Study of some phenomenological aspects of Neutrino Mixing

To be submitted in the partial fulfilment for the degree of

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BY

SRINU GOLLU

09PHPH10

Under the supervision of

Prof. RUKMANI MOHANTA

School of Physics

University of Hyderabad

Hyderabad 500 046, INDIA

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1) *Charged lepton correction to tribimaximal lepton mixing and its implications to neutrino phenomenology*, **Srinu Gollu**, K.N. Deepthi and R. Mohanta Mod. Phys. Lett. A **28**, 1350131 (2013), (ISSN No:1793-6632 (online)), Chapter 4.

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Prof. Rukmani Mohanta
Thesis Supervisor
School of Physics
University of Hyderabad

Prof. Bindu A. Bambah
Dean
School of Physics
University of Hyderabad

Date:

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Abstract

The study of neutrino physics is an indispensable limb of particle physics which can play a key role in addressing some of the unsolved problems of nature both theoretically and experimentally. In other words, the essential part of neutrino physics lies in how to explain the nine physical parameters which are related to neutrino masses, mixing angles and CP-violation. Neutrinos are being most abundant in nature, next to radiation, are extremely difficult to catch as they interact only through weak forces. Various neutrino oscillation experiments followed by theoretical work confirmed that neutrinos transform from one flavor to another during their propagation, which implies non-zero mass for neutrinos. As a result of of huge experimental efforts, most of the neutrino oscillation parameters are measured precisely and it is now confirmed that atleast two of the neutrinos are massive. But the smallness of the neutrino mass demanding different origin for neutrino mass compared to other fermions. Different types of see-saw mechanisms explain the smallness of neutrino mass but not the mixing.

In the case of SM symmetry, the parameters in the neutrino mass matrix M_ν are independent of each other, but if there is some flavor symmetry which relates different generations of fermions, the number of free parameters are reduced. There are some standard neutrino mixing patterns which favours the flavor symmetry such as Bimaximal mixing (BM),Tri-bimaximal mixing (TBM), Democratic mixing (DC), Hexagonal mixing (HG), Golden Ratio-A, Golden Ratio-B etc. All the above neutrino mixing patterns suggest reactor mixing angle θ_{13} is zero and the atmospheric mixing angle θ_{23} is maximal i.e., $(\pi/4)$. But the recent data from various dedicated neutrino oscillation experiments suggest non-vanishing value for θ_{13} and deviation of atmospheric mixing angle θ_{23} from its maximal value.

Hence, in order to accommodate the observed reactor mixing angle θ_{13}, we consider perturbation to the above neutrino mixing forms and observe that the perturbation to neutrino mixing patterns explains the observed non-zero θ_{13}. Here in this thesis, we consider perturbation to Tri-bimaximal mixing and studied the implications on neutrino phenomenology such as experimentally observed mixing angles and variation of effective neutrino mass M_{ee}^{ν} with light neutrino mass m_l for both normal hierarchy (NH) and inverted hierarchy (IH), Dirac CP-violation (δ_{CP}) and Jarlskog invariant J_{CP}.

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Dedicated to my family

Chapter 1

Introduction

Neutrinos are the most abundant and elusive fundamental particles next to radiation which are formed just after the Big Bang. Neutrinos are produced at various sources, such as in stars, supernovae, nuclear reactions on earth, etc. Neutrinos are quite different from other particles, as they have unmeasured mass and can travel close to speed of light. They are electrically neutral and react only through weak forces which makes them difficult to be detected in the detector. In the standard model (SM), neutrinos are strictly massless as they do not have any right-handed component. But the results from various dedicated neutrino oscillation experiments reveal that the flavor of neutrinos change as they propagate and such flavor change is possible only if neutrinos have finite non-zero mass. Thus, the observation of neutrino oscillation furnish the first clear evidence of physics beyond the scope of SM.

1.1 History of Neutrino

Wolfgang Pauli, an Austrian physicist first postulated the idea of neutrino [1], in an open letter to a physics conference at Tubingen on 4th December 1930. To explain the violation of energy-momentum conservation and angular momentum conservation in β-decay process, which was thought to proceed as $n \to p + e^-$, he proposed the existence of an weakly interacting neutral fermion. Pauli called this neutral fermion as neutron. After the discovery of neutron as we know it today by Chadwick in 1932, Fermi in 1934 [2] developed the theory of beta decay and renamed the particle proposed by Pauli as neutrino means "the little neutral one". In 1933, Perrin [3] and Fermi [2] separately concluded that neutrinos could be massless.

Neutrinos are not observed in the particle interactions, partly because of their negligible interaction rate with the matter. However, in the early 1950s, Reines and Cowan [4] searched for a new way to observe the neutrino in the inverse beta decay process in which anti-neutrinos are created in the reactor through beta decay interacts with protons in the target and produce positron and neutron. Thereafter, the positron combines with the electron in the target matter and annihilates quickly giving rise to two 0.5 MeV γ-rays, which can be detected by the scintillation detectors. The neutron is slowed down in the water and gets captured by the cadmium chloride and emits another photon. The interaction process is given by

$$\begin{aligned}
&\bar{\nu}_e + p \to n + e^+, \\
&e^+ + e^- \to \gamma + \gamma, \\
&n +^{108} Cd \to^{109} Cd + \gamma. \qquad (1.1)
\end{aligned}$$

Therefore, these experimental results confirm the existence of electron neutrino (ν_e) and this experiment is the first reactor based neutrino experiment. In 1995, Reines was awarded with Nobel prize for neutrino detection.

In 1956, to explain the famous $\theta - \tau$ puzzle, Lee and Yang [5] gave the idea of non-conservation of parity in weak interaction and suggested many number of ways to observe the parity violation. Later on parity violation was observed in β-decay of polarized ^{60}Co,

$$\begin{aligned}
&\pi^+ \to \mu^+ + \nu_\mu, \\
&\mu^+ \to e^+ + \nu_e + \bar{\nu}_\mu. \qquad (1.2)
\end{aligned}$$

In 1958, the $V - A$ theory of weak interaction was formulated [6, 7], which can be realized in the lepton sector by using the two-component framework of massless neutrino. This in turn implies neutrinos are left-handed particles and anti-neutrinos are right-handed.

In 1958, at Brookhaven National Laboratory (BNL), Goldhaber, Grodzins and

Sunyar [8] observed the polarization of neutrino in the process of electron capture

$$e^- +^{152} Eu \rightarrow^{152} Sm^* + \nu_e \ , \tag{1.3}$$

which subsequently decays into

$$^{152}Sm^* \rightarrow Sm^{152} + \gamma. \tag{1.4}$$

They observed the polarization of photon which gives the polarization of the electron neutrino (ν_e) is in a direction opposite to its motion (left-handed) which is in agreement with the two-component theory of massless neutrino.

The first accelerator based neutrino experiment was carried our at BNL by Lederman, Schwartz, Steinberger et.al. [9] and succeeded in establishing the existence of muon neutrino (ν_μ). In 1975, the tau lepton (τ) was discovered at SLAC by Perl et al. [10] and bottom (b), top (t) quarks were discovered at Fermilab in 1977 and 1995 respectively. In 2000, the tau neutrino (ν_τ) was first discovered at DONUT experiment [11].

In 1980s, the Large Electron-Positron Collider (LEP) experiment at CERN accurately measured the width of Z-resonance which provided the strong confirmation that there are only three active light neutrino flavors with $m_\nu < 45$ GeV.

The formulation of the SM by Glashow, Weinberg and Salam in 1967 [12] is the significant milestone in the theory of weak interactions which is based on the $SU(2)_L \times U(1)_Y$ gauge symmetry proposed by Glashow in 1961. Later Higgs mechanism [13] was developed in 1964 by Higgs-Englert-Brout-Guralnik-Hagen and Kibble. The observation of neutral-current (NC) neutrino interaction at Gargamelle experiment, CERN [14] stands as the first confirmation to the success of SM, which was subsequently confirmed at Fermilab. However, results from various dedicated neutrino oscillation experiments indicated that neutrinos change their flavor as they propagate. It is possible only if neutrinos have finite mass which evidently indicates the possibility of new physics beyond SM. Therefore, neutrinos act as a messenger for physics beyond the SM.

The concept of neutrino oscillation was first proposed by B.Pontecorvo in 1957

[15], motivated by the oscillation phenomenon $K^0 \leftrightharpoons \bar{K^0}$. Later in 1967, Maki-Nakagawa and Sakata [16] provided the complete picture of oscillations.

In 1969, Ray Davis [17] observed the solar neutrinos by using chlorine detector at Homestake solar mine in USA. According to Standard Solar Model (SSM), most of the electron neutrino (ν_e) flux is coming from the Sun through proton-proton ($p-p$) chain reactions in the nuclear fusion. The discrepancy between the experimentally measured solar neutrino flux by Gallium experiment and theoretically calculated neutrino flux by John Bachall [18] is known as "Solar neutrino problem". In 2002, SNO experiment [19], explained solar neutrino anomaly through the oscillations of ν_e's into ν_μ and ν_τ inside the sun through MSW resonance effect.

Atmospheric neutrinos are created in Earth's atmosphere by bombardment with cosmic rays gives ν_μ's and ν_e's through the processes $\pi^+ \to \mu^+ + \nu_\mu$ and $\mu^+ \to e^+ + \nu_e + \bar{\nu}_\mu$. The discrepancy in the expected and observed number of neutrinos is known as "Atmospheric neutrino anomaly". Data from the long baseline experiment K2K [20], confirmed that neutrinos oscillate and explained the atmospheric neutrino anomaly through the missing of atmospheric ν_μ's.

Therefore, over the years various dedicated experiments confirmed the phenomenon of neutrino oscillations that can explain the anomalies associated with solar and atmospheric neutrino sector.

The Chapter is organized as follows. The first section discusses the historical journey of neutrinos and in the following sections, we present a brief overview on SM of particle physics and Higgs mechanism which explains the generation of mass to gauge bosons and fermions sector in the SM. The last section contains a small discussion about the necessity of beyond the Standard Model scenarios.

1.2 Standard Model: Theory of Fundamental Interactions

The Standard model is a well established gauge theory which explains the interactions among the fundamental particles, i.e., strong, weak and electromagnetic interactions based on Quantum Field Theory (QFT) framework. It is constructed

	Field	$SU(3)_C \times SU(2)_L \times U(1)_Y$
Fermions	$Q_L \equiv (u, d)_L^T$	$(3, 2, 1/6)$
	u_R	$(3, 1, 2/3)$
	d_R	$(3, 1, -1/3)$
	$\ell_L \equiv (\nu,\ e)_L^T$	$(1, 2, -1/2)$
	e_R	$(1, 1, -1)$
Scalar	H	$(1, 2, 1/2)$

TABLE 1.1: Particle spectrum and their charges under the SM gauge group.

based on the gauge symmetry $SU(3)_C \times SU(2)_L \times U(1)_Y$, where the subscripts C, L, Y denote the color charge, left-handed parity and hypercharge respectively. The gauge group uniquely specifies the nature of interactions, properties and number of vector bosons that correspond to the generators of the gauge groups. In the SM, the elementary particles are classified into two major groups: fermions and bosons where fermions are the spin-$\frac{1}{2}$ particles that make up matter and bosons are the integral spin particles which transmit the forces.

1.2.1 Fermions

Fermions are the particles with $\frac{1}{2}$ integral spin and obey Fermi-Dirac statistics, which are again categorized into two classes: quarks and leptons.

1.2.1.1 Quarks

Quarks are the fundamental constituents of hadrons and exhibit confinement, i.e., quarks do not exist as free particles. There are six flavors of quarks, which are grouped into three generations and participate in all the four fundamental interactions. Quarks have fractional electric charge. Unlike leptons, quarks have color charge (red, blue, green) which causes them to participate in strong interaction. Quarks combine themselves to form hadrons in two different ways either in $q\bar{q}$ pairs of same color charge to form mesons or in qqq form, where each quark has different color to form color-less baryons.

1.2.1.2 Leptons

Leptons are the subatomic particles which participate in weak, electromagnetic and gravitational interactions and are not influenced by strong interaction. Leptons can exist as free particles in six flavors which are classified into three generations. There are three charged leptons: electron (e^-), muon (μ^-) and tau (τ^-) and three neutral leptons known as electron-neutrino (ν_e), muon-neutrino (ν_μ) and tau-neutrino (ν_τ).

1.2.2 Bosons

Bosons are the particles with integral spin which obey Bose-Einstein statistics. In the SM, gauge bosons mediates the fundamental interactions. Strong interactions are mediated by eight massless spin-1 bosons, corresponding to 8 generators of $SU(3)_C$. Weak interactions are mediated by three gauge bosons, which are massive corresponding to three $SU(2)_L$ generators, out of which two are charged (W^+, W^-) and one is neutral (Z^0) boson. Electromagnetic interaction is mediated by a neutral, massless particle: photon (γ) corresponding to one generator of $U(1)_Y$. The electroweak part of SM is based on $SU(2)_L \times U(1)_Y$ symmetry group. In addition, SM predicts the possibility of another scalar boson, known as Higgs boson, which is responsible for generation of mass to $W^\pm, Z^0$ and other fermions.

In the SM, the fermion sector depends on *thirteen* independent variables: *six* quark masses, *three* charged lepton masses (neutrinos are massless in SM), *three* quark mixing angles and *one* phase and all these variables must be measured from experiments. Phenomenologically, SM is extremely successful in describing almost all the known phenomena except neutrino oscillations, which will be discussed in Chapter-2.

1.3 Symmetries in Standard Model

Based on theoretical grounds, all the interactions are dictated by the symmetry principles. In particle physics context, SM is based on the so called local gauge symmetries that intimately related to conserved quantities such as electric charge, color etc.

1.3.1 Strong Interaction

Strong interactions are governed by $SU(3)_C$ local gauge symmetry. The physics has to be invariant under this non-abelian local symmetry transformation in color space. The transformation rule for the matter field $\psi(x)$ (which is a triplet under $SU(3)_C$ group) is symbolically represented as

$$\psi(x) \Rightarrow \psi'(x) = e^{i\alpha_a(x)\cdot T_a}\psi(x), \tag{1.5}$$

where T_a with $a = 1, 2, \cdots, 8$, are the set of eight traceless Gell-Mann matrices obeying the relation

$$[T_a, T_b] = i f_{abc} T_c. \tag{1.6}$$

Here, f_{abc} are known as structure constants. The $SU(3)_C$ gauge invariant Lagrangian is given by

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu \partial_\mu - m)\psi - g(\bar{\psi}\gamma^\mu T_a \psi)G^a_\mu - \frac{1}{4}G^a_{\mu\nu}G^{\mu\nu}_a, \tag{1.7}$$

where the first term denotes the kinetic term of the matter field, the second term denotes the interaction term and the last term gives the kinetic part for the mediating particles (gluons). The gluon field-strength tensor $G^a_{\mu\nu}$ is expressed in terms of gluon fields as

$$G^a_{\mu\nu} = \partial_\mu G^a_\nu - \partial_\nu G^a_\mu - g f_{abc} G^b_\mu G^c_\nu. \tag{1.8}$$

The gauge bosons G^a_μ transform as

$$G^a_\mu \to G^a_\mu - \frac{1}{g}\partial_\mu \alpha_a - f_{abc}\alpha_b G^c_\mu. \tag{1.9}$$

$SU(3)_C$ local gauge invariance requires the gluon fields to remain massless. The particles charged under the $SU(3)_C$ symmetry participate in strong interactions. Since quarks are color triples, the Lagrangian (1.7) describes the interactions between quarks and gluons.

1.3.2 Electroweak Interaction

Electroweak interactions can be incorporated under $SU(2_L) \times U(1)_Y$ symmetry. The subscript L denotes that only left-chiral fields which are charged under $SU(2)_L$ gauge symmetry. The transformation of a field under this symmetry is described by its weak-Isospin (I). All the left-handed fermion fields are isospin doublets (χ_L) and right-handed fields are isospin singlets (ψ_R). The subscript Y on $U(1)$ denotes the weak hypercharge. The isospin and hypercharge quantum numbers obey the relation

$$Q = I_3 + Y, \tag{1.10}$$

where I_3 denotes the third component of isospin, Q denotes the electric charge. The transformation rule for χ_L and ψ_R is

$$\begin{aligned} \chi_L(x) &\to \chi'_L(x) = e^{i\boldsymbol{\alpha}'_i(x)\cdot\boldsymbol{\tau}_i + i\beta'(x)Y}\chi_L(x), \\ \psi_R(x) &\to \psi'_R(x) = e^{i\beta'(x)Y}\psi_R(x), \end{aligned} \tag{1.11}$$

where $i = 1, 2, 3$ and $\boldsymbol{\tau_i} = \sigma_i/2$, σ_i denotes the Pauli matrices

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{1.12}$$

The left and right chiral fields are defined as

$$\chi_L = \frac{(1-\gamma_5)}{2}\chi\ , \quad \psi_R = \frac{(1+\gamma_5)}{2}\psi\ . \tag{1.13}$$

All the SM fermions and their charges are given in Table. (1.1). The invariant Lagrangian under $SU(2)_L \times U(1)_Y$ is given by

$$\begin{aligned} \mathcal{L} &= \overline{\chi}_L\gamma^\mu\left(i\partial_\mu - g\boldsymbol{\tau}_i\cdot\mathbf{W}^i_\mu - g'YB_\mu\right)\chi_L + \overline{\psi}_R\gamma^\mu\left(i\partial_\mu - g'YB_\mu\right)\psi_R \\ &\quad -\frac{1}{4}\mathbf{W}_{\mu\nu}\mathbf{W}^{\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu}\ , \end{aligned} \tag{1.14}$$

where the field tensors are given by

$$\begin{aligned} \mathbf{W}_{\mu\nu} &= \partial_\mu\mathbf{W}_\nu - \partial_\nu\mathbf{W}_\mu - g\mathbf{W}_\mu\times\mathbf{W}_\nu\ , \\ B_{\mu\nu} &= \partial_\mu B_\nu - \partial_\nu B_\mu\ . \end{aligned} \tag{1.15}$$

The gauge fields $\mathbf{W}_\mu$ and B_μ transform as

$$\begin{aligned} \mathbf{W}_\mu &\to \mathbf{W}_\mu - \frac{1}{g}\partial_\mu\boldsymbol{\alpha}'(x) - \boldsymbol{\alpha}'(x)\times\mathbf{W}_\mu\ , \\ B_\mu &\to B_\mu - \frac{1}{g'}\beta'(x)\ . \end{aligned} \tag{1.16}$$

Quarks and leptons being charged under the $SU(2)_L$ and $U(1)_Y$ gauge symmetries, and participate in the electroweak interactions. Weak interactions are short range, mediated by three heavy gauge bosons $W^{\pm}, Z^0$. A mechanism is needed to generate the masses to all these gauge fields. This mechanism is called "**Higgs mechanism**", named after by the renowned physicist Peter Higgs.

1.4 Higgs Mechanism

All the SM fields are massless, moving at relativistic speed at the early Universe. Later, a phase transition took place when all these massless fields (fermions and gauge bosons) became massive by interacting with a scalar field. To generate mass for all the SM particle content i.e., fermions and gauge bosons, we require an $SU(2)_L$ complex scalar doublet with $Y = +\frac{1}{2}$, to break the gauge symmetry spontaneously, which is denoted by

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} . \tag{1.17}$$

Here $\phi^+ = \phi_1 + i\phi_2$ and $\phi^0 = \phi_3 + i\phi_4$, the charged and neutral components respectively. In this mechanism, the neutral component ϕ_3 attains a vacuum expectation value (VEV). Thus the guage symmetry $SU(2)_L \times U(1)_Y$ is spontaneously breaks down to $U(1)_{\rm EM}$. The scalar potential of a complex doublet has the form

$$V(\phi) = \frac{1}{2}\mu_\phi^2(\phi^\dagger\phi) + \frac{1}{4}\lambda_\phi(\phi^\dagger\phi)^2 . \tag{1.18}$$

For $\mu_\phi^2 < 0$ and $\lambda_\phi > 0$, the potential attains two minima at $\phi_3 = \pm\, v$ with $v = \sqrt{-\frac{\mu_\phi^2}{\lambda_\phi}}$ and the shape of the potential $V(\phi)$ looks like a Mexican hat, shown in Fig 1.1. The scalar components ϕ_1, ϕ_2 and ϕ_4 do not acquire any VEV, remain as unphysical massless Goldstone modes. Under unitary gauge, these modes get eaten up by the massless gauge fields $W^{\pm}$, Z to become massive, while A_μ remains massless. In this gauge, the ϕ can be written as

$$\phi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ h + v \end{pmatrix} , \tag{1.19}$$

where h denotes the physical Higgs with $v = 246$GeV.

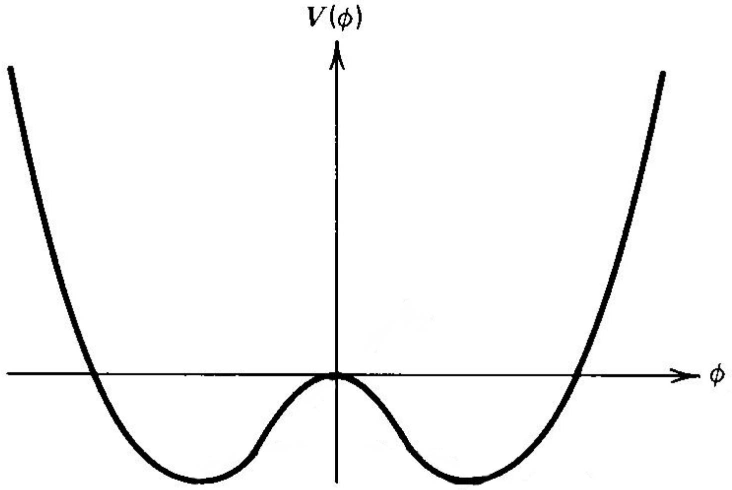

FIGURE 1.1: The form of Higgs potential for $\mu_\phi^2 < 0$ and $\lambda_\phi > 0$.

1.4.1 Masses to gauge fields

The masses to the gauge fields come from the kinetic term of the Lagrangian

$$\mathcal{L} = \left|\left(i\partial_\mu - g\boldsymbol{\tau}_i.\mathbf{W}^i_\mu - g'YB_\mu\right)\phi\right|^2 - V(\phi), \tag{1.20}$$

After spontaneous symmetry breaking, using Eqn. (1.19), we obtain

$$\mathcal{L} = \frac{1}{2}\partial_\mu h\, \partial^\mu h + \frac{(h+v)^2}{2}\left(\frac{g^2}{2}W^+W^- + \frac{g^2+g'^2}{4}Z_\mu Z^\mu\right) - V(\phi). \tag{1.21}$$

The charged $W^\pm$ bosons are represented by $W^\pm = (W^1 \mp iW^2)/\sqrt{2}$, the neutral fields W^3_μ and B_μ mix to give the physical fields Z_μ and A_μ as

$$\begin{aligned} Z_\mu &= -s_W B_\mu + c_W W^3_\mu, \\ A_\mu &= c_W B_\mu + s_W W^3_\mu, \end{aligned} \tag{1.22}$$

where $s_W = \sin\theta_W$ and $c_W = \cos\theta_W$ with $\theta_W = \tan^{-1}(g'/g)$, known as 'Weinberg mixing angle'. The mass spectrum of all the three gauge bosons and the Higgs can be derived as

$$
\begin{aligned}
M_W &= \frac{1}{2} v g \ , \\
M_Z &= \frac{1}{2} v \sqrt{g^2 + g'^2} \ , \\
M_A &= 0, \\
M_h &= \sqrt{2\lambda_\phi v^2} \ .
\end{aligned}
\tag{1.23}
$$

1.4.2 Masses to fermion fields

The fermion mass term $-m\overline{\psi}\psi$ violates the $SU(2)_L$ gauge invariance. So one can introduce the $SU(2)_L \times U(1)_Y$ invariant term after the guage symmetry is spontaneously broken by the Higgs doublet as

$$
\mathcal{L}_y = \sum_{i,j=1}^{3} (-y_l^{ij} \overline{\ell}_{iL} \phi e_{jR} - y_d^{ij} \overline{Q}_{iL} \phi d_{jR} - y_u^{ij} \overline{Q}_{iL} \tilde{\phi} u_{jR}) + h.c., \tag{1.24}
$$

where i, j denote the generations and $\tilde{\phi} = -i\tau_2\phi^*$. In Eqn. (1.24), y_d denotes Yukawa coupling for the down-type (d, s, b) quarks, y_u for the up-type (u, c, t) quarks and y_ℓ is for the leptons(e, μ, τ). Using Eqn. (1.19), we obtain the lepton mass as $M_l = y_l v/\sqrt{2}$ and quark mass $M_q = y_q v/\sqrt{2}$.

1.5 Neutrinos in Standard Model

The Standard Model is a remarkably successful theory, which provides an splendid description of interaction of particles. However, SM has not been up to experimental scrutiny in the description about neutrinos. The assumptions during the construction of SM are: Neutrinos are massless, there are only three flavors of neutrinos $(\nu_e, \nu_\mu, \nu_\tau)$, neutrinos and anti-neutrinos are distinct and all the neutrinos (anti-neutrinos) are left-handed (right-handed).

In SM, as the right-handed neutrinos don't exist, the Dirac mass term is forbidden in the SM Lagrangian. Furthermore, the Majorana mass term is also not allowed in the SM, as it violates lepton number $(\Delta L = 2)$, which is an accidental symmetry of the SM. Therefore, neutrinos are strictly massless in the SM. Nevertheless, we have a direct confirmation from neutrino oscillation data is that the neutrinos are not massless. Existence of mass for the neutrinos is a clear signal for beyond SM scenarios.

1.6 Beyond Standard Model

Even though the SM is phenomenologically successful in describing all the fundamental interactions of elementary particles, SM isn't the ultimate theory of nature and might be low-energy effective theory. It does not provide any satisfactory answer to some of the open issues, like like neutrino oscillations, dark matter, dark energy etc, which were established experimentally and comes under "Physics beyond the Standard Model". In general these scenarios incorporate all the possible extensions of SM regardless of whether the extensions explain any set of puzzles not resolved in the SM. BSM plays a key role in understanding the cosmological puzzles to go beyond SM to incorporate the new physics or suggest candidate suitable for dark matter, dark energy, inflation etc, which are not detected by the experiments till now, but the observed data put a constraint on these BSM candidates/theories.

1.7 Thesis Overview

The plan of the thesis is given as follows. In the first chapter, we introduce the neutrinos from a historical perspective followed by a brief discussion on Standard Model and Higgs mechanism that explains the generation of mass to gauge bosons and fermions. Then we briefly mention about why Standard model is not the complete fundamental theory and the necessity to go beyond it. In Chapter-2, we discuss about the theory of neutrino oscillation in vacuum in both 2ν and 3ν flavor cases and also the effect of matter on neutrino oscillation. The current status of oscillation parameters is also presented. Chapter-3 deals with the discussion about different see-saw mechanisms which explain the smallness of neutrino mass and various neutrino mixing patterns that are $\mu - \tau$ symmetric. In Chapter-4, we discuss charged lepton correction to TBM matrix and its phenomenological implications. Chapter-5, contains perturbation to TBM matrix and its implications on neutrino phenomenology. Summary and Conclusions are presented in Chapter-6.

Chapter 2

Neutrino Oscillation

2.1 Introduction

Neutrino oscillation, a quantum mechanical phenomenon where neutrino of a particular flavor changes into another flavor of neutrino, as it propagates out from the source. The flavor change in neutrinos clearly shows that the flavor eigenstates of neutrinos ($\nu_e,\ \nu_\mu,\ \nu_\tau$) are not same as the mass eigenstates ($\nu_1,\ \nu_2,\ \nu_3$). Neutrino flavor states can be written as the coherent superposition of mass eigenstates. When neutrino propagates, the quantum mechanical phases of the mass eigenstates move at different rates due to their different masses. This subsequently results the change in the combination of different mass eigenstate components, which corresponds to change in flavor eigenstates. Neutrino oscillation indicates, neutrinos to have non-zero mass and also mix among themselves, which gives the first experimental evidence of physics beyond SM.

Neutrino oscillations was first proposed theoretically by Bruno Pontecorvo [15] in terms of neutrino-antineutrino oscillations in analogy with the oscillations of $K^0 \rightleftharpoons \bar{K}^0$ as suggested by Gell-Mann and Pais [21]. The complete picture of mixing of neutrinos in the case of three flavor neutrinos was later developed by Maki, Nakagawa and Sakata in 1962 [16], based on Nagoya model in which nucleons are bound states of vector boson and neutrinos with definite mass. The experimental evidence for neutrino oscillation was first noticed at Super-Kamiokande [20], in the case of atmospheric neutrinos and at Sudbury Neutrino Observatory (SNO) in the case of solar neutrinos [19]. In this chapter, we will discus about neutrino oscillations in vacuum for both two flavor and three flavor cases and also neutrino oscillations in matter.

2.2 Theory

Neutrino of definite flavor eigenstate ν_α is created with the corresponding charged lepton flavor, i.e., electron (e), muon (μ), tau (τ). The flavor eigenstates of neutrinos ν_α, where $\alpha = e, \mu, \tau$, are not identical to the mass eigenstates.

Particular flavor eigenstate of neutrino $|\nu_\alpha\rangle$ can be written in terms of mass eigenstates $|\nu_r\rangle$ with $r = 1, 2, 3$, and is given by the expression

$$|\nu_\alpha\rangle = \sum_r V_{\alpha r} |\nu_r\rangle. \tag{2.1}$$

Thus, one can write the mass eigenstates in terms of the flavor eigenstates as

$$\begin{aligned} |\nu_r\rangle &= \sum_\alpha (V^\dagger)_{r\alpha} |\nu_\alpha\rangle \\ &= \sum_\alpha V^*_{\alpha r} |\nu_\alpha\rangle \ , \end{aligned} \tag{2.2}$$

where V is a lepton mixing matrix, which is unitary and known as Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix, which satisfies the relations

$$V^\dagger V = I, \qquad \sum_r V_{\alpha r} V^*_{\beta r} = \delta_{\alpha\beta} \ , \qquad \sum_\alpha V_{\alpha r} V^*_{\alpha s} = \delta_{rs} \ . \tag{2.3}$$

Analogously, one can express the flavor states for anti-neutrinos as

$$|\overline{\nu}_\alpha\rangle = \sum_r V^*_{\alpha r} |\bar{\nu}_r\rangle. \tag{2.4}$$

Since the neutrino states are orthonormal (both flavor and mass states), they satisfy the orthonormality conditions

$$\langle \nu_\beta | \nu_\alpha \rangle = \delta_{\alpha\beta} \ , \qquad \langle \nu_r | \nu_s \rangle = \delta_{rs} \ . \tag{2.5}$$

Let us assume that neutrino flux of particular flavor ν_α is produced by a given source at the origin, i.e., at $(x, t) = (0, 0)$ and propagates along x-direction. The

neutrino beam is aimed towards the detector which is at a distance L from the source along this axis. During the propagation through vacuum the neutrino is not bounded by any type of potential i.e., behaves like a free particle.
The time evolution of neutrino mass eigenstates is given by the time-dependent Schrodinger equation

$$i\frac{d}{dt}|\nu_r(x,t)\rangle = H|\nu_r(x,t)\rangle = E_r|\nu_r(x,t)\rangle \ , \tag{2.6}$$

where H denotes the Hamiltonian and E_r denotes the energy of neutrino with momentum $\vec{p_r}$ and mass m_r.

By solving the above equation, we get the plane wave solution for mass eigenstates as

$$\begin{aligned} |\nu_r(x,t)\rangle &= e^{-i(E_r t - \vec{p_r}\cdot\vec{x})}|\nu_r(0,0)\rangle \\ &= e^{-i\phi_r}|\nu_r(0,0)\rangle, \end{aligned} \tag{2.7}$$

where $\phi_r = (E_r t - \vec{p_r}\cdot\vec{x})$.
In the flavor basis, the flavor state ν_α at point (x,t) is given by

$$|\nu_\alpha(x,t)\rangle = \sum_{r=1}^{n} V_{\alpha r}|\nu_r(x,t)\rangle, \tag{2.8}$$

which with the substitution of $|\nu_r(x,t)\rangle$ value from Eqn. (2.7), can be read as

$$|\nu_\alpha(x,t)\rangle = \sum_r V_{\alpha r}e^{-i\phi_r}|\nu_r(0,0)\rangle. \tag{2.9}$$

Again expressing the mass state $|\nu_r(0,0)\rangle$ in terms of the flavor states as $|\nu_r(0,0)\rangle = \sum\limits_{\gamma=e,\mu\tau} V^*_{\gamma r}|\nu_\gamma(0,0)\rangle$, we get

$$\begin{aligned} |\nu_\alpha(x,t)\rangle &= \sum_r V_{\alpha r}e^{-i\phi_r}\sum_\gamma V^*_{\gamma r}|\nu_\gamma(0,0)\rangle \\ &= \sum_\gamma\sum_r V_{\alpha r}V^*_{\gamma r}e^{-i\phi_r}|\nu_\gamma(0,0)\rangle. \end{aligned} \tag{2.10}$$

Now the amplitude of transition in detecting the emerged flavor state $|\nu_\beta(x,t)\rangle$ at point (x,t), which is evolved from the state $|\nu_\alpha(0,0)\rangle$ is given by

$$\begin{aligned} A_{(\nu_\alpha(0,0)\to\nu_\beta(x,t))} &= \langle\nu_\beta(x,t)|\nu_\alpha(0,0)\rangle \\ &= \sum_\gamma\sum_r V_{\gamma r}V^*_{\beta r}e^{i\phi_r}\langle\nu_\gamma(0,0)|\nu_\alpha(0,0)\rangle \\ &= \sum_r V_{\alpha r}V^*_{\beta r}e^{i\phi_r}, \end{aligned} \quad (2.11)$$

where we have used $|\nu_\beta(x,t)\rangle = \sum_\gamma\sum_r V_{\beta r}V^*_{\gamma r}e^{-i\phi_r}|\nu_\gamma(0,0)\rangle$ and $\langle\nu_\gamma(0,0)|\nu_\alpha(0,0)\rangle = \delta_{\gamma\alpha}$.

Therefore, the transition probability of initial flavor state ν_α to final flavor state ν_β is given as

$$\begin{aligned} P_{(\nu_\alpha\to\nu_\beta)} &= |A_{(\nu_\alpha(0,0)\to\nu_\beta(x,t))}|^2 \\ &= \sum_{r=1}^{n} V_{\alpha r}V^*_{\beta r}e^{i\phi_r}\sum_{s=1}^{n} V_{\beta s}V^*_{\alpha s}e^{-i\phi_s} \\ &= \sum_{s=1}^{n}\sum_{r=1}^{n} V_{\alpha r}V^*_{\alpha s}V_{\beta s}V^*_{\beta r}e^{-i(\phi_s-\phi_r)}. \end{aligned} \quad (2.12)$$

Now let us consider the term present in the exponential part, i.e., $\phi_s - \phi_r$, where $\phi_r = E_r t - \vec{p}_r\cdot\vec{x}$. Thus, one can write

$$\phi_s - \phi_r = (E_s - E_r)t - (\vec{p}_s - \vec{p}_r)\cdot\vec{x}. \quad (2.13)$$

The energy of the neutrino can be written in a simple form as

$$E_r = \sqrt{p_r^2 + m_r^2} = p_r\sqrt{1 + \frac{m_r^2}{p_r^2}} \simeq p_r\left(1 + \frac{m_r^2}{2p_r^2}\right) \simeq p_r + \frac{m_r^2}{2p_r}. \quad (2.14)$$

As the speed of neutrino is close to speed of light, neutrinos are considered as relativistic particles. Hence, the time (t) required by the neutrino to travel the distance L, are equal. By assuming all the neutrinos have the momenta i.e., $|\bar{p}| = E$, Eqn. (2.13) becomes

$$\phi_s - \phi_r = \left[\left(p + \frac{m_s^2}{2p}\right) - \left(p + \frac{m_r^2}{2p}\right)\right]L = \left(\frac{m_s^2 - m_r^2}{2p}\right)L = \frac{\Delta m_{sr}^2 L}{2E}. \quad (2.15)$$

The probability of neutrino oscillation $\nu_\alpha \to \nu_\beta$ in terms of L (oscillation length) and E (energy of neutrino) as

$$\begin{aligned} P_{(\nu_\alpha\to\nu_\beta)}(L,E) &= \sum_{s=1}^{n}\sum_{r=1}^{n} V_{\alpha r}V_{\alpha s}^{*}V_{\beta s}V_{\beta r}^{*}e^{-i\frac{\Delta m_{sr}^2 L}{2E}} \\ &= \sum_{r=s}^{n}|V_{\alpha r}|^2|V_{\beta r}|^2 + \sum_{r\neq s}^{n} V_{\alpha r}V_{\alpha s}^{*}V_{\beta s}V_{\beta r}^{*}e^{-i\frac{\Delta m_{sr}^2 L}{2E}}. \end{aligned} \tag{2.16}$$

We know that

$$\left|\sum_{r=1}^{n} V_{\alpha r}V_{\beta r}^{*}\right|^2 = \sum_{r=s}^{n}|V_{\alpha r}|^2|V_{\beta r}|^2 + \sum_{r\neq s}^{n} V_{\alpha r}V_{\alpha s}^{*}V_{\beta s}V_{\beta r}^{*}, \tag{2.17}$$

and by using the unitary relation

$$\sum_{r=1}^{n} V_{\alpha r}V_{\beta r}^{*} = \delta_{\alpha\beta}\ , \tag{2.18}$$

the probability of transition $P_{(\nu_\alpha\to\nu_\beta)}(L,E)$ becomes

$$\begin{aligned} P_{(\nu_\alpha\to\nu_\beta)}(L,E) &= \delta_{\alpha\beta} - \sum_{r\neq s}^{n} V_{\alpha r}V_{\alpha s}^{*}V_{\beta s}V_{\beta r}^{*} + \sum_{r\neq s}^{n} V_{\alpha r}V_{\alpha s}^{*}V_{\beta s}V_{\beta r}^{*}e^{-i\frac{\Delta m_{sr}^2 L}{2E}} \\ &= \delta_{\alpha\beta} - \left\{\sum_{r<s}^{n} V_{\alpha r}V_{\alpha s}^{*}V_{\beta s}V_{\beta r}^{*} + \sum_{r<s}^{n} V_{\alpha s}V_{\beta r}V_{\alpha r}^{*}V_{\beta s}^{*}\right\} \\ &+ \left\{\sum_{r<s}^{n} V_{\alpha r}V_{\beta s}V_{\alpha s}^{*}V_{\beta r}^{*}e^{-i\frac{\Delta m_{sr}^2 L}{2E}} + \sum_{r<s}^{n} V_{\alpha r}^{*}V_{\beta s}^{*}V_{\alpha s}V_{\beta r}e^{-i\frac{\Delta m_{sr}^2 L}{2E}}\right\}. \end{aligned} \tag{2.19}$$

We know that for any complex number z, $(z+z^*) = 2\Re(z)$ and $(z-z^*) = 2\Im(z)$ and using the Euler identity ($e^{i\phi} = \cos\phi + i\sin\phi$), Eqn. (2.19) can be written as

$$\begin{aligned} P_{(\nu_\alpha\to\nu_\beta)}(L,E) &= \delta_{\alpha\beta} - 2\sum_{r<s}^{n}\Re(V_{\alpha r}V_{\beta s}V_{\alpha s}^{*}V_{\beta r}^{*}) + 2\sum_{r<s}^{n}\Re\left(V_{\alpha r}V_{\beta s}V_{\alpha s}^{*}V_{\beta r}^{*}\right)\cos\left(\frac{\Delta m_{sr}^2 L}{2E}\right) \\ &\quad + 2\sum_{r<s}^{n}\Im\left(V_{\alpha r}V_{\beta s}V_{\alpha s}^{*}V_{\beta r}^{*}\right)\sin\left(\frac{\Delta m_{sr}^2 L}{2E}\right) \\ &= \delta_{\alpha\beta} - 2\sum_{r<s}^{n}\Re\left(V_{\alpha r}V_{\beta s}V_{\alpha s}^{*}V_{\beta r}^{*}\right)\left(1-\cos\left(\frac{\Delta m_{sr}^2 L}{2E}\right)\right) \\ &\quad + 2\sum_{r<s}^{n}\Im\left(V_{\alpha r}V_{\beta s}V_{\alpha s}^{*}V_{\beta r}^{*}\right)\sin\left(\frac{\Delta m_{sr}^2 L}{2E}\right). \end{aligned} \tag{2.20}$$

Therefore, final expression for the transition probability of neutrino oscillation can be expressed as

$$\begin{aligned} P_{(\nu_\alpha \to \nu_\beta)}(L,E) &= \delta_{\alpha\beta} - 4\sum_{r<s}^{n} \Re(V_{\alpha r}V_{\beta s}V^*_{\alpha s}V^*_{\beta r}) \sin^2\left(\frac{\Delta m^2_{sr}L}{4E}\right) \\ &+ 2\sum_{r<s}^{n} \Im(V_{\alpha r}V_{\beta s}V^*_{\alpha s}V^*_{\beta r}) \sin\left(\frac{\Delta m^2_{sr}L}{2E}\right). \end{aligned} \tag{2.21}$$

If the PMNS mixing matrix V is real, then one can notice from Eqn. (2.21) that

$$P_{(\nu_\alpha \to \nu_\beta)} = P_{(\nu_\beta \to \nu_\alpha)}. \tag{2.22}$$

In the case of survival probability,which means the probability of finding the original flavor state ($\alpha = \beta$), the term $(V_{\alpha r}V_{\alpha s}V^*_{\alpha s}V^*_{\alpha r}) = |V_{\alpha r}|^2|V_{\alpha s}|^2$ becomes real and the last term in Eqn (2.21) becomes zero. Therefore, the survival probability is expressed as

$$P_{\nu_\alpha \to \nu_\alpha}(L,E) = 1 - 4\sum_{r>s} |V_{\alpha r}|^2|V_{\alpha s}|^2 \sin^2\left(\frac{\Delta m^2_{sr}L}{4E}\right). \tag{2.23}$$

Similarly, for anti-neutrinos, the oscillation probability can be obtained by replacing V with its complex conjugate V^* which can be expressed as

$$\begin{aligned} P_{(\bar\nu_\alpha \to \bar\nu_\beta)}(L,E) &= \delta_{\alpha\beta} - 4\sum_{r<s}^{n} \Re(V^*_{\alpha r}V^*_{\beta s}V_{\alpha s}V_{\beta r}) \sin^2\left(\frac{\Delta m^2_{sr}L}{4E}\right) \\ &+ 2\sum_{r<s}^{n} \Im(V^*_{\alpha r}V^*_{\beta s}V_{\alpha s}V_{\beta r}) \sin\left(\frac{\Delta m^2_{sr}L}{2E}\right). \end{aligned} \tag{2.24}$$

If the mixing matrix V is complex, it should be noted from Eqns. (2.21) and (2.24) that

$$P_{(\bar\nu_\alpha \to \bar\nu_\beta)} \neq P_{(\nu_\alpha \to \nu_\beta)} , \tag{2.25}$$

which hints the possibility of CP-violation in neutrino sector.

CP violation can be established itself, if the probability of oscillation for $(\nu_\alpha \to \nu_\beta)$ is different from its CP conjugate process $(\bar\nu_\alpha \to \bar\nu_\beta)$ and the observable is

$$\Delta P_{CP} = P_{(\nu_\alpha \to \nu_\beta)} - P_{(\bar\nu_\alpha \to \bar\nu_\beta)} \neq 0, \quad \text{for } (\alpha \neq \beta). \tag{2.26}$$

From the above expressions for the oscillation probabilities, it should be observed that for the possibility of neutrino oscillation to occur, some off-diagonal elements of the matrix V_{PMNS} should be non-zero and the mass eigenstates must be non-degenerate. If the neutrinos are massless, then $\Delta m^2_{sr}= 0$ and the probability of oscillation $P_{\nu_\alpha \to \nu_\beta}(L, E) = \delta_{\alpha\beta}$. This implies that observation of neutrino oscillation provides the direct indication of non-zero mass of neutrinos. Nevertheless, it can not give the absolute mass of neutrinos as the oscillation probabilities are sensitive only to mass square differences Δm^2_{rs}. In the following subsections, we will discuss the vacuum oscillations of neutrinos both in the case of two flavor and three flavor neutrinos.

2.2.1 Two flavor oscillations in vacuum

Two neutrino framework ($n = 2$) gives a simple and profound introduction to understand the phenomenon of neutrino oscillation. In fact many neutrino oscillation experiments have used two flavor neutrino case to analyze the data and to measure the oscillation parameters. Here we consider only two flavor eigenstates ν_α and ν_β and the corresponding two mass eigenstates are denoted as ν_1 and ν_2. The mass and flavor states are related by a orthogonal 2×2 rotation matrix which can be parametrised with a mixing angle ϕ.

Therefore, the flavor and mass states can be related as

$$\begin{pmatrix} \nu_\alpha \\ \nu_\beta \end{pmatrix} = \begin{pmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}. \tag{2.27}$$

In two flavor case, the oscillation probability is given by

$$\begin{aligned} P_{(\nu_\alpha \to \nu_\beta)} &= -4\Re(V_{\alpha 1} V^*_{\alpha 2} V_{\beta 2} V^*_{\beta 1}) \sin\left(\frac{\Delta m^2_{21} L}{4E}\right) \\ &= 4\cos^2\phi \sin^2\phi \sin\left(\frac{\Delta m^2_{21} L}{4E}\right) = \sin^2 2\phi \sin\left(\frac{\Delta m^2 L}{4E}\right). \end{aligned} \tag{2.28}$$

It should be noted that the above expression for transition probability is in natural units, where ($\hbar = c = 1$). By reinserting the light velocity (c) and the Plank's constant ($\hbar$), the phase part becomes

$$\left(\frac{\Delta m^2 L}{4E}\right) = \left(\frac{\Delta m^2 L c^3}{4E\hbar}\right). \tag{2.29}$$

If the mass square difference Δm^2 is measured in eV^2, energy of the neutrino (E) in GeV, L in kilometer and by substituting the values of $\hbar$ and c, the phase part becomes

$$\left(\frac{\Delta m^2 L c^3}{4E\hbar}\right) = \left(1.27\frac{\Delta m^2 L}{E}\right). \tag{2.30}$$

Therefore, the probability of oscillation becomes

$$P_{(\nu_\alpha \to \nu_\beta)} = \sin^2 2\phi \sin\left(1.27\frac{\Delta m^2 L}{E}\right) = \sin^2 2\phi \sin\left(\frac{\pi L}{L_{osc}}\right), \tag{2.31}$$

where

$$L_{osc} = 2.48\frac{E/\mathrm{MeV}}{\Delta m^2/\mathrm{eV}^2}\ \mathrm{met}, \tag{2.32}$$

is the minimum length below which the oscillations does not develop. The survival probability is given by

$$\begin{aligned} P_{(\nu_\alpha \to \nu_\alpha)} &= 1 - P_{(\nu_\alpha \to \nu_\beta)} \\ &= 1 - \sin^2 2\phi \sin\left(1.27\frac{\Delta m^2 L}{E}\right). \end{aligned} \tag{2.33}$$

From the above Eqns. (2.31) and (2.33), we can infer some note worthy points:

- **The mixing angle** ϕ: The mixing angle ϕ gives how the mass eigenstates are mixed in the flavor eigenstates. If the mixing angle ϕ =0, the flavor eigenstates are identical to mass eigenstates, i.e., the probability of oscillation between flavor states is zero. Therefore, for the possibility of neutrino oscillations the mixing angle ϕ should be non-zero ($\phi \neq 0$). If $\phi = \pi/4$, then the probability of oscillation is maximal, i.e., if a neutrino of definite flavor produced at source, it oscillates into different flavor during the propagation. The oscillation amplitude depends on the mixing angle ϕ and it increases as the value of $\sin^2 2\phi$ increases.

- **Mass square difference** Δm^2: It is obvious that if the neutrino mass square difference $\Delta m^2 = m_2^2 - m_1^2 = 0$, then the probability of oscillation

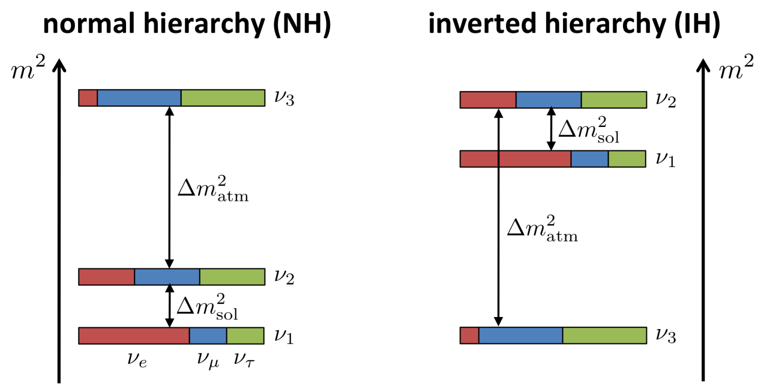

FIGURE 2.1: The schematic representation of neutrino mass orderings.

is zero, which implies the existence of at least one of the mass eigenstate is massive. Therefore, for the possibility of neutrino oscillations, the neutrino mass eigenstates must be non-degenerate, i.e., ($m_1 \neq m_2$). The mass square splitting Δm^2 can be measured by measuring the period of oscillation. The frequency of oscillation is large for the larger values of Δm^2.

For three-flavor case (which will be discussed in the next section), the two mass related experimental observables are: $|\Delta m^2_{31}|^2$ (sign is not yet known) and Δm^2_{21} (positive and definite) can help to understand the ordering of neutrino masses. The knowledge on the sign of $|\Delta m^2_{31}|$ is essential to understand the neutrino mass ordering, i.e., whether they follow normal or inverted ordering. For normal ordering $\Delta m^2_{31} > 0$ or $m_1 < m_2 \ll m_3$, and for inverted hierarchy $\Delta m^2_{31} < 0$ or $m_3 \ll m_2 < m_1$, which can be pictorially represented as in Fig. 2.1.

- **L/E ratio**: Neutrino oscillation probability changes with the ratio L/E for particular value of Δm^2. Generally experimentalists vary the L/E ratio to study the phenomenon of neutrino oscillations. In solar neutrino experiments L/E is fixed by the nature. Any experiment that designed to observe the neutrino oscillations, the oscillation probability should satisfy the condition

$$\begin{aligned} 1.27\frac{\Delta m^2 L}{E} &= \frac{\pi}{2} \\ \Rightarrow L/E &= \frac{\pi}{2.54\Delta m^2}\,. \end{aligned} \tag{2.34}$$

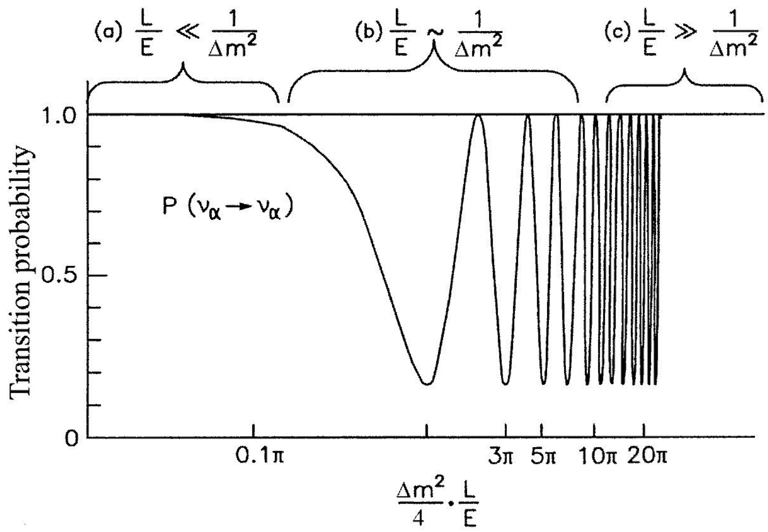

FIGURE 2.2: Logarithmic plot of Probability of oscillation $P_{\nu_\alpha \to \nu_\alpha}$ as function of $\frac{L}{E}$ for $\sin^2 2\phi = 0.83$. The brackets represent following cases: (a) no oscillations $\frac{L}{E} \ll \frac{1}{\Delta m^2}$; (b) oscillations $\frac{L}{E} \approx \frac{1}{\Delta m^2}$; (c) average oscillations for $\frac{L}{E} \gg \frac{1}{\Delta m^2}$.

i. If $\dfrac{L}{E} \ll \dfrac{4}{\Delta m^2}$, i.e., $L \ll L_{osc}$: In this case the neutrino oscillations do not have time to develop oscillation.

ii. If $\dfrac{L}{E} \gtrsim \dfrac{4}{\Delta m^2}$, i.e., $L \gtrsim L_{osc}$: It is the required condition to experimentally observe the neutrino oscillation.

iii. If $\dfrac{L}{E} \gg \dfrac{4}{\Delta m^2}$, i.e., $L \gg L_{osc}$: Here number of oscillations developed between source and detector is large. So here the oscillation experiments measure only average transition probability.

- **Types of experiments**:

  Neutrino oscillation experiments are classified into two categories:

  (a) Appearance experiments: Appearance experiments measure the transition probability $P_{\nu_\alpha \to \nu_\beta}$ for $(\nu_\alpha \to \nu_\beta)$ oscillations in which the initial flavor of neutrino after the oscillation changes into different flavor.

  (b) Disappearance experiments: Disappearance experiments measure the survival probability for $(\nu_\alpha \to \nu_\alpha)$ oscillation in which the initial flavor of the neutrino before and after the oscillation remains the same.

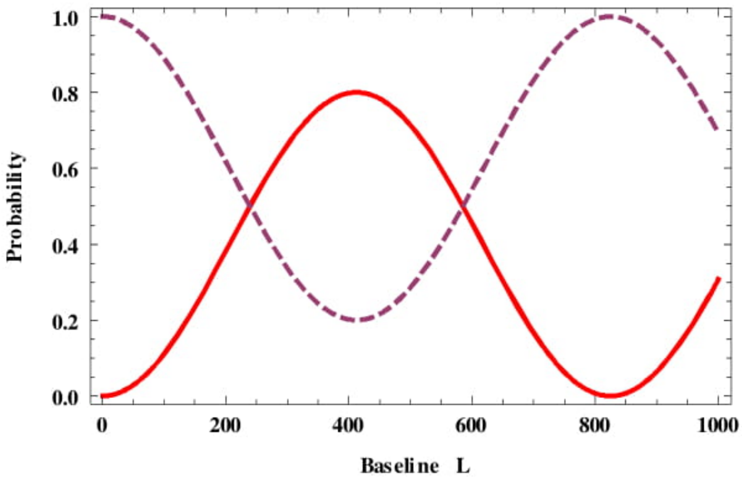

FIGURE 2.3: The transition probability (solid line) and the survival probability (dashed line) as a function of baseline L along with oscillation parameters $\sin^2 2\phi = 0.8$, $\Delta m^2 = 3 \times 10^{-3}$ eV^2, and the energy E=1 GeV.

The variation of transition probability with the baseline length (L) is depicted in Figure 2.3 for a representative set of values of neutrino oscillation parameters. It is obvious from the figure that at the source i.e., $L = 0$, only one particular flavor of neutrino say ν_α exists and as the neutrino propagates towards the detector, the oscillation probability increase and at one point the probability becomes maximum i.e., 0.8 (for $(1.27\Delta m^2 L/E) = \pi/2$ with L= 400 km). At this point, 80% of ν_α neutrinos converted into ν_β. As L increase further, the probability of oscillation decreases and at around L= 800 km, all the neutrinos oscillate into ν_α.

2.2.2 Three Flavor Neutrino Oscillation in Vacuum

In Standard Model physics there are three neutrino flavors ν_e, ν_μ and ν_τ. In the case of three flavor neutrino mixing ($n = 3$), the lepton mixing matrix V is a 3×3 unitary matrix with complex elements known as Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix [22]. The three flavor mixing of neutrinos is given by

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} V_{e1} & V_{e2} & V_{e3} \\ V_{\mu 1} & V_{\mu 2} & V_{\mu 3} \\ V_{\tau 1} & V_{\tau 2} & V_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}. \tag{2.35}$$

The unitarity of the matrix contains 9 independent parameters, out of which three are mixing angles and six are relative phases between the three neutrinos and three

charged leptons. Among the six phases, five phases can be absorbed by the six fields (3 neutrino fields and 3 charged lepton fields). Therefore, the V_{PMNS} matrix for three flavor neutrino can be parametrized with four parameters, i.e., (three mixing angles $\theta_{13},\ \theta_{12},\ \theta_{23}$ and one Dirac phase δ_{CP}).
In the standard parametrization [23], V_{PMNS} can be expressed as the product of three rotation matrices contains mixing angles between the mass eigenstates and complex phase factor $(e^{-i\delta_{CP}})$ as

$$\begin{aligned} V_{PMNS} &= R_{23}(\theta_{23})R_{13}(\theta_{13},\delta)R_{12}(\theta_{12}) \\ &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{CP}} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta_{CP}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \\ &= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{CP}} \\ -s_{12}c_{23} - c_{12}s_{13}s_{23}e^{i\delta_{CP}} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta_{CP}} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}s_{13}c_{23}e^{i\delta_{CP}} & -c_{12}s_{23} - s_{12}s_{13}c_{23}e^{i\delta_{CP}} & c_{13}c_{23} \end{pmatrix}, \end{aligned} \tag{2.36}$$

where $\cos\theta_{ij} = c_{ij}$ and $\sin\theta_{ij} = s_{ij}$. If neutrinos are Majorana particles, then a diagonal matrix containing two Majorana phases ρ, σ is required in the parametrization i.e., $P_\nu = \text{diag}(e^{i\rho}, e^{i\sigma}, 1)$. The first rotation matrix $R_{23}(\theta_{23})$ measures the effects of atmospheric neutrino oscillation. The third rotation matrix $R_{12}(\theta_{12})$ measures the effects of solar neutrino oscillation. The middle matrix $R_{13}(\theta_{13}, \delta_{CP})$ has the reactor mixing angle θ_{13} and Dirac CP phase δ_{CP}, the determination of which is one of the main goal of the current and future neutrino oscillation experiments.

In three flavor neutrino oscillation framework, there are three mass squared differences i.e, $\Delta m^2_{21}, \Delta m^2_{32}, \Delta m^2_{31}$. In general $\Delta m^2_{31} = \Delta m^2_{32}$ and we identify Δm^2_{32} is responsible for atmospheric oscillations ($|\Delta m^2_{32}| \approx |\Delta m^2_{31}| \approx |\Delta m^2_{atm}|$) and Δm^2_{21} accounts for solar neutrino oscillations ($\Delta m^2_{21} = \Delta m^2_{sol}$). The sign of Δm^2_{atm} gives the neutrino mass ordering or hierarchy either normal or inverted hierarchy.

There are three physical neutrinos, three mass eigenstates and so there will be two independent mass squared differences, for which

$$\Delta m^2_{12} + \Delta m^2_{23} + \Delta m^2_{31} = 0. \tag{2.37}$$

The general formula for oscillation probability in case of three flavor neutrino framework is quite complex. Therefore, by making some assumption that is $\Delta m^2_{21} = \Delta m^2_{sol}$ is small and considering the CP violating phase $\delta_{CP} = 0$, the oscillation probability for the transition $(\nu_\alpha \rightarrow \nu_\beta)$ is given by

$$\begin{aligned} P_{(\nu_\alpha \rightarrow \nu_\beta)} &= -4\sum_{r<s}^{n} \Re(V_{\alpha r} V_{\beta s} V_{\alpha s} V_{\beta r}) \sin^2\left(\frac{1.27\Delta m^2_{sr} L}{E}\right) \\ &= -4(V_{\alpha 1} V_{\beta 1} V_{\alpha 2} V_{\beta 2}) \sin^2\left(\frac{1.27\Delta m^2_{21} L}{E}\right) \\ &\quad -4(V_{\alpha 1} V_{\beta 1} V_{\alpha 3} V_{\beta 3}) \sin^2\left(\frac{1.27\Delta m^2_{31} L}{E}\right) \\ &\quad -4(V_{\alpha 2} V_{\beta 2} V_{\alpha 3} V_{\beta 3}) \sin^2\left(\frac{1.27\Delta m^2_{23} L}{E}\right). \end{aligned} \tag{2.38}$$

For the experiments in which $\left(\frac{1.27\Delta m^2_{sr} L}{E}\right) \ll 1$, implies

$$\sin^2\left(\frac{1.27\Delta m^2_{sr} L}{E}\right) \rightarrow 0. \tag{2.39}$$

In addition, using $\Delta m^2_{31} = \Delta m^2_{32}$, Eqn (2.38) becomes

$$P_{(\nu_\alpha \rightarrow \nu_\beta)} = 4\Big[V_{\alpha 1} V_{\beta 1} V_{\alpha 2} V_{\beta 2} + V_{\alpha 2} V_{\beta 2} V_{\alpha 3} V_{\beta 3}\Big] \sin^2\left(\frac{1.27\Delta m^2_{23} L}{E}\right). \tag{2.40}$$

By substituting the elements of PMNS matrix, the probabilities for specific transitions are given by

$$P_{(\nu_\mu \rightarrow \nu_\tau)} = \sin^2 2\theta_{23} \cos^2\theta_{13} \sin^2\left(\frac{1.27\Delta m^2_{23} L}{E}\right), \tag{2.41}$$

$$P_{(\nu_e \rightarrow \nu_\mu)} = \sin^2 2\theta_{13} \sin^2\theta_{23} \sin^2\left(\frac{1.27\Delta m^2_{23} L}{E}\right), \tag{2.42}$$

$$P_{(\nu_e \rightarrow \nu_\tau)} = \sin^2 2\theta_{13} \cos^2\theta_{23} \sin^2\left(\frac{1.27\Delta m^2_{23} L}{E}\right). \tag{2.43}$$

The three flavor neutrino framework is the most successful theoretical model that can accommodate the current oscillation data.

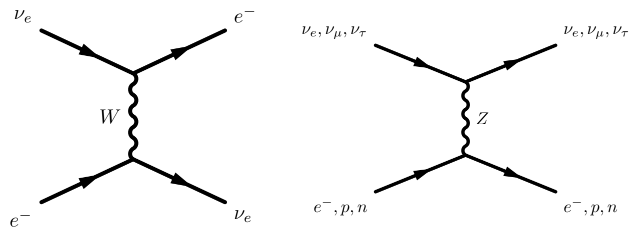

FIGURE 2.4: Feynman diagram for the coherent forward elastic scattering which generate the CC interaction potential through W exchange and NC interaction potential through Z exchange.

2.3 Neutrino oscillation in matter

The oscillations of neutrinos from one particular flavor to another flavor, when propagating through matter with varying density is known as matter effect or Mikheev-Smirnov-Wolfenstein (MSW) effect [24, 25], which explains the flavor changing of solar neutrinos as they propagate out of the sun. When neutrinos propagating through vacuum, they are not bounded by any type of potential, i.e., behave like free particles. But when the neutrinos propagating through matter, they are subjected to the interaction potential due to the coherent forward elastic scattering of neutrinos with electrons, protons, neutrons present in the matter.

If an electron neutrino (ν_e) propagating through matter, it can have both Neutral Current (NC) and Charged Current (CC) interactions with the electrons present in the matter. While the other type of neutrinos e.g., (ν_μ, ν_τ) can have only Neutral Current (NC) interactions. The Feynman diagrams for NC and CC scatterings are shown in figure 2.4.

The oscillation probability in matter does not get effected due to the NC contributions but the CC contributions which comes only with ν_e, affect the oscillation probability. As a result the probability of oscillation in matter can be rather different than vacuum. Depending on the variation of electron density in matter, the interaction potential can be constant or it may vary, for example the electron density varies inside the sun. In this section, we focus on the oscillation of neutrinos in matter with constant electron density both in two and three flavor neutrino case.

2.3.1 Two flavor oscillations in matter

Here we consider two flavor neutrino oscillations in matter. The time-dependent Schrödinger equation for mass eigenstates of neutrinos in vacuum is given by

$$i\frac{d}{dt}\begin{pmatrix}\nu_1\\ \nu_2\end{pmatrix} = H\begin{pmatrix}\nu_1\\ \nu_2\end{pmatrix}, \tag{2.44}$$

with Hamiltonian in vacuum is represented as

$$H = \frac{1}{2E}\begin{pmatrix} m_1^2 & 0\\ 0 & m_2^2\end{pmatrix}.$$

The flavor and mass eigensates of neutrinos are related by

$$\begin{pmatrix}\nu_\alpha\\ \nu_\beta\end{pmatrix} = U\begin{pmatrix}\nu_1\\ \nu_2\end{pmatrix}, \tag{2.45}$$

where the mixing matrix U is given as

$$U = \begin{pmatrix}\cos\phi & \sin\phi\\ -\sin\phi & \cos\phi\end{pmatrix}.$$

From Eqn. (2.45), one can write the mass states in terms of flavor states as

$$\begin{pmatrix}\nu_1\\ \nu_2\end{pmatrix} = U^\dagger\begin{pmatrix}\nu_\alpha\\ \nu_\beta\end{pmatrix}.$$

Thus, the time evolution equation (2.44) becomes

$$U^\dagger i\frac{d}{dt}\begin{pmatrix}\nu_\alpha\\ \nu_\beta\end{pmatrix} = HU^\dagger\begin{pmatrix}\nu_\alpha\\ \nu_\beta\end{pmatrix}. \tag{2.46}$$

Therefore, the time evolution equation in flavor states is given by

$$i\frac{d}{dt}\begin{pmatrix}\nu_\alpha\\ \nu_\beta\end{pmatrix} = UHU^\dagger\begin{pmatrix}\nu_\alpha\\ \nu_\beta\end{pmatrix}. \tag{2.47}$$

Hence, the Hamiltonian for the flavor states H_f is given by

$$\begin{aligned} H_f &= UHU^\dagger \\ &= \begin{pmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{pmatrix} \left(\frac{1}{2E}\right) \begin{pmatrix} m_1^2 & 0 \\ 0 & m_2^2 \end{pmatrix} \begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix} \\ &= \left(\frac{m_1^2+m_2^2}{4E}\right) I + \left(\frac{\Delta m^2}{4E}\right) \begin{pmatrix} -\cos 2\phi & \sin 2\phi \\ \sin 2\phi & \cos 2\phi \end{pmatrix} \\ &= H_0 + \left(\frac{\Delta m^2}{4E}\right) \begin{pmatrix} -\cos 2\phi & \sin 2\phi \\ \sin 2\phi & \cos 2\phi \end{pmatrix}, \end{aligned} \tag{2.48}$$

where $H_0 = (m_1^2 + m_2^2)/4E$ and $\Delta m^2 = m_2^2 - m_1^2$.

Thus, Eqn. (2.47) becomes

$$i\frac{d}{dt}\begin{pmatrix} \nu_\alpha \\ \nu_\beta \end{pmatrix} = \left[H_0 + \left(\frac{\Delta m^2}{4E}\right) \begin{pmatrix} -\cos 2\phi & \sin 2\phi \\ \sin 2\phi & \cos 2\phi \end{pmatrix} \right] \begin{pmatrix} \nu_\alpha \\ \nu_\beta \end{pmatrix}. \tag{2.49}$$

In the two flavor framework, the time evolution of neutrino flavor states in the presence of matter potential ($V' \neq 0$) with constant density is given by

$$i\frac{d}{dt}\begin{pmatrix} \nu_\alpha \\ \nu_\beta \end{pmatrix} = H_{eff} \begin{pmatrix} \nu_\alpha \\ \nu_\beta \end{pmatrix}, \tag{2.50}$$

where effective Hamiltonian $H_{eff} = (H_f + V')$.

During the propagation through matter, the electron neutrinos ν_e's undergo both Neutral and Charged current interactions with the particles in the matter. But the muon and tau neutrinos ($\nu_\mu,\ \nu_\tau$) undergo only Neutral current interactions. Hence ν_α state experiences the interaction potential V'_α is different from the interaction potential (V'_β) experienced by the state ν_β. Thus, one can write explicitly Eqn. (2.50) as

$$i\frac{d}{dt}\begin{pmatrix} \nu_\alpha \\ \nu_\beta \end{pmatrix} = \left[H_0 + \begin{pmatrix} -\left(\frac{\Delta m^2}{4E}\right)\cos 2\phi + V'_\alpha & \left(\frac{\Delta m^2}{4E}\right)\sin 2\phi \\ \left(\frac{\Delta m^2}{4E}\right)\sin 2\phi & \left(\frac{\Delta m^2}{4E}\right)\cos 2\phi + V'_\beta \end{pmatrix} \right] \begin{pmatrix} \nu_\alpha \\ \nu_\beta \end{pmatrix}. \tag{2.51}$$

As the addition of the constant V'_β does not affect the oscillation probability, the above equation can be rewritten as

$$i\frac{d}{dt}\begin{pmatrix}\nu_\alpha\\ \nu_\beta\end{pmatrix} = \left[H_0 + \begin{pmatrix} -\left(\frac{\Delta m^2}{4E}\right)\cos 2\phi + \Delta V & \left(\frac{\Delta m^2}{4E}\right)\sin 2\phi \\ \left(\frac{\Delta m^2}{4E}\right)\sin 2\phi & \left(\frac{\Delta m^2}{4E}\right)\cos 2\phi \end{pmatrix}\right]\begin{pmatrix}\nu_\alpha\\ \nu_\beta\end{pmatrix}, \quad (2.52)$$

where $\Delta V = (V'_\alpha - V'_\beta)$.

If $\alpha = e$, then ΔV has the form

$$\Delta V = (V'_e - V'_\beta) = 2\sqrt{2}G_F N_e E, \quad (2.53)$$

where G_F denotes the Fermi coupling constant, N_e denotes the electron number density in matter and E is the energy of the neutrinos.
The time evolution of mass eigenstates in presence of matter is given by

$$i\frac{d}{dt}\begin{pmatrix}\nu_1\\ \nu_2\end{pmatrix} = i\frac{d}{dt}U^\dagger\begin{pmatrix}\nu_\alpha\\ \nu_\beta\end{pmatrix}, \quad (2.54)$$

which can be simplified as

$$\begin{aligned} i\frac{d}{dt}\begin{pmatrix}\nu_1\\ \nu_2\end{pmatrix} &= U^\dagger\left[H_0 + \begin{pmatrix} -(\Delta m^2/4E)\cos 2\phi + \Delta V & (\Delta m^2/4E)\sin 2\phi \\ (\Delta m^2/4E)\sin 2\phi & (\Delta m^2/4E)\cos 2\phi \end{pmatrix}\right]\begin{pmatrix}\nu_\alpha\\ \nu_\beta\end{pmatrix} \\ &= U^\dagger\left[H_0 + \begin{pmatrix} -(\Delta m^2/4E)\cos 2\phi + \Delta V & (\Delta m^2/4E)\sin 2\phi \\ (\Delta m^2/4E)\sin 2\phi & (\Delta m^2/4E)\cos 2\phi \end{pmatrix}\right]U\begin{pmatrix}\nu_1\\ \nu_2\end{pmatrix} \\ &= \left[\frac{1}{2E}\begin{pmatrix} m_1^2 + \Delta V\cos^2\phi & \Delta V\cos\phi\sin\phi \\ \Delta V\cos\phi\sin\phi & m_2^2 + \Delta V\sin^2\phi \end{pmatrix}\right]\begin{pmatrix}\nu_1\\ \nu_2\end{pmatrix}. \end{aligned} \quad (2.55)$$

The above matter Hamiltonian in mass states has non-diagonal terms which means the mass eigenstates in matter are different from mass eigenstates in vacuum. We have to diagonalise the Hamiltonian matrix to obtain the mass eigenstates in matter $(\nu_{1m},\ \nu_{2m})$ and the mass eigenvalues $(m^2_{1m},\ m^2_{2m})$. Then

$$|H^i_m - \lambda I| = 0,$$

which can be explicitly written as

$$\begin{vmatrix} m_1^2 + \Delta V \cos^2\phi - \lambda & \Delta V \cos\phi \sin\phi \\ \Delta V \cos\phi \sin\phi & m_2^2 + \Delta V \sin^2\phi - \lambda \end{vmatrix} = 0. \tag{2.56}$$

By solving the above equation, we can get

$$\lambda_{1,2} = \frac{1}{2}\Bigg[\left(\Delta V + m_1^2 + m_2^2\right) \\ \pm \sqrt{\Delta V + (m_1^2+m_2^2)^2 - 4\left(m_1^2 m_2^2 + \Delta V \left(m_1^2 \sin^2\phi + m_2^2\cos^2\phi\right)^2\right)}\Bigg] ,$$

which after simplification becomes

$$\lambda_{1,2} = \frac{1}{2}\left[\Delta V + \left(m_1^2+m_2^2\right) \pm \sqrt{\left(\Delta m^2\right)^2 \sin^2 2\phi + \left(\Delta V - \Delta m^2 \cos 2\phi\right)^2}\right].$$

Then the modified mass eigenvalues $m_{1m}^2,\ m_{2m}^2$ can be expressed as

$$m_{2m,1m}^2 = \frac{1}{2}\left[\Delta V + \left(m_1^2+m_2^2\right) \pm \sqrt{\left(\Delta m^2\right)^2 \sin^2 2\phi + \left(\Delta V - \Delta m^2 \cos 2\phi\right)^2}\right],$$

and hence, the modified mass squared difference in matter is given by

$$\begin{aligned} \Delta m_m^2 = m_{2m}^2 - m_{1m}^2 &= \sqrt{\left(\Delta m^2\right)^2 \sin^2 2\phi + \left(\Delta V - \Delta m^2 \cos 2\phi\right)^2} \\ &= \Delta m^2 \sqrt{\left(\frac{\Delta V}{\Delta m^2} - \cos 2\phi\right)^2 + \sin^2 2\phi}. \end{aligned} \tag{2.57}$$

Thus, the diagonalised form of Hamiltonian can be expressed as

$$H_m^i = \left(\frac{1}{2E}\right) \begin{pmatrix} m_{1m}^2 & 0 \\ 0 & m_{2m}^2 \end{pmatrix} \tag{2.58}$$

Now let us consider a mixing matrix U_m which connects the flavor states $(\nu_\alpha,\ \nu_\beta)$ and the mass eigenstates $(\nu_{1m},\ \nu_{2m})$ in matter through a mixing angle (ϕ_m), where U_m has the form

$$U_m = \begin{pmatrix} \cos\phi_m & \sin\phi_m \\ -\sin\phi_m & \cos\phi_m \end{pmatrix}. \tag{2.59}$$

Then the modified Hamiltonian in matter for flavor state is given by

$$\begin{aligned} H_m^f &= U_m H_m^i U_m^\dagger \\ &= \frac{(m_{1m}^2 + m_{2m}^2)}{4E} I + \frac{\Delta m_m^2}{4E} \begin{pmatrix} -\cos 2\phi_m & \sin 2\phi_m \\ \sin 2\phi_m & \cos 2\phi_m \end{pmatrix} . \end{aligned} \tag{2.60}$$

By comparing the above Hamiltonian H_m^f with the Hamiltonian in equation (2.48), one can obtain

$$\Delta m_m^2 \sin 2\phi_m = \Delta m^2 \sin 2\phi \ . \tag{2.61}$$

With the substitution of Δm_m^2 from (2.57), one can obtain the mixing angle in matter as

$$\sin 2\phi_m = \frac{\sin 2\phi}{\sqrt{\left(\frac{\Delta V}{\Delta m^2} - \cos 2\phi\right)^2 + \sin^2 2\phi}} . \tag{2.62}$$

Then the probability of oscillation in matter in terms of mass eigenvalues and mixing angle is given by

$$P_m(\nu_\alpha \to \nu_\beta) = \sin^2 2\phi_m \sin^2\left(\frac{1.27\Delta m_m^2 L}{E}\right) , \tag{2.63}$$

and the survival probability is

$$P_m(\nu_\alpha \to \nu_\alpha) = 1 - P_m(\nu_\alpha \to \nu_\beta). \tag{2.64}$$

The oscillation length in matter is given by

$$\begin{aligned} L_m &= 2.48\left(\frac{E}{\Delta m_m^2}\right) \\ &= \frac{L_0}{\sqrt{\left(\frac{\Delta V}{\Delta m^2} - \cos 2\phi\right)^2 + \sin^2 2\phi}} = \left(\frac{\sin 2\phi_m}{\sin 2\phi}\right) L_0 . \end{aligned} \tag{2.65}$$

From the above set of equations, one can infer the following points due to the matter effect on neutrino oscillations:

- If $\Delta V \to 0$ then $m_{1m,2m}^2 \to m_{1,2}^2$, $\Delta m_m^2 \to \Delta m^2$ and $\sin^2 2\phi_m = \sin^2 2\phi$ i.e, the vacuum oscillation parameters remain unaffected.

- If $\sin^2 2\phi = 0 \Rightarrow \sin^2 2\phi_m = 0$, i.e., there is no oscillations in matter, which implies that neutrino oscillations in matter is possible only if there exists vacuum mixing.

- If $\Delta V \to \infty$, then $\sin^2 2\phi_m \to 0$, i.e., neutrino oscillations are not possible in matter with very high density.

- If $\cos 2\phi = (\Delta V/\Delta m^2)$, then $\sin^2 2\phi_m = 1$, which means the mixing angle in matter (ϕ_m) is maximal. Therefore, regardless of the mixing angle value in vacuum (ϕ), the probability of oscillation is enhanced significantly, which is known as MSW Resonance.

- The values of $\sin^2 2\phi_m,\ \Delta m_m^2$ depend on the sign of Δm^2. If $\Delta m^2 \to -\Delta m^2$, then mixing angle in matter (ϕ_m) and Δm_m^2 would change, which is not possible in the case of vacuum oscillations. Therefore, matter effect plays important role in the evolution of neutrino mass squared differences.

2.4 Current status of neutrino mixing parameters

Here we discuss the present status of neutrino mixing parameters in the three flavor neutrino framework. The mixing parameters that represent the neutrino mass matrix are: three mixing angles ($\theta_{12}, \theta_{23}, \theta_{13}$) and three CP-violating phases (one Dirac, and two Majorana) and three neutrino masses. These mixing parameters are classified into three sectors.

- **Solar neutrino sector ($\theta_{12}, \Delta m_{21}^2$):**

  Solar neutrino experiments are paramount to determine the solar mass squared difference (Δm_{21}^2) and solar mixing angle ($\sin^2 2\theta_{12}$) of V_{PMNS} matrix, representing the neutrino oscillation in solar neutrino sector. Solar neutrinos are coming from the Sun and propagate through matter with varying density. Solar mixing parameters ($\sin^2 2\theta_{12}, \Delta m_{21}^2$) are determined by the solar

neutrino experiments (SNO, Canada, Super-Kamiokande, Japan) and the reactor experiment KAMLAND. Solar neutrino experiments measure the electron neutrino (ν_e) survival probability, whereas the reactor experiments measure the electron anti-neutrino ($\bar{\nu}_e$) survival probability.
The survival probability in two flavor case is given by

$$P(\nu_e \to \nu_e) = 1 - \sin^2 2\theta_\odot \sin^2\left(1.27\Delta m^2_\odot \frac{L}{E}\right). \tag{2.66}$$

- **Atmospheric neutrino sector** ($\theta_{23}, \Delta m^2_{31} = \Delta m^2_{32}$):

  The atmospheric mixing angle (θ_{23}) and the atmospheric mass squared difference (Δm^2_{32}) are constrained by the atmospheric neutrino experiments like K2K, MINOS which measure the muon-neutrino (ν_μ) survival probability,

$$P(\nu_\mu \to \nu_\mu) = 1 - \sin^2 2\theta_{atm} \sin^2\left(1.27\Delta m^2_{atm} \frac{L}{E}\right). \tag{2.67}$$

- **Reactor neutrino sector** (θ_{13}):

  The reactor neutrino experiments measure the $\bar{\nu}_e$ survival probability to determine the reactor mixing angle (θ_{13}) and the survival probability is given by

$$P(\bar{\nu}_e \to \bar{\nu}_e) = 1 - \sin^2 2\theta_{13} \sin^2\left(1.27\Delta m^2_{atm} \frac{L}{E}\right). \tag{2.68}$$

Initially the reactor neutrino experiments did not detect any type of oscillation i.e, $\theta_{13} = 0$. But the recent data from T2K [26], MINOS [27], Double Chooz [28], Daya Bay [29] and RENO [30] experiments measured the non vanishing reactor mixing angle i.e, $\theta_{13} \neq 0$.

The present data from various neutrino oscillation experiments and their global analysis done by several groups [31–33], and it has been inferred that three neutrino framework is sufficient to explain the observed neutrino oscillation phenomenology. The current global fit values of neutrino oscillation parameters from [31] are given in the Table 2.1.

Neutrino oscillation experiments measured the oscillation parameters with great

Mixing Parameters	Best Fit value	3σ Range
$\sin^2\theta_{12}$	0.323	$0.278 \rightarrow 0.375$
$\sin^2\theta_{23}$ (NH)	0.567	$0.392 \rightarrow 0.643$
$\sin^2\theta_{23}$ (IH)	0.573	$0.403 \rightarrow 0.640$
$\sin^2\theta_{13}$ (NH)	0.0234	$0.0177 \rightarrow 0.0294$
$\sin^2\theta_{13}$ (IH)	0.0240	$0.0183 \rightarrow 0.0297$
$\Delta m^2_{21}/10^{-5}$ eV^2	7.6	$7.11 \rightarrow 8.18$
$\|\Delta m_{31}\|^2/10^{-3}$ eV^2 (NH)	2.48	$2.30 \rightarrow 2.65$
$\|\Delta m_{31}\|^2/10^{-3}$ eV^2 (IH)	2.38	$2.20 \rightarrow 2.54$

TABLE 2.1: The current best-fit values with their 3σ ranges of neutrino oscillation parameters.

accuracy except the sign of $|\Delta m^2_{31}|$, which is very essential to understand the neutrino mass hierarchy, i.e., for normal hierarchy ($\Delta m^2_{31} > 0$, $m_1^2 < m_2^2 < m_3^2$), and for inverted hierarchy ($\Delta m^2_{31} < 0$, $m_3^2 < m_2^2 < m_1^2$), the Octant of atmospheric mixing angle θ_{23}, i.e., lower octant ($\theta_{23} < \pi/4$) or higher octant ($\theta_{23} > \pi/4$), CP violating phase δ_{CP}, absolute mass of neutrino and the nature of neutrinos (Dirac or Majorana). The determination of these unknown oscillation parameters which play important role in mass model building are the challenging goals for the current and upcoming neutrino oscillation experiments.

Chapter 3

Neutrino Mass - Mechanisms and Models

3.1 Neutrino mass models

Neutrino mass has become the major inspiration for both theoretical and experimental investigation in neutrino sector. Neutrinos are massless particles in the SM, which is constructed based on the gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$, as there are no right-handed neutrinos present in the SM. If the SM is enriched with three right-handed neutrinos, then the standard Higgs mechanism can generate the mass terms for neutrinos similar to the mass generation of other fermions. However, this extension can not give the explanation for the smallness of neutrino mass in comparision with other SM particles. Alternatively, this small mass can be incorporated in different realizations of seesaw, i.e., Type-I [34, 35], Type-II [36, 37], and Type-III [38] etc.

3.2 See-saw mechanism

See-saw mechanism is remarkably successful and elegant way to describe the smallness of neutrino mass. There are three different realizations at tree level for the seesaw mechanism. The existence of three different mechanisms of seesaw can be explained in the basis that two doublets can be decomposed into a singlet and a triplet ($2 \otimes 2 = 1 \oplus 3$). Then the SM Higgs and left handed leptons can be coupled

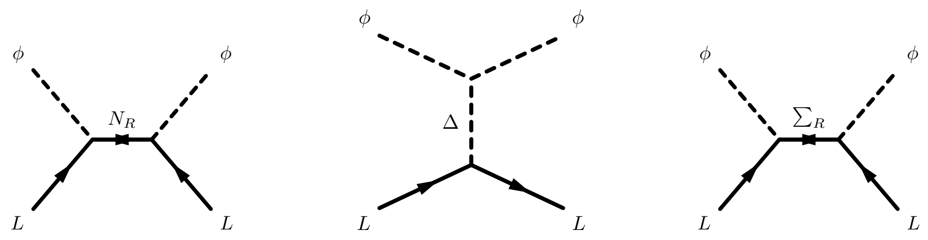

FIGURE 3.1: Different realizations of seesaw at tree level.

to a singlet and a triplet. The three different realizations of seesaw mechanism are pictorially illustrated in figure 3.1. In the figure ϕ is the Higgs filed and L is the left-handed lepton doublet with SM neutrinos and N_R, Δ, Σ are the corresponding heavy mediators and the coupling constant similar to Yukawa couplings in the SM is generally denoted by Y.

3.2.1 Type-I See-saw mechanism

In Type-I seesaw [34, 35], a right-handed neutrino $N_{\ell R}$ is introduced per generation of fermions in the SM. The right-handed neutrinos are assumed to be Majorana particles, electrically neutral and they are singlets under $SU(2)_L$, like the other right-handed fermions. In principle, right-handed neutrinos are allowed by the quark-lepton symmetries of the SM. Then from Gell-Mann Nishijima formula, the electric charge $Q = I_3 + Y/2$, one can say that right-handed neutrinos have zero hypercharge ($Y = 0$). The leptonic sector in the SM in the presence of right-handed neutrinos is

$$\begin{pmatrix} \nu_L \\ e_L \end{pmatrix}, \quad \nu_R,\ e_R. \tag{3.1}$$

The gauge invariant Yukawa interaction with right-handed neutrinos is given by

$$\mathcal{L}_Y = -\sum_{\ell,\ell'} Y^{\nu}_{\ell\ell'} \overline{L}_{\ell L} \tilde{H} N_{\ell' R} + \text{h.c.}, \tag{3.2}$$

where $Y^{\nu}_{\ell\ell'}$ are the new coupling constants and $L_{\ell L}$ are the lepton doublets. This Yukawa interaction introduces Dirac type mass through Higgs mechanism for neutrinos. Since $N_{\ell R}$'s are singlets under $SU(2)_L \times U(1)_Y$, then the bare mass term is

$$-\mathcal{L}_{bare} = \frac{1}{2} \sum_{\ell,\ell'=1}^{3} M_{\ell\ell'} \overline{\hat{N}}_{\ell L} N_{\ell' R} + \text{h.c.}, \tag{3.3}$$

where $\hat{N}_{\ell L}$ is the conjugate of $N_{\ell R}$. By using the following identity

$$\overline{\nu}_{\ell L} N_{\ell' R} = \overline{\hat{N}}_{\ell' L} \hat{\nu}_{\ell R}, \tag{3.4}$$

one can write the mass terms of neutrinos in the Majorana basis as

$$\mathcal{L}_{mass} = -\frac{1}{2}(\overline{\nu}_L \;\; \overline{\hat{N}}_L) \begin{pmatrix} 0 & M_D \\ M_D^T & B \end{pmatrix} \begin{pmatrix} \hat{\nu}_R \\ N_R \end{pmatrix} + \text{h.c.}. \tag{3.5}$$

For N generations, B and M_D are $N \times N$ matrices and ν_L, N_R are N-element column matrices. Upon diagonalizing the mass matrix M_ν, which decouples the light and heavy neutrino mass terms, one can obtain $2N$ Majorana fermions per generation, and the corresponding mass matrix can be expressed as

$$M_\nu = \begin{pmatrix} 0 & M_D \\ M_D^T & B \end{pmatrix}. \tag{3.6}$$

for single generation case ($N = 1$). The mass matrix M_ν is 2×2 and can be written as

$$M_\nu = \begin{pmatrix} 0 & M_D \\ M_D & B \end{pmatrix}. \tag{3.7}$$

Thus, for one generation ($N = 1$), M_D and B are simply numbers, i.e., M_D, and B are real and if we assume $B > 0$, then the diagonalization of the mass matrix M_ν is given by the orthogonal transformation

$$M_\nu^{'} = O M_\nu O^T. \tag{3.8}$$

The orthogonal matrix O can be expressed as

$$O = \begin{pmatrix} \cos\varphi & \sin\varphi \\ -\sin\varphi & \cos\varphi \end{pmatrix}, \tag{3.9}$$

with

$$\tan 2\varphi = \frac{2M_D}{B}. \tag{3.10}$$

Then the diagonal matrix $M_\nu^{'}$ become

$$M_\nu^{'} = O M_\nu O^T = \begin{pmatrix} -m_1 & 0 \\ 0 & m_2 \end{pmatrix}. \tag{3.11}$$

One can get the eigenvalues of the matrix as

$$m_{1,2} = \frac{1}{2}\left[\sqrt{B^2 + 4M_D^2} \mp B\right]. \tag{3.12}$$

In the seesaw mechanism, M_D is taken in the order of electroweak scale and right-handed neutrino fields are massive. Hence under the limit $B \gg M_D$, known as seesaw limit, the eigenvalues become

$$m_1 = m_{light} = \frac{-M_D^2}{B}, \tag{3.13}$$

$$m_2 = m_{heavy} = B. \tag{3.14}$$

The negative sign in the light neutrino mass (m_1) can be removed by using the transformation

$$K^T M_\nu^{'} K = \text{diag}\left(\frac{M_D^2}{B}, B\right), \tag{3.15}$$

where $K = \text{diag}(i, 1)$.
Then for single generation, the two eigenstates are expressed as

$$\begin{aligned} n_1 &= i\cos\varphi(\nu_L - \hat{\nu}_R) - i\sin\varphi(\hat{N}_L - N_R), \\ n_2 &= \sin\varphi(\nu_L + \hat{\nu}_R) + \cos\varphi(\hat{N}_L - N_R). \end{aligned} \tag{3.16}$$

Since $n_1 = \hat{n}_1$ and $n_2 = -\hat{n}_2$, n_1, n_2 are Majorana particles.
Since $B \gg M_D$, one can notice from Eqn. (3.13), that $m_1 \ll M_D$, i.e, the light neutrino mass (m_1) is lighter than the mass of charged fermions and the other neutrino mass (m_2) is nearly equal to B, which is larger than the masses of charged fermions. Thus, making of one particle light at the expense of making other particle heavy is known as see-saw mechanism. Therefore, in Type-I seesaw, the smallness of light neutrino mass is ensured by the presence of heavy right-handed neutrino.

For three generations ($N = 3$) seesaw, the mass matrix M_ν Eqn. (3.6) will be 6×6 and each element is a 3×3 matrix. The matrix M_ν can be block diagonalized by using the transformation

$$\Theta^T M_\nu \Theta = M_\nu^{'}, \tag{3.17}$$

where unitary matrix Θ (upto second order in ρ) is given by

$$\Theta = \begin{pmatrix} 1 - \frac{\rho\rho^\dagger}{2} & \rho \\ -\rho^\dagger & 1 - \frac{\rho^\dagger\rho}{2} \end{pmatrix}, \tag{3.18}$$

with $\rho = \frac{M_D}{B} \ll 1$. The block diagonalization of M_ν gives the mass of light neutrinos as

$$M_{light} = -M_D^T B^{-1} M_D, \tag{3.19}$$

and the heavy mass matrix

$$M_{heavy} = B. \tag{3.20}$$

The diagonalization of M_{light} gives the light neutrino mass and the mass eigenstates. Similarly the diagonalization of B gives the heavy neutrino mass and the mass states. But in this case ($N = 3$), the diagonalization can be done in two ways starting with flavor basis and end up in the mass basis.

Therefore, flavor and mass eigenstates of neutrinos are related by

$$\nu_f \approx U_\nu \nu_k + \rho U_N N_k \ , \tag{3.21}$$

$$N_f \approx -\rho^\dagger U_\nu + U_N N_k \ , \tag{3.22}$$

where U_f (U_N)diagonalize the light (heavy) neutrino mass matrix. ν_f, ν_k are the column matrices contains flavor and mass eigenstates of light neutrinos

$$\nu_f = \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix},$$

$$\nu_k = \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}.$$

Similarly N_f, N_k are the column matrices contains flavor and mass eigen states of heavy neutrinos.

3.2.2 Type-II see-saw mechanism

In Type-II seesaw [36, 37], the SM Lagrangian is expanded solely by a $SU(2)_L$ scalar triplet $\mathbf{\Delta}$ with hypercharge $Y = -2$. The mass of neutrinos may be obtained through the VEV of neutral Higgs in a isospin triplet depiction. The smallness of triplet VEV of the order less than a GeV assures the smallness of neutrino mass.

The relevant Yukawa interaction term beyond the SM is given by

$$-\mathcal{L}_Y = \sum_{\ell',\ell''} y^l_{\ell'\ell''} \overline{\hat{L}}_{\ell' L} \frac{1}{\sqrt{2}} \boldsymbol{\tau} \cdot \mathbf{\Delta} L_{\ell'' L} + \text{h.c.}, \tag{3.23}$$

where $\boldsymbol{\tau}$ represents the Pauli matrices and $\boldsymbol{\tau} \cdot \mathbf{\Delta}$ is the 2-dimensional representation of $\mathbf{\Delta}$ given by

$$\boldsymbol{\tau} \cdot \mathbf{\Delta} = \begin{pmatrix} \frac{1}{\sqrt{2}}\Delta^+ & \Delta^{++} \\ \Delta^0 & -\frac{1}{\sqrt{2}}\Delta^+ \end{pmatrix}. \tag{3.24}$$

As $\langle \Delta^0 \rangle = \frac{1}{\sqrt{2}} \upsilon_\Delta$, then the Yukawa interaction term (Eqn.3.23) generates the Majorana mass

$$\mathcal{L}_{mass} = \sum_{\ell'\ell''} \overline{\hat{\nu}}_{\ell' L} m^\nu_{\ell'\ell''} \nu_{\ell'' L} + \text{h.c.}, \tag{3.25}$$

where

$$m^\nu_{\ell'\ell''} = y^l_{\ell'\ell''} \frac{\upsilon_\Delta}{\sqrt{2}}. \tag{3.26}$$

With the normal SM Higgs doublet ϕ, the Higgs triplet scalar potential of this model is given by [39]

$$\begin{aligned} V(\phi,\Delta) = & - \mu^2 \phi^\dagger \phi + m^2_\Delta \text{Tr}\left((\boldsymbol{\tau} \cdot \mathbf{\Delta})^\dagger \boldsymbol{\tau} \cdot \mathbf{\Delta} \right) + \left(\mu_\Delta \tilde{\phi}^\dagger (\boldsymbol{\tau} \cdot \mathbf{\Delta})^\dagger \phi + \text{h.c.} \right) \\ & - \lambda \left(\phi^\dagger \phi \right)^2 + \lambda^\Delta_1 \left[\text{Tr}\left((\boldsymbol{\tau} \cdot \mathbf{\Delta})^\dagger \boldsymbol{\tau} \cdot \mathbf{\Delta} \right) \right]^2 + \lambda^\Delta_2 \left[\text{Tr}\left((\boldsymbol{\tau} \cdot \mathbf{\Delta})^\dagger \boldsymbol{\tau} \cdot \mathbf{\Delta} \right) \right]^2 \\ & + \lambda^{\phi\Delta}_1 \phi^\dagger \phi \text{Tr}\left((\boldsymbol{\tau} \cdot \mathbf{\Delta})^\dagger \boldsymbol{\tau} \cdot \mathbf{\Delta} \right) + \lambda^{\phi\Delta}_2 \phi^\dagger (\boldsymbol{\tau} \cdot \mathbf{\Delta})(\boldsymbol{\tau} \cdot \mathbf{\Delta})^\dagger \phi \,. \end{aligned} \tag{3.27}$$

The triplet VEV (υ_Δ) can be obtained from the minimization of the potential $V(\phi,\Delta)$, is given by

$$\upsilon_\Delta = \frac{\mu_\Delta \upsilon^2}{\sqrt{2} m^2_\Delta} \tag{3.28}$$

where υ denotes the SM Higgs VEV.

The Higgs triplet VEV υ_Δ varies inversely with the square of its mass m_Δ and the above relation leads to the smallness of v_Δ. However in Type-II seesaw Higgs triplets would be heavy to be observed experimentally.

3.2.3 Type-III seesaw

In Type-III seesaw [38], fermion triplet with hypercharge (Y) zero is added to SM particle content. To explain Type-III seesaw, minimum two fermion triplets are required. With the addition of fermion triplet, the Lagrangian is given by

$$\mathcal{L} = -\frac{1}{2}\mathrm{Tr}\left[\overline{\Sigma} M_\Sigma \Sigma^c\right] - \tilde{\phi}^\dagger \overline{\Sigma}\sqrt{2} Y_\Sigma L + \mathrm{h.c.}, \tag{3.29}$$

where

$$\Sigma = \begin{pmatrix} \frac{\Sigma^0}{\sqrt{2}} & \Sigma^+ \\ \Sigma^- & \frac{-\Sigma^0}{\sqrt{2}} \end{pmatrix}. \tag{3.30}$$

Below the electroweak scale, the mass term is given by

$$\mathcal{L}_{mass} = -v\left(\frac{\overline{\Sigma^0}}{\sqrt{2}} Y_\Sigma \nu_L + \overline{\Sigma^-} Y_\Sigma l_L + \mathrm{h.c.}\right). \tag{3.31}$$

Hence, neutrino mass term can be written as

$$\mathcal{L}^\nu_{mass} = -\frac{1}{2}\begin{pmatrix} \overline{\nu}_L & \overline{\Sigma}^{0c} \end{pmatrix}\begin{pmatrix} 0 & \sqrt{2}Y_\Sigma v \\ \sqrt{2}Y_\Sigma^T v & M_\Sigma \end{pmatrix}\begin{pmatrix} \nu_L^c \\ \Sigma^0 \end{pmatrix}. \tag{3.32}$$

For $M_\Sigma \gg v$, the above equation gives light neutrino mass as

$$m_\nu = Y_\Sigma^T M_\Sigma^{-1} Y_\Sigma v^2, \tag{3.33}$$

similar to Type-I seesaw formula. Hence, in Type-III seesaw the smallness of ratio of Dirac type mass to Majorana type mass ensures tiny mass for active neutrinos as in Type-I seesaw framework.

3.3 Neutrino mixing models

All types of seesaw mechanism predicted that neutrinos are Majorana type and hence, the mass matrix is symmetric with $N(N+1)$ parameters. In case of SM symmetry, which governs the interactions between the elementary particles, the parameters in the matrix are independent of each other. But if there exists flavor symmetry between the generations, the number of free parameters are reduced. The neutrino mixing patterns obtained from the experiments support the flavor symmetry because the mixing patterns are close to standard mixing patterns, such as TBM, BM, DC etc., which can be obtained if there exists flavor symmetry such as A_4, S_4, between the generations.

3.3.1 Bimaximal mixing pattern

In Bi-maximal mixing [40–44], the neutrino mixing matrix assumes atmospheric and solar neutrinos are maximally mixed in vacuum. This pattern implies CP conservation, which in turn hints there is no disappearance of atmospheric neutrinos ν_e.

Assuming solar neutrinos (ν_e) maximally oscillates into equal numbers of ν_μ's and ν_τ's in vacuum and the unitarity conditions can be applied to procure the complete mixing pattern in three flavor, which is given as

$$U_{BM} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix}. \tag{3.34}$$

Bimaximal mixing is one of the popular mixing patterns of interest till date. However, the present updated oscillation data from various oscillation experiments indicate that the BM pattern has to be modified to accommodate the current results.

3.3.2 Tri-Bimaximal mixing pattern (TBM)

Tribimaximal mixing matrix [45–53], is one of the special neutrino mixing patterns, which is suggested to account for wide range of experimental data. TBM is considered as specific case of Alterelli-Feruglio scheme [54, 55] of discrete flavor model.

Results from numerous atmospheric neutrino experiments like Super-Kamiokande [56], strongly suggest the 2-fold maximal mixing of $\nu_\mu - \nu_\tau$, i.e $|U_{\mu 3}|^2 = \frac{1}{2}$, which is in aggrement with the results of intermediate baseline experiments like CHOOZ [57, 58], PALOVERDE [59]. They give $|U_{e3}|^2 \lesssim 0.03$ at 95%CL which indicates

$U_{e3} = 0$ and hence consistent with the 2-fold maximal mixing of $\nu_\mu - \nu_\tau$. In addition, solar neutrino experiments like SNO [60], HOMESTAKE etc. obtained $|U_{e2}|^2 \approx \frac{1}{3}$, which is regarded as an adequate illustration of the solar neutrino data.

Thus, in conclusion, the combined data from neutrino oscillations suggest that there is an effective bimaximal mixing between $\nu_\mu - \nu_\tau$ and an effective trimaximal mixing of ν_e with ν_μ and ν_τ at the atmospheric and solar scale respectively suggests the tri-bimaximal mixing matrix with the form

$$U_{TBM} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \end{pmatrix}. \tag{3.35}$$

3.3.3 Democratic mixing pattern

Democratic (DC) mixing pattern [61–63], is a three flavor framework constructed the under an assumption that the neutrinos are nearly degenerate in mass, atmospheric and solar mixing angles are nearly maximal. This ansatz is also accommodated with cosmological requirement for hot dark matter and the recent neutrinoless double beta decay data at the time of its proposal. The form of the mixing matrix is

$$U_{DC} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{6}} & -\sqrt{\frac{2}{3}} \\ -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{3}} \end{pmatrix}. \tag{3.36}$$

DC pattern is one of the well motivated candidate for the texture of charged lepton and quark mass matrices, since it explains why the first two generation particle are lighter than third generation.

After having the idea about various neutrino mass mechanisms and models, we now proceed to discuss the implications of tri-bimaximal neutrino mixing pattern on neutrino phenomenology.

Chapter 4

Charged lepton correction to tribimaximal lepton mixing and its implications to neutrino phenomenology

4.1 Introduction

It is now firmly established from the results of various neutrino oscillation experiments that neutrinos oscillate from one flavour to other as they propagate. Such change in neutrino flavour or neutrino oscillation is possible, only if they tiny but finite nonzero mass. This in turn implies that analogous to the mixing in the down-quark sector, the three flavor eigenstates of neutrinos ($\nu_e,\ \nu_\mu,\ \nu_\tau$) are related to the corresponding mass eigenstates ($\nu_1,\ \nu_2,\ \nu_3$) by the unitary transformation

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} V_{e1} & V_{e2} & V_{e3} \\ V_{\mu 1} & V_{\mu 2} & V_{\mu 3} \\ V_{\tau 1} & V_{\tau 2} & V_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} , \qquad (4.1)$$

where V is the 3×3 unitary PMNS matrix, which contains three mixing angles and three CP violating phases (one Dirac type and two Majorana type). In the standard parametrization [23], $V_{\rm PMNS}$ is expressed in terms of the solar, atmospheric and reactor mixing angles θ_{12}, θ_{23}, θ_{13} and three CP-violating phases $\delta_{\rm CP}, \rho, \sigma$ as

$$\begin{aligned} V_{\rm PMNS} &= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{\rm CP}} \\ -s_{12}c_{23} - c_{12}s_{13}s_{23}e^{i\delta_{\rm CP}} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta_{\rm CP}} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}s_{13}c_{23}e^{i\delta_{\rm CP}} & -c_{12}s_{23} - s_{12}s_{13}c_{23}e^{i\delta_{\rm CP}} & c_{13}c_{23} \end{pmatrix} P_\nu \\ &\equiv U_{\rm PMNS} P_\nu \ , \end{aligned} \tag{4.2}$$

where $c_{ij} \equiv \cos\theta_{ij}$, $s_{ij} \equiv \sin\theta_{ij}$ and $P_\nu \equiv \{e^{i\rho}, e^{i\sigma}, 1\}$ is a diagonal matrix with CP violating Majorana phases ρ and σ. It should be noted that the mixing matrix $U_{\rm PMNS}$ describes the mixing of Dirac type neutrinos, similar to the CKM matrix of quark sector. The neutrino oscillation data from various experiments allow us to determine mixing parameters related to the solar and atmospheric neutrinos, with very high precision. In recent past, the value of the reactor mixing angle θ_{13} has been measured by the Daya Bay [29] and RENO Collaborations [30] with the best fit (1σ) result as

$$\begin{aligned} \sin^2 2\theta_{13} &= 0.089 \pm 0.010({\rm stat}) \pm 0.005({\rm syst}), \quad \text{Daya Bay} \\ \sin^2 2\theta_{13} &= 0.113 \pm 0.013({\rm stat}) \pm 0.019({\rm syst}). \quad \text{RENO} \end{aligned} \tag{4.3}$$

which is equivalent to $\theta_{13} \simeq 8.8^\circ \pm 0.8^\circ$. It is further supported by the recent data from T2K [64] and NOνA [65] experiments. The global analysis of the recent results of various neutrino oscillation experiments has been performed by several groups [31–33], and the parameters which are used in this analysis are taken from Ref. [31], are presented in Table-2.1.

The observation of the reactor mixing angle θ_{13} has ignited a lot of interest to understand the mixing pattern in the lepton sector [66–78]. Furthermore, it also opens up promising perspectives for the observation of CP violation in the lepton sector. The precise determination of θ_{13} in addition to providing a complete picture of neutrino mixing pattern, could be a signal of underlying physics responsible for lepton mixing and for the physics beyond standard model. It has been shown that if one includes some perturbative corrections to the leading order neutrino mixing patterns, such as bi-maximal (BM) [40–44], tri-bimaximal (TBM) [45–53] and democratic (DC) [61–63], it is possible to explain the observed neutrino mixing angles [79–82]. However, it should be noted that among these leading order mixing patterns i.e., BM, TBM and DC, the tri-bimaximal pattern, whose explicit form as given below

$$U_{\rm TBM} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0 \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & -\sqrt{\frac{1}{2}} \end{pmatrix}, \tag{4.4}$$

is particularly very interesting. It corresponds to the three mixing angles of the standard parametrization as $\theta_{12} = \arctan(1/\sqrt{2}) \simeq 35.3^\circ$, $\theta_{13} = 0^\circ$ and $\theta_{23} = 45^\circ$. Clearly, to accommodate the large value of θ_{13}, one has to consider possible perturbations to the TBM mixing matrix. In this chapter [83], we would like to study the possible corrections arising from the charged lepton sector. The essential features of our analysis are as follows. We assume the charged lepton mixing matrix to be of the same form as the CKM quark mixing matrix and the neutrino mixing matrix to be of the tri-bimaximal form. Furthermore, we use the Wolfenstein-like parametrization for the charged lepton mixing matrix and study its implications on various phenomenological observables. It should be noted that there have been several attempts made recently to understand the nonzero θ_{13} due to charged lepton correction [72, 84–90] and in the past also corrections to the leptonic mixing matrix due to charged leptons were considered in Ref. [91–95].

The chapter is organized as follows. The methodology of our analysis is presented in Section-4.2 and the Results and Conclusion are discussed in Section 4.3 .

4.2 Methodology

It is well known that the leptonic mixing matrix arises from the overlapping of the matrices that diagonalize charged lepton and neutrino mass matrices

$$U_{\rm PMNS} = U_l^\dagger U_\nu \; . \tag{4.5}$$

Here we are focussing only the component of the mixing matrix which describes the mixing of Dirac type neutrinos. For the study of leptonic mixing it is generally assumed that the charged lepton mixing matrix as an identity matrix and the neutrino mixing matrix U_ν has a specific form dictated by a symmetry which fixes the values of the three mixing angles in U_ν. The small deviations of the mixing angles from those measured in the PMNS matrix, are considered, in general, as perturbative corrections arising from symmetry breaking effects. A variety symmetry forms of U_ν have been explored in the literature e.g., BM/TBM/DC and so on. In this work we will consider the situation wherein the neutrino mixing matrix is described by the TBM matrix, i.e.,

$$U_\nu = U_{\rm TBM} \ , \tag{4.6}$$

and that the mixing angles induced by the charged leptons can be considered as corrections. Furthermore, we will neglect possible corrections to $U_{\rm TBM}$ from higher dimensional operators and from renormalization group effects. In this approximation we will derive formulae which allow us to include corrections to neutrino mixing angles and to constrain the CP violating phase (δ_{CP}) conveniently.

In our study, we use a simple *ansatz* for the charged lepton mixing matrix U_l, i.e., we assume that U_l has the same structure as the CKM matrix connecting the weak eigenstates of the down type quarks to the corresponding mass eigenstates. This approximation is quite reasonable as we know that the CKM matrix is almost diagonal with the off diagonal elements strongly suppressed by the small expansion parameter $\lambda = \sin\theta_C$ (θ_C, being the Cabibbo angle). Hence, such an assumption can naturally provide the small perturbations to the tri-bimaximal mixing pattern for neutrino mixing matrix. Furthermore, as discussed in Ref. [96], this approximation is quite acceptable as the mass spectrum of charged leptons exhibits similar hierarchical structure as the down type quarks, i.e., $(m_e, m_\mu) \approx (\lambda^5, \lambda^2) m_\tau$ and $(m_d, m_s) \approx (\lambda^4, \lambda^2) m_b$. This may imply that the charged lepton mixing matrix has a structure similar to the down type quark mixing and is governed by the CKM matrix.

To illustrate the things more explicitly, let us recall the values of the quark mixing angles in the standard PDG parametrization for the CKM matrix within 1σ range as [97]

$$\theta_{13}^q = 0.20^\circ \pm 0.01^\circ, \quad \theta_{23}^q = 2.35^\circ \pm 0.07^\circ, \quad \theta_{12}^q \equiv \theta_C = 13.02^\circ \pm 0.04^\circ. \tag{4.7}$$

However, the leptonic sector is described by two large mixing angles θ_{23}^l and θ_{12}^l and the third mixing angle θ_{13}^l, was expected to be very small. Recently, the third mixing angle θ_{13}^l has been measured by T2K, Double CHOOZ, Daya Bay and

RENO Collaborations yielding the following mixing patterns in the lepton sector:

$$\theta^l_{13} = 8.8^\circ \pm 1.0^\circ, \qquad \theta^l_{23} = 40.4^\circ \pm 1.0^\circ, \qquad \theta^l_{12} = 34.0^\circ \pm 1.1^\circ \ . \tag{4.8}$$

The different nature of the quark and lepton mixing angles can be inter-related in terms of the quark lepton complementarity (QLC) relations [98–104], as

$$\theta^q_{12} + \theta^l_{12} \simeq 45^\circ, \qquad \theta^q_{23} + \theta^l_{23} \simeq 45^\circ. \tag{4.9}$$

The QLC relations indicate that it could be possible to have a quark-lepton symmetry based on some flavor symmetry. The experimental result of this not-so-small reactor mixing angle θ^l_{13} has triggered a lot of interest in the theoretical community. Given the rather precise measurement of θ^l_{13}, one may wonder whether θ^l_{13} numerically agrees well with the QLC relation, i.e.,

$$\theta^l_{13} = \frac{\theta_C}{\sqrt{2}} \approx 9.2^\circ. \tag{4.10}$$

In particular, it is quite interesting to see whether this specific connection to θ_C can be a consequence of some underlying symmetry, which may provide a clue to the nature of quark-lepton physics beyond the standard model.

Starting from the fact that the mixing matrix of the up type quark sector can be almost diagonal and so the CKM matrix is mainly generated from the down type quark mixing matrix, we assume that the mixing matrix of the charged lepton sector is basically of the same form as that of down type quark sector. Consequently the lepton mixing matrix appears as the product of CKM like matrix (induced by charged lepton sector) and the TBM pattern matrix induced from the neutrino sector. As discussed before, in the limit of diagonal charged lepton mass matrix i.e., $U_l = \mathbf{1}$, and $U_\nu = U_{\rm TBM}$, which gives the mixing angles at the leading order as

$$\theta^{l0}_{12} = \arctan(1/\sqrt{2}) \simeq 35.3^\circ, \quad \theta^{l0}_{13} = 0^\circ, \quad \text{and} \qquad \theta^{l0}_{23} = 45^\circ \ , \tag{4.11}$$

deviate significantly from their measured values as

$$|\theta^l_{12} - \theta^{l0}_{12}| \simeq 2^\circ, \quad \theta^l_{13} - \theta^{l0}_{13} \approx 9^\circ \quad \text{and} \quad |\theta^l_{23} - \theta^{l0}_{23}| \simeq 5^\circ \ . \tag{4.12}$$

These deviations are attributed to the corrections arising from the charged lepton sector. We assume the charged lepton mixing matrix to have the form as the CKM matrix in the standard parametrization, i.e.,

$$U_l = R_{23} U_{13} R_{12} \; , \tag{4.13}$$

where the matrices R_{23}, U_{13} and R_{12} are defined by

$$R_{12} = \begin{pmatrix} \cos\theta^l_{12} & \sin\theta^l_{12} & 0 \\ -\sin\theta^l_{12} & \cos\theta^l_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \qquad R_{23} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta^l_{23} & \sin\theta^l_{23} \\ 0 & -\sin\theta^l_{23} & \cos\theta^l_{23} \end{pmatrix}$$

$$U_{13} = \begin{pmatrix} \cos\theta^l_{13} & 0 & \sin\theta^l_{13}\, e^{-i\delta} \\ 0 & 1 & 0 \\ -\sin\theta^l_{13}\, e^{i\delta} & 0 & \cos\theta^l_{13} \end{pmatrix}. \tag{4.14}$$

Furthermore, as the mixing angle θ_{13} receives maximum deviation from the TBM pattern, we assume $\sin\theta^l_{13} = \sin\theta_C = \lambda$, where, λ is a small expansion parameter analogous to the expansion parameter of Wolfenstein parametrization of the CKM matrix. The other two angles are assumed to be of the form

$$\sin\theta^l_{23} = A\lambda^2, \qquad \sin\theta^l_{12} = A\lambda^3 \; , \tag{4.15}$$

where the parameter $A = \mathcal{O}(1)$. With these values, one can obtain the Wolfenstein-like parametrization for U_l (upto order λ^3) as

$$U_l = \begin{pmatrix} 1-\lambda^2/2 & A\lambda^3 & \lambda\, e^{-i\delta} \\ -A\lambda^3(1+e^{i\delta}) & 1 & A\lambda^2 \\ -\lambda e^{i\delta} & -A\lambda^2 & 1-\lambda^2/2 \end{pmatrix} \tag{4.16}$$

Thus, with the help of Eqs. (4.5), (4.6) and (4.16), one can schematically obtain the PMNS matrix up to order of λ^3 as

$$U_{\rm PMNS} = U_{\rm TBM} + \Delta U \; , \tag{4.17}$$

with

$$\Delta U = \begin{pmatrix} \frac{\lambda e^{-i\delta}-\lambda^2+A\lambda^3(1+e^{-i\delta})}{\sqrt{6}} & -\frac{\lambda e^{-i\delta}+\lambda^2/2+A\lambda^3(1+e^{-i\delta})}{\sqrt{3}} & -\frac{\lambda e^{-i\delta}-A\lambda^3(1+e^{-i\delta})}{\sqrt{2}} \\ \frac{A\lambda^2(1+2\lambda)}{\sqrt{6}} & -\frac{A\lambda^2(1-\lambda)}{\sqrt{3}} & -\frac{A\lambda^2}{\sqrt{2}} \\ \frac{2\lambda e^{i\delta}-A\lambda^2+\lambda^2/2}{\sqrt{6}} & \frac{\lambda e^{i\delta}+A\lambda^2-\lambda^2/2}{\sqrt{3}} & -\frac{A\lambda^2+\lambda^2/2}{\sqrt{2}} \end{pmatrix} + \mathcal{O}(\lambda^4), \tag{4.18}$$

which allows one to obtain the elements of the PMNS matrix as

$$
\begin{aligned}
|U_{e1}| &= \sqrt{\frac{2}{3}}\left[1+\frac{1}{2}\lambda\cos\delta-\frac{1}{8}\lambda^2(3+\cos^2\delta)+\frac{1}{16}\lambda^3\left(8A(1+\cos\delta)-\cos\delta\sin^2\delta\right)\right],\\
|U_{e2}| &= \frac{1}{\sqrt{3}}\left[1-\lambda\cos\delta-\frac{1}{2}\lambda^2\cos^2\delta-\frac{1}{2}\lambda^3(2A(1+\cos\delta)-\cos\delta\sin^2\delta)\right],\\
|U_{e3}| &= \frac{\lambda}{\sqrt{2}}\left[1-A\lambda^2(1+\cos\delta)\right],\\
U_{\mu1} &= -\frac{1}{\sqrt{6}}\left[1-A\lambda^2-2A\lambda^3\right],\\
U_{\mu2} &= \frac{1}{\sqrt{3}}\left[1-A\lambda^2+A\lambda^3\right],\\
|U_{\mu3}| &= \frac{1}{\sqrt{2}}\left(1+A\lambda^2\right),\\
U_{\tau1} &= -\frac{1}{\sqrt{6}}\left(1-2\lambda e^{i\delta}+\frac{1}{2}\lambda^2(2A-1)\right),\\
U_{\tau2} &= \frac{1}{\sqrt{3}}\left(1+\lambda e^{i\delta}+\frac{1}{2}\lambda^2(2A-1)\right),\\
|U_{\tau3}| &= \frac{1}{\sqrt{2}}\left(1-\frac{1}{2}\lambda^2-A\lambda^2)\right).
\end{aligned}
\tag{4.19}
$$

From Eq. (4.2), one can express the neutrino mixing parameters in terms of the PMNS mixing matrix elements as

$$
\begin{aligned}
\sin^2\theta_{12} &= \frac{|U_{e2}|^2}{1-|U_{e3}|^2}, \qquad \sin^2\theta_{23}=\frac{|U_{\mu3}|^2}{1-|U_{e3}|^2},\\
\sin\theta_{13} &= |U_{e3}|.
\end{aligned}
\tag{4.20}
$$

Thus, from Eqs. (4.19) and (4.20), one can obtain the solar neutrino mixing angle θ_{12}, up to order λ^3, as

$$\sin^2\theta_{12}\simeq\frac{1}{3}\left(1-2\lambda\cos\delta+\frac{\lambda^2}{2}-\lambda^3[2A(1+\cos\delta)+\cos^3\delta]\right), \tag{4.21}$$

Clearly, when $\cos\delta$ approaches zero we observe a tiny deviation from $\sin^2\theta_{12}=1/3$. Following similar approach, one can obtain the atmospheric neutrino mixing angle θ_{23} as

$$\sin^2\theta_{23}\simeq\frac{1}{2}\left(1+\frac{\lambda^2}{2}(1+4A)\right), \tag{4.22}$$

which also shows a small deviation from the maximal mixing pattern i.e., $\sin^2\theta_{23} = 1/2$. The reactor mixing angle θ_{13} can be obtained as

$$\sin\theta_{13} \;=\; \frac{\lambda}{\sqrt{2}}\left(1 - A\lambda^2(1+\cos\delta)\right) \;. \tag{4.23}$$

Thus, we have a non-vanishing large θ_{13}. This in turn implies that it could be possible to observe CP violation in the lepton sector analogous to the quark sector, which could be detected through long base-line neutrino oscillation experiments. The Jarlskog invariant, which is a measure of CP violation, for the lepton sector has the expression

$$J^{\ell}_{\rm CP} \equiv {\rm Im}[U_{e1}U_{\mu 2}U^*_{\mu 1}U^*_{e2}] = -\frac{\lambda\sin\delta}{6}\left(1 - \frac{\lambda^2}{2} - A\lambda^2\right) + \mathcal{O}(\lambda^4)\;, \tag{4.24}$$

which is sensitive to the Dirac CP violating phase.

The Dirac CP phase δ_{CP} can be deduced by using the PMNS matrix elements and the neutrino mixing parameters as [96]

$$\delta_{\rm CP} = -\arg\left(\frac{\frac{U^*_{e1}U_{e3}U_{\tau 1}U^*_{\tau 3}}{c_{12}c^2_{13}c_{23}s_{13}} + c_{12}c_{23}s_{13}}{s_{12}s_{23}}\right)\;. \tag{4.25}$$

With Eqs. (4.4), (4.17) and (4.18), this yields the correlation between the two CP violating phases (Dirac type CP violating phase and the phase δ introduced in the charged lepton mixing)

$$\delta_{\rm CP} = -\arctan\left[\frac{-\lambda(1-(A+\frac{1}{2}\lambda^2))\sin\delta}{\lambda\Big[(A(1-\lambda^2) - \frac{5}{2}\lambda^2)\cos\delta - (\frac{3}{2}\lambda + A\lambda^2)\Big] + \lambda^2(1+\lambda\cos\delta)}\right]. \tag{4.26}$$

Three mass-dependent neutrino observables are probed in different types of experiments. The sum of absolute neutrino masses $\sum_i m_i$ is probed in cosmology, the kinetic electron neutrino mass in beta decay (M_β) is probed in direct search for neutrino masses, and the effective mass (M_{ee}) is probed in neutrino-less double beta decay experiments with the decay rate for the process $\Gamma \propto |M_{ee}|^2$. In terms of the bare physical parameters m_i and $U_{\alpha i}$, the observables are given by [105]

$$\begin{aligned}
&\sum_i m_i = m_1 + m_2 + m_3, \\
&M_{ee} = \sum_i U_{ei}^2 m_i, \\
&M_\beta = \sqrt{\sum_i |U_{ei}|^2 m_i^2}\ .
\end{aligned} \tag{4.27}$$

The absolute values of neutrino masses are currently unknown. Recently the Planck experiment on Cosmic Microwave Background (CMB) [106] has reported an interesting result for the sum of three neutrino masses with an assumption of three species of degenerate neutrinos as

$$\sum_i m_i \leq 0.23 \text{ eV} . \quad (\text{Planck} + \text{WP} + \text{highL} + \text{BAO}) . \tag{4.28}$$

The most stringent upper bound on electron-antineutrino mass has been measured in the Troitsk experiment [107] on the high precision measurement of the end-point spectrum of tritium beta decay as

$$M_\beta < 2.05 \text{ eV} \quad 95\% \text{ C.L.} \tag{4.29}$$

In our analysis we ignore the Majorana phases (ρ, σ) and consider the normal hierarchy scenario for the neutrino mass spectrum in which the neutrino masses m_2 and m_3 can be expressed in terms of the lightest neutrino mass m_1 and the measured solar and atmospheric mass-squared differences Δm^2_{sol} and Δm^2_{atm} as

$$m_2 = \sqrt{m_1^2 + \Delta m^2_{\text{sol}}}, \qquad m_3 = \sqrt{m_1^2 + \Delta m^2_{\text{sol}} + \Delta m^2_{\text{atm}}}\ . \tag{4.30}$$

4.3 Results and Discussion

For numerical estimation we need to know the values of the three unknown parameters A, λ and δ. In this analysis we assume the small expansion parameter λ to have the same value as that of the quark sector [97]:

$$\lambda = 0.22535 \pm 0.00065\ . \tag{4.31}$$

Now with Eq. (4.22) and using the experimental value of $\sin^2\theta_{23}$ as input param-

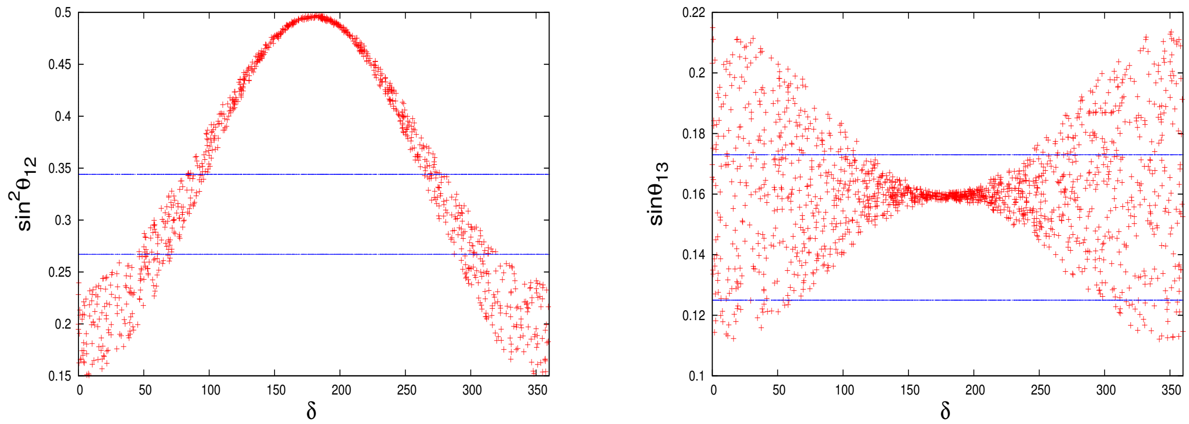

FIGURE 4.1: Variation of $\sin^2\theta_{12}$ with the CP violating phase δ (left panel) and $\sin\theta_{13}$ on the right panel. The horizontal lines (in both panels) represent the 3 σ allowed range.

eter, we obtain the 1σ (3σ) range of A as

$$\begin{aligned} A &= (-2.4 \to -1.2) \quad (1\sigma) \\ &= (-3.4 \to 3.0), \quad (3\sigma) \end{aligned} \tag{4.32}$$

and we treat the CP violating phase δ as a free parameter, i.e., we allow it to vary in its entire range $0 \le \delta \le 2\pi$. Now varying these input parameters in their 3σ ranges, and using Eqs. (4.21) and (4.23), we present the variation of the solar and reactor mixing angles ($\sin^2\theta_{12}$ and $\sin\theta_{13}$) with the CP violating phase δ in figure-4.1. From the figure, it can be seen that in this formalism, it is possible to accommodate simultaneously the observed value of the reactor mixing angle θ_{13} and solar mixing angle θ_{12}. The correlation plots between the solar and atmospheric mixing angles with θ_{13} is shown in figure 4.2. In figure 4.3, we show the variation of the Jarlskog Invariant J_{CP} with δ and θ_{13}. From the figure it can be seen that it could be possible to have large CP violation $\mathcal{O}(10^{-2})$ in the lepton sector. The correlation between the Dirac CP violating phase δ_{CP} and the CP violating parameter δ of the charged lepton mixing matrix is shown in figure 4.4. The variation of M_{ee} with the lightest neutrino mass m_1 (for Normal Hierarchy) and the variation of M_β with $\sum m_i$ (where the parameters are varied in their 1 σ range) are shown in figure 4.5. Thus, for m_1 below $\mathcal{O}(10^{-2})$ eV, one can get $M_{ee} \le 1.2 \times 10^{-2}$ eV and $M_\beta \le 1.4 \times 10^{-2}$ eV.

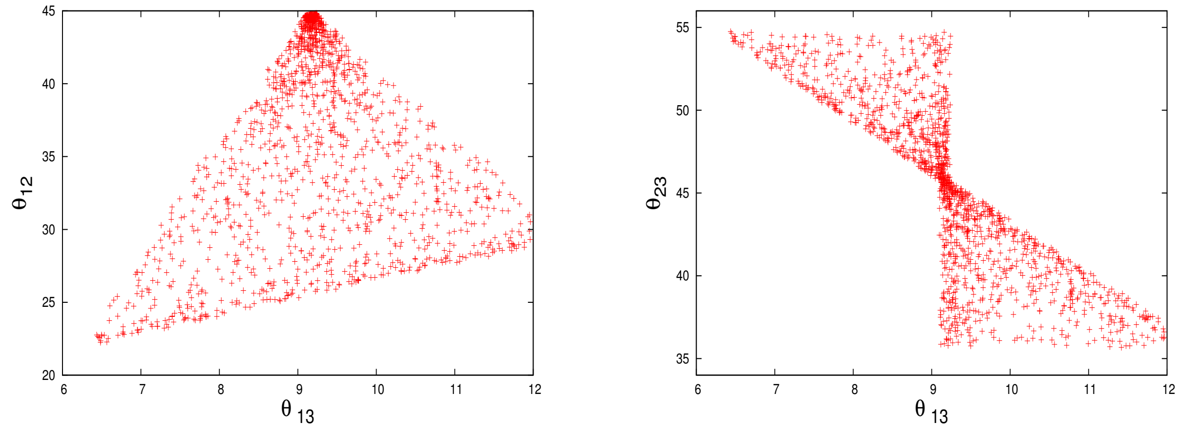

FIGURE 4.2: Correlation plot between solar (left panel) and the atmospheric mixing angle (right panel) with θ_{13}.

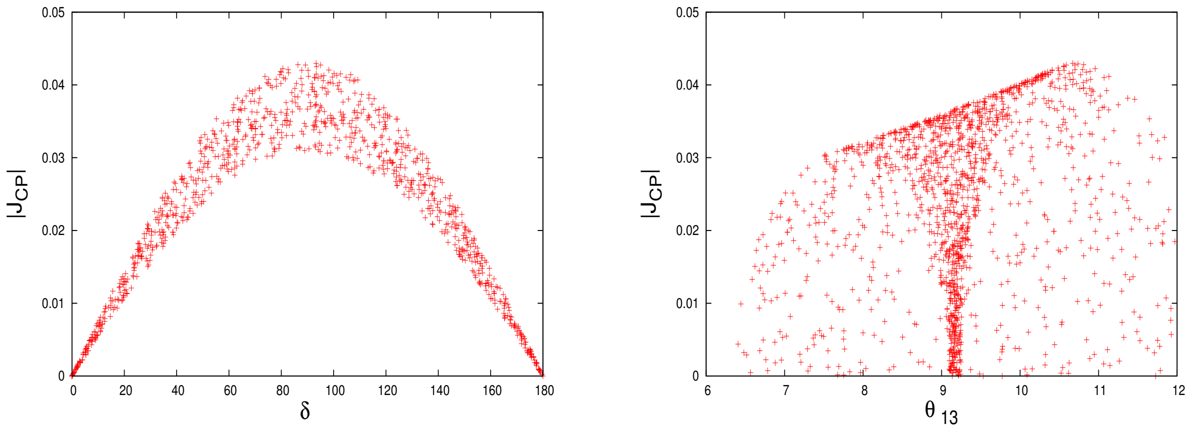

FIGURE 4.3: Variation of J_{CP} with δ (left panel) and with θ_{13} (right panel).

To summarize, to accommodate the observed value of relatively large θ_{13}, we consider the corrections due to the charged lepton mixing matrix to the TBM pattern of neutrino mixing matrix. Based on the possible inter-relation between the charged lepton and the quark mixing structures we constructed the lepton mixing matrix to have the form of the CKM-like matrix (induced from the charged lepton sector) times the TBM matrix induced from the neutrino sector. Our result showed that in this formalism, it is possible to accommodate the observed reactor mixing angle θ_{13} along with the other mixing parameters within their experimental range. We have also found that sizable leptonic CP violation characterized by the Jarlskog invariant J_{CP}, i.e., $|J_{CP}| \leq 10^{-2}$ could be possible in this scenario. The observation of CP violation in the upcoming long base-line neutrino experiments would be a smoking gun signal of this formalism. We have also shown that the

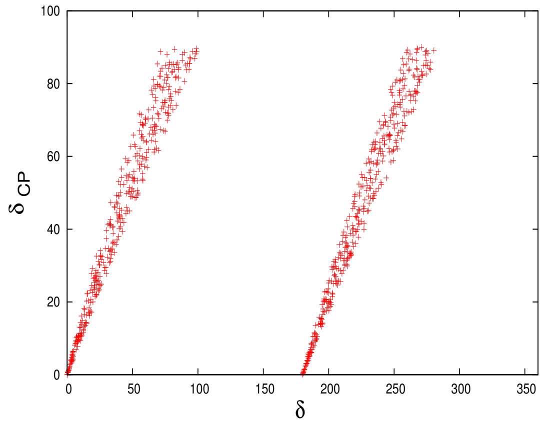

FIGURE 4.4: The correlation plot between the Dirac CP violating phase δ_{CP} and δ.

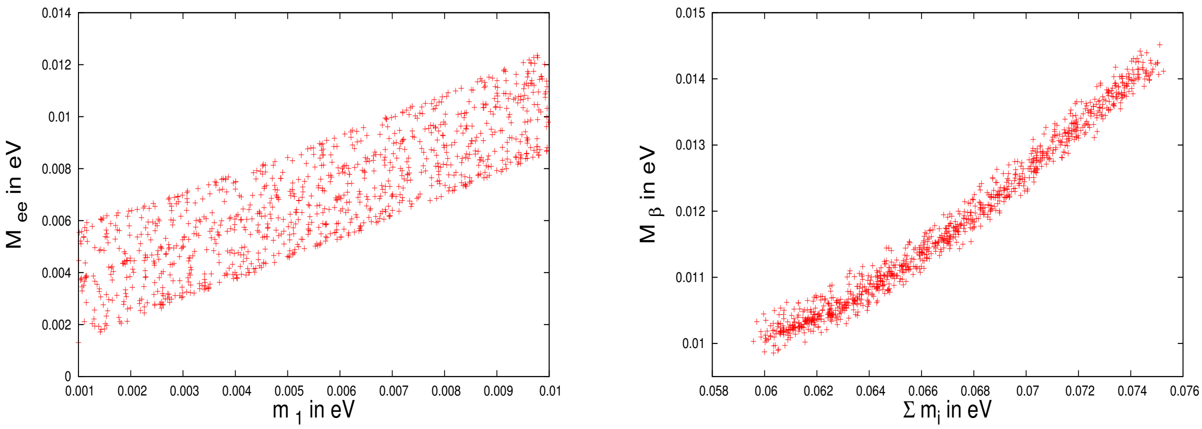

FIGURE 4.5: Variation of M_{ee} with the lightest neutrino mass m_1 (left panel) and the variation of M_β with $\sum m_i$ (right panel).

measured value of θ_{13} along with other mixing parameters can be used for constraining the value of the Dirac CP violating phase $\delta_{\rm CP}$. The upper limits on M_{ee} and M_β are found to be $\mathcal{O}(10^{-2})$, if the mass of the lightest neutrino $m_1 \leq 0.01$ eV.

Chapter 5

Perturbation to TBM mixing and its phenomenological implications

5.1 Introduction

From the observation of neutrino oscillation, it is now well-known that neutrinos have non-zero masses. This in turn, indicates at least one of the mass eigenstates is non-degenerate and the neutrino flavour eigenstates are admixture of mass states, ν_1, ν_2, and ν_3, i.e.,

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}, \tag{5.1}$$

where U is the well-known lepton mixing matrix, which can be parametrized in terms of three mixing angles and one CP violating phase (δ_{CP}), if neutrinos are Dirac particles. There will also be two more additional phases known as Majorana phases, if neutrinos are of Majorana type. In the standard parametrization [23], PMNS matrix is represented as

$$V_{PMNS} = U_{PMNS}.P_\nu = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{13}s_{23}e^{i\delta} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}s_{13}c_{23}e^{i\delta} & -c_{12}s_{23} - s_{12}s_{13}c_{23}e^{i\delta} & c_{13}c_{23} \end{pmatrix} P_\nu \,, \tag{5.2}$$

where $c_{ij} = \cos\theta_{ij}$ and $s_{ij} = \sin\theta_{ij}$, θ_{12}, θ_{23} and θ_{13} are the three mixing angles, δ_{CP} is the Dirac phase and the other two Majorana phases come in P_ν as

$$P_\nu = \mathrm{diag}(e^{i\rho}, e^{i\sigma}, 1)\ .$$

The best-fit values and 3σ ranges of neutrino oscillation parameters taken from reference [31] are given in Table 2.1. One can reconstruct neutrino mass matrix once all the mixing parameters and three masses are known. Neutrino oscillation experiments probe mass squared differences ($\Delta m^2_{ij} = m^2_i - m^2_j$) and all the mixing parameters except Majorana phases. Results from neutrino oscillation experiments show two masses are close to each other, while the third mass is comparatively far away than the other two (Δm^2_{21} is of the order of 10^{-5} eV^2 and $|\Delta m^2_{32}|$ is of the order of 10^{-3} eV^2) and this results two possible mass hierarchies, either $m_1 < m_2 \ll m_3$ (normal hierarchy) or $m_3 \ll m_1 < m_2$ (inverted hierarchy). There are ongoing neutrino oscillation experiments like NOνA and T2K etc., which are expected to resolve mass hierarchy. Beta decay experiments and cosmological bound on sum of neutrino masses ($\Sigma_i m_i$) give the absolute scale of neutrino masses while neutrino less double beta decay ($0\nu\beta\beta$) experiments can test Majorana nature of neutrinos. The tritium beta decay experiment KATRIN [108] shows absolute scale of neutrino mass is less than 0.35 eV and cosmological bound on $\Sigma_i m_i$ from PLANCK data is 0.23 eV [109].

Initially neutrino oscillation experiments indicated the atmospheric mixing angle, θ_{23} is maximal i.e., $\theta_{23} = \pi/4$ and reactor mixing angle θ_{13} is vanishingly small and motivated by such anticipation many models for neutrino mixing were proposed such as Bimaximal mixing (BM) [40–44], Tri-bimaximal mixing (TBM) [45–53], Golden ratio type-A (GRA), type-B (GRB) [89, 110] and Hexagonal mixing (HG), etc. All such models are based on some discrete symmetries such as A_4, S_4 [111, 112] etc. and can be represented as

$$\begin{pmatrix} \cos\theta_{12} & \sin\theta_{12} & 0 \\ \frac{-\sin\theta_{12}}{\sqrt{2}} & \frac{\cos\theta_{12}}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \\ \frac{-\sin\theta_{12}}{\sqrt{2}} & \frac{\cos\theta_{12}}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix},$$

where $\sin\theta_{12}$ takes the values $\frac{1}{\sqrt{2}}$, $\frac{1}{\sqrt{3}}$, $\frac{1}{\sqrt{2+r}}$ ($r = \frac{1+\sqrt{5}}{2}$ is the golden ratio), $\frac{\sqrt{3-r}}{2}$, and $\frac{1}{2}$ for BM, TBM, GRA, GRB and HG respectively. Recently Daya Bay [29, 113] RENO [30] and T2K [26, 114] experiments measured non-zero reactor mixing angle and hence, the above mentioned symmetry forms can't explain the experimental results. But various studies show that these models can be modified suitably to accommodate the observed mixing angles by adding perturbations [66–82, 115–121]. Among above mentioned symmetry forms TBM is of great interest because of its prediction to solar mixing angle, $\sin^2\theta_{12} = \frac{1}{3}$ against the experimental best fit value 0.323 and it can be explained on the basis of A_4 [111] symmetry, the smallest non abelian discrete symmetry with three dimensional irreducible representation. The perturbations can be incorporated in various ways and one possible form for example is the $Z_2 \times Z_2$ symmetry in neutrino sector and Z_3 symmetry in charged lepton sector. In this chapter [122], we study a possible form of perturbation which modifies TBM to make it compatible with the recent experimental results. We also study the variation of electron neutrino mass (m_{ν_e}) and the 11 element of the Majorana neutrino mass matrix ($|M^{\nu}_{ee}|$), observables of β decay and $0\nu\beta\beta$ decay experiments respectively with the lightest neutrino mass in order to verify the model.

The chapter is organized as follows. In section 5.2, we will discuss briefly about the lepton mixing matrix and in section 5.3, we present the perturbation in the neutrino sector and its effect on the observables like mixing angles, δ_{CP}, m_{ν_e} and $|M^{\nu}_{ee}|$. We conclude our discussion in section 5.4.

5.2 The Lepton mixing matrix

The lepton mixing matrix commonly known as PMNS matrix arises from the overlapping of the matrices that diagonalize charged lepton and neutrino mass matrices, hence PMNS matrix is given by

$$U_{PMNS} = U_l^{\dagger} U_{\nu} \ , \tag{5.3}$$

where U_l and U_{ν} are the matrices which diagonalize charged lepton and neutrino mass matrices respectively. But it is always possible to work in a basis where charged lepton mass matrix is diagonal so that $U_l = I$ and $U_{PMNS} = U_{\nu}$. Hence, one can write

$$U_{PMNS} = U_{\nu} \ , \tag{5.4}$$

without loss of generality. So here we consider $U_l = I$ and U_ν as TBM in the leading order, and hence the PMNS matrix is given as

$$U_{PMNS} = U_{TBM} = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{2}} \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix} , \tag{5.5}$$

Since TBM predicts $\theta_{13} = 0$, it can't accommodate the recent observation of largish θ_{13} by the reactor experiments. So it has to be modified suitably for being in agreement with the experimental results. It is reasonable to assume that such modifications can come from perturbative corrections due to higher dimensional operators. We will discuss a possible form of perturbation in next section and show that it can consistently accommodate all the measured mixing angles.

5.3 Perturbation in neutrino sector

In this section we consider the deviations from TBM mixing angles due to perturbation in the neutrino sector so that the obtained mixing angles satisfy experimental results. The perturbation is taken as a rotation in 23 plane followed by a rotation in 13 plane. Existence of Dirac CP phase is ensured by the complex phase in the 13 rotation matrix. Such a perturbation is quite reasonable, as it will give correction to the atmospheric mixing angle θ_{23} which deviates from its maximal value and large correction to the reactor mixing angle θ_{13}. With this perturbation the lepton mixing matrix will be of the form

$$U = U_{TBM} \cdot X \ , \tag{5.6}$$

where U_{TBM} is the TBM mixing matrix given in Eqn. (5.5) and X is the perturbation matrix given as

$$X = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c'_{23} & s'_{23} \\ 0 & -s'_{23} & c'_{23} \end{pmatrix} \begin{pmatrix} c'_{13} & 0 & s'_{13}e^{-i\phi} \\ 0 & 1 & 0 \\ -s'_{13}e^{i\phi} & 0 & c'_{13} \end{pmatrix} = \begin{pmatrix} c'_{13} & 0 & s'_{13}e^{-i\phi} \\ -s'_{23}s'_{13}e^{i\phi} & c'_{23} & s'_{23}c'_{13} \\ -c'_{23}s'_{13}e^{i\phi} & -s'_{23} & c'_{23}c'_{13} \end{pmatrix} . \tag{5.7}$$

In general the leading order mixing matrix can receive corrections from both charged lepton and neutrino sector. For example, in Ref. [123], TBM mixing is realized based on A_4 symmetry which breaks to one of its subgroup Z_3 in the charged lepton sector while neutrino sector preserves $Z_2 \times Z_2$ symmetry. They have shown that charged lepton and neutrino sector form a parallel world of flavour symmetry breaking and both the charged lepton and neutrino sectors receive corrections due to interaction between the sectors after symmetry breaking. But one can always go to charged lepton mass diagonal basis so that only neutrino sector contributes to lepton mixing. We obtained mixing angles and Jarlskog invariant in terms of elements of U by equating it with PMNS matrix as

$$\begin{aligned} \sin^2\theta_{12} &= \frac{|U_{12}|^2}{1-|U_{13}|^2}, \quad \sin^2\theta_{23} = \frac{|U_{23}|^2}{1-|U_{13}|^2}, \quad \sin^2\theta_{13} = |U_{13}|^2, \\ J_{CP} &= Im\left[U_{11}U_{22}U^*_{21}U^*_{12}\right] , \end{aligned} \tag{5.8}$$

where U_{ij} is the ij element of the lepton mixing matrix U. Now comparing Eqns (5.6) and (5.8), we obtain

$$\sin^2\theta_{13} = \frac{1}{3}\left[2s'^2_{13} + 2\sqrt{2}s'_{23}c'_{13}s'_{13}\cos\phi + s'^2_{23}c'^2_{13}\right] , \tag{5.9}$$

$$\sin^2\theta_{12} = \frac{1-s'^2_{23}}{3-\left(2s'^2_{13} + 2\sqrt{2}s'_{23}c'_{13}s'_{13}\cos\phi + s'^2_{23}c'^2_{13}\right)} , \tag{5.10}$$

$$\sin^2\theta_{23} = \frac{1}{2} - \frac{\sqrt{6}c'_{23}c'_{13}(s'_{23}c'_{13} - \frac{1}{\sqrt{2}}s'_{13}\cos\phi)}{3-\left(2s'^2_{13} + 2\sqrt{2}s'_{23}c'_{13}s'_{13}\cos\phi + s'^2_{23}c'^2_{13}\right)} , \tag{5.11}$$

and

$$J_{CP} = \frac{-1}{\sqrt{3}}\left(\frac{c'^2_{23}}{3} - \frac{s'^2_{23}}{2}\right)c'_{23}c'_{13}s'_{13}\sin\phi . \tag{5.12}$$

In standard parameterization the value of J_{CP} is

$$J_{CP} = \frac{1}{8}\sin 2\theta_{12}\sin 2\theta_{23}\sin 2\theta_{13}\cos\theta_{13}\sin\delta_{CP} . \tag{5.13}$$

Comparing equations (5.12) and (5.13) we obtain

$$\sin\delta_{CP} = \frac{3(\frac{c'^2_{23}}{3} - \frac{s'^2_{23}}{2})c'_{23}c'_{13}s'_{13}\sin\phi}{\sqrt{X(\frac{2-X+s'^2_{23}}{3-X})(\frac{1}{2} - \frac{Y}{3-X})(\frac{1}{2} + \frac{Y}{3-X})(3-X)}}, \tag{5.14}$$

where

$$X = \left[2s_{13}'^2 + 2\sqrt{2}s_{23}'c_{13}'s_{13}'\cos\phi + s_{23}'^2c_{13}'^2\right] ,$$
$$Y = \sqrt{6}c_{23}'c_{13}'(s_{23}'c_{13}' - \frac{1}{\sqrt{2}}s_{13}'\cos\phi) . \quad (5.15)$$

Next, we obtain the allowed parameter space by varying these parameters s_{23}', s_{13}' and $\cos\phi$ in their allowed ranges i.e., between -1 to 1 and choosing those set of values for which the mixing angles fall within their 3σ ranges, which are shown in Fig. 5.1. Using the allowed parameter space we show in Fig. 5.2, the correlation

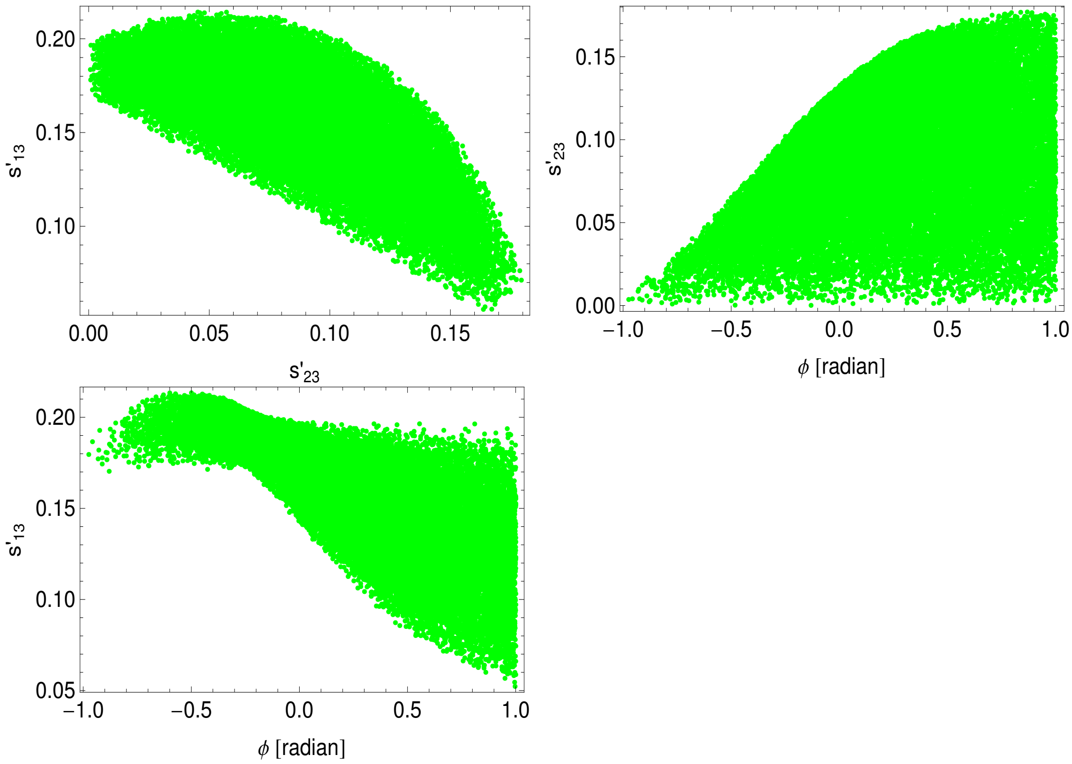

FIGURE 5.1: Allowed parameter space in $s_{13}' - s_{23}'$, $s_{23}' - \phi$ and $s_{13}' - \phi$ planes compatible with the observed data.

plots between the mixing angles, which are found to lie within their 3σ allowed ranges.

Neutrino oscillation experiments do not give any idea about the absolute mass of neutrinos as they only measure mass square differences. We will get the absolute scale of neutrino mass from Tritium beta decay experiments, which measure electron neutrino mass defined by

$$m_{\nu_e} = \Sigma_i|U_{1i}|^2m_i \quad (5.16)$$

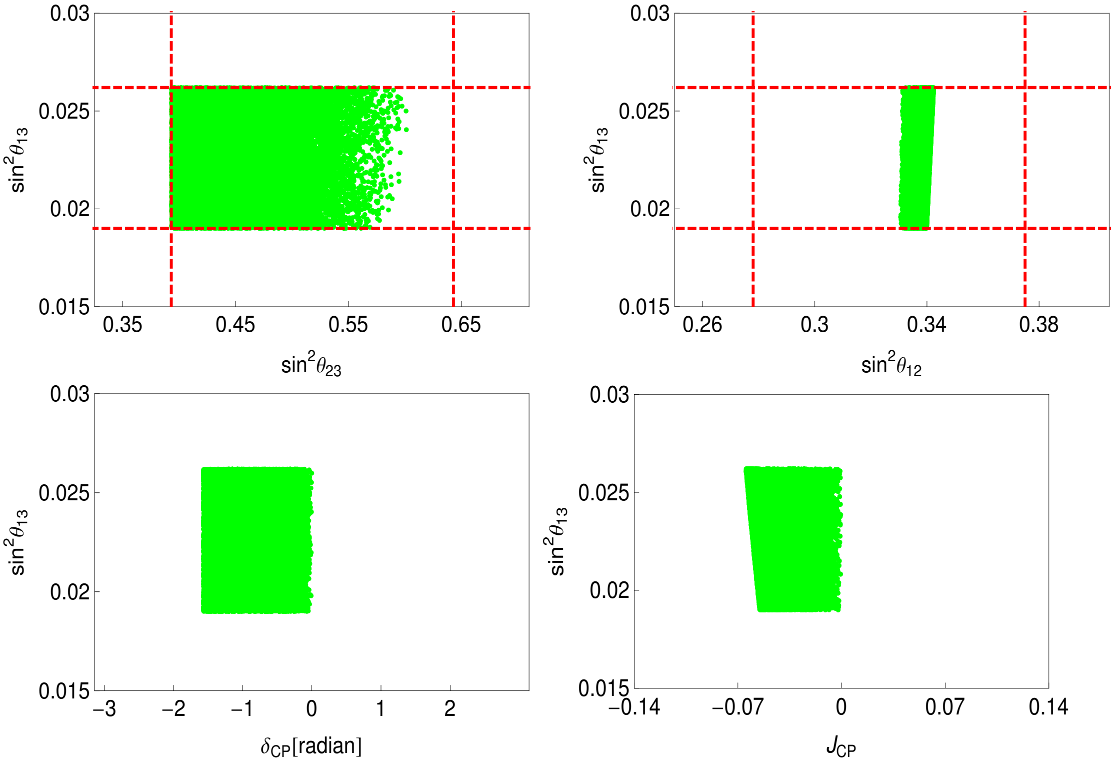

FIGURE 5.2: Correlation plot between $\sin^2\theta_{13}$ and $\sin^2\theta_{23}$ (top left panel), $\sin^2\theta_{13}$ and $\sin^2\theta_{12}$ (top right panel), δ_{CP} and $\sin^2\theta_{13}$ (bottom left panel) and between J_{CP} and $\sin^2\theta_{13}$ (bottom right panel).

where i varies from 1 to 3 and m_1, m_2, and m_3 are light neutrino masses and U_{1i}'s are elements of first row of the lepton mixing matrix U, which are given by

$$\begin{aligned} U_{11} &= \frac{2}{\sqrt{6}}c'_{13} - \frac{1}{\sqrt{3}}s'_{23}s'_{13}e^{i\phi} , \\ U_{12} &= \frac{1}{\sqrt{3}}c'_{23} , \\ U_{13} &= \frac{2}{\sqrt{6}}s'_{13}e^{-i\phi} + \frac{1}{\sqrt{3}}s'_{23}c'_{13} . \end{aligned} \tag{5.17}$$

We now proceed to study the variation of m_{ν_e} with lightest neutrino mass in the case of inverted and normal hierarchy and the results are shown in the right panel of Fig 5.3. In our calculations we used the relations

$$\begin{aligned} m_2 &= \sqrt{\Delta m^2_{21} + m_1^2} , \\ m_3 &= \sqrt{\Delta m^2_{31} + m_1^2} \end{aligned} \tag{5.18}$$

for NH and

$$\begin{aligned} m_1 &= \sqrt{m_3^2 - \Delta m_{31}^2}\ , \\ m_2 &= \sqrt{m_3^2 - \Delta m_{31}^2 + \Delta m_{21}^2} \end{aligned} \tag{5.19}$$

for IH and obtained upper limit on m_1 (m_3) as 0.071 eV (0.065 eV) in the case of normal (inverted) hierarchy taking cosmological upper bound on $\Sigma_i m_i$ as 0.23 eV [109].

Neutrinos are very light compared to other fermions. The smallness of neutrino are addressed by different types of seesaw mechanisms such as, type-I, type-II and inverse seesaw. All those seesaw mechanisms treat neutrino as majorana particle. Hence they predict neutrinoless double beta decay, a process in which two neutrons inside a nucleus convert to two protons without emitting neutrinos.

$$(A, Z) \rightarrow (A, Z + 2) + 2e$$

The observation of neutrinoless double beta ($0\nu\beta\beta$) decay will be a consistency test for all those models. The half life of $0\nu\beta\beta$ decay is proportional to $|M_{ee}^{\nu}|^2$ [124], (1,1) element of neutrino mass matrix in flavor basis. Several on going experiments like KamLAND-Ze [125], EXO [126] and GERDA [127] to observe neutrino-less double beta decay put upper bound on $|M_{ee}^{\nu}|$. The lowest upper bound on $|M_{ee}^{\nu}|$ is 0.22eV came from GERDA Phase-I data so only those mass models are valid which predicts $|M_{ee}^{\nu}| < 0.22eV$. Hence we studied the variation of $|M_{ee}^{\nu}|$ with the lightest neutrino mass which is m_1 in the case of normal hierarchy and m_3 otherwise. We have calculated $|M_{ee}^{\nu}|$ using the relation

$$|M_{ee}^{\nu}| = |m_1 U_{11}^2 + m_2 U_{12}^2 + m_3 U_{13}^2|. \tag{5.20}$$

The variation of $|M_{ee}^{\nu}|$ with lightest neutrino mass in the case of normal and inverted hierarchies is shown in the left panel Fig 5.3.

5.4 Conclusions

In this chapter, we have studied deviation from Tribimaximal mixing (TBM) due to perturbation in neutrino sector in the form of combined rotation in 13 and

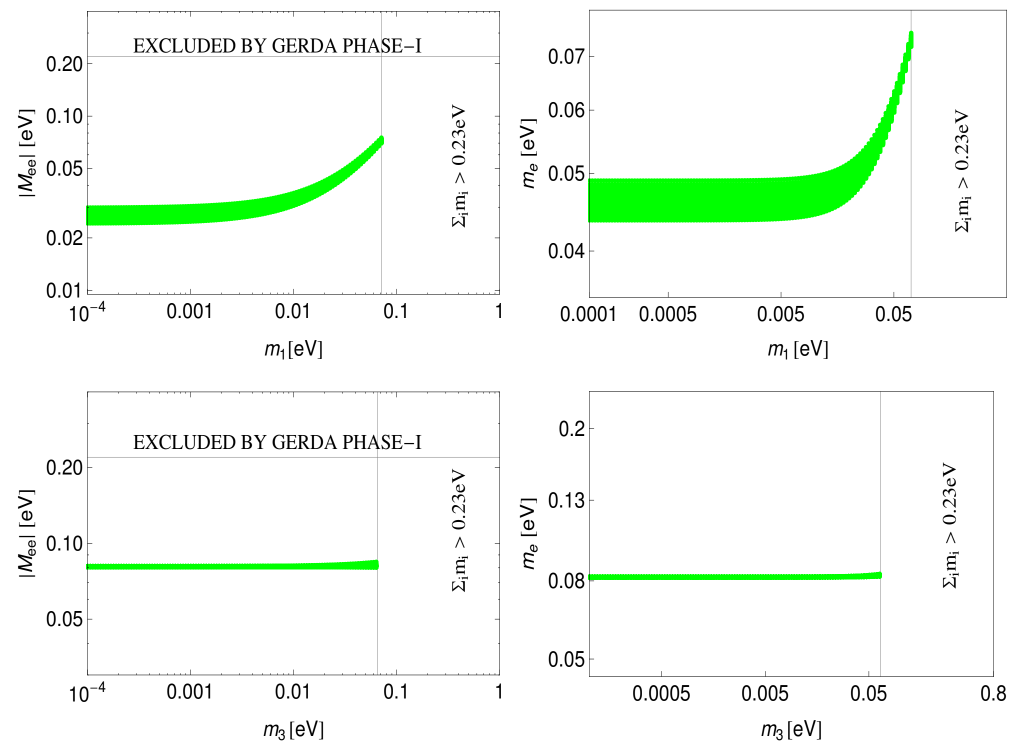

FIGURE 5.3: variation of $|M^{\nu}_{ee}|$ with lightest neutrino mass m_1 (m_3) in the left panel and m_{ν_e} with m_1 (m_3) in the right panel for normal (inverted) hierarchy.

23 plane. It is found that such perturbation can explain the recent experimental results on neutrino mixing angles. We obtained the parameter space for which mixing angles fall within their 3σ ranges and calculated possible values of Dirac CP phase (δ_{CP}) and found that all values between 0 to $-\pi/2$ is possible for δ_{CP}. We studied the variation of $|M^{\nu}_{ee}|$ with the lightest neutrino mass in the case of normal and inverted hierarchy and found that it falls bellow the upper bound (0.22 eV) for all values of the lightest neutrino mass below its upper bound, which we obtained as 0.071 eV for normal hierarchy and 0.065 for inverted hierarchy by taking cosmological upper bound on $\Sigma_i m_i$ as 0.23 eV. We also studied the variation of electron neutrino mass, m_{ν_e} with lightest neutrino mass for normal and inverted mass hierarchies.

Chapter 6

Summary and Conclusion

Neutrino physics is relatively large research area and it comprises a lot of research including the basic properties of neutrinos. Neutrino, one of the elementary particles which interested the physicists with its puzzling nature and strange properties. In the SM, neutrinos are massless and interact through weak forces only which makes quite difficult to detect them in the detector. But the results from various dedicated neutrino oscillation experiments accompanied by theoretical results reveal that neutrinos change their flavor during propagation and such flavor changing is possible only if the neutrinos have finite non-zero mass, which implies physics beyond the SM.

Data from many oscillation experiments provide a very clear idea about the oscillation parameters and the unknown parameters may be fixed by the future experiments. Hence, one can reconstruct the neutrino mass matrix (M_ν), and according to the current oscillation data the neutrino mass matrix is nearly mu-tau ($\mu - \tau$) symmetric. $\mu - \tau$ symmetry draws remarkable attention in the search of explicit form for the neutrino mixing pattern. There are many special mixing patterns i.e, Tri-bimaximal mixing (TBM), Bi-maximal mixing (BM), Democratic mixing (DC), Hexagonal mixing, Golden Ratio-I, Golden Ratio-II etc, which obey the $\mu - \tau$ symmetry. But due to non vanishing value for θ_{13} and deviation of θ_{23} from $\pi/4$, the neutrino mixing matrix is deviated from $\mu - \tau$ symmetry. The mass matrix (M_ν) can be $\mu - \tau$ symmetric at leading order but breaks due to higher order corrections to the matrix. Hence the observed form for the lepton mixing matrix can be due to the flavor symmetry between the generation of fermions at higher energies. This thesis focused on the study of some phenomenological aspects of neutrino mixing.

In Chapter-1 of the thesis, we have discussed the historical journey of neutrino from the discovery and its elusive properties and also discussed the neutrino oscillation, solar neutrino problem, atmospheric neutrino problem in the section-1.1. In section 1.2, we presented the brief overview of SM of particle physics, which includes the particle content,symmetries,types of interactions and Higgs mechanism to generate masses to SM particles. In the last section, we discussed the neutrinos in the SM and physics beyond the SM . At the end of chapter-1, we have given the overview of the thesis.

In Chapter-2 of the thesis, we have discussed the theory of neutrino oscillation. In section 2.1, we discussed the neutrino oscillation in vacuum in both two and three flavor cases and obtained the generalized expression for the oscillation probability, discussed the dependence of oscillation probability on mixing angles (θ_{ij}), mass square difference (Δm^2) and L/E ratio. In three neutrino mixing case, we discussed the PMNS matrix and obtained the probability of oscillation between different flavors of neutrinos. In section 2.2, the effect of matter on neutrino oscillations is discussed in two flavor case, the oscillation probability in matter and the dependence on various parameters such as $\phi_m, \Delta V$ are presented. In the last section, we have given the current status of neutrino mixing parameters.

Chapter-3 presents about the different models of neutrino mass. In section 3.1, we discussed various see-saw mechanisms which include Type-I, II, III see-saw mechanisms, which account for the generation of mass to neutrino and the smallness of neutrino mass in the context of physics beyond SM. In the next section, we discussed the different neutrino mixing patterns such as TBM, BM, DC etc. The current data from various neutrino experiments suggest the modification to the above models to accommodate the current oscillation data. In this thesis we have mainly studied the modification to tri-bimaximal mixing matrix to explain the current oscillation data.

In Chapter-4, we have studied the charged lepton correction to tri-bimaximal mixing and its implications to neutrino phenomenology. Motivated by the data from Daya-Bay and RENO experiments which established the reactor mixing angle θ_{13}

is non-vanishing with relatively large value ($\theta_{13} = 9^0$) at 5σ level. By adding some perturbation to neutrino mixing models such as TBM, BM one can explain the current data. In this chapter, we considered charged lepton correction to TBM pattern and studied the implications of these corrections on phenomenological observables such as mixing angles, CP-violating phase δ_{CP}, Jarlskog invariant J_{CP} and the light neutrino mass m_1. Here we assumed U_l has the form of CKM matrix of quark sector based on QLC relations and we found that this modification $U_{PMNS} = U_l^\dagger U_\nu$ where $U_\nu = U_{TBM}$, it is possible to explain the non-vanishing reactor mixing angle θ_{13} and other mixing parameters are within the experimental limit. We also found that Jarlskog invariant $|J_{CP}| \leq 10^{-2}$ and the upper bounds on effective majorana mass M_{ee} and M_β as $\mathcal{O}(10^{-2})$ if m_1=0.01 eV.

In Chapter-5, we have studied perturbation to tri-bimaximal mixing and its phenomenological implications. The non-zero value for reactor mixing θ_{13} is very important for the measurement of CP-violation in the neutrino sector. To accommodate the recently observed θ_{13}, we considered the tri-bimaximal mixing pattern(TBM) with a perturbation in the neutrino sector. Here the perturbation matrix X is taken as rotation in 23 plane followed by rotation in 13 plane i.e $R_{23}(\theta_{23})R_{13})(\theta_{13}, \phi)$. Here the complex phase ϕ in 13 rotation matrix ensures the existence of Dirac CP phase. This perturbation is reasonable because it modifies the atmospheric mixing angle (θ_{23}) from its maximal value ($\pi/4$) and accommodate the non-zero θ_{13}. The lepton mixing matrix has the form $U_{PMNS} = U_{TBM} \cdot X$ where $X = R_{23}(\theta_{23}R_{13})(\theta_{13}, \phi)$. With this modification it is possible to accommodate the observed mixing angles consistently and we found that the Dirac CP phase δ_{CP} can have values between 0 to $\pi/2$. We have studied the variation of effective Majorana mass $|M^\nu_{ee}|$ with light neutrino mass m_1 in the case of NH, IH and found that 0.071eV for NH and 0.065eV for IH by taking $\sum\limits_i m_i$ =0.23 eV. We also studied the variation of m_{ν_e} with m_1 for both NH and IH.

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List of Publications

1. *Neutrino mixing matrices with relatively large θ_{13} and with texture one-zero*, K.N. Deepthi, **Srinu Gollu**, R. Mohanta, Eur. Phys. J. C **72**, 1888 (2012) [arXiv:1111.2781 [hep-ph]].

2. *Charged lepton correction to tribimaximal lepton mixing and its implications to neutrino phenomenology*, **Srinu Gollu**, K.N. Deepthi, R. Mohanta, Mod. Phys. Lett. A **28**, 1350131 (2013) [arXiv:1303.3393 [hep-ph]].

3. *Perturbation to TBM mixing and its phenomenological implications*, M. Sruthilaya and **Srinu Gollu**, Mod. Phys. Lett. A **31** no.38, 1650207 (2016) [arXiv:1609.09609 [hep-ph]].

Study of some phenomenological aspects of Neutrino Mixing

by Srinu G

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