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University of Hyderabad,
Entrance Examination, 2012

Ph.D. (Statistics-OR)

Hall Ticket No.	

Answer Part A by **circling** the correct letter in the array below:

1	a	b	c	d
2	a	b	c	d
3	a	b	c	d
4	a	b	c	d
5	a	b	c	d
6	a	b	c	d
7	a	b	c	d
8	a	b	c	d
9	a	b	c	d
10	a	b	c	d
11	a	b	c	d
12	a	b	c	d
13	a	b	c	d
14	a	b	c	d
15	a	b	c	d
16	a	b	c	d
17	a	b	c	d
18	a	b	c	d
19	a	b	c	d
20	a	b	c	d
21	a	b	c	d
22	a	b	c	d
23	a	b	c	d
24	a	b	c	d
25	a	b	c	d

Time- **2** Hours

Max. Marks: **75**
Part A:**25**
Part B:**50**

Instructions

1. calculators are not allowed.
2. Part A carries 25 marks. Each correct answer carries **1 mark** and each wrong answer carries **-0.33 mark**. If you want to change any answer, cross out the old one and circle the new one. Over written answers will be ignored.
3. Part B carries 50 marks. instructions for answering Part B are given at the beginning of Part B.
4. Use a separate booklet for Part B.

Part-A

- Find the correct answer and mark it on the OMR sheet. Each correct answer gets 1 (one) mark and wrong answer gets -0.33 marks..

1. Two squares are chosen at random on a chess board. What is the probability that they have one common side?

   (a) $\frac{1}{18}$ (b) $\frac{32}{2016}$ (c) $\frac{49}{64}$ (d) $\frac{1}{36}$

2. Let $X_1, X_2 X_3, X_4$ be independent random variables. X_2, X_3, X_4 have Poisson distribution with mean 5, further $Y = \sum_{i=1}^{4} X_i \sim$ Poisson(25). The distribution of X_1 is

   (a) Binomial (b) Exponential (c) Rectangular (d) Poisson

3. Let X_1, X_2 be *i.i.d.* random variables with pdf $f(x)$, define $Y_1 = \min\{X_1, X_2\}$ and $Y_2 = \max\{X_1, X_2\}$, the joint pdf of Y_1 and Y_2 is

   (a) $f(y_1)f(y_2)$ (b) $f^2(y_1)f(y_2)$ (c) $2f(y_1)f(y_2)$ (d) $f(y_1)f^2(y_2)$

4. For two events A and B, $P(A|B) = 1$, so

   (a) $P(B^c|A^c) = 0$ (b) $P(B^c|A^c) = 1$ (c) $P(B^c|A^c) = \frac{1}{2}$ (d) $P(B^c|A^c) = \frac{3}{4}$

5. Let x_1 and x_2 be two independent observations of a Bernoulli random variable that takes values 1 or 0 with probabilities θ and $(1-\theta)$ respectively. If $\theta \in [\frac{1}{3}, \frac{2}{3}]$, the maximum likelihood estimate of θ is ?

   (a) $\frac{x_1 + x_2}{2}$ (b) $\frac{2 + x_1 + x_2}{6}$ (c) $\frac{x_1 + 2x_2}{6}$ (d) $\frac{3 + 2x_1 + 2x_2}{6}$

6. X_1, X_2, X_3 is a random sample from the $N(0,1)$, define $Y_1 = X_1 + X_2 + X_3$, then $V(Y_1|X_2)$ is

   (a) 6 (b) 3 (c) 2 (d) 1

7. Five numbers are drawn from the set $\{1, 2, \ldots, 100\}$ by SRSWOR, the probability p that their median is at least 20 is

   (a) less than 0.25 (b) $0.25 \leq p < 0.5$ (c) $0.5 \leq p < 0.8$ (d) $0.8 \leq p < 0.99$

8. $X \sim U(0, 1)$ and $Y \sim B(10, X)$, then $V(Y)$ is

   (a) 5 (b) 6 (c) 10 (d) 12

9. If for a random variable X, $E(X) = 1$ and $E(X^2) = 3$, then

   (a) $P(-3 \leq X \leq 3) < 0.5$ (b) $P(-3 \leq X \leq 3) > 0.75$

   (c) $P(-3 \leq X \leq 3) = 0.6$ (d) $P(-3 \leq X \leq 3) = 0.5$

10. $\lim_{n\to\infty} \left(1 - \frac{a_n}{n}\right)^2$ where $a_n = \left(1 + \frac{1}{n}\right)^n$ is equal to

    (a) 1 (b) e^e (c) e^{-e} (d) $e^{e^{-1}}$

11. Based on a random sample of size 16 from the $N(\mu, \sigma^2)$ population. The 95% confidence interval for μ was $[39, 52]$. It means

    (a) the mean of this random variable is certainly in the interval $[39, 52]$

    (b) one is 95% sure that the mean is in the interval $[39, 52]$

    (c) the mean is greater than 52 with probability 0.025

    (d) none of the above

12. In a simple regression of Y on X, what is the correlation coefficient between Y and $\hat{Y}$- the predicted value of Y if the correlation between X and Y is $\frac{1}{2}$.

    (a) 0 (b) $\frac{1}{2}$ (c) $-\frac{1}{2}$ (d) 1

13. $X_1, \ldots, X_n$ is a random sample from the $U(a - \theta, b + \theta)$, $a < b$, $\theta \geq 0$. Let $X_{(1)} = \min(X_1, \ldots, X_n)$, $X_{(n)} = \max(X_1, \ldots, X_n)$ and $\overline{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$, the joint sufficient statistic for (a, θ, b) is

    (a) $(-X_{(1)}, X_{(n)})$ (b) $(X_{(1)}, \overline{X}, X_{(n)})$ (c) $(X_{(1)} + X_{(n)})$ (d) $(X_1, \ldots, X_n)$

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14. X_1 and X_2 are *i.i.d.* $N(0,1)$ random variables, then $X_1 + X_2$ and $X_1 - X_2$

(a) have different expected values (b) are uncorrelated but not independent

(c) are independent (d) have different variances

15. If the characteristic function of *i.i.d.* random variables X_1 and X_2 is $\phi(t)$ where $\phi : \mathbb{R} \to \mathbb{R}$, then the characteristic function of $X_1 - X_2$ is

(a) $\phi(t)$ (b) $\phi^2(t)$ (c) $-\phi(t)$ (d) $\phi(\frac{1}{t})$

16. $X_1, \ldots, X_n$ is a random sample from the $N(\mu, \sigma^2)$ population. If $T_1 = X_1 +, \ldots, +X_n$ and $T_2 = X_1^2 +, \ldots, +X_n^2$ then which of the following statements is correct regarding sufficient statistics for μ and σ^2

(a) T_1 is sufficient statistic for μ

(b) T_2 is sufficient statistic for σ^2

(c) $T_2 - T_1$ is sufficient for (μ, σ^2)

(d) (T_1, T_2) is sufficient for (μ, σ^2)

17. A is an interval $(2, 5]$ and B is an interval $[3, 7)$, then $A \triangle B$ is

(a) a closed interval

(b) an open interval

(c) an open set that is not an interval

(d) a finite set

18. The one step transition probability matrix of a homogeneous Markov chain $\{X_n,\ n \geq 0\}$ is

$$\begin{bmatrix} 1/3 & 1/6 & 1/6 & 1/3 \\ 1/4 & 1/4 & 1/4 & 1/4 \\ 1/5 & 1/4 & 1/4 & 3/10 \\ 1/6 & /1/6 & 1/3 & 1/3 \end{bmatrix}.$$

Then which of the following is not correct?

(a) $P(X_7 = 1|X_5 = 2) = P(X_{11} = 1|X_9 = 2)$ (b) this Markov chain is irreducible (c) the states 1 and 2 are the only recurrent states (d) it is a recurrent Markov chain

19. Each of the seven treatments have to appear in blocks of sizes four each. Which of the following choices on "number of blocks" and "number of blocks in which a pair of treatment appear" respectively, gives a valid BIBD.

(a) $6, 3$ (b) $7, 2$ (c) $8, 3$ (d) $7, 1$

20. In a factorial design, it is decided to confound the effect $ABCD$. Blocks 1 and 2 in a replication contain the following combinations or treatments

Block 1: a b c abc d abd

Block 2: ab ac bc bd cd abcd

What are the other treatments in blocks 1 and 2 respectively?

(a) $\{(1), acd\}$ and $\{bcd, ad\}$ (b) $\{(1), bcd\}$ and $\{acd, ad\}$ (c) $\{(1), ad\}$ and $\{acd, bcd\}$ (d) $\{acd, bcd\}$ and $\{(1), ad\}$

21. $\{X_n\}$ is a sequence of independent random variables with pmf $P(X_n = -n) = P(X_n = n) = \dfrac{1}{n^2+1}$, $P(X_n = 0) = 1 - \dfrac{2}{n^2+1}$. Let $S_n = X_1 + \ldots + X_n$. Then

(a) $P(|S_n/n| > \epsilon \text{ for infinitely many n }) = 0$

(b) $P(|S_n/n| > \epsilon \text{ for infinitely many n }) = 1$

(c) $E(S_n/n) \nrightarrow 0$

(d) $\lim_{n\to\infty} P(|S_n/n| > \epsilon\ ) = 1/2$

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22. The dispersion matrix of a random vector $(X_1, X_2, X_3)'$ is $\begin{bmatrix} 10 & 5 & 5 \\ 5 & 9 & a \\ 5 & a & 16 \end{bmatrix}$. The value of a, so that $X_1 + X_2 + X_3$ and $X_1 - 2X_2 + X_3$ are uncorrelated, is

(a) 8 (b) 21 (c) 13 (d) 16

23. In a complete randomized design, for three treatments whose effects are $\alpha_1, \alpha_2, \alpha_3$, which of the following is testable?

(a) $\alpha_1 = 2$

(b) $\alpha_1 + \alpha_2 - \alpha_3 = 0$

(c) $\frac{\alpha_1+\alpha_2}{2} = \alpha_3$

(d) $\alpha_1 + \alpha_2 + \alpha_3 = 0$

24. Shoppers arrive at a mall in accordance with a homogeneous Poisson Process, if the expected number of arrivals in an hour is 600, the expected time between consecutive arrivals is

(a) 1 min

(b) 10 second

(c) 6 second

(d) different between different consecutive pairs of arrivals

25. $X_n \sim U(-\frac{1}{n}, \frac{1}{n})$, $n = 1, 2, \ldots$ then

(a) $V(X_n) \nrightarrow 0$

(b) $X_n \nrightarrow 0$ in probability but $X_n \to 0$ weakly

(c) $X_n \nrightarrow 0$ weakly

(d) $X_n \to 0$ in probability

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Part-B

- Answer as much as you can,the maximum marks that you can score is **50**.

1. a) 7 girls and 8 boys are randomly arranged in a row. Determine the probability of the event that no two girls are sitting together.

   b)Two numbers are drawn from the set $\{1, 2, \ldots, 100\}$ by SRSWOR, determine the probability that the largest of the two is a prime number. (5+5)

2. X_1 and X_2 are *i.i.d.* $\exp(\lambda)$ random variables. Determine $E(X_1|X_1+X_2=10)$ and $V(X_1|X_1+X_2=10)$. (5+5)

3. $2.8, 3.2, 4.1, 2.2, 1.8, 2.7$ are six independent observations of a random variable X with pdf

$$f(X, \mu, \lambda) = \frac{1}{\lambda}e^{(-\frac{x-\mu}{\lambda})},\ x \geq \mu$$

   where $\lambda \geq 0$ and $-\infty < \mu < \infty$. Assume $\mu = 1$ and construct a 90% confidence interval for λ based on the given sample. 5

4. Let $X_1, \ldots, X_n$ be *i.i.d.* with pdf

$$f(x; \theta) = \theta x^{\theta-1},\ 0 < x < 1,\ \theta > 0$$

   a) Find the MLE of θ.

   b) Find the asymptotic distribution of the MLE. (5+5)

5. In a bag there are N slips numbered $1, 2, \ldots, N$, N not known. Draw a SRSWR of size n. Let $X_1, \ldots, X_n$ denote the numbers drawn, obtain the most powerful level α test for $H_0 : N = N_0$ vs. (i)$H_1 : N > N_0$, (ii)$H_1 : N \neq N_0$ 5

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6. A school is preparing a trip for 400 students. the company who is providing the transportation has 10 buses of 50 seats each and 8 buses of 40 seats, but only 9 drivers available. The rental cost for a large bus is $Rs.$ 8000 and $Rs.$ 6000 for small bus. Calculate how many buses of each type should be used for the trip for at least possible cost. 10

7. Find the optimal solution of the dual of the following LPP(P1) and hence the solution to this P1.

   P1: maximize $x_1 + x_2 + 2x_3$

   Subject to $x_1 + 2x_2 \leq 3$, $2x_1 + x_2 + 2x_3 \leq 1$ and all $x_i \geq 0$ 10

8. $X_1, \ldots$ are independent random variables with the following probability distribution:

$$P(X_j = j) = P(X_j = -j) = \frac{1}{j+1}, \; P(X_j = 0) = 1 - \frac{2}{j+1}, \; j = 1, 2, \ldots$$

   a) Can you say that i) $X_n \to 0$ in probability?

   ii) $X_n \to 0$ with probability 1?

   Explain or prove as the case may be.

   b) What can you say about the limiting distribution (as $n \to \infty$), of $T_n = \frac{1}{n\sqrt{2}} \sum_{j=1}^{n} X_j \sqrt{1 + \frac{1}{j}}$? (5+5)