M and B W Silverman (1997), "Wavelet Threshold Estimators for Data with Correlated Noise", *Journal of Royal Statistical Society,* B, 59, 319-51.

Joyeux, R (1979), "Harmonizable Processes in Economics", *Ph.D Thesis, Economics Department, University of California, San Diego.*

Kargill, T F and R A Meyer (1972), "A Spectral Approach to Estimating the Distributed Relationship Between Long and Short Term Interest Rates", *International Economic Review,* 13, 223 - 238.

Karolyi, G.A. (1995), "A Multivariate GARCH Model of International Transmissions of Stock Returns and Volatility: The Case of the U.S. and Canada", *Journal of Business & Economic Statistics*, 13, 11-25.

Kawaller, I G, P D Koch, and T W Koch (1990), "Intraday Relationship Between Volatility in S & P 500 Futures Prices and Volatility in the S & P 500 Index", *Journal of Banking and Finance,* 14, 373-397.

Kim, S.W. and J.H. Rogers (1995), "International Stock Price Spillovers and Market Liberalization: Evidence from Korea, Japan and the United States", *Journal of Empirical Finance*, 2, 117-133.

King, M.A. and S. Wadhwani (1990), "Transmission of Volatility between Stock Markets", *Review of Financial Studies*, 3, 5-33.

Klotz, B P and L Neal (1973), "Spectral and Cross Spectral Analysis of the Long Swing Hypothesis", *Review of Economics and Statistics,* 55, 291 - 298.

Knif, J and M Luoma (1992), "Spectral Characteristics of Common Stock Return Series", *Discussion Papers - 142, Proceedings of the University of Vaasa,* 1-15.

Knif, J, S Pynnonen, and M Luoma (1995), "An Analysis of Lead-Lag Structures Using a Frequency Domain Approach: Empirical Evidence From the Finnish and Swedish Stock Markets", *European Journal of Operational Research,* 81, 259 - 270.

Koopmans, L H (1974), The Spectral Analysis of Time Series. Academic Press Inc., New York.

Koopmans, L H (1983), "A Spectral Analysis Primer", in D R Brillinger and P R Krishnaiah, ed., *Handbook of Statistics,* Vol. III, Elsevier Science Publishers Amsterdam, 169-183.

Kulkarni, S N (1978), "Share Price Behaviour in India: A Spectral Analysis of Random Walk Hypothesis", *Sankhya,* 40, series D, 135-62.

Lee, H S (2000), "Price and Volatility Spillovers in Stock Markets: A Wavelet Analysis", Department of Economics, Sogang University.

Lee, H S (2001), "International Transmission of Stock Market Movements: A Wavelet Analysis of MENA Stock Markets", *ERF's 8th Annual Conference,* Manama, Bahrain.

LeRoy, S F (1973), "Risk Aversion and the Martingale Model of Stock Prices", ***International Economic Review,*** **14, 436 - 446.**

LeRoy, S F and W R Parke (1992), "Stock Price Volatility: Tests Based on Implied Variance Bounds", *Econometrica,* 49, 555 - 574.

Lin, W., R.F. Engle and T. Ito. (1994), "Do Bulls and Bears Move across Borders? International Transmission of Stock Prices and Volatility", *Review of* ***Financial Studies*****, 7, 507-538.**

Lo, A W and A C MacKinlay (1988), "Stock Market Prices Do Not Follow Random Walks: Evidence From a Specification Test", *Review of Financial Studies,* 1,41-66.

Lo, A. W. (1991), "Long Term Memory in Stock Market Prices", *Econometrica,* 59, 1279-1313.

Lobato, I N and N E Savin (1997), "Real and Spurious Long Memory Properties of Stock Market Data", *Discussion Paper,* Department of Economics, University of Iowa.

Lobato, I N and N E Savin (1998), "Spurious and Long Memory Properties of **Stock Market Data",** ***Journal of Business and Economic Statistics,*** **16, 261-83.**

Lucas, R E (1978), "Asset Prices in an Exchange Economy", *Econometrica,* 46, 1429- 1445.

Mallat, S (1989), "A Theory for Multiresolution Signal Decomposition: The **Wavelet Representation",** ***IEEE Transactions on Pattern Analysis and Machine Intelligence,*** **11, 674-93.**

Malliaris, A. G. and Urrutia, J. L. (1992), "The International crash of October 1987: **Causality tests,"** ***Journal of Financial and Quantitative Analysis*****, 27, 3,** 353-364.

Mandelbrot, B. B. (1963) "The Variation of Certain Speculative Prices", *Journal of Business,* 36, 394-419.

Mandelbrot, B. B. (1971), "When Can a Price be Arbitraged Efficiently? A Limit to the Validity of the Random Walk and Martingale Models", *Review of* ***Economics and Statistics,*** **53, 225-36.**

Mandelbrot, B. B. (1997) "*Multifractals & 1/f noise",* New York, Springer Verlag.

Mandelbrot, B. B.(1977), Fractals: Form. Chance. and Dimension. Free Press, New York.

Mankiw, M. G., D. Romer and M. D. Shapiro (1991) "Stock Market Forecastability and Volatility: A Statistical Appraisal", *Review of Economic Studies*, 58, 455-77.

Mantegna, R. N. and H. E. Stanley (2000) An Introduction to Econophysics: Correlations and Complexity in Finance. Cambridge University Press.

Masih, Abul M. M. and Masih, Rumi (1997), "A Comparative Analysis of the Propagation of Stock Fluctuations in Alternative Models of Dynamic Causal **Linkages,"** ***Applied Financial Economics,*** **7, 59-74.**

McCoy E J and A T Walden (1996), "Wavelet Analysis and Synthesis of Stationary **Long Memory Processes,** ***Journal of Computational and Graphical Statistics,*** 5, 26-56.

Melnick, A and A Kraus (1969), "Short-Run Interest Rates Cycles in the US- 1954-1967", ***Journal of Financial and Quantitative Analysis,*** **4,** 291-299.

Ng, A. (2000), "Volatility Spillover Effects from Japan and the U.S. to the Pacific **Basin",** ***Journal of International Money and Finance***, **19, 207-233.**

Ng, V., R.P. Chang and R. Chow. (1991), "An Examination of the Behavior of International Stock Market Volatility" in S.G. Rhee and R.P. Chang (eds.), Pacific-basic Capital Markets Research. 245-260, Amsterdam: North-Holland.

Oppenheimer, P and T V Grissom (1998), "Frequency Space Correlation Between REITs and Capital Market Indices", *Journal of Real Estate Research,* 16, 291 - 309.

Pan, Z and Wang X (1998), "A Stochastic Nonlinear Regression Estimator Using **Wavelets",** ***Computational Economics,*** 11, **89-102.**

Park, Jinwoo and Fatemi, Ali M. (1993), "The Linkages between the Equity Markets of Pacific-Basin Countries and those for the U.S., U.K., and Japan: a Vector Autoregression Analysis," *Global Finance Journal,* 4, 1, 49-64.

Percival, D P (1995), "On Estimation of the Wavelet Variance", ***Biometrica*****,** 82, 619-31.

Plerou, V., P. Gopikrishnan, B. Rosenow, L. A. N. Amaral and H. E. Stanley (1999), "Universal and nonuniversal properties of cross correlations in financial time series", *Physical Review Letters,* 83, 7, 1471-74.

Poraat, B (1997), A Course in Digital Signal Processing. John Wiley and Sons, New York.

Porsius, P (1977), "Spectral Analysis of the Structure of Interest Rates in the Netherlands", *Journal of Monetary Economics,* 3, 191-215.

Poterba, J and L Summers (1987), "Mean Reversion in Stock Prices: Evidence and Implications", *Journal of Financial Economics,* **12,** 27 - **59.**

Praetz, P D (1973), "A Spectral Analysis of Australian Share Prices", *Australian Economic Papers,* 12, 70 - 78.

Priestley, M B (1981), *Spectral Analysis and Time Series,* Academic Press Inc., London.

Ramsey, J B (1999), "The Contribution of Wavelets to the Analysis of Economic and Financial Data", *Philosophical Transactions of the Royal Society of London,* 357, 2593 - 2606.

Ramsey, J B and C Lampart (1998a), "The Decomposition of Economic Relationships by Time Scale Using Wavelets: Expenditure and Income", *Studies in Nonlinear Dynamics and Economics,* 3, 23-42.

Ramsey, J B and C Lampart (1998b), "Decomposition of Economic Relationships by Time Scale Using Wavelets: Money and Income", *Journal of Macroeconomic Dynamics,* 2, 49-71.

Ramsey, J B and D J Thomson (1994), "A Reanalysis of the Spectral Properties of Some Economic and Financial Time Series", *Department of Economics, Working Paper Series, New York University.*

Ramsey, J B and D J Thomson (1998), "A Reanalysis of Spectral Properties of Some Economic and Financial Time Series", in Nonlinear Time Series Analysis of Economic and Financial Data, (ed.) P Rothman, MA: Kluwer, Boston, 45-85.

Ramsey, J B and Z Zhang (1996), "The Application of Wave Form Dictioneraies to Stock Market Data", in Predictability of Complex Dynamical Systems, (eds.) Y A Kravtsov and J B Kadtke, Springer, 189-205.

Ramsey, J B, D Usikov and G Zaslavsky (1995), "An Analysis of US Stock Price Behaviour Using Wavelets", *Fractals,* 3, 377-89.

Ranganatham, M and V Subramanian (1993), "Weak Form of Efficient Markets Hypothesis: A Spectral Analytic Investigation", *Vikalpa,* 18, 25 - 30.

Robinson, P (1995a), Log-Periodogram Regression of Time Series with Long Range Dependence", *Annals of Statistics,* 13, 1048-72.

Robinson, P (1995b), "Gaussian Semiparametric Estimation of Long Range Dependence", *Annals of Statistics,* 13, 1630-1661.

Robinson, P M (1994), "Time Series with Strong Dependence" in Advances in Econometrics, (ed.) C A Sims, Cambridge University Press, 1, 47-95.

Sargent, T J (1968), "Interest Rates in the Nineteen-Fifties", *Review of Economics and Statistics,* 50, 164 - 172.

Scheinkman, J A and B LeBaron (1987), "Nonlinear Dynamics and GNP Data", in Economic Complexity, Chaos, Sunspots, Bubbles, and Nonlinearity, (eds.) W Bernett, J Geweke and K Shells, Cambridge University Press, Cambridge.

Schmidt, C M and R Tschernig (1995), "The Identification of Fractional ARIMA Models", *Discussion Paper 8*, Humboldt-Universitat Zu, Berlin.

Schulz, M., S. Trimper, and B. Schluz (2001) "Variations of Asset Prices", *Physical Review E*, 64, 026104-1:026104-5.

Shachmurove, Y. (1995), "Dynamic Linkages among Latin American and other Major World Stock Markets," Presented at North American Economic and Finance Association Annual Meeting, Washington, DC, January.

Sharma, J L and R E Kennedy (1977), "A Comparative Analysis of Stock Price Behaviour on the Bombay, London, and New York Stock Exchanges", *Journal of Financial and Quantitative Analysis,* 391 -413.

Smith, K L (1999), "Major World Equity Market Interdependence a Decade After the 1987 Crash: Evidence From Cross Spectral Analysis," *Journal of Business, Finance and Accounting,* 26, 3 & 4.

Smith, V K and R G Marcis (1972), "A Time Series Analysis of Post-accord Interest Rates", *Journal of Finance,* 27, 589-605.

Strang, G and T Nguyen (1996), Wavelets and Filter Banks, Wellesley-Cambridge Press, Wellesley, Massachusetts.

Walden A T (1990), "Variance and Degrees of Freedom of a Spectral Estimator Following Data Tapering and Spectral Smoothing", *Signal Processing,* 20, 67-75.

Wang, Y (1995), "Jump and Sharp Cusp Detection by Wavelets", *Biometrica*, 82, 385-97.

Wilson, P J and J Okunev (1998), "Cycles and Regime Switching Models in Real Estate and Financial Assets Market", *14th ARES Conference Monterey, California*, 1-25.

Yajima, Y. (1985), "On the Estimation of Long Memory Time Series Models", *Australian Journal of Statistics*, 27, 303-320.

Ying, C C (1966), "Stock Market Prices and Volumes of Sales", *Econometrica*, 34, 676 - 685.