Entrance Examination : M.Sc. Statistics, 2012

Hall Ticket Number | | | | | | | |

Time : 2 hours

Max. Marks. 100

Part A : 25 marks

Part B : 75 marks

Instructions

1. Write your Hall Ticket Number on the OMR Answer Sheet given to you. Also write the Hall Ticket Number in the space provided above.
2. Answers are to be marked on the OMR answer sheet.
3. Please read the instructions carefully before marking your answers on the OMR answer sheet.
4. Hand over the question paper along with the OMR answer sheet after the examination.
5. No additional sheets will be provided. Rough work can be done in the question paper itself/space provided at the end of the booklet.
6. Calculators are not allowed.
7. There are a total of 50 questions in Part A and Part B together.
8. **Each question in Part A has only one correct option and there is negative marking.**
9. **In Part, B some questions have more than one correct option. All the correct options have to be marked in the OMR answer sheet otherwise zero marks will be credited**
10. The appropriate answer(s) should be coloured with either a blue or a black ball point or a sketch pen. DO NOT USE A PENCIL.
11. THE MAXIMUM MARKS FOR THIS EXAMINATION IS 100 AND THERE WILL BE NO INTERVIEW.

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Part-A

- Find the correct answer and mark it on the OMR sheet. Each correct answer gets 1 (one) mark and wrong answer gets -0.33 marks..

1. The mean of 15 distinct numbers is 31 and the median is 28, suppose the largest is reduced by 4 and the smallest is increased by 1 and the next one by 3,then which of the statements is correct regarding the mean and median of the modified numbers?

   [A] The mean and median remain the same as before.

   [B] The mean decreases but the median increases.

   [C] Both the median and mean increase.

   [D] the mean increases but the median decreases.

2. Let $f(x)$ be the pdf of a continuous random variable defined on $\mathbb{R}$,

   [A] f is an increasing function on $\mathbb{R}$.

   [B] f is a continuous function.

   [C] $f \geq 0$ for all x.

   [D] f is always bounded.

3. A and B are two independent events with equal probabilities, if $P(A \bigcup B) = 0.75.$, then

   [A] $P(A) = P(B) = 1/4$. [B] $P(A) = P(B) = 3/4$.

   [C] $P(A) = P(B) = 1/2$. [D] $P(A) = P(B) = 1$.

4. Let $\overline{X}$ be the sample mean,based on a sample $X_1, \ldots, X_n$, the maximum likelihood estimator for σ^2 (variance) of a normally distributed random variable with mean 1 is

   [A] $\frac{\sum_{i=1}^{n} X_i^2}{n} - 1$. [B] $\frac{\sum_{i=1}^{n}(X_i - \overline{X})^2}{n}$.

   [C] $\frac{\sum_{i=1}^{n}(X_i - 1)^2}{n}$. [D] $(\overline{X} - 1)^2$.

5. Let X be normally distributed with mean 10 and variance 100, the value of b for which $E|X - b|$ is least is

   [A] 0. [B] 5.

   [C] 10. [D] none of the above.

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6. 4 dice have been rolled. What is the probability that the sum is less than 25?

   [A] 0. [B] 1/2.

   [C] 1/6. [D] 1.

7. One out of 3 balls in a bag is red and the two are blue, one ball is removed, the probability that one of the remaining two is red is

   [A] 1/4. [B] 1/3.

   [C]1/2. [D] 2/3.

8. The probability that at least one of A and B occurs is 0.9 and probability that both A and B occur is 0.4, the probability that atmost one of A and B occurs

   [A] 0.6. [B] 0.5.

   [C] 0.4. [D] cannot be determined from the given data.

9. How many of 30 balls should be red and how many blue so that the number of possible arrangements is maximized?

   [A] Red–10, Blue–20. [B] Red–0, Blue–30.

   [C] Red–25, Blue–5. [D] None of the previous options.

10. The probability density function of a real valued random variable X is $f_X(x) = 0.5\exp(-|x|)$, $-\infty < x < \infty$. If for some $a > 0$, $P(X > a) = \alpha$, then

    [A] $P(X < -a) = \alpha$. [B] $P(X < -a) = 1 - \alpha$.

    [C] $P(|X| > a) = \alpha$. [D] $P(X > -a) = \alpha$.

11. The sum of 10 real numbers is hundred, the sum of their reciprocals can be

    [A] 1/3. [B] 1/2.

    [C] 3/4. [D] 2.

12. If the rank of 3×3 real matrix A is 3, it means that

    [A] the determinant of A is zero.

    [B] one of the rows of A is a linear combination of other rows.

    [C] one of the columns of A is a linear combination of other columns.

    [D] the rows of A form a basis in $\mathbb{R}^3$.

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13. The expected value of a random variable X taking positive integer values is 2, further $P(X = 1) = 0.1$, $P(X = 2) = 0.2$, $P(X = 3) = 0.3$, we can say that

[A] $P(X = 4)$ can be equal to 0.16.

[B] $P(X = 4)$ is certainly less than 0.16.

[C] $P(X = 5)$ can be equal to 0.2.

[D] $P(X = 5)$ is more than 0.16.

14. The sum of three numbers is equal to 4, then their product is

[A] at least 4. [B] less than 3.

[C] equal to 3. [D] equal to 4.

15. The correlation coefficient between two random variables X and Y- $\rho_{X,Y}$ is 1/2, define $U = 2X - 1$, and $V = -Y + 18$ so $\rho_{U,V}$ is equal to

[A] 1/2. [B] $-1/2$.

[C] 0. [D] $-1/18$.

16. Let X and Y be binomial random variables with parameters $(n, 1/2)$ and $(n, 1/3)$ respectively, then

[A] $P(X \leq j) \leq P(Y \leq j)$, for $j = 0, 1, 2, \ldots, n$. [B] $E(X) < E(Y)$.

[C] $V(X) < V(Y)$ [D] none of the previous options.

17. Which of the following random variables is bimodal?

[A] $X_1 \sim P(10)$ Poisson random variable with parameter 10.

[B] $X_2 \sim B(15, 1/2)$ Binomial random variable with parameters (15, 1/2).

[C] $X_3 \sim N(10, 25)$ normal random variable with mean 10 and variance 25.

[D] $X_4 \sim \exp(5)$ the exponential random variable with parameter 5.

18. 6 girls $G_1, G_2, \ldots, G_6$ and 10 boys $B_1, B_2, \ldots, B_{10}$ are randomly made to sit in a row. What is the probability that none of the girls is at either end?

[A] 1/4 [B] 3/8 [C] 1/2 [D] 5/8

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19. Given below are marks of 6 students in English and Mathematics

English	62	63	64	65	66	67
Mathematics	92	92	92	92	92	97

[A] Variances of marks in the two subjects are the same.

[B] Variance of marks in English is less than the variance of marks in Mathematics.

[C] The correlation between the marks in the two subjects is zero.

[D] The correlation between the marks is almost 1.

20. T_1 and T_2 are two unbiased estimators of a parameter θ, however, $V(T_1) < V(T_2)$. So to estimate θ we should use the

[A] mean of the observed values of T_1 and T_2 because we will be using more information then.

[B] observed values of T_1 and T_2 by random selection to be unbiased.

[C] observed value of T_1 because it will be certainly closer to θ than the observed value of T_2.

[D] observed value of T_1 because it is more likely to be closer to θ than the observed value of T_2.

21. Consider the following linear programming problem.

$$\begin{aligned} \text{maximize} \quad & 4x_1 + 6x_2 \\ \text{subject to} \quad & x_1 + 2x_2 \leq 31 \\ & 2x_1 + x_2 \leq 25 \\ & 3x_1 + x_2 \leq 30 \\ & x_1, x_2 \geq 0. \end{aligned}$$

Which of the following is an optimal solution.

[A] $x_1 = 6.2$, $x_2 = 12$. [B] $x_1 = 5.8$, $x_2 = 12.6$.

[C] $x_1 = 7.1$, $x_2 = 12$. [D] $x_1 = 10$, $x_2 = 12$.

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22. Based on a sample of size n from the $N(\mu, 9)$ population to test $H_0 : \mu = \mu_0$ vs $H_1 : \mu > \mu_0$, the most powerful level α test criterion is: reject H_0 if $\overline{X}_n > a$ where $\overline{X}_n$ is the sample mean. Now suppose variance is not 9 but 16, let n' be the sample size required so that the test criterion is: reject H_0 if $\overline{X}_{n'} > a$ is the most powerful level α test then

[A] is equal to n. [B] is equal to $2n$.

[C] is more than n but less than $2n$. [D] is less than n.

23. The series $\sum_{n=1}^{\infty} \frac{10^n}{n!}$.

[A] diverges. [B] equal to a positive number greater than 20.

[C] equal to 3. [D] equal to 10.

24. the heights of adult ladies in our country are normally distributed with mean 150cm and variance 49cm^2, if $P(0 < Z \leq a) = 0.25$ where $Z \sim N(0, 1)$, then half the ladies have heights between

[A] $(150 - a)$cm and $(150 + a)$cm.

[B] 150cm and $(150 + 2a)$cm.

[C] $(150 - 7a)$cm and $(150 + 7a)$cm.

[D] $(150 - 7a)$cm and $(150 + a)$cm.

25. The statement "Ashok and Bharat saw Chandru at the cinema hall" is not true. It means

[A] Chandru was not at the cinema hall.

[B] neither Ashok nor Bharat saw Chandru at the cinema hall.

[C] Bharat was not at the cinema hall.

[D] none of the above.

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Part-B

- Questions (26)-(37) may have **more than** one correct option. For the answer to be right **all the correct options** have to be marked on the OMR sheet. **No credit** will be given for partially correct answers.
- Questions (38)-(50) may have **only one** correct option.
- Find the correct answers and mark them on the OMR sheet. Correct answers (marked in OMR sheet) to a question get 3 marks and zero otherwise.

26. Two events A and B satisfy the following conditions : $0 < P(A) < 1; 0 < P(B) < 1$ and $P(A|B) = P(A)$. Then

    [A] A and B are mutually exclusive.

    [B] $P(B|A) = P(B)$.

    [C] $P(A^c|B^c) = P(A^c)$.

    [D] A and B are independent events.

27. Pick up a number from the set $\{1, 2, \ldots, 100\}$. Let A and B denote the events that the selected number is odd and that the selected number is a prime respectively, so

    [A] $P(A) = P(A^c)$. [B] $P(B|A) > P(B)$.

    [C] $P(B^c|A^c) = P(B^c)$. [D] A and B are independent events.

28. The random variable X is normally distributed with mean 35 and variance 81. So

    [A] $E(X) = 45$. [B] $P(X > 55) = P(X < -55)$.

    [C] $P(X > 45) > P(X < 15)$. [D] $P(X > 45) = P(X < 25)$.

29. Bag 1 contains 60 balls, 20 each of them numbered 0,1 and 2. Bag 2 also contains 60 balls. But 25 each are numbered 0 and 1 and 10 are numbered 2. A ball is drawn from each of the bags, let X_1 and X_2 denote the numbers on the balls drawn from Bag 1 and bag 2 respectively. Then

    [A] $P(X_1 > 0) > P(X_2 > 0)$.

    [B] $P(X_1 > 1) > P(X_2 > 1)$.

    [C] $E(X_1) > E(X_2)$.

    [D] $E\left(\frac{1}{X_1+1}\right) > E\left(\frac{1}{X_2+1}\right)$.

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30. The arithmetic mean and median of 5 distinct natural numbers are both 7, the maximum of the 5 numbers

[A] could be 18. [B] is at least 19.

[C] is at most 17. [D] is at least 9.

31. 10 numbers are drawn from the set $\{1, 2, \ldots, 100\}$ without replacement. The probability that

[A] the mean of the selected numbers is more than 5.5 is greater than 0.99.

[B] the median of the selected numbers is 6 is less than 0.0001.

[C] the variance of the selected numbers is more than 8 is 1.

[D] the maximum of the selected numbers is more than 90 is greater than 0.5.

32. Consider the function

$$f(x) = \begin{cases} 0 & x < 0 \\ \sum_{j=0}^{n-1} \frac{\lambda^j}{j!} & (n-1) \leq x < n, n = 1, 2, \ldots \end{cases}$$

[A] f is continuous everywhere.

[B] f is bounded.

[C] f is differentiable everywhere.

[D] f is non-decreasing in x.

33. X_1, X_2, X_3 are independent and identically distributed random variables with mean θ, and variance 1. Let $Y_1 = \frac{X_1+X_2+X_3}{3}$ and $Y_2 = \frac{X_1+2X_2+3X_3}{6}$, so

[A] Y_1 and Y_2 are unbiased estimators for θ.

[B] Y_1 is an unbiased but Y_2 is not an unbiased estimator for θ.

[C] The correlation coefficient ρ_{Y_1,Y_2} between Y_1 and Y_2 is greater than 1/2.

[D] Variance of Y_1 is less than variance of Y_2.

34. Let m_n, M_n and v_n be the mean, median and variance respectively of the first $n(\geq 5)$ natural numbers, remove 1 and n from this set, let m'_n, M'_n and v'_n denote the same for the new set, then

[A] $m'_n = m_n$, $v'_n < v_n$. [B] $M'_n = M_n$, $v'_n < v_n$.

[C] $m'_n = m_n$, $M'_n = M_n$, $v'_n = v_n$. [D] $m'_n < m_n$, $M'_n < M_n$.

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35. A_1, A_2, A_3 and A_4 are independent events and $P(A_i) = 1/4$, for $i = 1, 2, 3, 4$ then

[A] the probability that at least one of them occurs is 1.

[B] the probability that at least one of them occurs is less than 3/4.

[C] the probability that at most one of them occurs is more than 1/4.

[D] the probability that exactly two of them occur is more than 1/4.

36. $X \sim B(60, \frac{1}{3}))$. So

[A] $P(X = 48) \leq P(X = 45)$. [B] $P(X = 20) \leq P(X = 48)$.

[C] $P(X = 6) \leq P(X = 54)$. [D] $P(X = 30) \leq P(X = 25)$.

37. X is a random variable with probability mass function $P(X = x) = pq^{x-1}, x = 1, 2, \ldots,$ $0 < p < 1, p + q = 1$. Then

[A] $P(X > j) = q^j$. [B] $E(X) > 1$.

[C] $V(X) > E(X)$. [D] $V(X) < E(X)$.

The next 13 questions have only one correct option.

38. A and B are two events with $P(A) = 2/3; P(B) = 2/3; P(A|B) = 3/4$. Then $P(A|B^c)$

[A] is equal to 1/3. [B] is equal to 1/2.

[C] is equal to 2/3. [D] cannot be determined from the given data.

39. Let (X, Y) denote a randomly selected point in a square of area 1. What is the probability of the event that $|X - Y| \leq 1/3$?

[A] 1/3. [B] 2/9.

[C] 4/9. [D] 5/9.

40. From a bag containing 6 white balls (all alike), either 1 or 2 or 3 or 4 or 5 or 6 are taken out with probabilities 1/6 each , then the balls drawn are painted red and returned to the bag (now this bag contains at least one red ball). Now a ball is taken from this bag and it is red. What is the probability that the number of balls taken out and painted red in the beginning was 4?

[A] 4/21. [B] 6/21.

[C] 1/3. [D] 8/21.

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41. A bag contains 100 slips. n_1 of them are numbered 1, n_2 are numbered 2, n_3 are numbered 3, n_4 are numbered 4 and the remaining n_5 are numbered 5. 50 draws are made with replacement. The frequency distribution of these 50 numbers is given below :

number	1	2	3	4	5
frequency	9	10	11	12	8.

To test the hypothesis that n_1, n_2, n_3, n_4 and n_5 are equal, the χ^2 goodness of fit statistic is

[A] 0. [B] 1. [C] 1/2. [D] 1/4.

42. The probability distribution of a random variable is as follows :

X	1	2	3	4
P(X=x)	p_1	$2p_1$	p_2	p_2

where $0 < p_2 < 1/2$. From the above distribution, we have the following sample of size 6 : 1, 3, 3, 4, 2, 2. The maximum likelihood estimate of p_1 is

[A] 1/4. [B] 1/5.

[C] 1/6. [D] 1/7.

43. Consider a data set of marks in a public exam. It is found that 75% of them are within 10 from the mean marks. According to Chebychev's inequality the standard deviation of the marks is

[A] less than 4. [B] equal to 5.

[C] equal to 4. [D] more than 5.

44. A coin for which the probability of heads showing up on tossing it is p is tossed 15 times. The first head appeared in the 3rd toss and 6 heads showed up in the 15 tosses, unbiased estimates for p and $1/p$ are respectively

[A] 6/15 and 15/6. [B] 1/3 and 3.

[C] 6/15 and 3. [D] 1/3 and 15/6.

45. A coin is tossed 7 times and the outcomes are HTTHHTH, if the probability of the coin showing up heads upon tossing is p, an unbiased estimate for p^2 is

[A] 1/2. [B] 2/7.

[C] 5/7. [D] 16/49.

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46. Let R_1, R_2 and R_3 be the rows of 3×3 non singular real matrix A, let the first row of B be $2R_1$, the second row of B be $R_2 + R_3$and suppose the third row of B is R_2, then

[A] B is singular. [B] rank(B)=3.

[C] rank(A+B)=2. [D] rank(B-A)=2.

47. Let X and Y be independent and identically distributed random variables with probability distributions given by $P(X = j) = \frac{1}{j(j+1)}$, $j = 1, 2, \ldots$ then the value of $P(X = Y)$ is in the interval

[A] $(0, 1/4]$. [B] $(1/4, 1/2]$.

[C] $(1/2, 3/4]$. [D] $(3/4, 1]$.

48. $X_1, \ldots, X_n$ is a random sample from the $N(\mu, \sigma^2)$ population. Let $\overline{X}$ be the sample mean and s^2 be the statistic $\frac{1}{n-1}\sum_{i=1}^{n}(X_i - \overline{X})^2$ an unbiased estimator for μ^3 is

[A] $\overline{X}^3$.

[B] $\overline{X}^3 - 3s^2$.

[C] $\overline{X}^3 - \frac{3 \times \overline{X} \times s^2}{n}$.

[D] $(\overline{X} - s^2/n)^3$.

49. X_1, X_2 are a random sample from the variable X with probability mass function $P(X = x) = p^x(1-p)^{1-x}$, $x = 0, 1$ and $0 < p < 1$ which of the following statements is correct?

[A] $X_1 - X_2$ is not a sufficient statistic for p.

[B] $(X_1 + X_2)/2$ is not a sufficient statistic for p.

[C] $(X_1 + X_2)/2$ is not an unbiased estimator for p.

[D] X_2 is a sufficient statistic for p.

50. Suppose X is a random variable following exponential distribution with mean $1/\lambda$ if its median is 0.6 the mean is

[A] $1/\log 2$. [B] $1/\log 4$.

[C] $0.6/\log 2$. [D] $\log 2/0.6$.

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