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Entrance Examination: M.Sc. Mathematics, 2014

Hall Ticket Number: ☐☐☐☐☐☐☐☐

Time: 2 hours
Max. Marks: 100

Part-A: 25 marks
Part-B: 75 marks

Instructions

1. Write your Hall Ticket Number on the OMR Answer Sheet given to you. Also write the Hall Ticket Number in the space provided above.
2. In Part-A a right answer gets **1 mark** and a wrong answer gets **−0.33 mark**.
3. **In Part-B, some questions have MORE THAN ONE correct option. All the correct options have to be marked in the OMR answer sheet, otherwise ZERO marks will be credited.**
4. Answers are to be marked on the OMR answer sheet following the instructions provided there upon.
5. Handover the OMR answer sheet at the end of the examination.
6. No additional sheets will be provided. Rough work can be done in the question paper itself/space provided at the end of the booklet.
7. There are a total of 50 questions in Part-A and Part-B together.
8. The appropriate answer should be coloured in either a blue or black ball point or sketch pen. DO NOT USE A PENCIL.
9. The question paper can be retained by the candidates at the end of the examination, after handing over the OMR sheet.

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Part A

In Part-A a right answer gets **1 mark** and a wrong answer gets **−0.33 mark.**

1. Let $0 \neq \overline{v} \in \mathbb{R}^2$. For $0 \leq \theta < \pi$, let $A = \begin{pmatrix} \sin\theta & \cos\theta \\ -\cos\theta & \sin\theta \end{pmatrix}$. Then the angle between $\overline{v}$ and $A\overline{v}$ is

   (A) $\pi - \theta$. **(B)** θ. **(C)** $\frac{\pi}{2} - \theta$. **(D)** 0.

2. Let $A \in M_n(\mathbb{R})$. If $A^2 = -I$ (where I is the identity matrix), then

   (A) n is even.
   (B) $A = \pm I$.
   (C) all the eigen values of A are in $\mathbb{R}$.
   (D) A is a diagonal matrix.

3. Let $\overline{\mathbf{f}} = (u, v, w)$ be a vector field which is solinoidal. If curl(curl $\overline{\mathbf{f}}$) = 0, then

   (A) $\text{curl}(\overline{\mathbf{f}}) = 0$. **(B)** $\text{grad}(\overline{\mathbf{f}}.\overline{\mathbf{f}}) = 0$.

   (C) $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 1$. **(D)** $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0$.

4. Let $\overline{\mathbf{v}}, \overline{\mathbf{w}} \in \mathbb{R}^3$. Then a sufficient condition for $\overline{\mathbf{v}} \times \overline{\mathbf{w}} \neq 0$, is

   (A) both $\overline{\mathbf{v}}$ and $\overline{\mathbf{w}}$ are non-zero.
   (B) dimension of the linear span of $\{\overline{\mathbf{v}}, \overline{\mathbf{w}}, \overline{\mathbf{v}} \times \overline{\mathbf{w}}\}$ is ≥ 2.
   (C) either $\overline{\mathbf{v}}$ or $\overline{\mathbf{w}}$ is non-zero.
   (D) none of the above.

5. Let $f : \mathbb{R} \to \mathbb{R}$ be a function such that $|f(x)| \leq |x|^2$ for all $x \in \mathbb{R}$. Then

   (A) f is continuous but not differentiable at $x = 0$.
   (B) f is differentiable at $x = 0$.
   (C) f is an increasing function.
   (D) f is a decreasing function.

6. The number of points of continuity of the function $f = \begin{cases} |x^2 - 1| & \text{if } x \text{ is irrational,} \\ 0 & \text{if } x \text{ is rational} \end{cases}$ is

   (A) 0. **(B)** 1. **(C)** 2. **(D)** infinite.

7. The number of words formed by permuting the letters L,O,C,K,U,P such that neither 'LOCK' nor 'UP' appears in any such arrangement is

(A) $6! - 4! - 2! + 1$. (B) $6! - 5! - 3! + 2$.
(C) $6! - 5! - 3! + 1$. (D) $6! - 2! + 1$.

8. The domain of the real-valued function $f(x) = \frac{1}{\sqrt{x+1}} + \frac{1}{\sqrt{6-x}}$ is

(A) $(-\infty, \infty) \setminus \{-1, 6\}$.

(B) $\mathbb{R}$.

(C) $(-\infty, 6) \cap (-1, \infty)$.

(D) $(-1, 6)$

9. Let G be a group with identity element e, and N be a normal subgroup. Let the index of N in G be 12, *i.e.*, $[G : N] = 12$. Then

(A) $x^{12} = e$ for all $x \in N$.

(B) $x^{12} = e$, the identity element in G, for all $x \in G$.

(C) $x^{24} \in N$ for all $x \in G$.

(D) none of the above.

10. If $\{a_n\}$ is a sequence converging to l. Let $b_n = \begin{cases} a_{2n}, & \text{if } n \text{ is odd,} \\ a_{3n}, & \text{if } n \text{ is even.} \end{cases}$ Then the sequence $\{b_n\}$

(A) need not converge.

(B) should converge to 0.

(C) should converge to $2l$ or to $3l$.

(D) should converge to l.

11. Two fair dice 1 red and 1 blue are rolled. The probability that the sum of the numbers that show up on the two dice is a prime number is

(A) 7/18. (B) 7/36. (C) 15/36. (D) 29/72.

12. Let V be a vector space of dimension n over $\mathbb{R}$ and $\{v_1, \ldots, v_n\}$ be a basis of V. Let σ be a permutation of the numbers $\{1, \ldots, n\}$, *i.e.*, $\sigma : \{1, \ldots, n\} \to \{1, \ldots, n\}$ is a bijective map. Then the linear transformation defined by $T(v_i) = v_{\sigma(i)}$ is

(A) 1-1 but not onto.

(B) onto but not 1-1.

(C) neither 1-1 nor onto.

(D) an isomorphism on V.

13. Let $\{x_n\}$ and $\{y_n\}$ be two sequences in $\mathbb{R}$ such that $\lim_{n\to\infty} x_n = 2$ and $\lim_{n\to\infty} y_n = -2$. Then

(A) $x_n \geq y_n$ for all $n \in \mathbb{N}$.

(B) $x_n^2 \geq y_n$ for all $n \in \mathbb{N}$.

(C) there exists an $m \in \mathbb{N}$ such that $|x_n| \leq y_n^2$ for all $n > m$.

(D) there exists an $m \in \mathbb{N}$ such that $|x_n| = |y_n|$ for all $n > m$.

14. Let $\{x_n\}$ be an increasing sequence of irrational numbers in $[0, 2]$. Then

(A) $\{x_n\}$ converges to 2.

(B) $\{x_n\}$ converges to $\sqrt{2}$.

(C) $\{x_n\}$ converges to some number in $[0, 2]$.

(D) $\{x_n\}$ may not converge.

15. Let X be a set. For $A \subset X$, let $A^c = X \setminus A$. The correct statement for $A, B \subset X$ is:

(A) $A \setminus B = B^c \setminus A^c$, always.

(B) If $A \setminus B = B^c \setminus A^c$ then $A \subset B$ or $B \subset A$.

(C) If $A \setminus B = B^c \setminus A^c$ then $A \cap B = \emptyset$.

(D) If $A \setminus B = B^c \setminus A^c$ then $A = X$ or $B = X$.

16. The value of $\int_0^{2\sqrt{\pi}} |\pi - x^2| dx$ is

(A) $2\pi\sqrt{\pi}$. (B) $2\sqrt{\pi}$. (C) 2π. (D) none of the above.

17. Let $\bar{\mathbf{r}} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$ and ϕ be a homogeneous function with degree 3. Then $\operatorname{div}(\bar{\mathbf{r}}\phi)$ is

(A) 3ϕ. (B) 6ϕ. (C) 9ϕ. (D) 27ϕ.

18. Let $f : [-2, 5] \to \mathbb{R}$ be the function given by $f(x) = x^6 + 3x^2 + 60$. Then

(A) f is a bounded function.

(B) there exists a $c \in [-2, 5]$ such that $f(c) = 0$.

(C) f is increasing.

(D) f is decreasing.

19. Write the logical negation of the following statement about a sequence $\{a_n\}$ of real numbers:

" For all $n \in \mathbb{N}$ there exists an $m \in \mathbb{N}$ such that $m > n$ and $a_m \neq a_n$. "

(A) There exists an $n \in \mathbb{N}$ such that $a_m = a_n$ for all $m > n$.

(B) For all $n \in \mathbb{N}$ there exists $m \in \mathbb{N}$ such that $m > n$ and $a_m = a_n$.

(C) There exists $n \in \mathbb{N}$ such that $a_m \neq a_n$, for all $m > n$.

(D) There exists $n \in \mathbb{N}$ such that $a_m = a_n$, for all $m \leq n$.

20. Let G be a group of order 6. Then

(A) G has 2 possibilities (upto isomorphism).

(B) G is cyclic.

(C) G is abelian but not cyclic.

(D) there is not sufficient information to determine G.

21. The value of $\int_0^1 (1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \cdots + \frac{(-1)^n x^n}{n!} + \cdots) e^x dx$ is

(A) 0. **(B)** e. **(C)** 1. **(D)** not defined.

22. The general solution of the differential equation $y''' - 5y'' + 8y - 4 = 0$ is

(A) $ae^{2x} - bxe^{2x} - ce^x$. **(B)** $e^{2x} + xe^{2x} + e^x$.

(C) $ae^{2x} + be^x$. **(D)** $ae^x + b(e^{2x} + xe^{2x})$.

23. The number of common tangents to the spheres $x^2 + y^2 + z^2 - 2x - 4y + 6z + 13 = 0$ and $x^2 + y^2 + z^2 - 6x - 2y + 2z - 5 = 0$ is

(A) 0. **(B)** 1. **(C)** 3. **(D)** 4.

24. Let $f(x) = \begin{cases} \frac{\sin x}{x}, & \text{if } x > 0, \\ \cos x, & \text{if } x \leq 0. \end{cases}$ Then,

(A) f is continuous but not differentiable at 0.

(B) f is differentiable at 0.

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(C) f is not continuous at 0.

(D) f is neither integrable nor continuous 0.

25. The least positive integer n such that every integer is greater than n is of the form $2a + 11b$ for some positive integers a and b is

(A) 13. (B) 23. (C) 35. (D) 44.

Part B

In Part-B, each question carries **3 marks.** Some questions have MORE THAN ONE correct option. All the correct options for each question have to be marked in the OMR answer sheet to score 3 marks, otherwise ZERO marks will be credited.

26. Let $f(x) = \max\{\sin x, \cos x\}$, for $x \in \mathbb{R}$. Then

    (A) f is discontinuous at $(2n+1)\pi/4$, $n \in \mathbb{Z}$.

    (B) f is continuous everywhere.

    (C) f is differentiable everywhere.

    (D) f is differentiable everywhere except at $(4n+1)\pi/4$, $n \in \mathbb{Z}$.

27. $\lim_{n\to\infty} \left(\frac{1}{n} + \frac{1}{n+1} + \cdots + \frac{1}{3n}\right) =$

    (A) 0. **(B)** $\log 2$. **(C)** $\log 3$. **(D)** ∞.

28. Let $f : \mathbb{R} \to \mathbb{R}$ such that $(f(x))^2 = 1 + \int_0^x f(t)dt$. Then $f(x) =$

    (A) $\frac{x}{2} - 1$. **(B)** $\frac{x}{2} + 1$.

    (C) $\frac{x}{2}$. **(D)** $\frac{x}{2} \pm 1$.

29. Let $A \neq \pm I$ be a 2×2-matrix over $\mathbb{R}$ whose square is I. Then which of the following statements are correct:

    (A) A is a diagonal matrix.

    (B) sum of the diagonal elements of A is 0.

    (C) there are infinitely many such matrices.

    (D) 1 must be an eigen value.

30. Let $f : (0, \pi/2) \to \mathbb{R}$ given by $f(x) = \sin x + \cos 2x$ is

    (A) increasing in $(0, \pi/4)$.

    (B) decreasing in $(0, \pi/4)$.

    (C) has a minimum in $(0, \pi/4)$.

    (D) has a maximum in $(0.\pi/4)$.

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31. The numbers $0, 1, 2, \ldots, 9$ are arranged randomly (without repetitions) in a row to get a 10-digit number greater than 10^9. What is the probability that the number so obtained is a multiple of 5?

(A) $\frac{8 \times 8!}{9 \times 9!}$. (B) $\frac{2 \times 9!}{9 \times 9!}$.

(C) $\frac{2 \times 9!}{10!}$. (D) $\frac{8 \times 8!}{9 \times 9!} + \frac{9!}{9 \times 9!}$.

32. Let f, g be Riemann integrable on $[a, b]$. Define, $h(x) := \min(f(x), g(x))$ and $l(x) := \int_a^x f(t)dt$ for $x \in [a, b]$. Then

(A) h need not be Riemann integrable but l always is.

(B) l need not be Riemann integrable but h always is.

(C) h, l are Riemann integrable, always.

(D) whenever h and l are Riemann integrable, $\int_a^x h(t)dt \leq l(x)$ for all $x \in (a, b)$.

33. Let $f : \mathbb{R} \to \mathbb{R}$ be a continuous function. Which of the following are sufficient conditions for f to have a fixed point in $[0, 1]$?

(A) $f(0) = f(1)$. (B) $f(1) < 0 < f(0)$.

(C) $0 < f(1) < f(0)$. (D) $f(0) < 0 < 1 < f(1)$.

34. Let $x_n \in \mathbb{R}$ such that $\sum_{n=1}^{\infty} x_n = -5$. Then

(A) $\lim_{n\to\infty} x_n = 0$.

(B) there exists an $m \in \mathbb{N}$ such that $x_n \leq 0$ for all $n > m$.

(C) $\sum_{n=1}^{\infty} |x_n| = 5$.

(D) $|x_n| \leq 5$ for all $n \in \mathbb{N}$.

35. Which of the following series are convergent?

(A) $\sum_{n=1}^{\infty} \frac{(-1)^n + \frac{1}{2}}{n}$.

(B) $\sum_{n=1}^{\infty} e^{-n} n^2$.

(C) $\sum_{n=1}^{\infty} \frac{1+2+\cdots+n}{1^2+2^2+\cdots+n^2}$.

(D) $\sum_{n=1}^{\infty} \frac{1.2.3}{4.5.6} + \frac{7.8.9}{10.11.12} + \cdots$.

36. In a class with 200 students, all the students know either Hindi, English or Telugu (and no other language). Of them, 100 know English, 150 know Telugu, 80 know Hindi, 50 know both Telugu and Hindi, 40 know only Telugu and no other language, 10 know all the three languages. Which of the following statements are correct:

(A) 30 students know only English and no other language.

(B) 110 know atleast two languages.

(C) 60 know Hindi but not English.

(D) 110 know exactly two languages.

37. For a group G, which of the following statements are true?

(A) If $x,\ y \in G$ such that order of x is 3, order of y is 2 then order of xy is 6.

(B) If every element is of finite order in G then G is a finite group.

(C) If all subgroups are normal in G then G is abelian.

(D) If G is abelian then all subgroups of G are normal.

38. Let V be the vector space of all polynomials with coefficients in $\mathbb{R}$, *i.e.*, $V = \mathbb{R}[X]$. Then which one of the following $T : V \to V$ are **not** linear transformations: for $f(X)$ in V, define $T(f(X))$ as

(A) $f(X^2)$. (B) $f(X)^2$.

(C) $X^2 f(X)$. (D) $f(X^2+1)$.

39. Let S, T and U be three sets of horses. Let U be the set of all white horses. If all the horses in the set S are white and if no horse in the set T is black, then it necessarily follows that:

(A) S and T are disjoint. (B) $S \subset U$.

(C) $U \subset T$. (D) $S \cap T = U$.

40. For the real number system, which of the following statements are true:

(A) let $x, y \in \mathbb{R}$ such that $0 < x < y$ then there exists an $n \in \mathbb{N}$ such that $y < nx$.

(B) let $x, y \in \mathbb{R}$ such that $x < y$ then there exists an $r \in \mathbb{Q}$ such that $x < r < y$.

(C) let $x, y \in \mathbb{R}$ such that $x < y$ then there exists an $\alpha \in \mathbb{R} \setminus \mathbb{Q}$ such that $x < \alpha < y$.

(D) For $y \in \mathbb{R}$ such that $y > 0$ there exists an $n \in \mathbb{N}$ such that $n \leq y < n + 1$.

41. The value of α such that the sum of the squares of the roots of $x^2 - (\alpha - 2)x - \alpha - 1$ is minimum is

(A) 0. **(B)** 1. **(C)** $1/\sqrt{2}$. **(D)** 4.

42. An integrating factor for $ydx + (x - 2x^2y^3)dy = 0$ is

(A) $\dfrac{1}{x^2 + y^2}$. **(B)** $\dfrac{1}{x + y}$.

(C) $e^{\frac{1}{x^2y^2}}$. **(D)** $\dfrac{1}{x^2y^2}$.

43. Let $\overline{r} = x\hat{i} + y\hat{j} + z\hat{k}$, which of the following statements are true:

(A) There exists a non constant vector valued function $\overline{f}$ such that $\overline{f}$ is both irrotational and solinoidal.

(B) $\text{div}(\text{curl}(\overline{r}|\overline{r}|^2) = 0$.

(C) $\text{curl}(\text{grad}(|\overline{r}|^6)) = 0$.

(D) If $\overline{f}$ is solinoidal then $\text{div}(|\overline{r}|^2\overline{f}) = 2\overline{r} \cdot \overline{f}$.

44. Let B be the unit sphere in $\mathbb{R}^3$. The value of $\displaystyle\iint_B (x^2 + 2y^2 - 3z^2)dS$ is

(A) 4π. **(B)** $\frac{4}{3}\pi$.
(C) 6π. **(D)** none of the above.

45. Let $A, B \subset [0, 1]$ be two uncountable sets. Which of the following are **false** statements:

(A) If $A \cap B = \phi$, then either $\sup(A) \leq \inf B$ or $\sup B \leq \inf A$.

(B) If $A \cap B = \phi$, then $[0, 1] \setminus (A \cap B)$ is countable.

(C) If $\inf A = \inf B$ and $\sup A = \sup B$, the $A \cap B \neq \phi$.

(D) If $A \subset B$, then $B \setminus A$ is countable.

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46. The value of λ such that the plane $2x - y + \lambda z = 0$ is a tangent plane to the sphere $x^2 + y^2 + z^2 - 2x - 2y - 2z + 2 = 0$ is

(A) 4. (B) 1. (C) 2. (D) -2.

47. The distance between the straight lines

$$\frac{x}{2} = \frac{y-1}{1} = \frac{z+1}{1} \quad \text{and} \quad \frac{x+1}{2} = \frac{y+1}{1} = \frac{z+1}{1}$$

is

(A) $\frac{\sqrt{21}}{3}$. (B) $\frac{21}{9}$.

(C) 0. (D) impossible to find from the given data.

48. Let V be a 3-dimensional vector space over $\mathbb{C}$. Let $T : V \to V$ be a linear transformation whose characteristic polynomial is $(X-2)(X-1)(X+1)$. Let B be a basis of V. Then which of the following are correct?

(A) The matrix of T w.r.t B is conjugate to $\begin{pmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$.

(B) The matrix of T^{-1} w.r.t B is conjugate to $\begin{pmatrix} 1/2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$.

(C) The matrix of T w.r.t B is conjugate to $\begin{pmatrix} 2 & 1 & 0 \\ 0 & -1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$.

(D) The matrix of T w.r.t B is conjugate to $\begin{pmatrix} 2 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$.

49. Let S be the group of all permutation of the letters U,N,I,V,E,R,S,I,T,Y such that the letter 'I' is fixed. Then

(A) There exists elements of order 21 and of order 11.

(B) There exists an element of order 21 and no element of order 11.

(C) There is an element of order 11 and no element of order 21.

(D) There are no elements of order 11 or of order 21.

50. The least positive integer r such that $\binom{2014}{r}$ is a multiple of 10 is

(A) 5. (B) 10. (C) 11. (D) 14.