# ENTRANCE EXAMINATIONS – 2020

## (Ph.D. Admissions - January 2021 Session)

## Ph.D. (PHYSICS)

Marks: 70

Time: 2.00 hrs.

Hall Ticket No.:

I. Please enter your **Hall Ticket Number on Page 1 of this question paper and on the OMR sheet** without fail.

II. Read carefully the following instructions:

1. This Question paper has two parts: **PART - A** and **PART - B**.
2. **PART-A** consists of 20 objective type questions related to Research Methods.
3. **PART-B** consists of 20 objective type questions related to Physics.
4. All questions carry 1.75 marks each.
   **There is no negative marking**
5. Answers are to be marked on the OMR answer sheet following the instructions provided there upon. An example is shown below

   100. (A) (B) (C) ●

6. Only Scientific Calculators are permitted. Mobile phone based calculators are not permitted. Logarithmic tables are not allowed.
7. Hand over the OMR sheet at the end of the examination.

This book contains 18 pages

III. Values of physical constants:
$c = 3 \times 10^8$ m/s; $h = 6.63 \times 10^{-34}$ J.s; $k_B = 1.38 \times 10^{-23}$ J/K
$e = 1.6 \times 10^{-19}$ C; $\mu_\circ = 4\pi \times 10^{-7}$ Henry/m; $\epsilon_\circ = 8.85 \times 10^{-12}$ Farad/m

This page intentionally left blank.
Blank sheets are provided for rough work at the end of this booklet

## Part-A

1. Consider seven individuals, identified as L, M, N, O, P, Q, and R sitting equispaced on the circumference of a circle, facing the centre. If O is sitting between L and R, Q is second to the right of R, P is second to the right of Q, and N is not an immediate neighbor of R, which of the following is **NOT** correct:

   A. R is second to the right of L

   B. M is second to the left of N

   C. P sits to the opposite of N

   D. L sits between O and P

2. The value of $f(t) = \sum_{n=0}^{\infty}\left[\frac{(1+t)^n}{2^n n!}\right]$, at $t = 1$ is

   A. $e$

   B. $\pi$

   C. $\cos(1)$

   D. $\sin(1)$

3. Out of a total of $N$ students in a class, $1/4^{th}$ of the total want to specialize in Condensed Matter Physics (CMP) and $1/3^{rd}$ of the remaining want to study Optical Physics (OP). Half of the remaining want to study Biophysics (BP) and the remaining 20 want to pursue Theoretical Physics (TP). The number distribution of students among the four specializations (CMP,OP,BP,TP) is

   A. 40, 10, 10, 20

   B. 20, 20, 20, 20

   C. 30, 30, 30, 20

   D. 10, 20, 40, 20

4. Area of a regular hexagon is $24\sqrt{3}$. Its perimeter is

   A. $24\sqrt{3}$

   B. 24

   C. $16\sqrt{3}$

   D. 8

5. The operations $\Diamond, \#, \&$ are defined as $a \Diamond b = (a-b)/(a+b)$, $a\#b = (a+b)/(a-b)$, $a\&b = ab$. The values of the expressions (i) $(54\#36)\&(54\Diamond36)$ and (ii) $(108\#72)\&(108\Diamond72)$ respectively are

   A. $-1, -1$

   B. $-1, +1$

   C. $+1, -1$

   D. $+1, +1$

6. Consider a hypothetical situation wherein 10% of the inhabitants of a village have died of Corona virus infection. Panic sets in, during which 20% of the remaining inhabitants leave the village. The population is now reduced to 3600. The total number of inhabitants in the village, at the beginning, is

   A. 4000

   B. 4500

   C. 5000

   D. 5500

7. Puneet was 3 times as old as his son 6 years ago. After 6 years, Puneet will be twice as old as his son. The present ages of Puneet and his son respectively are

   A. 18, 6

   B. 42, 18

   C. 42, 12

   D. 36, 12

8. Area of the geometric shape (in square units) formed by connecting points $(1,1), (-1,5), (7,9)$ and $(9,5)$, with straight lines, in $(x,y)$ plane, is

   A. 40

   B. 80

   C. 12

   D. 20

9. If the slope of the tangent at every point of a curve in $(x,y)$ plane is $-2x/y$, the curve is

   A. a straight line

   B. a parabola

   C. a circle

   D. an ellipse

10. For real $x$, the upper bound of $\cos x + \sin x$ is

    A. $1/\sqrt{2}$

    B. 1

    C. 2

    D. $\sqrt{2}$

11. Given the function $f(x) = Dx^2 + vx$ where $D > 0, v > 0$, the minimum value attained by $f$, as $x$ takes real values, is

    A. $v^2/(2D)$

    B. $v^2/(4D)$

    C. $-v^2/(2D)$

    D. $-v^2/(4D)$

12. A die with faces numbered by integers from 1 to 6 is thrown twice randomly. The probability, that the sum of the answers obtained in the two throws is less than 5, is

    A. 1/3

    B. 2/5

    C. 1/6

    D. 5/36

13. If angle $\theta = 45^o \pm 3^o$, then the value of $\cos\theta$ is

    A. $\frac{1}{\sqrt{2}} \mp 0.037$

    B. $\frac{1}{\sqrt{2}} \mp 0.05$

    C. $\frac{1}{\sqrt{2}} \mp 0.074$

    D. $\frac{1}{\sqrt{2}} \mp 0.37$

14. If the word "CAB" is mapped to the number 14, the word "BACK" should be mapped to number

   A. 31

   B. 135

   C. 86

   D. 17

15. Given $f(x) = x^3/(1 - e^{-x^2})$, $f(x)$ for small $|x|$ is approximately equal to

   A. $x^3$

   B. $x^2$

   C. $\infty$

   D. $x$

16. A box contains red, green and white balls. The probability of pulling out a red ball at random from this box is 1/4 and the probability of pulling out a green ball at random is 3/8. If there are 12 white balls in the box, the total number of balls in the box is

   A. 32

   B. 40

   C. 48

   D. 24

17. A tower in a town does not cast any shadow at exact noon of an equinox day. If, at 3pm, the tower casts a 15m long shadow, considering the equinox is an exact 12 hour day, the actual height of the tower is

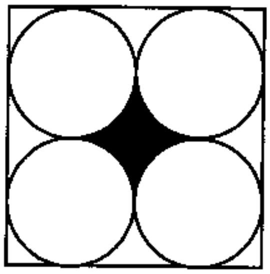
   A. 10 m

   B. 15 m

   C. 25 m

   D. 30 m

18. Four circles, each of diameter 2 units, are inscribed inside a square box as shown in figure. The probability of picking a point randomly from the shaded region of the figure, when one is allowed to pick any point within the square, is



A. $1/4$

B. $\pi/16$

C. $1/4 - \pi/16$

D. $1 - \pi/4$

19. In the congruence relation, if $5 := 1$, $16 := 0$, $7 := 3$, then $13 :=$

A. 0

B. 1

C. 2

D. 3

20. The denominator of a fraction is 1 more than twice the numerator. If the sum of the fraction and its reciprocal is $2\frac{16}{21}$, the fraction is

A. $\frac{3}{7}$

B. $\frac{7}{3}$

C. $\frac{4}{3}$

D. $\frac{4}{9}$

## Part-B

21. If $z = (q + ip)/\sqrt{2}$, $\bar{z} = (q - ip)/\sqrt{2}$ where $q, p$ are canonically conjugate position and momentum, then the Hamilton's equations of motion can be stated as

A. $\frac{dz}{dt} + i\frac{\partial H}{\partial \bar{z}} = 0$

B. $\frac{dz}{dt} - i\frac{\partial H}{\partial \bar{z}} = 0$

C. $\frac{dz}{dt} + i\frac{\partial H}{\partial z} = 0$

D. $\frac{dz}{dt} - i\frac{\partial H}{\partial z} = 0$

22. For the Lagrangian $L = \alpha\sqrt{\dot{q_1}^2 + \dot{q_2}^2}$ where $\alpha$ is a constant and $q_1, q_2$ are generalised coordinates, the corresponding Hamiltonian is

A. $H = \alpha(p_1^2 + p_2^2)$

B. $H = \alpha\sqrt{p_1^2 + p_2^2}$

C. $H = \alpha\sqrt{p_1^2 + p_2^2 + p_1 p_2}$

D. $H = 0$

23. The transformation $(q, p) \to (Q, P)$ from one set of phase space variables to another set of phase space variables given by $Q = \alpha\sqrt{q}e^t \cos p$, $P = \beta\sqrt{2q}e^{-t} \sin p$ is a canonical transformation if and only if

A. $\alpha\beta = 1$

B. $\alpha = \beta = \sqrt{2}$

C. $\alpha\beta = \sqrt{2}$

D. $\alpha\beta = -\sqrt{2}$

24. The electric field associated with an electromagnetic wave propagating in a conducting medium is $\tilde{E}_0 e^{K_I x} e^{i(K_R x - \omega t)}\hat{z}$, where $K_R$ and $K_I$ are real and imaginary parts of the wavenumber respectively. The corresponding magnetic field is

A. $\vec{B} = -\frac{\tilde{E}_0}{\omega}(K_R - iK_I)e^{K_I x}e^{i(K_R x - \omega t)}\hat{y}$

B. $\vec{B} = \omega\tilde{E}_0(K_R + iK_I)e^{K_I x}e^{i(K_R x - \omega t)}\hat{y}$

C. $\vec{B} = \frac{\tilde{E}_0}{\omega}(K_R + iK_I)e^{K_I x}e^{i(K_R x - \omega t)}\hat{y}$

D. $\vec{B} = -\tilde{E}_0\omega(K_R - iK_I)e^{K_I x}e^{i(K_R x - \omega t)}\hat{y}$

25. Consider a coaxial cable made from two coaxial cylindrical shells. The inner cylinder is of radius $r_1$, charge per unit length $-\lambda$ and carries a current $I$ along $-z$ direction. The outer cylinder is of radius $r_2$ ($> r_1$), charge per unit length $\lambda$ and carries a current $I$ along $+z$ direction. The rate at which energy flows through a cross section of the cable is

A. $\frac{\lambda I}{2\pi\epsilon_0}\ln\left(\frac{r_2}{r_1}\right)$

B. $\frac{\lambda I}{4\pi\epsilon_0}\left(\frac{r_2}{r_1}\right)$

C. $\frac{\lambda I}{2\pi\epsilon_0}\ln\left(\frac{r_1}{r_2}\right)$

D. $\frac{\lambda I}{4\pi\epsilon_0}\left(\frac{r_1}{r_2}\right)$

26. The current density in a region is given by $J(x,y) = J_0[\hat{i}\exp(-ax^2)+\hat{j}\exp(-by^2)]$. The rate of change of charge density at a point $(1,1,0)$ in that region due to the current density is given by

A. $2J_0\left[\frac{a}{e^a} + \frac{b}{e^b}\right]$

B. $2J_0\left[\frac{a}{e^a} - \frac{b}{e^b}\right]$

C. $\frac{J_0}{e}(a-b)$

D. $\frac{J_0}{e}(a+b)$

27. Let $N$ be the average number of particles in a gas contained in a volume $V$ with a pressure $P$ at temperature $T$. The relative fluctuation in number density in grand canonical ensemble, in the thermodynamic limit, is nonzero and finitely large, then the isothermal compressibility (defined as $\kappa_T = -(1/v)(\partial v/\partial P)|_{T,V}$, where $v$ is molar volume), is proportional to

A. $N^{-1}$

B. $N^0$

C. $N^1$

D. $N^2$

28. Consider a three level system which has a ground state energy $-\Delta$, and the two excited states with same energy $\Delta$. If this system is in equilibrium with a reservoir at temperature $\Delta/k_B$, the probability of finding the system in ground state is

A. 1/3

B. 1/2

C. $e^2/(2+e^2)$

D. $e^2/(1+e^2)$

29. The Hamiltonian for a particle is $H = \hbar\omega(a^\dagger a + a^\dagger a^\dagger a a)$, where $a, a^\dagger$ satisfy $[a, a^\dagger] = 1$. The energy eigenvalue $E_n$ are (with $n = 0, 1, ..$):

A. $\hbar\omega(n + n^2)$

B. $\hbar\omega(n - n^2)$

C. $\hbar\omega(-n + n^2)$

D. $\hbar\omega n^2$

30. Consider a wavefunction $\psi(x, t = 0) = [u_1(x) + \alpha\ u_2(x)]/\sqrt{2}$, where $\alpha$ is a constant and $u_1$, $u_2$ are nondegenerate orthonormal eigenfunctions of a Hamiltonian. The expectation value $\langle x \rangle$ for this state is $X$ at time $t = 0$, with the property that $\int dx\ u_1^*(x) x u_2(x) \neq 0$. If we desire $\langle x \rangle$ to remain equal to $X$ at any other time $t$, then $\alpha$ should be equal to

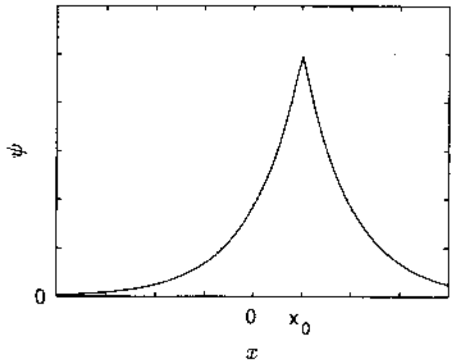
A. 1

B. 0

C. for all choice

D. for no choice it is possible

31. The ground state eigenfunction for a particle in a potential is as shown in the figure which has a derivative discontinuity at $x_0$. The potential must then have:




A. finite discontinuity at $x_0$

B. finite discontinuity anywhere along $x$

C. infinite discontinuity at $x_0$

D. infinite discontinuity anywhere along $x$

32. The matrix representation of a linear operator $T$ on $\mathbb{R}^3$ has the property

$$T\begin{bmatrix}1\\1\\0\end{bmatrix} = \begin{bmatrix}2\\5\\3\end{bmatrix},\ T\begin{bmatrix}1\\0\\1\end{bmatrix} = \begin{bmatrix}5\\2\\3\end{bmatrix} \text{ and } T\begin{bmatrix}0\\1\\1\end{bmatrix} = \begin{bmatrix}3\\-3\\0\end{bmatrix}.$$

Then, $T\begin{bmatrix}1\\1\\1\end{bmatrix}$ is

A. $\begin{bmatrix}5\\2\\3\end{bmatrix}$

B. $\begin{bmatrix}0\\0\\0\end{bmatrix}$

C. $\begin{bmatrix}2\\2\\3\end{bmatrix}$

D. $\begin{bmatrix}3\\-3\\0\end{bmatrix}$

33. The value of the integral

$$I = \oint_C \frac{3z^2+2}{(z-4)(z^2+9)}dz,$$

where $C$ is a circle given by $|z+1| = 4$ and taken in the anticlockwise direction, is

A. $2\pi i$

B. $-2\pi i$

C. $6\pi i$

D. 0

34. Which one of the following is the real part of an analytic function?

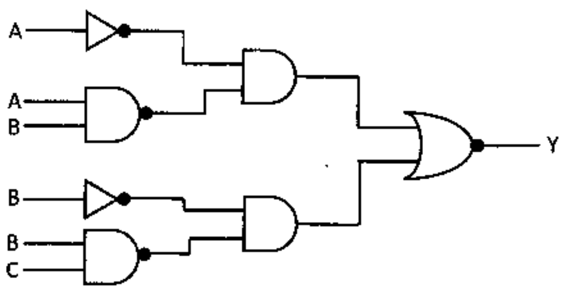
A. $x^2 - y^2 - 5x + y + 2$

B. $x^2 + y^2 - x + y + 2$

C. $-x^2 - y^2 + 5x - y + 2$

D. $x^2 + y^2 - 5x + 5y + 2$

35. The output $(Y)$ of the circuit given below is



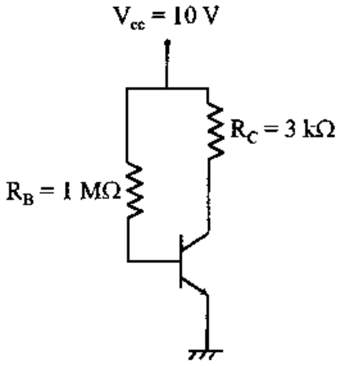

A. $AC$

B. $BC$

C. $AB$

D. $A\overline{C}$

36. In the circuit shown below the base current is




A. $10\mu A$

B. $9.3\mu A$

C. $3.3\mu A$

D. $0.7\mu A$

37. The measured values of quantities $x$, $y$ and $z$ respectively are 1, 2 and 3. If the measured errors are respectively 0.01, 0.02 and 0.01, the fractional error in the function $2x^2y + 5z/y$ is approximately

A. 2%

B. 4%

C. 1%

D. 3%

38. Consider the process $\gamma + p \to p + \pi^0$ with the proton initially at rest. The threshold energy of the photon $E_\gamma$ for this process to occur, in terms of the particle rest masses, is

A. $\left(\frac{m_p^2 - m_\pi^2}{2m_p}\right)c^2$

B. $\left(\frac{m_p^2 + m_\pi^2}{2m_p}\right)c^2$

C. $\left(\frac{m_\pi^2 + 2m_p m_\pi}{2m_p}\right)c^2$

D. $\left(\frac{m_p^2 + 2m_p m_\pi}{2m_\pi}\right)c^2$

39. The $2P \to 1S$ transition in the hydrogen atom corresponds to an angular frequency $\omega = 1.5 \times 10^{16} Hz$. If the lifetime of the $2P$ state for spontaneous emission is $1.6 \times 10^{-9} s$, assuming refractive index $n = 1$, the Einstein $A_{21}$ coefficient is

 A. $6.25 \times 10^{8}/s$
 B. $2.40 \times 10^{7}/s$
 C. $0.94 \times 10^{25}/s$
 D. $1.01 \times 10^{-25}/s$

40. Consider the phonon spectra of a two-dimensional lattice with three atoms per unit cell. The number of acoustic and optical phonon branches respectively are

 A. 2 and 3
 B. 2 and 4
 C. 4 and 2
 D. 3 and 2

# PHD PHYSICS – JANUARY 2021
## ANSWER KEY – Y 64

| PART A | | PART B | |
|---|---|---|---|
| Q # | KEY | Q # | KEY |
| 1. | C | 21. | A |
| 2. | A | 22. | D |
| 3. | B | 23. | C |
| 4. | B | 24. | A |
| 5. | D | 25. | A |
| 6. | C | 26. | A |
| 7. | B | 27. | C |
| 8. | A | 28. | C |
| 9. | D | 29. | D |
| 10. | D | 30. | B |
| 11. | D | 31. | C |
| 12. | C | 32. | A |
| 13. | A | 33. | A |
| 14. | B | 34. | A |
| 15. | D | 35. | C |
| 16. | A | 36. | B |
| 17. | B | 37. | A |
| 18. | C | 38. | C |
| 19. | B | 39. | A |
| 20. | A | 40. | B |