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ENTRANCE EXAMINATIONS – 2019
(Ph.D. Admissions - January 2020 Session)

Ph.D. Physics

Marks: 70

Time: 2.00 hrs. Hall Ticket No.: [ ]

I. Please enter you **Hall Ticket Number on Page 1 of this question paper and on the OMR sheet** without fail.

II. Read carefully the following instructions:

1. This Question paper has two parts: **PART - A** and **PART - B**
2. **PART - A** consists of 20 objective type questions, each carrying **1.75** marks for correct answer and **a negative mark of 0.5** for wrong answer.
3. **PART - B** consists of 35 objective type questions, each carrying 1 mark for the correct answer. **There is no negative marking for questions in PART-B**
4. Answers are to be marked on the OMR answer sheet following the instructions provided there upon. An example is shown below

   100. (A) (B) (C) ●

5. Only Scientific Calculators are permitted. Mobile phone based calculators are not permitted. Logarithmic tables are not allowed.
6. Hand over the OMR sheet at the end of the examination to the invigilator.

This book contains 24 pages

III. Values of physical constants:
$c = 3 \times 10^8$ m/s; $h = 6.63 \times 10^{-34}$ J.s; $k_B = 1.38 \times 10^{-23}$ J/K
$e = 1.6 \times 10^{-19}$ C; $\mu_o = 4\pi \times 10^{-7}$ Henry/m; $\epsilon_o = 8.85 \times 10^{-12}$ Farad/m
$m_e = 9.1 \times 10^{-31}$Kg

PART-A

1. Given an arbitrary complex number Z,

$$\frac{arg(Z)}{arg(2Z^*)} \quad \text{is}$$

   A. $\frac{1}{2}$

   B. $-\frac{1}{2}$

   C. -1

   D. 1

2. The real and imaginary parts of the complex number $Z = \frac{1}{12+i5}$ are

   A. $\frac{12}{169}, -\frac{5}{169}$

   B. $-\frac{12}{169}, \frac{5}{169}$

   C. $\frac{12}{13}, -\frac{5}{13}$

   D. $\frac{1}{12}, -\frac{1}{5}$

3. The function $f(x) = 2x^4e^{2x}$ in the interval $-4 < x < 4$, has

   A. exactly one maxima

   B. no maxima or minima.

   C. many maxima.

   D. exactly one minimum.

4. If $(\sin\theta - \cos\theta) = \sqrt{2}$, then $\sin 2\theta$ is

   A. 1

   B. -1

   C. $\frac{1}{2}$

   D. $-\frac{1}{2}$

5. A real, anti-symmetric matrix M is

   A. always invertible

   B. always non-invertible

   C. $det M = 1$

   D. $Tr M < 0$

6. The probability of finding an electron at the Fermi energy in a metal at non-zero temperature is

   A. dependent on the temperature of the sample

   B. always half $(\frac{1}{2})$

   C. always one (1)

   D. zero (0) at any temperature

7. The transition from the normal to the superconducting state is accompanied by

   A. an increase in entropy and decrease in specific heat at T_c

   B. a decrease in both entropy and specific heat at T_c

   C. an increase in both entropy and specific heat at T_c

   D. a decrease in entropy and an increase in specific heat at T_c

8. If $4 \times 1 = 4$, $4 \times 2 = 3$, $4 \times 3 = 2$ and $4 \times 4 = 1$, then 4×6 is equal to

   A. 0

   B. 1

   C. 6

   D. 4

9. If x and y are two independent random numbers drawn from the normal distribution with the variances $\Delta x^2 =< x^2 > - < x >^2$ and $\Delta y^2 =< y^2 > - < y >^2$. The variances of $x + y$ and $x - y$ are

A. $\Delta x^2 + \Delta y^2$, $\Delta x^2 - \Delta y^2$

B. $\Delta x^2 - \Delta y^2$, $\Delta x^2 + \Delta y^2$

C. $\Delta x^2 - \Delta y^2$, $\Delta x^2 - \Delta y^2$

D. $\Delta x^2 + \Delta y^2$, $\Delta x^2 + \Delta y^2$

10. 35 people are siting in a room, with their names are displayed in English. Then,

A. there are at least four people whose names start with the same letter

B. there are at least two people whose names start with the same letter

C. there are at least three people whose names start with the same letter

D. there are no two people whose names start with the same letter

11. A laser beam with wavelength 500 nm has an emission bandwidth of 10^6 Hz. The longitudinal coherence length of the laser is

A. 300 m

B. 30000 m

C. 500 m

D. 50000 m

12. In the reaction ${}^{238}_{92}U \rightarrow {}^{4}_{2}He + {}^{234}_{90}U + X$, X represents

A. 2γ

B. 3γ

C. γ

D. e^-

13. The number of distinct normal modes of vibration of a linear triatomic molecule is

A. 4

B. 2

C. 3

D. 1

14. A simple pendulum is used to estimate the acceleration due to gravity g by measuring time period of oscillation $T = 2\pi\sqrt{\frac{l}{g}}$ where l is the length of the pendulum. In this experiment, measured length and time period are 1 m with uncertainty of 1×10^{-3} m and 2 s with an uncertainty of 4×10^{-3} s, respectively. The value of g along with the uncertainty is

A. $980 \pm 10\ cm/s^2$

B. $988 \pm 4\ cm/s^2$

C. $162 \pm 2\ cm/s^2$

D. $9.8 \pm 1\ cm/s^2$

15. Two water waves of wavelength 0.99 cm and 1.01 cm are superimposed to form beats whereby a series of envelops are formed. The number of cycles present in a single envelop is

A. 25

B. 50

C. 75

D. 100

16. To solve the equation $\frac{df}{dx} = f(x)$ numerically, the method used is

    A. Newton-Raphson method

    B. Secant method

    C. Euler's method

    D. Bisection method

17. Three numerical methods to solve a polynomial $f(x) = 0$ whose convergence relations are (i) $|\epsilon_{n+1}| = M\epsilon_n^2$, (ii)$|\epsilon_{n+1}| = M\epsilon_n$ and (iii) $|\epsilon_{n+1}| = M\epsilon_n^{1.618}$, respectively. The order in which they converge , from slowest to fastest is

    A. (i), (ii) and (iii)

    B. (ii), (iii) and (i)

    C. (i), (iii) and (ii)

    D. (iii), (ii) and (i)

18. The minimum number of cuts required to slice a cylindrical shaped birthday cake into eight equal pieces is

    A. 4

    B. 5

    C. 7

    D. 3

19. Quark model explains the existence of

    A. a meson with charge +1 and strangeness −1

    B. a baryon of spin 1

    C. a meson with same signs of charm and strangeness

D. an anti-baryon of charge +2

20. A student tosses a coin twenty times. She then plots a histogram of her results. The distribution of the plot will be a

A. beta distribution

B. Gaussian distribution

C. Lorentzian distribution

D. binomial distribution

PART-B

21. Laplace transform of the function

$$f(t) = \sin t, \ 0 \leq t < \pi, \qquad 0, \quad t \geq \pi$$

A. $\frac{1}{s^2+1} + \frac{e^{-\pi s}}{s^2+1}$

B. $\frac{1}{s^2+1} + \frac{se^{-\pi s}}{s^2+1}$

C. $\frac{s}{s^2+1} + \frac{se^{-\pi s}}{s^2+1}$

D. $\frac{1}{s^2+1} + \frac{e^{\pi s}}{s^2+1}$

22. A conserved quantity associated with the Lagrangian $L = \frac{m}{2}(\dot{q}_1^2 + q_3\dot{q}_2^2) + mq_1^2\dot{q}_3^2 + V(q_1, q_3)$, is

A. $m\dot{q}_1$.

B. $2mq_1^2\dot{q}_3$.

C. mq_2.

D. $mq_3\dot{q}_2$.

23. The electric field lines associated with an uncharged solid metal sphere placed in a uniform electric field is represented by the figure

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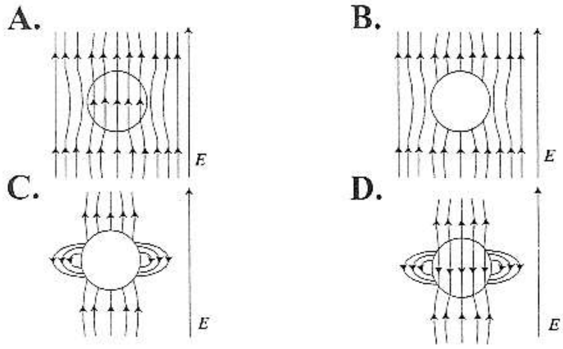


24. The dielectric polarisation of a dielectric material

A. varies exponentially with temperature

B. is independent of the temperature

C. varies linearly with the temperature

D. varies inversely with the temperature

25. Consider a container of volume $3V$, which is fitted with a removable partition that divides it into two chambers of volumes V and $2V$, respectively. Initially, particle A is in the chamber with volume V and particle B is in the other chamber. The change in entropy of systems A (B) is ΔS_A (ΔS_B), when the partition is removed. Then,

A. $\frac{\Delta S_A}{\Delta S_B} > 1$

B. $\frac{\Delta S_A}{\Delta S_B} < 1$

C. $\Delta S_A \Delta S_B < 1$

D. $\Delta S_A \Delta S_B = 0$

26. If a thin layer of water whose surface tension is T wets the two glass plates separated by a distance d over a circular area of radius r, the force required to separate the glass plates is

A. $\frac{2\pi r^2 T}{d}$ N

B. $\frac{2\pi dT}{r^2}\ N$

C. $\frac{\pi r^2 T}{d}\ N$

D. $\frac{\pi d^2 T}{r}\ N$

27. A thin convex lens of focal length $f = 50\ cm$ projects image of an object on a screen kept at a distance of 300 cm from the lens. If the screen is moved towards the lens by 50 cm from its initial position, the obejct should be shifted by

A. 2.5 cm towards the lens

B. 1.5 cm towards the lens

C. 2.5 cm away from the lens

D. 1.5 cm away from lens

28. Consider composite states of three electrons. The number of these composite states, having a total spin zero, is

A. 1

B. 3

C. 0

D. 2

29. The Lagrangian, describing a particle of mass $2m$, moving under the action of a conservative force $F(x) = -2x - \frac{5}{x}$, is

A. $m\dot{x}^2 - x^2 - 5lnx.$

B. $m\dot{x}^2 + 2x + \frac{5}{x}.$

C. $m\dot{x}^2 + x^2 + 5lnx.$

D. $m\dot{x}^2 - 2x^2 + \frac{5}{x^2}.$

30. A particle moving under the action of a central potential is described in terms of canonical variables q and p. A transformation to $X = \alpha q - \beta p$ and $Y = -\lambda q + \sigma p$, where $\alpha, \beta, \lambda, \sigma$ are constants is a canonical transformation if and only if

A. $\alpha = 0, \beta = 0.$

B. $\beta = 0, \lambda = 0.$

C. $\begin{pmatrix} \alpha & \beta \\ \lambda & \sigma \end{pmatrix}$ is a real matrix with unit determinant.

D. $\begin{pmatrix} \alpha & \beta \\ \lambda & \sigma \end{pmatrix}$ is a $SU(2)$ matrix.

31. A monochromatic electromagnetic wave of amplitude E_0, angular frequency ω and phase angle 0(zero), is traveling along the direction of a line joining origin to $(1, 1, 1)$ with polarisation in the xy-plane. The real part of the corresponding electric field is (here, $\hat{x}, \hat{y}$ are the unit vectors)

A. $\frac{E_0}{\sqrt{2}} \cos(\frac{\omega}{c\sqrt{3}}(x + y + z) - \omega t)(\hat{x} + \hat{y})$

B. $E_0 \cos(\frac{\omega}{c}(x + y + z) - \omega t)(\hat{x} - \hat{y})$

C. $\frac{E_0}{\sqrt{2}} \cos(\frac{\omega}{c\sqrt{3}}(x + y + z) - \omega t)(\hat{x} - \hat{y})$

D. $E_0 \cos(\frac{\omega}{c\sqrt{3}}(x + y + z) - \omega t)(\hat{x} + \hat{y})$

32. The Poynting vector associated with an electromagnetic plane wave with $\vec{E} = A_0 \cos(kx) \cos(\omega t)\hat{y}$ and $\vec{H} = A_0 \sin(kx) \sin(\omega t)\hat{z}$ is

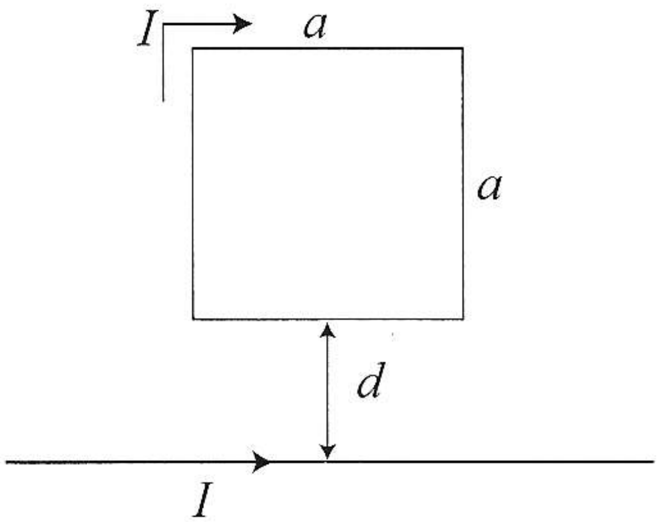
A. $\frac{A_0^2}{4} \sin(2kx) \sin(2\omega t)\hat{x}$

B. $\frac{A_0^2}{4} \sin(kx) \sin(\omega t)\hat{x}$

C. $A_0^2 \sin(kx) \sin(2\omega t)\hat{x}$

D. $A_0^2 \sin(2kx) \sin(\omega t)\hat{x}$

33. Consider a square loop with side a, placed such that one side is parallel to a straight, infinite wire and at a distance d from it. If both the loop and the straight wire are carrying a steady current I, as shown in the figure, the force between them is



A. $\frac{\mu_0 I^2 a}{2\pi} \frac{2d+a}{d(d+a)}$

B. $\frac{\mu_0 I^2 a}{2\pi d}$

C. $\frac{\mu_0 I^2 a}{2\pi(d+a)}$

D. $\frac{\mu_0 I^2 a^2}{2\pi d(d+a)}$

34. The value of $\oint_C \frac{dz}{z^3+1}$, where the contour is of radius $\frac{1}{2}$, taken in the anti-clockwise direction is

A. 0

B. $\frac{2\pi i}{3}$

C. $\frac{\pi i - \pi\sqrt{3}}{3}$

D. $\frac{-\pi i + \pi\sqrt{3}}{3}$

35. The matrix $A = \begin{pmatrix} 2 & 3 & 5 \\ 0 & 2 & 13 \\ 0 & 0 & 2 \end{pmatrix}$ satisfies

A. $(2\mathbb{I} - A)^3 = 0$

B. $(2\mathbb{I} + A)^3 = 0$

C. $(\mathbb{I} - A)^2 = 0$

D. $(\mathbb{I} - 2A)^2 = 0$

36. In $\mathbb{C}^3$ with inner product defined as $(x, y) = x^\dagger y$, the action of an operator A is given by the relation $A \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} x_1 + x_2 \\ x_2 + x_3 \\ x_3 + x_1 \end{pmatrix}$. The action of adjoint of A on $\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$ gives

A. $\begin{pmatrix} x_1 + x_3 \\ x_1 + x_2 \\ x_2 + x_3 \end{pmatrix}$

B. $\begin{pmatrix} x_1 + x_3 \\ x_2 + x_3 \\ x_1 + x_2 \end{pmatrix}$

C. $\begin{pmatrix} x_2 + x_3 \\ x_1 + x_2 \\ x_1 + x_3 \end{pmatrix}$

D. $\begin{pmatrix} x_1 + x_2 \\ x_1 + x_3 \\ x_2 + x_3 \end{pmatrix}$

37. If α, β, γ are the elements of a given basis of a 3-dimensional real vector space and $(5x + y)\alpha + (2 - 8x + z)\beta + (7 - x + y)\gamma = x\alpha + 2y\beta + z\gamma$, (where x, y, z are real numbers), the solution is

A. $x = 1, \quad y = 4, \quad z = -2$

B. $x = -1, y = -4, \quad z = 2$

C. $x = -1, y = -4, z = -2$

D. $x = 1, \quad y = -4, z = -2$

38. A state of a quantum mechanical system is given by $|\psi >= x|1 > +ye^{i\phi}|2 >$, where $|1 >$ and $|2 >$ are orthonormal basis states and x, y, ϕ are real numbers. The state orthogonal to $|\psi >$ is

A. $x|1 > -ye^{i\phi}|2 >$

B. $y|1 > -xe^{i\phi}|2 >$

C. $x|1 > -ye^{-i\phi}|2 >$

D. $y|1 > -xe^{-i\phi}|2 >$

39. State of a particle moving in a harmonic oscillator potential is given by $|\psi >= \frac{1}{\sqrt{21}}(|0 > +2|1 > +4|2 >)$, where $|n >$ is the eigen state of the number operator $\hat{N} = a^\dagger a$ where the annihilation operator is given by $a = \frac{1}{\sqrt{2m\omega\hbar}}(m\omega\hat{x} + i\hat{p})$. The expectation value of position operator x in this state is

A. $\frac{4}{21}\sqrt{\frac{\hbar}{m\omega}}(8 + \sqrt{2})$

B. $\frac{3}{21}\sqrt{\frac{\hbar}{m\omega}}(8 + \sqrt{2})$

C. $\frac{2}{21}\sqrt{\frac{\hbar}{m\omega}}(8 + \sqrt{2})$

D. $\frac{5}{21}\sqrt{\frac{\hbar}{m\omega}}(8 + \sqrt{2})$

40. An electric dipole $\vec{p} = p_1\hat{i} + p_2\hat{j}$ ($\hat{i}, \hat{j}$ are unit vectors in the x and y directions, respectively) is placed in an external electric field $\vec{E} = \frac{\rho}{3\epsilon_0}\vec{r}$. Given $\vec{F}$ and $\vec{\tau}$ are, respectively, the force and torque acting on the dipole, the value of $\vec{F} \cdot \vec{\tau}$ is

A. $\frac{\rho^2}{9\epsilon_0}p_1p_2(x+y)$

B. 0

C. $\frac{\rho^2}{9\epsilon_0}(x^2+y^2)$

D. $\frac{\rho^2}{9\epsilon_0}(x-y)$

41. A quantum mechanical harmonic oscillator described by $H = \hbar\omega(\hat{N}+\frac{1}{2})$ is kept at a finite temperature T. Here $\hat{N}$ is the number operator. The relation between the partition function Z of this system and the expectation value $<N>$ of the number operator is (with $\beta = \frac{1}{k_BT}$)

A. $Z = <N> e^{-\beta\hbar\omega}$.

B. $Z = <N> e^{\beta\hbar\omega}$.

C. $Z = <N> e^{-\frac{\beta\hbar\omega}{2}}$.

D. $Z = <N> e^{\frac{\beta\hbar\omega}{2}}$.

42. Consider a system of N Ising spins each of which can take values $s = \pm 1$. The magnetisation of the system is given by $m = \frac{N_+ - N_-}{N}$ where $N_\pm$ are the number of spins with $s = \pm 1$. The value of the entropy of the system $S(m)$, for $m = 1, 0, -1$, are respectively

A. $1, 0, 1$

B. $Nln2$, 0, $Nln2$

C. 0, $Nln2$, 0

D. 1, $Nln2$, 1

43. The equation of motion of a system, described by the Lagrangian $\mathcal{L} = \frac{1}{2}e^{\alpha t}(\dot{q}^2 - \omega^2 q^2)$, is

A. $\ddot{q} - \omega^2 q = 0$

B. $e^{\alpha t}\ddot{q} - e^{-\alpha t}\omega^2 q = 0$

C. $\ddot{q} + \alpha\dot{q} + \omega^2 q = 0$

D. $\ddot{q} - \alpha\dot{q} - \omega^2 q = 0$

44. Consider a laser cavity with plane parallel mirrors, and separated by a distance d, each with reflectivity R. For a laser light reflecting back and forth between these mirrors to retain same intensity, the gain coefficient of the medium must be

A. $\frac{R}{d}$

B. $\frac{lnR}{d}$

C. $\frac{2lnR}{d}$

D. $\frac{d}{R}$

45. Which of the following is an allowed process

A. $p + n \rightarrow \Xi^- + k^+ + \Sigma^+$

B. $p \rightarrow e^+ + \pi^0$

C. $\pi^- + p \rightarrow \pi^0 + n$

D. $\Xi^- \rightarrow n + \pi^-$

46. A proton beam impinges on a stationary proton leading to the reaction $p + p \rightarrow H + k^+ + k^+$. The minimum value of the momentum of incident proton beam to produce the state H of mass $m_H = 2150\ MeV$ is ($m_p = 938\ MeV, m_{k^+} = 494\ MeV$)

A. $2.104\ GeV$

B. 4.208 GeV

C. 8.416 GeV

D. 16.832 GeV

47. Two spherical soap bubbles of radii a and b, coalesce to form a single spherical bubble of radius c, without any leakage of air. If P is the external pressure, the surface tension of the liquid, from which the bubble is formed, is

A. $\frac{P}{4}\frac{(\sqrt{a+b+c})^3}{(a+b+c)^2}$

B. $\frac{P}{4}\frac{c^3-b^3-a^3}{a^2+b^2-c^2}$

C. $\frac{P}{4}\frac{a^3+b^3+c^3}{a^2+b^2+c^2}$

D. $\frac{P}{4}\frac{a^2+b^2+c^2}{c^3-b^3-a^3}$

48. If the change in the volume of a cylinder made up of rubber when stretched (within the elastic limit) along its axis compared to its shape is negligible, the value of Poisson ratio is

A. 0

B. 0.2

C. 0.5

D. 1

49. The microwave power transmission component that acts as a power transformer is

A. frequency meter

B. horn antenna

C. magic Tee

D. directional coupler

50. Regular singular points of the differential equation $(x-2)\frac{d^2y}{dx^2}+\frac{x^2}{(x+1)}\frac{dy}{dx}+\frac{1}{x(x-2)^2}y=0$ are

A. $x = 2, -1$

B. $x = 2, 0$

C. $x = -1, 0$

D. $x = 0, -1, 2$

51. Fourier transform of $f(t)\cos\omega_0 t$, where

$$f(t) = 1, \quad for \ |t| < \frac{l}{2}$$
$$0, \quad for \ |t| > \frac{l}{2}, \quad is$$

A. $\frac{2}{\omega-\omega_0}\sin\frac{l(\omega-\omega_0)}{2}$

B. $\frac{2}{\omega-\omega_0}\sin\frac{l(\omega+\omega_0)}{2}$

C. $\frac{\sin\frac{l(\omega-\omega_0)}{2}}{\omega-\omega_0}-\frac{\sin\frac{l(\omega-\omega_0)}{2}}{\omega+\omega_0}$

D. $\frac{\sin\frac{l(\omega-\omega_0)}{2}}{\omega-\omega_0}+\frac{\sin\frac{l(\omega-\omega_0)}{2}}{\omega+\omega_0}$

52. A system of two identical spin-0 bosons with two energy levels of energy 0 and ϵ, ($\epsilon > 0$) is in thermal equilibrium at temperature T. The Helmholtz free energy of this system is

A. $k_BTln(1+e^{-\frac{\epsilon}{K_BT}}+e^{-\frac{2\epsilon}{K_BT}})$

B. $-k_BTln(1+e^{-\frac{\epsilon}{K_BT}}+e^{-\frac{2\epsilon}{K_BT}})$

C. $k_BTln(1+2e^{-\frac{\epsilon}{K_BT}}+e^{-\frac{2\epsilon}{K_BT}})$

D. $-k_BTln(1+2e^{-\frac{\epsilon}{K_BT}}+e^{-\frac{2\epsilon}{K_BT}})$

53. The term symbol of Dy^{3+} (with outer electronic configuration of $4f^9$) is

A. ${}^6I_{\frac{5}{2}}$

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B. $^4I_{\frac{15}{2}}$

C. $^8S_{\frac{7}{2}}$

D. $^6H_{\frac{15}{2}}$

54. Given that F is a function of $(\vec{r} \cdot \vec{r})^2$, the correct statement is

A. it is invariant under rotations in 3-dimensions

B. it is invariant only under rotations in 2-dimensions

C. it is invariant under translations in 3-dimensions

D. it is invariant under rotations and translations in 3-dimensions

55. The frequency of radiation emitted by the Josephson junction when a voltage of 10 μV is applied across is

A. $2.41 \times 10^9\ Hz$

B. $0.61 \times 10^9\ Hz$

C. $1.21 \times 10^9\ Hz$

D. $4.82 \times 10^9\ Hz$