ENTRANCE EXAMINATION, JUNE 2011
QUESTION PAPER
Ph.D. (ACRHEM)

Marks: 75
Time: 2.00 Hrs.

Hall Ticket No. 

Please confirm that
(a) This booklet has all 28 pages (including 3 blank pages) printed clearly and numbered
(b) You are given a clean and clear OMR answer sheet.

Read carefully all the instructions given below & on the OMR sheet.

1. Please enter your Hall Ticket Number on Page 1 (this sheet) of this booklet without fail.
2. Please enter your Hall Ticket Number on the OMR Answer Sheet without fail.
3. All answers are to be marked on the OMR answer sheet following the instructions provided on the OMR answer sheet.
4. No additional sheets will be provided. Rough work is to be done in the booklet itself/space provided at the end of the booklet on pages 27 & 28.
5. Hand over both the question paper and OMR answer sheet at the end of the examination.
6. Question paper has two parts: Part-A and Part-B.
7. Part-A consists of 25 objective type questions of one mark each. There is negative marking of 0.33 marks for every wrong answer. The marks obtained by the candidate in this part will be used for resolving tie cases.
8. Part-B consists of three sections P (26-50), M (51-75) & C (76-100) each containing 25 questions.
9. One needs to answer any 25 questions from Part – B. Each correct answer carries two marks. There is no negative marking in these sections.
10. In case the number (N) of answered questions is greater then 25, in Part-B, then marks per question shall be 50/N.
11. Non programmable calculators are permitted.
12. All the symbols used in text have their usual meanings.
PART A

1. For a system with two linearly independent states $\left(\begin{array}{c} 0 \\ 1 \end{array}\right)$ and $\left(\begin{array}{c} 1 \\ 0 \end{array}\right)$ the eigenvalues and eigenvectors of the operator $\frac{1}{\sqrt{2}}\left(\begin{array}{cc} 1 & 0 \\ 0 & -1 \end{array}\right)$ are

A) $\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}; \frac{1}{\sqrt{2}} \left(\begin{array}{c} 1 \\ 1 \end{array}\right), -\frac{1}{\sqrt{2}} \left(\begin{array}{c} 1 \\ 1 \end{array}\right)$
B) $\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}; \frac{1}{\sqrt{2}} \left(\begin{array}{c} -1 \\ 1 \end{array}\right), -\frac{1}{\sqrt{2}} \left(\begin{array}{c} 1 \\ 1 \end{array}\right)$
C) $\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}; \frac{1}{\sqrt{2}} \left(\begin{array}{c} 1 \\ 0 \end{array}\right), -\frac{1}{\sqrt{2}} \left(\begin{array}{c} 0 \\ 1 \end{array}\right)$
D) $\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}; \frac{1}{\sqrt{2}} \left(\begin{array}{c} 1 \\ 1 \end{array}\right), -\frac{1}{\sqrt{2}} \left(\begin{array}{c} 1 \\ -1 \end{array}\right)$

2. To reduce the resolution and increase the sensitivity of a spectrometer the following needs to be done with the entrance slits

A) Widening the slits
B) Bring the slits closer
C) Make the slits closer to the wavelength of light
D) Remove the slits completely

3. The pole and order of the pole for $\frac{e^x}{1+e^x}$ are

A) $(2n+1)\pi, 0$
B) $(n+\frac{1}{2})\pi, 1$
C) $(n+\frac{1}{2})\pi, 1$
D) $(2n+1)\pi, 1$

4. EM wave is of the form $\vec{E} = E_0 \hat{x} \exp(ikz - i\omega t), \vec{B} = \frac{E_0}{c} \hat{y} \exp(ikz - i\omega t)$,

where $\hat{x}, \hat{y}$ are the unit vectors in the x, y directions and $E_0$ is the amplitude of the electric field. This represents

A) Circulary polarized plane wave
B) Linearly polarized spherical wave
C) Elliptically polarized cylindrical wave
D) Linearly polarized plane wave
5. For the Zener diode network shown below, $V_L$ and $P_Z$ are given respectively by

\[ 16V \quad \begin{array}{c}
1 \text{ KΩ} \\
V_L=10V
\end{array} \quad 1.2\text{ KΩ} \quad V_L \\
\text{P}_{zm}=30\text{mW} \]

A) 2.34 V, 26.7 mW  
B) 8.73V, 0 mW  
C) 2.34 V, 0 mW  
D) 10 V, 0 mW

6. A sphere of mass $m$ rolls from rest from the top at height $h$, of an inclined plane (at angle $\alpha$ to the horizontal). Its final velocity when it reaches the bottom is

A) $V = (2gh)^{1/2}$  
B) $V = (10/7gh)^{1/2}$  
C) $V = (mgh/5)^{1/2}$  
D) $V = (5/2 mgh)^{1/2}$

7. The Volume of the primitive cell in the hexagonal space lattice is

A) $\frac{\sqrt{3}}{2} a^2 c$  
B) $\frac{3}{\sqrt{2}} a^2 c$  
C) $a^2 c$  
D) $a^2 c / 2$

8. The interaction between magnetic atoms in a ferromagnetic material is called

A) Double exchange  
B) Super exchange  
C) Interaction exchange  
D) No exchange
9. The coefficient of $a^5b^6c^7$ in the expansion of $(a+b+c)^{12}$ is

A) \( \frac{18!}{5!6!7!} \)

B) \( \binom{12}{6} 30 \)

C) \( \binom{12}{6} 6! \)

D) \( \binom{12}{6} 6 \)

10. Which of the following molecule exhibit greater hydrogen bonding with phenol?

<table>
<thead>
<tr>
<th>A)</th>
<th>B)</th>
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<tbody>
<tr>
<td><img src="image1" alt="Molecule A" /></td>
<td><img src="image2" alt="Molecule B" /></td>
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<tr>
<td>C)</td>
<td>D)</td>
</tr>
<tr>
<td><img src="image3" alt="Molecule C" /></td>
<td><img src="image4" alt="Molecule D" /></td>
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</table>

11. The decreasing order of percentage of enol form in the following cis fused diketones is

<table>
<thead>
<tr>
<th>I</th>
<th>II</th>
<th>III</th>
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<tbody>
<tr>
<td><img src="image5" alt="Diketone I" /></td>
<td><img src="image6" alt="Diketone II" /></td>
<td><img src="image7" alt="Diketone III" /></td>
</tr>
</tbody>
</table>

A) $III > II > I$
B) $II > III > I$
C) $I > II > III$
D) $II > I > III$

12. Alkalides are

A) Alkali metals
B) Alkali metal cations
C) Alkali metal anions
D) Alkali metal halides
13. 18-Crown-6 has strong affinity towards
   A) Cl
   B) K
   C) NH
   D) Mn

14. The incorrect statement about Zeolites is
   A) They have a very rigid framework
   B) Contain channels and pores
   C) Form intercalated complexes
   D) Easily soluble in organic solvents

15. Number of nodal planes in s, p, d, and f – orbitals, respectively, are
   A) 0, 0, 0, 0
   B) 1, 1, 1, 1
   C) 1, 2, 3, 4
   D) 0, 1, 2, 3

16. Ferrocene is a neutral compound of iron where the oxidation state of Fe is
   A) -1
   B) 0
   C) +2
   D) +3

17. The function \( f(z) = |z|^2 \) is
   A) Everywhere analytic
   B) No where analytic
   C) Analytic at \( z = 0 \)
   D) None of these

18. Which of the following differential equations represents a bell shaped curve?
   A) \( y' = -3x \)
   B) \( y' = -3xy \)
   C) \( y' = -3x^2 y \)
   D) \( y' = -3y \)
19. The vectors \((a,1,0)\), \((1,a,1)\), \((0,1,a)\) in \(\mathbb{R}^3\) are linearly dependent when the scalar \(a\) is
   
   A) 0  
   
   B) \(\pm\sqrt{2}\)  
   
   C) \(\pm1\)  
   
   D) \(\pm\sqrt{3}\)

20. The order of secant method is
   
   A) 1.618  
   
   B) 1.5  
   
   C) 1.718  
   
   D) 1.318

21. The geometrical distribution is a special case of
   
   A) \(n = 1\) of Gamma distribution  
   
   B) \(n = 1\) of Binomial distribution  
   
   C) \(n = 1\) of Negative Binomial distribution  
   
   D) \(n = 1\) of Exponential distribution

22. The probability mass function of random variable \(X\) is \(f(x) = \frac{k}{x!}\), \(x = 0, 1, 2, \ldots\)

Then the value of \(k\) is
   
   A) \(E\)  
   
   B) \(\frac{1}{e}\)  
   
   C) \(1 - \frac{1}{e}\)  
   
   D) \(1 + \frac{1}{e}\)
23. The function \( y = f(x) \) satisfying the condition \( f\left(x + \frac{1}{x}\right) = x^2 + \frac{1}{x^2}, x \neq 0 \) is given by

A) \( y = x^2 + 2 \)
B) \( y = x^2 - 2 \)
C) \( y = x^2 + 1 \)
D) \( y = x^2 - 1 \)

24. The type I error in hypothesis testing is

A) The hypothesis is true and test accepts it
B) The hypothesis is true but test rejects it
C) The hypothesis is false but test accepts it
D) None of the above

25. The variance of first \( n \) natural numbers is

A) \( \frac{n^2(n+1)^2}{12} \)

\[ \left( \frac{n + 1}{2} \right)^2 \]

B) \( \frac{n^2 + 1}{12} \)
C) \( \frac{n^2 - 1}{12} \)
D) \( \frac{n^2}{12} \)
PART B
Physics

26. The voltage across a 1\mu F capacitor is 10V for t < 0. At t = 0, 1M\Omega resistor is connected across the capacitor terminals. Find the time constant \( \tau \) and \( V(t) \) at \( t = 5s \).

A) 1 s, 0.067 V  
B) 10 s, 0.67 V  
C) 1 s, 6.7 V  
D) 0.1 s, 0.067 V

27. The emitted power coming from a surface of temperature 300K and area 0.02 m\(^2\) over a wavelength interval of 0.1 \mu m at a wavelength of 1 \mu m is

A) \( 1.06 \times 10^{-15} \) W  
B) \( 1.06 \times 10^{-7} \) W  
C) \( 1.06 \times 10^{-2} \) W  
D) \( 1.06 \times 10^{18} \) W

28. The wave function \( \psi_{200}(r) \) for a hydrogen atom is

A) \( \left( \frac{1}{32\pi a_0^2} \right)^{1/2} \left( \frac{r}{a_0} \right) e^{r/a_0} \cos\theta \)

B) \( \left( \frac{1}{32\pi a_0^2} \right)^{1/2} \left( 2 - \frac{r}{a_0} \right) e^{-r/a_0} \)

C) \( \left( \frac{1}{\pi a_0^2} \right)^{1/2} \frac{-r}{a_0} e^{r/a_0} \)

D) \( \left( \frac{1}{2\pi a_0^2} \right)^{1/2} \frac{-r}{e^{2r/a_0}} \)

29. A beta particle, a gamma ray and an alpha particle all have the same momentum. Which has the larger wavelength?

A) beta particle  
B) gamma ray  
C) alpha particle  
D) all the same
30. A quantum particle in a box is in an excited quantum state. The momentum of the particle is uncertain because

A) The quantum state is a superposition of two different momenta
B) The quantum state is entangled
C) The particle is not in the quantum ground state
D) The particle could tunnel out of the box

31. Which of the following statements is NOT true for an ideal gas

A) The change in the internal energy in a constant pressure process from temperature \( T_1 \) to \( T_2 \) is equal to \( nC_v \) \( (T_2 - T_1) \), where \( C_v \) = molar heat capacity at constant value and ‘\( n \)’ is the number of mole of gas
B) Heat is added or removed in an adiabatic process
C) The change in the internal energy of the gas and the work done by the gas are equal
D) The internal energy does not change in an isothermal process.

32. The internal energy (\( U \)) of a Van der Waals gas (per mole) with temperature remaining constant is given by

A) \( C_v \) \( dT + a/V \)
B) \( dQ + dW \)
C) \( \int C_v dT - a/V + \text{constant} \)
D) \( PV \)

33. In the Einstein model for low temperature specific heat of a solid, \( C_v \) is proportional to

A) \( e^{-\hbar \omega E/K_BT} \)
B) \( 3NL_B \)
C) \( K_BT \)
D) \( K_BT^3 \)

34. A simple pendulum of mass \( m_2 \) has a mass \( m_1 \) at its point of support, which can move on a horizontal line lying in the plane of oscillation and \( \phi \) is the angle between the string and the vertical, the length of pendulum is \( l \). The Lagrangian of the system is

A) \( L = \frac{1}{2} (m_1 + m_2) \dot{x}^2 + \frac{1}{2} m_2 (l^2 \dot{\phi}^2 + 2l \dot{x} \dot{\phi} \cos \phi) + m_2 g \cos \phi \)
B) \( L = \frac{1}{2} (m_1 + m_2) + \frac{1}{2} (m_1 + m_2) \dot{x} \dot{\phi} \cos \phi \)
C) \( L = \frac{1}{2} (m_1 + m_2) - \frac{1}{2} (m_1 + m_2) \dot{x} \dot{\phi} \cos \phi \)
D) \( L = \frac{1}{2} (m_1 + m_2) \dot{x}^2 + \frac{1}{2} m_2 l^2 \dot{\phi}^2 - m_1 l \dot{\phi} \dot{x} \sin \phi + m_2 g \cos \phi \)
15. A rectangular thin plate of dimensions 'a' and '2a' spins about an axis along the diagonal at a constant angular velocity 'ω'. The principal moments of inertia and angular momentum (with δ = angle between $\omega$ and $l$) are:

A) $I_x = 1/12 M a^2, I_y = 1/12 M a^2, I_z = 1/12 M a^2, L = 1/12 M a^2 \omega (\cos \delta \Gamma + \sin \delta \Gamma)$

B) $I_x = 5/12 M a^2, I_y = 5/12 M a^2, I_z = 5/12 M a^2, L = 5/12 M a^2 \omega (\cos \delta \Gamma + \sin \delta \Gamma)$

C) $I_x = 1/12 M a^2, I_y = 1/3 M a^2, I_z = 5/12 M a^2, L = 1/12 M a^2 \omega \cos \delta \Gamma + 1/3 M a^2 \omega \sin \delta \Gamma$

D) $I_x = 5/12 M a^2, I_y = 1/3 M a^2, I_z = 3/4 M a^2, L = 5/12 M a^2 \omega \cos \delta \Gamma + 1/3 M a^2 \omega \sin \delta \Gamma$

36. Evaluate $(\vec{A} \times \vec{B}) \cdot \vec{\nabla} \times (\vec{C}(\vec{F}, \vec{G}))$

A) $(\vec{B} \cdot \vec{C}) (\vec{A} \cdot \vec{\nabla}) (\vec{F} \cdot \vec{G}) - (\vec{A} \cdot \vec{C}) (\vec{B} \cdot \vec{\nabla}) (\vec{F} \cdot \vec{G}) + (\vec{F} \cdot \vec{G}) [\vec{B} \cdot ((\vec{A} \cdot \vec{B}) \vec{C}) - \vec{A} \cdot ((\vec{B} \cdot \vec{D}) \vec{C})]$

B) $((\vec{A} \times \vec{B}) \cdot \vec{F} + ((\vec{A} \times \vec{B}) \cdot \vec{C})(\vec{F} \cdot \vec{G}))$

C) $(\vec{\nabla} \cdot (\vec{A} \times \vec{B}) + (\vec{F} \cdot \vec{G})) \vec{C}$

D) $(\vec{A} \cdot \vec{C}) (\vec{B} \cdot \vec{D}) (\vec{F} \cdot \vec{G}) - (\vec{B} \cdot \vec{C}) (\vec{A} \cdot \vec{\nabla}) (\vec{F} \cdot \vec{G}) - (\vec{F} \cdot \vec{G}) [\vec{B} \cdot ((\vec{A} \cdot \vec{\nabla}) \vec{C}) - \vec{A} \cdot ((\vec{B} \cdot \vec{\nabla}) \vec{C})]$

37. Evaluate $\iiint_V \vec{F} \, dV$ where $\vec{F} = 2xz \hat{i} -xy^2 \hat{j} + y^2 \hat{k}$, and V is the region bounded by the surfaces $x = 0, y = 0, y = 6, z = x^2$ and $z = 4$.

A) $24\hat{i} + 12\hat{j} + 4\hat{k}$

B) $282\hat{i} + 17\hat{j} + 62\hat{k}$

C) $60\hat{i} - 72\hat{j} - 24\hat{k}$

D) $128\hat{i} - 24\hat{j} + 384\hat{k}$

38. Which of the following numerical methods is NOT used to solve $\int_0^\pi \frac{\sin(x)}{x} \, dx$?

A) Newton-Cotes Method

B) Simpson’s method

C) Newton- Pafston method

D) Trapezoid method

39. A closed curve is defined by $Y^2 + x^2 = a^2$. The total length of the curve is given by

A) a

B) $\pi$

C) $\pi a$

D) $2 \pi a$
40. The Franck-Hertz experiment demonstrates that
A) The number of electrons in an orbit is quantized
B) The radius of an electron orbital is fixed
C) The electrons occupy only discrete, quantized energy states
D) Atoms consist of electrons and photons

41. Which of the following is not true for the common emitter configuration of a BJT?
A) it has medium input impedance
B) it has high voltage gain
C) it has high current gain
D) it has unity (or less) current gain

42. If a bridge rectifier with a filter capacitor of 100μF is fed with a 60Hz sinusoidal voltage with voltage peak of 100V, then for a load of 10kΩ the ripple voltage is
A) 1.66 V
B) 100 V
C) 70.72 V
D) 10 V

43. A first order diffraction maximum is observed for a chromium crystal at a glancing angle of 30° using a beam of x-rays of wavelength 1.67Å. If the diffraction is due to the (111) plane, find the crystal structure of chromium and give the relation between the atomic radius 'r' and the lattice constant 'a'. Given the atomic radius of Cr atom is 1.25 Å
A) a/2, FCC
B) √3a/4, BCC
C) 2a, FCC
D) a/2, BCC

44. Calculate the approximate value for the fraction of electrons that are excited from below the Fermi level to above EF, when Cu is heated from 0K to 300K
A) 0.23%
B) 23%
C) 0.12%
D) 0.55%
45. Unpolarized light of intensity $I_0$ is incident on a series of three polarizing filters. The axis of the second filter is oriented at 45° to that of the first filter, while the axis of the third filter is oriented at 90° to that of the first filter. What is the intensity of the light transmitted through the third filter?

A) 0
B) $I_0/8$
C) $I_0/4$
D) $I_0/2$

46. We have $F(n) = 1 + \frac{1}{2}C_1 + \frac{1}{3}C_2 + \ldots + \frac{1}{n+1}C_n$. Where, $\binom{n}{k} = \frac{n!}{k!(n-k)!}$

$F(n)$ equals

A) $\frac{2(2^n-1)}{n+1}$
B) $\frac{(2^n-1)}{n}$
C) $\frac{2(2^{n+1}-1)}{n+1}$
D) $\frac{(2^{n+1}-1)}{n+1}$

47. For a system with a fixed number of particles, the reciprocal of a absolute temperature is given by

A) $\left(\frac{\partial V}{\partial T}\right)_U$
B) $\left(\frac{\partial S}{\partial V}\right)_T$
C) $\left(\frac{\partial U}{\partial S}\right)_V$
D) $\left(\frac{\partial S}{\partial T}\right)_U$
48. A given quantity of a mono-atomic ideal gas \( \gamma = \frac{C_p}{C_v} = 5/3 \) expands to twice its volume reversibly. Let \( W_I \) be the work done if the expansion is isothermal and \( W_A \) be the work done if the expansion is adiabatic. Employ the convention that work done on the system is positive and work done by the system is negative. Which of the following statements is true.

A) \( W_I = W_A \)
B) \( 0 = W_I < W_A \)
C) \( W_I < W_A < 0 \)
D) \( 0 < W_A < W_I \)

49. The chemical potential of an ideal gas is

A) Negative
B) Positive
C) Zero
D) Negative at high temperatures and positive at low temperatures

50. Which of the following is not true for a ferromagnetic substance?

A) It exhibits magnetization in the absence of an applied magnetic field
B) It becomes diamagnetic above the Curie temperature
C) It becomes paramagnetic above the Curie temperature
D) It has large non-zero coercivity.

**Mathematics**

51. If \( X \) is a Poisson variate such that \( 3P(X=2)=P(X=4) \). Its mean is

<table>
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<tr>
<th>A</th>
<th>B</th>
<th>C</th>
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<tbody>
<tr>
<td>-4</td>
<td>2</td>
<td>4</td>
<td>6</td>
</tr>
</tbody>
</table>
52. The differential equation \((ay^2+x+x^5)dx+(y^8-y+bxy)dy=0\) is exact if

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<tbody>
<tr>
<td>(b=a)</td>
<td>(b=2a)</td>
<td>(a=1,b=3)</td>
<td>(b \neq a)</td>
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53. The value of \(\log i^i\) is

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<td>complex number</td>
<td>real number</td>
<td>irrational</td>
<td>none of these</td>
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54. Evaluate \(\oint_C \frac{e^{z^2}}{(z-1)^2} \, dz\), where \(C\) is the circle \(|z|=2\)

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<th>C</th>
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<td>(8\pi i)</td>
<td>(16\pi i)</td>
<td>(4\pi i)</td>
<td>(\pi i)</td>
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55. The value of the wronskian of the functions \(f(x) = 9\cos 2x\); \(g(x) = 2\cos^2 x - 2\sin^2 x\) is

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<th>A</th>
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<th>D</th>
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<tr>
<td>(-4)</td>
<td>4</td>
<td>8</td>
<td>0</td>
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56. The eigen values for the boundary value problem

\[ x'' + \lambda x = 0, x(0) = 0, x(\pi) + x'(\pi) = 0 \]

satisfy

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<th>D</th>
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<tr>
<td>\lambda + \tan \lambda \pi = 0</td>
<td>\sqrt{\lambda} + \tan \lambda \pi = 0</td>
<td>\sqrt{\lambda} + \tan \sqrt{\lambda} \pi = 0</td>
<td>\sqrt{\lambda} + \tan \sqrt{\lambda} = 0</td>
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57. If \( f(x) = \frac{1}{1-x} \), then the function \( y = f(f(x)) \), \( x > 1 \) represents

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<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>A circle</td>
<td>An ellipse</td>
<td>A pair of straight lines</td>
<td>A straight line</td>
</tr>
</tbody>
</table>

58. Let \( X_1, X_2, \ldots, X_n, \ldots \) are iid random variables with mean \( m \) and standard deviation \( s \). Then \( S_n \), sum of \( X_1, X_2, \ldots, X_n \), follows distribution with mean \( \_\_\_\_\_\_\_\_\_ \) and variance \( \_\_\_\_\_\_\_\_\_ \) for large \( n \).

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<th>B</th>
<th>C</th>
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<tr>
<td>Mean m ,</td>
<td>Mean nm ,</td>
<td>Mean m^2 ,</td>
<td>Mean nm ,</td>
</tr>
<tr>
<td>Variance n^2 s^2</td>
<td>Variance s^2</td>
<td>Variance ns^2</td>
<td>Variance ns^2</td>
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</table>
59. The characteristic equation \( \det(A - \lambda I) = 0 \) is \( \lambda^3 + 2\lambda^2 - 3\lambda + 4 = 0 \). The characteristic equation for \( A^{-1} \) is

\[
\begin{array}{c|c|c|c|c}
A & B & C & D \\
\hline
\lambda^3 - 2\lambda^2 + 3\lambda + 4 = 0 & \lambda^3 + \frac{1}{2} \lambda^2 - \frac{1}{3} \lambda + \frac{1}{4} = 0 & \lambda^3 - \lambda^2 - \frac{3}{2} \lambda - 2 = 0 & \lambda^3 - \frac{3}{4} \lambda^2 + \frac{1}{2} \lambda + \frac{1}{4} = 0 \\
\end{array}
\]

60. If the regression coefficient of \( X \) on \( Y \) is \(-1/6\) and that of \( Y \) on \( X \) is \(-3/2\). What is the correlation coefficient between \( X \) and \( Y \)?

\[
\begin{array}{c|c|c|c|c}
A & B & C & D \\
\hline
0.5 & -0.5 & 0.25 & -0.75 \\
\end{array}
\]

61. If \( A \) and \( B \) are idempotent matrices, then \( A+B \) will be idempotent iff

\[
\begin{array}{c|c|c|c|c}
A & B & C & D \\
\hline
AB = Zero matrix & BA = Zero matrix & AB+BA & None of these \\
& & Zero matrix & \\
\end{array}
\]

62. The following series \( \sum_{n=0}^{\infty} \frac{(1100 + 75i)^n}{n!} \) is

\[
\begin{array}{c|c|c|c|c}
A & B & C & D \\
\hline
Convergent & Divergent & Non-zero & Absolutely convergent \\
\end{array}
\]
63. If \( 1, \omega, \omega^2 \) are the cube roots of unity, then the roots of \((x-1)^3 + 8 = 0\) are

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1,-1,-1</td>
<td>-1,1-2(\omega),1-2(\omega^2)</td>
<td>-1,1+2(\omega),1+2(\omega^2)</td>
<td>1,(\omega),2(\omega)</td>
</tr>
</tbody>
</table>

64. Value of \( f(z) = \oint_C \frac{costz}{z^2-1} \, dz \) around rectangle with vertices \(2 \pm i, -2 \pm i\)

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>6</td>
<td>0</td>
<td>2(\pi i)</td>
</tr>
</tbody>
</table>

65. The generators of a group \( G = \{a,a^2,a^3,a^4,a^5,a^6=e\} \)

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a,a^5)</td>
<td>(a^3,a^5)</td>
<td>(a^2,a^4)</td>
<td>(a^2,a^3)</td>
</tr>
</tbody>
</table>

66. The estimator provided by Hit or Miss method for a definite integral is an

(i) unbiased estimator

(ii) consistent estimator

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>Only (i) not (ii)</td>
<td>Only (ii) not (i)</td>
<td>Neither (i) nor (ii)</td>
<td>(i) as well as (ii)</td>
</tr>
</tbody>
</table>
67. The continuous random variable $X$ is randomly distributed with 
$E(X) = \mu$ and $V(X) = \sigma^2$. If $Y = cX + d$, then $V(Y)$ is

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma^2$</td>
<td>$c^2\sigma^2$</td>
<td>$c^2\sigma^2 + d^2$</td>
<td>$c\sigma^2$</td>
</tr>
</tbody>
</table>

68. The weighted sum of independent identical random variables $X_i$'s for $i=1,2,\ldots,n$,

$S = \sum w_iX_i$ is an unbiased estimator for population mean if $\sum w_i =$

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n$</td>
<td>1</td>
<td>$1/n$</td>
<td>0</td>
</tr>
</tbody>
</table>

69. The solution of $y'' - y = t, y(0) = 1, y'(0) = 1$ is given by

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e^t \cdot \sinht + t$</td>
<td>$e^t \cdot \sinht - t$</td>
<td>$e^t \cdot \cosht + t$</td>
<td>$e^t \cdot \cosht - t$</td>
</tr>
</tbody>
</table>

70. If $A = (1 \ 2 \ 3 \ 4 \ 5), B = (2 \ 3)(4 \ 5)$ are permutations, then, product of permutations $AB =$

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(1 \ 2 \ 3)$</td>
<td>$(2 \ 3 \ 4)$</td>
<td>$(1 \ 3 \ 5)$</td>
<td>$(2 \ 4 \ 6)$</td>
</tr>
</tbody>
</table>
71. The characteristic roots of a skew-symmetric matrix are

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>Zero or real</td>
<td>Zero</td>
<td>Zero or</td>
<td>None of these</td>
</tr>
<tr>
<td></td>
<td></td>
<td>pure imaginary</td>
<td></td>
</tr>
</tbody>
</table>

72. A linear transformation $T$ from $V$ to $W$ is isomorphism if it is

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>Into</td>
<td>Onto</td>
<td>One-one</td>
<td>None of these</td>
</tr>
</tbody>
</table>

73. The Hermite interpolation polynomial $p(x)$ which approximates a function in an interval $[a, b]$ such that $p(x_i) = f(x_i)$, $p'(x_i) = f'(x_i)$ where $i = 0, 1, 2, 3, 4$ is of degree

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>8</td>
<td>9</td>
<td>10</td>
</tr>
</tbody>
</table>

74. Let $f: A \rightarrow B$, $g: B \rightarrow C$ be functions, then

i) $gof: A \rightarrow C$ is onto $\Rightarrow g: B \rightarrow C$ is onto

ii) $gof: A \rightarrow C$ is one-one $\Rightarrow g: B \rightarrow C$ is one-one

iii) $gof: A \rightarrow C$ is one-one and $f: A \rightarrow B$ is onto $\Rightarrow g: B \rightarrow C$ is one-one

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>i, ii, iii are true</td>
<td>i, ii, iii are false</td>
<td>i, iii are true, ii is false</td>
<td>Only iii is true</td>
</tr>
</tbody>
</table>
75. The mean and variance of $X \sim b(16,0.75)$ are

\[
\begin{array}{cccc}
A & B & C & D \\
12,3 & 15,3 & 12,4 & 15,4 \\
\end{array}
\]

**Chemistry**

76. The effect of the substituents (X) on the rate of thermal rearrangement of 3-aryl-2,2-dimethylmethylene-cyclopropanes (E) is in the decreasing order of:

\[
\begin{array}{c}
\text{CH}_2 \\
\text{Me}
\end{array}
\xrightarrow{X}
\begin{array}{c}
\text{Me} \\
\text{Me}
\end{array}
\]

[A] 3-CH$_3$O, H, 4-NO$_2$, 4-(CH$_3$)$_2$N  
[B] 4-(CH$_3$)$_2$N, 4-NO$_2$, H, 3-CH$_3$O  
[C] 4-(CH$_3$)$_2$N, 3-CH$_3$O, H, 4-NO$_2$  
[D] H, 3-CH$_3$O, 4-NO$_2$, 4-(CH$_3$)$_2$N

77. A 0.01 molal K$_4$Fe(CN)$_6$ freezes at $-0.06^\circ$C. Its degree of dissociation is (Cryoscopic constant ($k_t$) value is 1.86.)

[A] 60.2%  
[B] 30.56%  
[C] 70.2%  
[D] 74.2%
78. The molecule undergo faster hydrolysis is.

[A] ![Molecule A]  [B] ![Molecule B]

[C] ![Molecule C]  [D] ![Molecule D]

79. Predict the product of the following reaction?

\[ \text{C}_2\text{H}_5\text{N} \xrightarrow{200^\circ\text{C}} \text{A} \]

[A] ![Molecule A]  [B] ![Molecule B]

[C] ![Molecule C]  [D] ![Molecule D]

80. The decreasing order of hardness of the following bases is

[A] \( \text{H}^+ > \text{F}^- > \text{Br}^- > \text{CN}^- \)

[B] \( \text{OH}^- > \text{CN}^- > \text{Br}^- > \text{F}^- \)

[C] \( \text{F}^- > \text{OH}^- > \text{Br}^- > \text{CN}^- \)

[D] \( \text{F}^- > \text{OH}^- > \text{CN}^- > \text{Br}^- \)

81. In metal complexes containing bent nitrosyl group, the NO acts as

[A] 1 electron donor

[B] 1.5 electron donor

[C] 2 electron donor

[D] 3 electron donor
82. Predict the product B obtained in the following transformation.

\[
\text{H}_2\text{C} \begin{array}{c} \text{N} \\ \text{H} \end{array} \xrightarrow{\text{HNO}_3} \begin{array}{c} \text{A} \\ \text{H}_2\text{SO}_4 \end{array} \xrightarrow{\text{PhCHO}} \text{B}
\]

\[\text{[A]}\]

\[\text{[B]}\]

\[\text{[C]}\]

\[\text{[D]}\]

83. Predict the suitable product of the following transformation?

\[
\text{Me}_2\text{N} \begin{array}{c} \text{O} \\ \text{Ts} \end{array} \xrightarrow{\text{Et}_3\text{N}} \begin{array}{c} \text{A} \\ \text{EtOH, H}_2\text{O} \end{array}
\]

\[\text{[A]}\]

\[\text{[B]}\]

\[\text{[C]}\]

\[\text{[D]}\]

84. Carbonyl compounds on protonation shows bathochromic shift in UV spectra. This is due to

[A] decrease in \(\pi\) and \(\pi^*\) energy gap on protonation
[B] decrease in \(n\) and \(\pi^*\) energy gap on protonation
[C] decrease in \(\sigma\) and \(\pi^*\) energy gap on protonation
[D] decrease in \(\sigma\) and \(\sigma^*\) energy gap on protonation
85. The major product obtained in the reaction between Z-Boron enolate and an aldehyde is,
   [A] the anti isomer through cyclic TS
   [B] the anti isomer through open TS
   [C] the syn isomer through cyclic TS
   [D] the syn isomer through open TS

86. The criteria not essential for a reaction in order to carry out Gravimetric analysis is
   [A] reaction should undergo completion very quickly
   [B] desired product should not be soluble in the reaction medium
   [C] product should be colored
   [D] particle size of the desired product should be large

87. The incorrect statement about Superacids is
   [A] they are aqueous solution of strong acids
   [B] their pH is less than zero
   [C] they are non-aqueous in nature
   [D] they can dissolve organic compounds to generate cations

88. Fluxional molecule among the following is
   [A] XeF₄
   [B] Mn₂(CO)₁₀
   [C] cis-Pt(NH₃)₂Cl₂
   [D] (η⁶-C₆H₆)Cr(CO)₃

89. The force constant (in Nm⁻¹) of the bond in ^1H^¹₂⁷I molecule with vibrational wave number 2308.09 cm⁻¹ is
   [A] 313.8
   [B] 574.9
   [C] 813.3
   [D] 133.8

90. The energetic ordering of the term symbols that arise from the configuration 1s¹2p¹ is given by
   [A] ³P₀ < ³P₁ < ³P₂ < ¹P₁
   [B] ³D₀ < ³D₁ < ³D₂ < ¹D₁
   [C] ³S₀ < ³S₁ < ³S₂ < ³S₁
   [D] ³P₀ > ³P₁ > ³P₂ > ¹P₁
91. The mean kinetic energy of an electron in the 1s orbital of hydrogen atom is

[A] $2E_{1s}$
[B] $E_{1s}$
[C] 0.0
[D] $-E_{1s}$

92. Calculate the pressure at an altitude of 500 miles above the earth assuming that the atmosphere (80% N₂ + 20% O₂) is isothermal with temperature at 0 °C.

[A] $3.336 \times 10^{-44}$ atm
[B] $4.336 \times 10^{-24}$ atm
[C] $0.92 \times 10^{-48}$ atm
[D] $2.32 \times 10^{30}$ atm

93. A steel ball of density 7.90 gm/cc and 4 mm diameter requires 55 seconds to fall through a distance of 1 meter in a liquid of density 1.10 gm/cc. Calculate the viscosity of the liquid.

[A] 10.29 poise
[B] 130.19 poise
[C] 20.5 poise
[D] 230.29 poise

94. Calculate the longest wavelength of X-ray that may be used to determine a lattice spacing of 1 Å by the Bragg’s reflection method.

[A] 10 Å
[B] 2 Å
[C] 5 Å
[D] 1.52 Å

95. Equivalent conductance at infinite dilution of HCl, NaCl, and CH₃COONa are 462.2, 126.5, and 91 (in Ohm⁻¹cm²eqv⁻¹), respectively. The equivalent conductance of CH₃CO₂H is

[A] 390.7
[B] 410.8
[C] 480.2
[D] 380.1
96. A $10^{-3}$ M solution of A contains B as impurity. The solution absorbs 80% of incident light when placed in a cell of 2 cm path length. The molar extinction co-efficient of A and B are 250 and 1000, respectively in the usual unit. The concentration of B in the solution is

[A] $2.25 \times 10^{-5}$ M
[B] $9.95 \times 10^{-5}$ M
[C] $3.25 \times 10^{-3}$ M
[D] $1.25 \times 10^{-10}$ M

97. The ‘heat of reaction’ of $\text{H}_2\text{O}_2$ (l) + $\text{N}_2\text{H}_4$ (l) $\rightarrow$ ? is (Given: heats of formation of $\text{H}_2\text{O}_2$, $\text{N}_2\text{H}_4$ and $\text{H}_2\text{O}$ are $-187.78$, $+50.63$ and $-241.82$ kJ/mole respectively)

[A] $-967.28$
[B] $+642.35$
[C] $-642.35$
[D] $+967.28$

98. The precession frequency in MHz for $^1\text{H}$ and $^{31}\text{P}$ nuclei in magnetic field strength of 7.05 Tesla in a Nuclear Magnetic Resonance Spectroscopy, respectively, are (gyromagnetic ratio for $^1\text{H}$ and $^{31}\text{P}$ are 267.53 and 108.3 radians/tesla, respectively).

[A] 121.5 MHz and 300 MHz
[B] 300 MHz and 300 MHz
[C] 300 MHz and 121.5 MHz
[D] 121.5 MHz and 121.5 MHz

99. How many total resonance structures can be drawn for the following anion (include those without separation of charges)?

\[
\text{[\text{cation}]} \quad \begin{array}{c}
\text{[\text{anion}]} \\
\text{[\text{structure}]} \\
\text{[\text{A}] 1} \\
\text{[\text{B}] 2} \\
\text{[\text{C}] 3} \\
\text{[\text{D}] 4}
\end{array}
\]

100. Consider that the 1,3,5-trinitro-1,3,5-triazacyclohexane (RDX) is completely burnt to form carbon dioxide, water and nitrogen gas. The oxygen balance for the RDX is

[A] $+21.6$
[B] $-21.6$
[C] $0.0$
[D] $-78.4$