ENTRANCE EXAMINATIONS – 2019

(Ph.D. Admissions - January 2020 Session)

Ph.D. Mathematics/Applied Mathematics

Hall Ticket No.	

Time : 2 hours
Max. Marks : 70

PART A: 35 Marks
PART B: 35 Marks

Instructions

1. Write your Hall Ticket Number on the OMR Answer Sheet given to you. Also write the Hall Ticket Number in the space provided above.
2. Answers are to be marked on the OMR sheet.
3. Please read the instructions carefully before marking your answers on the OMR answer sheet.
4. Hand over the OMR answer sheet at the end of the examination to the Invigilator.
5. No additional sheets will be provided. Rough work can be done in the question paper itself/space provided at the end of the booklet.
6. Calculators are not allowed.
7. There are a total of 70 questions in **PART A** and **PART B** together.
8. Each correct carries **1 mark**.
9. The appropriate answer(s) should be coloured with either a blue or black ball point or a sketch pen. DO NOT USE A PENCIL.
10. This book contains **15 pages** including this page and excluding pages for the rough work. Please check that your paper has all the pages.
11. $\mathbb{R}$ denotes the set of real numbers, $\mathbb{C}$ the set of complex numbers, $\mathbb{Z}$ the set of integers, $\mathbb{Q}$ the set of rational numbers, and $\mathbb{N}$ the set of all natural numbers.

V-59

Part-A

1. A proposition is a tautology that is

    (A) true at least once.
    (B) always true.
    (C) false at least once.
    (D) always false.

2. Suppose "If P then Q" and "If Q then R". Then it follows "If P then R". This is known as

    (A) Modus ponens.
    (B) Modus tollens.
    (C) Syllogism.
    (D) None of the above.

3. The last two digits of the number 9^{9^9} are

    (A) 89
    (B) 91
    (C) 87
    (D) 97

4. For propositions A and B, the formula (Not A and B) is logically equivalent to

    (A) Not (A implies B)
    (B) Not (B implies A)
    (C) (A and Not B)
    (D) (A or Not B)

5. Let $X = A \cup B \cup C$. Then, the complement of the set $A \cap B \cap C$ equals to

    (A) $(A \setminus B) \cup (B \setminus C) \cup (C \setminus A)$
    (B) $(A \setminus B) \cup (B \setminus C)$
    (C) $(A \cup B) \setminus C$
    (D) None of the above.

6. Let A, B be two sets with $|A| = 6$, $|B| = 7$ and $A \not\subset B$. Then the minimum value of $|(A \times (B \setminus A)) \cup (B \times (A \setminus B))|$ is

   (A) 19
   (B) 20
   (C) 21
   (D) 24

7. Let $Y_n \subset X$ for $n \in \mathbb{N}$. Then the set $\{x \in X : x \in Y_n$ for infinitely many $n \in \mathbb{N}\}$ is equal to

   (A) $\bigcup_{k=1}^{\infty} \bigcap_{n=k}^{\infty} Y_n$
   (B) $\bigcup_{k=1}^{\infty} \bigcap_{n=1}^{k} Y_n$
   (C) $\bigcap_{k=1}^{\infty} \bigcup_{n=k}^{\infty} Y_n$
   (D) $\bigcap_{k=1}^{\infty} \bigcup_{n=1}^{k} Y_n$

8. The number of equivalence relations on a set with 4 elements is:

   (A) 16
   (B) 15
   (C) 12
   (D) 10

9. Let A, B be non-empty sets, $f : A \to B$ be injective and $g : B \to A$ be surjective. Then which of the following is TRUE?

   (A) If f is bijective, then g is bijective.
   (B) If g is bijective, then f is bijective.
   (C) If $|A| = |B| = \infty$ and f is bijective, then g is bijective.
   (D) None of the above.

10. If $D \subset B \subset A \subset C \subset X$, then $(C \triangle A) \cup (A \triangle B)$ is equal to (where $\triangle$ denotes the symmetric difference):

   (A) $X \setminus B$
   (B) $C \setminus B$
   (C) $X \setminus A$
   (D) $C \setminus D$

V-59

11. In how many distinct ways one can rearrange the letters of the word **HIJKKLM** so that no two adjacent letters are the same?

(A) $5! \times 15$

(B) $\frac{7!}{2!}$

(C) $\frac{6!}{2!}$

(D) $5! \times 10$

12. If A, B, C are subsets of a set X, then $(A \cup B \cup C) \cap (B \setminus C) \cap (A \setminus B)$ is equal to

(A) $X \setminus (B \cup C)$

(B) $(B \cap C) \setminus A$

(C) $A \setminus (B \cup C)$

(D) The empty set.

13. The number of 6-digit numbers such that sum of the digits is at most 51 is

(A) $9 \times 10^5 - 20$

(B) $9 \times 10^5 - 22$

(C) $9 \times 10^5 - 28$

(D) None of the above.

14. The number of square-free integers x with $65 < x \leq 105$ is

(A) 23

(B) 21

(C) 20

(D) 25

15. A bus has to visit three cities, each of them four times. The number of ways it can be done if bus is not allowed to start and end in the same city is

(A) 1260

(B) 1120

(C) 980

(D) None of the above.

V-59

16. Let $|A| = 4$ and $|B| = 6$. Then the number of injective functions from A to B is

(A) 120
(B) 24
(C) 360
(D) None of the above.

17. 10 people in a party offered gifts to one another. What was the total number of gifts?

(A) 35
(B) 60
(C) 40
(D) 90

18. In a family A and E are husband and wife. B is brother-in-law of E. D is father-in-law of E. How is D related to B?

(A) Brother
(B) Mother
(C) Father
(D) Father-in-law

19. A is 3 years older to B and 3 years younger to C, while B and D are twins. How many years older is C to D?

(A) 3
(B) 6
(C) 9
(D) 12

20. In an examination, a candidate gets 2 marks for the right answer and loses 1 mark for the wrong answer. He got 80 marks by answering 100 questions. How many of his answers were right?

(A) 30
(B) 35
(C) 40
(D) 60

21. A man has Rs. 480 in the denominations of one rupee notes, five rupee notes and ten rupee notes. The number of each denomination is equal. What is the total number of notes that he has?

(A) 45
(B) 75
(C) 90
(D) 120

22. The number of boys in a class is three times the number of girls. Which one of the following numbers cannot represent the total number of children in the class?

(A) 40
(B) 42
(C) 44
(D) 48

23. Arrange the words given below in a meaningful and logical sequence:
1. Table 2. Tree 3. Wood 4. Seed 5. Plant

(A) 4,5,3,2,1
(B) 1,3,2,4,5
(C) 4,5,2,3,1
(D) 1,2,3,4,5

24. The smallest number when increased by " 1 " is exactly divisible by 12, 18, 24, 32 and 40 is

(A) 1439
(B) 1440
(C) 1449
(D) 1459

25. Five children are sitting in a row. S is sitting next to P but not T. And K is sitting next to R who is sitting on the extreme left, and T is not sitting next to K. Who are sitting adjacent to S?

(A) R and P
(B) Only P
(C) P and T
(D) K and P

26. If A means +, B means $-$, C means $\times$ and D means $\div$ then 18 C 14 A 6 B 16 D 4=?

    (A) 258
    (B) 254
    (C) 238
    (D) 188

27. If the equation $24 * 6 * 12 * 16 = 0$ has to be balanced, then the sequence of signs to be used in the place of the three stars is

    (A) $-$, $+$, and $+$
    (B) $\div$, $+$, and $\div$
    (C) $\div$, $+$, and $-$
    (D) $-$, $-$, and $-$

28. Which of the following interchange of signs will make the equation $5+6\div3-12\times2 = 17$ correct?

    (A) $\div$ and $\times$
    (B) $+$ and $\times$
    (C) $+$ and $\div$
    (D) $+$ and $-$

29. The number of 4-digit numbers that can be formed using the digits 1,2,3,4,5 and 6 that are divisible by 3 is

    (A) 864
    (B) 648
    (C) 216
    (D) 432

30. There are five books A,B,C,D and E. Book C lies above D, book E is below A, D is above A and B is below E. Which book is at the bottom?

    (A) E
    (B) B
    (C) A
    (D) C

31. The two missing letters in the sequence 'A, I, B, J, C, K, _, _' are

(A) E, M
(B) E, L
(C) D, L
(D) D, M

32. If BCFE is coded as HILK, then the code for NORQ is

(A) TXWU
(B) TUXW
(C) TXUW
(D) TVWX

33. Find out the odd one:

(A) NOPQ
(B) JKLM
(C) FGHI
(D) ABCE

34. How many letters are there in the word CATEGORY each of which is as far away from the beginning of the word as when these letters are arranged in the alphabetical order?

(A) one
(B) two
(C) three
(D) four

35. In a class 40% of the students enrolled for Mathematics and 70% enrolled for Statistics. If 15% of the students enrolled for both Mathematics and Statistics, what % of the students of the class did not enroll for either of the two subjects?

(A) 5%
(B) 10%
(C) 15%
(D) 20%

V-59

Part-B

36. If a complex power series $\sum_{n=0}^{\infty} a_n z^n$ converges at $4-3i$, then which of the following is FALSE?

    (A) $\sum_{n=0}^{\infty} a_n z^n$ converges for $z = 2+4i$.

    (B) $\sum_{n=0}^{\infty} a_n z^n$ converges for $z = 3+3i$.

    (C) $\sum_{n=0}^{\infty} a_n z^n$ converges uniformly on $\{z \in \mathbb{C} : |z| < 5\}$

    (D) $\sum_{n=0}^{\infty} a_n z^n$ converges absolutely and uniformly on $\{z \in \mathbb{C} : |z| \leq 4\frac{1}{2}\}$

37. Let $f : \mathbb{C} \to \mathbb{C}$ be an entire function and suppose both 1 and i belong to $f'(\mathbb{C})$, where f' is the derivative of f. Then, the TRUE statement is :

    (A) $f(\mathbb{C}) = \mathbb{C}$.

    (B) f has infinitely many zeroes.

    (C) Both 1 and i belong to $f''(\mathbb{C})$.

    (D) f cannot be injective.

38. Let $f : \mathbb{C} \setminus \{0\} \to \mathbb{C}$ be an analytic function. If there is a sequence of polynomials converging to f uniformly on the unit circle, then the best conclusion is:

    (A) f has an analytic extension to the whole of $\mathbb{C}$.

    (B) f must be a polynomial.

    (C) f must be constant.

    (D) There is an analytic function $g : \mathbb{C} \setminus \{0\} \to \mathbb{C}$ with $f = e^g$.

39. Let $\mathcal{A}$ be the σ-algebra on $\mathbb{R}$ generated by the collection $\{\mathbb{Q}, \mathbb{R} \setminus \mathbb{Q}\}$. Then,

    (A) $\mathcal{A}$ is countably infinite.

    (B) $\mathcal{A}$ is finite.

    (C) $\mathcal{A}$ is uncountable and is different from $\mathbb{P}(\mathbb{R})$.

    (D) $\mathcal{A} = \mathbb{P}(\mathbb{R})$.

40. Let μ be a measure defined on $(\mathbb{R}, \mathbb{P}(\mathbb{R}))$. If $\mu(\mathbb{R} \setminus \mathbb{Q}) = 0$, then the best conclusion is:

    (A) $\mu \equiv 0$.

    (B) $\mu(\mathbb{R}) < \infty$.

    (C) $\mu(\{x\}) = 0$ for every $x \in \mathbb{R}$.

    (D) None of the above.

V-59

41. Let μ be the Lebesgue measure on $\mathbb{R}^2$. For a measurable set $A \subset \mathbb{R}^2$, the FALSE statement is:

(A) If A is countable, then $\mu(A) = 0$.

(B) If $\mu(\overline{A}) = 0$, then A is nowhere dense in $\mathbb{R}^2$.

(C) If A is compact, then $\mu(A) < \infty$.

(D) If A is open and dense in $\mathbb{R}^2$, then $\mu(A) = \infty$.

42. Let X be a normed linear space, $A, B \subset X$ be non-empty sets, and let $A+B = \{a+b : a \in A \text{ and } b \in B\}$. Which of the following is TRUE?

(A) If A is compact, then $A + B$ is compact.

(B) If A is open in X, then $A + B$ is open in X.

(C) If A is connected, then $A + B$ is connected.

(D) None of the above.

43. The closure of l^1 in the Banach space l^∞ is equal to:

(A) $\{(x_n) \in l^\infty : \lim_{n\to\infty} x_n = 0\}$

(B) l^1

(C) l^2

(D) l^∞

44. Let $T : \mathbb{R}^2 \to \mathbb{R}$ be $T(x, y) = 3x - 4y$. Then, with respect to the Euclidean norm,

(A) $\|T\| = 3$

(B) $\|T\| = 4$

(C) $\|T\| = 5$

(D) $\|T\| = 7$

45. Let $A \subset \mathbb{R}^2$. With respect to the Euclidean metric, which of the following is FALSE?

(A) If A is sequentially compact, then A is bounded.

(B) If A is bounded, then A is totally bounded.

(C) If A is totally bounded, then A is limit point compact.

(D) If A is limit point compact, then A is sequentially compact.

V- 59

46. Let X be a metric space and $Y \subset X$ be a countably infinite set. Then the TRUE statement is:

(A) Y cannot be compact.

(B) Y cannot be connected.

(C) Y cannot be homeomorphic to $X \setminus Y$.

(D) Y cannot be second countable.

47. Let X be a topological space and $Y \subset X$. Then the TRUE statement is:

(A) If Y is dense in X, then the interior of Y is non-empty.

(B) If Y is dense in X, then $X \setminus Y$ is nowhere dense in X.

(C) If $X \setminus Y$ is nowhere dense in X, then Y is dense in X.

(D) None of the above.

48. Let X, Y be metric spaces, and $f : X \to Y$ be continuous. If (x_n) is a Cauchy sequence in X, then:

(A) $(f(x_n))$ converges in Y.

(B) $(f(x_n))$ is a Cauchy sequence but may not converge.

(C) $(f(x_n))$ is a bounded sequence but may not be a Cauchy sequence.

(D) None of the above.

49. About the improper integral $\int_0^1 \log x dx$, the TRUE statement is:

(A) $\int_0^1 \log x dx = -\infty$

(B) $-\infty < \int_0^1 \log x dx < -1$

(C) $\int_0^1 \log x dx = -1$

(D) $-1 < \int_0^1 \log x dx < 0$

50. The limit $\lim_{x \to 0+} (\cos x)^{1/x^2}$:

(A) is equal to 0

(B) is equal to $e^{-1/2}$

(C) is equal to 1

(D) does not exist.

V-59

51. For a function $g : [0,1] \to \mathbb{R}$, which of the following is TRUE?

    (A) If $g([0,1])$ is a finite set, then there is $f : [0,1] \to \mathbb{R}$ with $f' = g$.

    (B) If g is Riemann integrable, then there is $f : [0,1] \to \mathbb{R}$ with $f' = f$.

    (C) If g is continuous, then there is $f : [0,1] \to \mathbb{R}$ with $f' = g$.

    (D) None of the above.

52. Let $f : \mathbb{R} \to \mathbb{R}$ be a continuous bijection. Then,

    (A) f is of the form $x \mapsto ax + b$.

    (B) f is an open map.

    (C) f is differentiable.

    (D) f is Lipschitz continuous.

53. Let $0 < m < n$ be integers, A be an $m \times n$ complex matrix, and B be an $n \times m$ complex matrix. Which of the following is TRUE?

    (A) AB is never invertible, but BA can be invertible.

    (B) BA is never invertible, but AB can be invertible.

    (C) AB is never invertible, and BA is never invertible.

    (D) None of the above.

54. Let A be a square matrix with $trace(A^t A) = 0$, where A^t denotes the transpose of A. Then which of the following is TRUE?

    (A) $trace(A) = 0$, but $det(A)$ can be non-zero.

    (B) $det(A) = 0$, but $trace(A)$ can be non-zero.

    (C) $trace(A) = 0 = det(A)$.

    (D) None of the above.

55. Let A be a symmetric real matrix. Then which of the following is TRUE?

    (A) A is diagonalizable, and all eigenvalues of A are real.

    (B) A is diagonalizable, but A can have eigenvalues in $\mathbb{C} \setminus \mathbb{R}$.

    (C) All eigenvalues of A are real, but A need not be diagonalizable.

    (D) None of the above.

V-59

56. Let $X = \{(x_n) \in \mathbb{R}^{\mathbb{N}} : x_n \in \{-1, 1\} \text{ for every } n \in \mathbb{N}\}$ and $Y = \{(x_n) \in \mathbb{R}^{\mathbb{N}} : x_n \in \{-1, 1\} \text{ only for finitely many } n \in \mathbb{N}\}$. Then

(A) Both X and Y are countable.

(B) X is countable but Y is uncountable.

(C) Y is countable but X is uncountable.

(D) Both X and Y are uncountable.

57. Which of the following is TRUE?

(A) The map $f(x) = 55x$ is a group homomorphism from $\mathbb{Z}_{121}$ to $\mathbb{Z}_{120}$.

(B) The map $f(x) = 10x$ is a group homomorphism from $\mathbb{Z}_{121}$ to $\mathbb{Z}_{120}$.

(C) The map $f \equiv 0$ is the only group homomorphism from $\mathbb{Z}_{121}$ to $\mathbb{Z}_{120}$.

(D) None of the above.

58. Which of the following is TRUE?

(A) Any group of order 250 is simple.

(B) Any group of order 169 is simple.

(C) Any group of order 99 is simple.

(D) None of the above.

59. The number of distinct maximal ideals of the ring $\mathbb{Z}_{10625}$ is

(A) 2

(B) 5

(C) 1

(D) 8

60. Suppose R is a ring with 1, and x is not a zero divisor in R. Then which of the following is TRUE?

(A) x is a unit if there is $n \in \mathbb{N}$ such that the principal ideals $(x^n) = (x^{n+1})$.

(B) x is not a unit if R is infinite.

(C) $x = 1$ if there is $n \in \mathbb{N}$ such that the principal ideals $(x^n) = (x^{n+1})$.

(D) None of the above.

V-59

56. Let $X = \{(x_n) \in \mathbb{R}^{\mathbb{N}} : x_n \in \{-1, 1\}$ for every $n \in \mathbb{N}\}$ and $Y = \{(x_n) \in \mathbb{R}^{\mathbb{N}} : x_n \in \{-1, 1\}$ only for finitely many $n \in \mathbb{N}\}$. Then

(A) Both X and Y are countable.

(B) X is countable but Y is uncountable.

(C) Y is countable but X is uncountable.

(D) Both X and Y are uncountable.

57. Which of the following is TRUE?

(A) The map $f(x) = 55x$ is a group homomorphism from $\mathbb{Z}_{121}$ to $\mathbb{Z}_{120}$.

(B) The map $f(x) = 10x$ is a group homomorphism from $\mathbb{Z}_{121}$ to $\mathbb{Z}_{120}$.

(C) The map $f \equiv 0$ is the only group homomorphism from $\mathbb{Z}_{121}$ to $\mathbb{Z}_{120}$.

(D) None of the above.

58. Which of the following is TRUE?

(A) Any group of order 250 is simple.

(B) Any group of order 169 is simple.

(C) Any group of order 99 is simple.

(D) None of the above.

59. The number of distinct maximal ideals of the ring $\mathbb{Z}_{10625}$ is

(A) 2

(B) 5

(C) 1

(D) 8

60. Suppose R is a ring with 1, and x is not a zero divisor in R. Then which of the following is TRUE?

(A) x is a unit if there is $n \in \mathbb{N}$ such that the principal ideals $(x^n) = (x^{n+1})$.

(B) x is not a unit if R is infinite.

(C) $x = 1$ if there is $n \in \mathbb{N}$ such that the principal ideals $(x^n) = (x^{n+1})$.

(D) None of the above.

V- 59

61. If $f \in \mathbb{F}_p[x] \setminus \{0\}$, then which of the following is a sufficient condition for f to divide $x^{p^n} - x$ for some $n \in \mathbb{N}$?

    (A) The degree of f is not a multiple of p.

    (B) The degree of f is a multiple of p.

    (C) f has no multiple roots.

    (D) None of the above.

62. Let k be a field and $f, g \in k[X] \setminus \{0\}$ be irreducible polynomials of degree n, m respectively. Also, let α be a root of f. Which of the following is TRUE?

    (A) g is irreducible over $k[\alpha]$ if $g.c.d.(n, m) = 1$.

    (B) g is never irreducible over $k[\alpha]$.

    (C) g is irreducible over $k[\alpha]$ if $g.c.d.(n, m) \neq 1$.

    (D) None of the above.

63. Let $u(x, y) = \frac{1}{2}\log(x^2 + y^2)$. Then which of the following is TRUE?

    (A) u is harmonic on $\mathbb{R}^2 \setminus \{(0,0)\}$, and u is the real part of an analytic function $f : \mathbb{C} \setminus \{0\} \to \mathbb{C}$.

    (B) u is harmonic on $\mathbb{R}^2 \setminus \{(0,0)\}$, but u is not the real part of any analytic function $f : \mathbb{C} \setminus \{0\} \to \mathbb{C}$.

    (C) u is the real part of an analytic function $f : \mathbb{C} \setminus \{0\} \to \mathbb{C}$, but u is not harmonic on $\mathbb{R}^2 \setminus \{(0,0)\}$.

    (D) None of the above.

64. Assume $p : [0,1] \to [1,2]$ is a C^1-map, and $q, s : [0,1] \to [1,2]$ are continuous. Consider the eigenvalue problem

$$\begin{cases} -(py')' + (q + \lambda s)y = 0, \ x \in (0,1) \\ y'(0) = 0 = y'(1). \end{cases}$$

    Then the set of all eigenvalues of this problem is a subset of

    (A) the imaginary axis of $\mathbb{C}$

    (B) $[1, 2]$

    (C) $[0, 1]$

    (D) $(-\infty, 0]$

65. Consider $J[y] = \int_0^\pi ((y')^2 - y^2)dx$, $y(0) = 0 = y(\pi)$. Then which of the following is TRUE?

    (A) There are no extremals of J.

    (B) There are more than one but only finitely many extremals of J.

    (C) There are infinitely many extremals of J.

    (D) There is a unique extremal of J.

V-59

66. Let u be a solution of
$$\begin{cases} u_t(x,t) = u_{xx}(x,t), \ x \in \mathbb{R}, \ t > 0, \\ u(x,0) = 1 \text{ if } x \geq 0, \text{ and } u(x,0) = 0 \text{ if } x < 0. \end{cases}$$

Then $\lim_{t \to 0+} u(0,t)$

(A) is equal to $1/2$

(B) is equal to 1

(C) is equal to 0

(D) does not exist.

67. Let Ω be an open disc in $\mathbb{R}^2$ and $\partial\Omega$ be its boundary. Then the problem
$$\begin{cases} -\Delta u = f \text{ in } \Omega, \\ \dfrac{\partial u}{\partial n} = g \text{ on } \partial\Omega, \end{cases}$$
has

(A) a unique solution for every continuous f and g.

(B) only finitely many solutions for every continuous f and g.

(C) only finitely many solutions whenever $\int_\Omega f dxdy + \oint_{\partial\Omega} g dc = 0$.

(D) infinitely many solutions whenever $\int_\Omega f dxdy + \oint_{\partial\Omega} g dc = 0$ and f, g are continuous.

68. Let $f : \mathbb{R} \to \mathbb{R}$ be Lipschitz continuous. Assume that the set $\{x \in \mathbb{R} : f(x) = 0\}$ is neither bounded above nor bounded below. Then every solution of $y' = f(y)$ is

(A) bounded above but not bounded below.

(B) bounded below but not bounded above.

(C) bounded.

(D) neither bounded above nor bounded below.

69. Consider the system $Y'(t) = A(t)Y(t), t > 0$, where $A : [0, \infty) \to M_n(\mathbb{R})$ is continuous. Let Φ and Ψ be a solution matrix and fundamental matrix of the system respectively. Then which of the following is TRUE?

(A) $\Psi = C\Phi$ for some $C \in GL_n(\mathbb{R})$

(B) $C\Psi$ is also a fundamental matrix whenever $C \in GL_n(\mathbb{R})$

(C) Φ is a non-singular matrix

(D) $\Phi\Psi$ is a solution matrix

70. A complete integral of $xu_x(x,y)u_y(x,y) + yu_y^2(x,y) = 1$ is

(A) $(u+b)^2 = 4(ax+y)$

(B) $u^2 = 4y + ax + c$

(C) $4u^2 + ax + by = 0$

(D) $(u+ax)^2 = bx$

V-59