I. Please enter your Hall Ticket Number on Page 1 of this question paper and on the OMR sheet without fail.

II. Read carefully the following instructions:

1. This Question paper has two Sections: Section A and Section B
2. Section A consists of 25 objective type questions of one mark each. There is negative marking of 0.33 mark for every wrong answer. The marks obtained by the candidate in this Section will be used for resolving the tie cases.
3. Section B consists of 50 objective type questions of one mark each. There is no negative marking in this Section.
4. Answers are to be marked on the OMR answer sheet following the instructions provided there upon. An example is shown below.

   100. A B C

5. Calculators are permitted. Logarithmic tables are not allowed.
6. All questions are to be answered.
7. Hand over both question booklet and the OMR sheet at the end of the examination.

This book contains 26 pages

III. Values of physical constants:

\[ c = 3 \times 10^8 \text{ m/s} ; \quad h = 6.63 \times 10^{-34} \text{ J.s} ; \quad k_B = 1.38 \times 10^{-23} \text{ J/K} \]

\[ e = 1.6 \times 10^{-19} \text{ C} ; \quad \mu_0 = 4\pi \times 10^{-7} \text{ Henry/m} ; \quad \varepsilon_0 = 8.85 \times 10^{-12} \text{ Farad/m} \]
1. $\lim_{x\to0} \left( \frac{\sinh x - x}{\sin x - x \cos x} \right)$ is equal to
   
   A. 0
   B. $-1/2$
   C. $1/2$
   D. $1/3$

2. If $A$ and $B$ are $n \times n$ matrices, which of the following does not equal $(A + B)^2$?
   
   A. $A^2 + 2AB + B^2$
   B. $A^2 + AB + BA + B^2$
   C. $(A + B)A + (A + B)B$
   D. $(B + A)^2$

3. [Graph of a hyperbola]

Which of the following functions represents the graph shown above?

   A. $y = 1/x$
   B. $y = -\frac{1}{x}$
   C. $y = e^{-x} + 1$
   D. $y = -x^2 + 1$
4. The function \[ u(x, t) = \frac{1}{\sqrt{t}} e^{-x^2/2Dt} \]
satisfies

A. \[ \frac{\partial^2 u}{\partial t^2} = \frac{1}{D} \frac{\partial^2 u}{\partial x^2} \]

B. \[ 2 \frac{\partial u}{\partial t} = D \frac{\partial^2 u}{\partial x^2} \]

C. \[ \frac{\partial u}{\partial x} = D \frac{\partial^2 u}{\partial t^2} \]

D. \[ \frac{\partial u}{\partial t} + D \frac{\partial^2 u}{\partial x^2} = 0 \]

5. Which of the following can represent a conservative force? (\(\hat{i}, \hat{j}, \hat{k}\) are unit vectors)

A. \( C(i\hat{y} + \hat{j}z + \hat{k}z) \)

B. \( C(i\hat{x} + \hat{j}y + \hat{k}y) \)

C. \( C \cdot xyz(i\hat{j} + \hat{j} + \hat{k}) \)

D. \( C(i\hat{x} + \hat{j}y + \hat{k}z) \)

6. Consider a body of mass 500 gm connected to two springs as shown in the figure below. The springs are of equal length and spring constant. The body is displaced vertically upward by a small distance \(x\) and then left. Which of the following statements is true about the oscillations executed by the body?

A. Motion is simple harmonic and periodic.

B. Motion is simple harmonic but not periodic.

C. Motion is not simple harmonic but periodic.

D. Motion is neither simple harmonic nor periodic.

\[ \sqrt{10} \]
7. A particle moves in a potential $V = A/r^2$. The corresponding force is given by

A. $\frac{2AF}{r^4}$
B. $\frac{AF}{r^4}$
C. $\frac{AF}{r}$
D. $\frac{AF}{r^3}$

8. A wave traveling on a string is given by

$$y(x, t) = 10 \sin(0.1x - 22t).$$

The wavelength and frequency of the wave are given by

A. $\lambda = 62.9$ cm, $\nu = 3.5$ Hz
B. $\lambda = 31.4$ cm, $\nu = 7$ Hz
C. $\lambda = 0.002$ cm, $\lambda = 0.3$ Hz
D. $\lambda = 6.28$ cm, $\lambda = 35$ Hz

9. Lissajous figure produced by two sine waves of equal frequency $\omega$ and equal amplitude $A$ but $90^\circ$ out of phase will be

A. a diagonal line of length $A$
B. a circle with radius $A$
C. an ellipse with major axis $A$
D. a sine wave

10. As temperature increases, the viscosity of a liquid in general

A. decreases
B. increases
C. initially decreases, passes through a minimum and then increases
D. remains constant

11. Blood flows through our arteries of radius $R$ with a flow rate $Q$. If the arteries become clogged so that the radius is reduced by a factor of two, the flow rate $Q$ is reduced by

A. a factor of two
B. a factor of four
C. a factor of eight
D. a factor of sixteen
12. A gas has an equation of state $PV = \Theta$, where $P$ denotes pressure, $V$ denotes volume and $\Theta$ is a constant. Work done $W$ is obtained by integrating $-PdV$ between the initial volume, $V_i$ and the final volume, $V_f$. $W$ is proportional to

A. $\frac{1}{V_f} - \frac{1}{V_i}$
B. $(V_iV_f)^{1/2}$
C. $\ln \left( \frac{V_i}{V_f} \right)$
D. $\frac{V_i - V_f}{V_i + V_f}$

13. Let $\Delta W$ be the work done during a reversible process. Then $\Delta W$ is

A. a perfect differential for any process.
B. a perfect differential only for an isothermal process.
C. a perfect differential only for an adiabatic process.
D. not a perfect differential for any process.

14. The partial derivative of temperature ($T$) with respect to volume ($V$) keeping entropy ($S$) constant, $\left( \frac{\partial T}{\partial V} \right)_S$, equals

A. $-\left( \frac{\partial V}{\partial S} \right)_P$
B. $-\left( \frac{\partial P}{\partial S} \right)_V$
C. $\left( \frac{\partial V}{\partial P} \right)_S$
D. $\left( \frac{\partial P}{\partial V} \right)_S$

15. Helmholtz free energy, usually denoted by the symbol $F$ or $A$ is a natural function of

A. temperature $T$ and volume $V$.
B. entropy $S$ and pressure $P$.
C. entropy $S$ and volume $V$.
D. temperature $T$ and pressure $P$. 
16. A plane electromagnetic sinusoidal wave of frequency 40 MHz travels in the X-direction. At some point and at some instant, the electric field ($\vec{E}$) has its maximum value of 750 N/C and is along the Y-axis. What is the magnetic field $\vec{B}$ in magnitude and direction when $\vec{E} = 750\, \text{N/C}$?

A. $2.5 \times 10^{-6} \, \text{Wb/m}^2$ and along Z-axis
B. $2.5 \times 10^{-6} \, \text{Wb/m}^2$ and along X-axis
C. $2.5 \times 10^{-6} \, \text{Wb/m}^2$ and along Y-axis
D. $2.5 \times 10^{11} \, \text{Wb/m}^2$ and along Z-axis

17. Dominated by diffraction effects, what is the angular resolution of a normal human eye (of pupil diameter 2 mm) at a wavelength 600 nm and what is the separation between two points that the human eye can resolve from a distance of 20 m?

A. 0.3 rad and 6 μm
B. 3 mrad and 6 μm
C. 3 rad and 60 μm
D. $3 \times 10^{-4}$ rad and 6 mm

18. Which of the following optical components is critical to differentiate between an unpolarized light and a circularly polarized light?

A. Half wave plate
B. Quarter wave plate
C. Polarizer
D. Half wave plate and polarizer

19. A cylinder of radius $R$ with the right side cut at an angle 30° as shown in the figure below is kept in a uniform electric field $\vec{E}$, with its axis along the direction of electric field.

The total electric flux linked with the entire cylinder is

A. 0
B. $-(\pi R^2 E \sqrt{3})/2$
C. $-\pi R^2 E/2$
D. $\pi R^2 E$
20. A parallel plate capacitor is filled with two dielectrics of dielectric constant $k_1$ and $k_2$ as shown in the figure below. $A$ is the area of each plate and $d$ is the separation between the plates.

![Diagram of a parallel plate capacitor with dielectrics $k_1$ and $k_2$ and separation $d$.]

The capacitance of the capacitor is given by

A. $C = \frac{\varepsilon_0 A}{d} \left( \frac{k_1 + k_2}{2} \right)$

B. $C = \frac{\varepsilon_0 A}{d} (k_1 - k_2)$

C. $C = \frac{\varepsilon_0 A}{d} \left( \frac{k_1 - k_2}{2} \right)$

D. $C = \frac{\varepsilon_0 A}{d} (k_1 + k_2)$

21. An electron travelling horizontally enters a region as shown in the figure below where a uniform magnetic field is applied perpendicular to the plane of the paper and directed into. Which one of the following phrases most accurately describes the motion of the electron after it enters the field?

![Diagram of an electron moving horizontally into a magnetic field with grid representing field lines.]

A. Helical
B. Circular clockwise
C. Circular anticlockwise
D. Elliptical clockwise
22. What is the equivalent resistance for the figure given below?

A. $5\ \Omega$
B. $10\ \Omega$
C. $15\ \Omega$
D. $20\ \Omega$

23. An alternating voltage $V = 50\sin(30t)$ is applied to the primary of a 10:1 step-down transformer and the secondary of this transformer is connected to the input of a half-wave rectifier. The average output voltage ($V_{DC}$) of this rectifier is

A. $5\ \text{V}$
B. $5/\pi\ \text{V}$
C. $10/\pi\ \text{V}$
D. $5/\sqrt{2}\ \text{V}$

24. The ground state energy of a one-dimensional harmonic oscillator is 6.2 eV. If the oscillator undergoes a transition from its $n = 3$ level to $n = 2$ level by emitting a photon, what is the wavelength associated with the photon?

A. $50\ \text{nm}$
B. $62\ \text{nm}$
C. $100\ \text{nm}$
D. $200\ \text{nm}$

25. The de-Broglie wavelength of a neutron (kinetic energy $3/2\ kT$) at $0^\circ\text{K}$ is

A. $1.5\ \text{Å}$
B. $2\ \text{Å}$
C. $0\ \text{Å}$
D. $273\ \text{Å}$
26. Let \( m \) and \( n \) be positive integers with \( m > n \). When \( 8m + 7n \) is divided by 5, we get remainder 1. When \( 7m + 8n \) is divided by 5, we get remainder 2. If \( m - n \) is divided by 5, the remainder has to be necessarily equal to

A. 2
B. 3
C. 4
D. 1

27. If

\[(\log_2 x)(\log_4 x)(\log_6 x) = (\log_2 x)(\log_4 x) + (\log_4 x)(\log_6 x) + (\log_6 x)(\log_2 x)\]

and \( x \neq 1 \), then \( x \) is equal to

A. 12
B. 24
C. 44
D. 48

28. The solution of the differential equation \( \frac{dy}{dx} = x + xy \) is to be obtained under the condition that \( y = 0 \) at \( x = 0 \). The solution is given by

A. \( y = \frac{x^2}{2} \)
B. \( y = e^{-x^2/2} - 1 \)
C. \( y = e^{x^2/2} - 1 \)
D. \( y = \frac{e^{x^2}}{2} - 1 \)

29. What should be the value of \( a \) such that the straight line \( x + y = a \) will be a tangent to the circle \( x^2 + y^2 = 4 \)?

A. \( \frac{1}{\sqrt{2}} \)
B. \( \sqrt{2} \)
C. \( 2\sqrt{2} \)
D. 4
30. $\nabla^2(\ln r)$ is equal to

A. $\frac{1}{r}$
B. $\frac{1}{r^2}$
C. $\frac{1}{r^3}$
D. $\frac{1}{r} + \frac{1}{r^2}$

31. At very small $x$, the expression

\[
\left(2 - \frac{1}{\sqrt{1+x}} - \frac{1}{\sqrt{1-x}}\right)
\]

behaves as

A. $\frac{3}{4}x^2$
B. $1 + \frac{x}{2}$
C. $1 - \frac{x}{2}$
D. $x$

32. $|z| = 2$, where $z = x + iy$, represents a

A. straight line
B. circle
C. ellipse
D. parabola

33. What should be the value of $\alpha$ so that the matrix

\[
\begin{pmatrix}
1 & 0 & 0 \\
0 & \alpha & -\alpha \\
0 & \alpha & \alpha
\end{pmatrix}
\]

can become orthogonal?

A. 1
B. $\frac{1}{2}$
C. $\frac{1}{\sqrt{2}}$
D. $\sqrt{2}$
34. The eigenvalues of the matrix 
\[
\begin{pmatrix}
1 & 0 & 1 \\
0 & 1 & 0 \\
1 & 0 & 1 \\
\end{pmatrix}
\]
are given by
A. 1,0,1  
B. 0,1,2  
C. 1,2,1  
D. 1,1,1  

35. The commutator of two hermitian matrices is
A. hermitian  
B. unitary  
C. anti-hermitian  
D. orthogonal

36. A mouse of mass \( m \) jumps on the outside edge of a freely turning ceiling fan of rotational inertia \( I \) and radius \( R \). The ratio of the new angular velocity to the old one is given by
A. \( \frac{I}{mR^2} \)  
B. \( \frac{I + mR^2}{I} \)  
C. \( \frac{I + mR^2}{I - mR^2} \)  
D. \( \frac{I}{I + mR^2} \)

37. A particle of mass \( m \) has a speed \( v = A/x \), where \( x \) is the displacement at time \( t \). Then the force on the particle at time \( t \) is proportional to
A. \( t^{1/3} \)  
B. \( t^{-1/2} \)  
C. \( t^{-2/3} \)  
D. \( t^{-3/2} \)
38. A boat of mass \( m \) with initial speed \( v_0 \) is launched on a lake. The boat is slowed by the water by a force \( F = -m \frac{ev}{A} \), where \( A \) is a constant. What is the distance the boat sails before it stops?

A. \( A \)
B. Less than \( A \)
C. Greater than \( A \)
D. \( A(1 + v_0) \)

39. A projectile is fired with a velocity \( v_0 \) such that it passes through two points \( P \) and \( Q \) both at a distance \( h \) above the horizontal. The gun has been adjusted for maximum range. The separation of the points \( P \) and \( Q \) is given by

A. \( \frac{v_0}{g} \sqrt{v_0^2 - 4gh} \)
B. \( \frac{v_0}{g} \sqrt{2gh} \)
C. \( \frac{(v_0^2 - 2gh)}{g} \)
D. \( \frac{v_0^2}{g} \)

40. A particle of mass \( m \) starts from rest in a constant gravitational field under a frictional force equal to \( kmv^2 \), where \( k \) is a constant and \( v \) is the instantaneous velocity. The distance the particle falls in accelerating from the velocity \( v_1 \) to \( v_2 \) is given by

A. \( \frac{1}{k} e^{-g(v_2-v_1)^2} \)
B. \( \frac{1}{2k} \ln \left( \frac{g - kv_2^2}{g - kv_1^2} \right) \)
C. \( kg \frac{e^{\frac{1}{2k}g(v_2-v_1)^2}}{\ln[g(v_2^2 - v_1^2)]} \)
D. \( \frac{g}{2kv_1^2} \ln \left( \frac{g - kv_1^2}{g + kv_2^2} \right) \)

41. A particle at rest is attracted towards a center of force \( F \) according to the relation \( F = -\alpha m/x^3 \), where \( x \) is the distance from the centre of force and \( \alpha \) is a constant. The time required for the particle to reach the force centre from a distance \( d \) is proportional to

A. \( d^{1/2} \)
B. \( d^2 \)
C. \( d \)
D. \( d^5 \)
42. The density of a thin rod of length $l$ varies with the distance $x$ from one end as
\[ \rho = \rho_0 \frac{x^2}{l^2}. \] The centre of mass is at a distance

A. \( \frac{3l}{4} \)
B. \( \frac{4l}{3} \)
C. \( \frac{l}{3} \)
D. \( \frac{l}{2} \)

43. How much work is done around the path shown below by the force \( \vec{F} = A(\hat{i}y^2 + \hat{j}2x^2) \), where $A$ is a constant?

![Diagram](image)

A. \( Ad^3 \)
B. \( Ad^2 \)
C. \( Ad \)
D. \( 0 \)

44. A spring-mass system of mass $M$ suspended from a spring of spring constant $k$, and equilibrium stretching $h$, is equivalent to a simple pendulum of mass $m$ and length

A. \( \frac{h}{4\pi^2} \)
B. \( h \)
C. \( \frac{h}{2} \)
D. \( 4\pi^2h \)

45. If a pendulum bob is suspended by a string of length $L$ inside a fluid (the bob is $\eta$ times more dense than the fluid), the period of small oscillations of the pendulum will be given by

A. \( 2\pi \sqrt{\frac{\eta L}{g}} \)
B. \( 2\pi \sqrt{\frac{\eta L}{g(\eta-1)}} \)
C. \( 2\pi \sqrt{\frac{(\eta-1)L}{g\eta}} \)
D. \( 2\pi \sqrt{\frac{L}{g\eta}} \)
46. The dispersion relation for deep water waves is given approximately by

\[ \omega^2 = gk + \frac{T}{\rho}k^3. \]

The phase velocity and the group velocity can be equal only when

A. \( g \) and \( \frac{T}{\rho}k^2 \) are equal.
B. \( g \) is greater than \( \frac{T}{\rho} \).
C. \( g \) is equal to \( \frac{2T}{\rho}k^2 \).
D. \( g \) is negligible compared to \( \frac{T}{\rho}k^2 \).

47. Standing waves are generated in a pipe with one end closed while the other being open. If the length of the pipe is \( l \) and speed of sound in air is given by \( v_s \), the fundamental frequency of the standing wave pattern is given by

A. \( v_s/2l \)
B. \( v_s/2\pi l \)
C. \( v_s/l \)
D. \( v_s/4l \)

48. A metallic cube of length \( l = 1 \) m is stretched out uniformly by a force \( F \) normal to each of its faces. If \( Y \) and \( \sigma \) are the Young’s modulus and Poisson’s ratio for the material, the deformation of the cube is given by

A. \( \frac{\Delta V}{V} = \frac{3F}{Y}(1 + 2\sigma) \)
B. \( \frac{\Delta V}{V} = \frac{F}{Y}(1 - 2\sigma) \)
C. \( \frac{\Delta V}{V} = \frac{3F}{Y}(1 - 2\sigma) \)
D. \( \frac{\Delta V}{V} = \frac{F}{Y}(1 + 2\sigma) \)

49. A soap bubble of radius \( R \) has a uniformly distributed charge \( q \) on its surface. The excess pressure inside the bubble is

A. \( \frac{4T}{R} - \frac{q^2}{32\pi^2\varepsilon_0 R^4} \)
B. \( \frac{4T}{R} + \frac{q^2}{32\pi^2\varepsilon_0 R^4} \)
C. \( \frac{4T}{R} + \frac{q^2}{16\pi R^2} \)
D. \( \frac{2T}{R} - \frac{q^2}{32\pi^2\varepsilon_0 R^4} \)
50. Water has a surface tension of 0.4 N/m. In a 3 mm diameter vertical tube if the liquid rises 6 mm above the liquid outside the tube, what is the contact angle?

A. \( \cos(11) \)
B. \( \sin(11) \)
C. \( \cos^{-1}(0.11) \)
D. \( \sin^{-1}(0.11) \)

51. One thousand water droplets with radius \( r = 10^{-6} \text{m} \) coalesce to form a single droplet. If the surface tension of water is \( T = 7.5 \times 10^{-2} \text{N m}^{-1} \), how much energy is liberated when the droplets coalesce?

A. \( 8.5 \times 10^{-10} \text{ J} \)
B. \( 8.5 \times 10^{-2} \text{ J} \)
C. \( 8.5 \times 10^{-5} \text{ J} \)
D. \( 4.5 \times 10^{-7} \text{ J} \)

52. A given quantity of gas is taken from state \( A \) to state \( C \) quasi-statically and reversibly by two paths: \( A \rightarrow C \) and \( A \rightarrow B \rightarrow C \) (see figure below).

During the process \( A \rightarrow C \), work done by the gas is 60 J and the heat absorbed equals 90 J. During the process \( A \rightarrow B \rightarrow C \), the work done by the gas equals 45 J. The heat absorbed in this process equals

A. 105 J
B. 75 J
C. 15 J
D. 45 J

\( \sqrt{-10} \)
53. Which of the following is a correct statement of the second law of thermodynamics? 
\( S = \text{entropy}; \ dQ = \text{energy gained by the system in the form of heat}; \ dW = \text{energy gained by the system in the form of work}; \ T = \text{temperature}; \ F = \text{Helmholtz free energy} \)

A. \( dS \leq \frac{dQ}{T} \)
B. \( dS \leq 0 \)
C. \( dU = dQ + dW \)
D. \( dF \leq W \)

54. Consider the Maxwell - Boltzmann distribution of the velocity of a molecule of an ideal gas. Let the most probable value of the speed of the molecule be denoted by \( v_m \). Let the root-mean-square velocity of the molecule be denoted by \( v_{rms} = \sqrt{\langle v^2 \rangle} \). The ratio \( v_m/v_{rms} \) is given by

A. \( \frac{2}{3} \)
B. \( \frac{3}{2} \)
C. 1
D. \( \sqrt{\frac{2}{3}} \)

55. The entropy \( S \) of a system at fixed volume \( V \) depends on the absolute temperature as
\[
S = -aT^3 - bT^2 + S_o
\]
where \( a, b, S_o \) are constants. The heat capacity at constant volume \( C_V \) is then given by

A. \( -(aT^2 + bT) + S_o/T' \)
B. \( 3aT^2 + 2bT \)
C. \( -3aT^3 - 2bT^2 \)
D. \( -6aT - 2b \)

56. An insulated chamber is divided into two equal volumes by a partition. The left half contains an ideal gas at temperature \( T_0 \) and the right half is evacuated. A small hole is then opened between the two halves, allowing the gas to flow through, and the system comes to equilibrium. What is the final temperature of the system?

A. \( T_o \)
B. \( \frac{T_o}{2} \)
C. \( \sqrt{\frac{T_o}{2}} \)
D. \( \frac{T_o}{\sqrt{2}} \)
57. A quantity \( q \) of energy is transferred in the form of heat from a reservoir at absolute temperature \( T_1 \) to another reservoir at absolute temperature \( T_2 \), quasi-statically and reversibly. The change in entropy of the combined system is

A. \( q \left( \frac{1}{T_1} - \frac{1}{T_2} \right) \)

B. \( q \left( \frac{1}{T_2} - \frac{1}{T_1} \right) \)

C. \( \frac{q}{\sqrt{T_1 T_2}} \log \left( \frac{T_2}{T_1} \right) \)

D. \( q \left( \frac{1}{T_1} + \frac{1}{T_2} \right) \)

58. What is the group velocity for an electromagnetic wave in water if the refractive index of water at 20°C is 1.3311 for \( \lambda = 656.3 \) nm and is 1.3314 for \( \lambda = 643.8 \) nm?

A. \( 2.28 \times 10^8 \) m/sec

B. \( 1.14 \times 10^8 \) m/sec

C. \( 4.56 \times 10^8 \) m/sec

D. \( 7.60 \times 10^7 \) m/sec

59. An electromagnetic wave is obtained by superposition of two waves having the same amplitude and right and left-hand circular polarizations as given below:

\[
\vec{E}_L = iE_o \cos \omega t + j \sin \omega t \\
\vec{E}_R = iE_o \cos(\omega t - \delta) - j \sin(\omega t - \delta).
\]

If the initial phase difference between the waves is \( \delta \), what are the characteristic of the resultant wave?

A. Linearly polarized with \( \vec{E} \) forming angle \( \delta \) with the \( x \)-axis

B. Linearly polarized with \( \vec{E} \) forming angle \(-\delta/2\) with the \( x \)-axis

C. Linearly polarized with \( \vec{E} \) forming angle \( \delta/2 \) with the \( x \)-axis

D. Right circularly polarized

60. The ordinary and extraordinary refractive indices of quartz are \( n_o = 1.54424 \) and \( n_e = 1.552335 \) at \( \lambda = 589.3 \) nm. What should be the thickness of the quartz plate if it has to function as a quarter wave plate?

A. \( 64.68 \) \( \mu \)m

B. \( 32.34 \) \( \mu \)m

C. \( 8.085 \) \( \mu \)m

D. \( 16.17 \) \( \mu \)m
61. A 633 nm He-Ne laser light passes through a narrow slit and the resulting diffraction pattern is observed on a screen kept 6 m away from the source. If the distance between the centers of the first minima on either side of the central maximum on the screen is 32 mm, how wide is the slot?

A. 0.24 mm  
B. 0.12 mm  
C. 0.36 mm  
D. 0.48 mm

62. A cylindrical glass rod of refractive index \( n_g = 1.52 \) is immersed in water (refractive index \( n_w = 1.33 \)). One end of the glass rod has a polished hemi-spherical surface with radius \( R = 2 \) cm. A small object is placed at a distance 8 cm to the left of the vertex of the hemi-spherical surface. The image distance \( s' \) and the magnification \( m \) are given by

![Diagram of object and glass rod with radius 2 cm and distance 8 cm]

A. \( s' = -10.65 \) cm ; \( m = +4.66 \)  
B. \( s' = -21.3 \) cm ; \( m = +2.33 \)  
C. \( s' = -31.95 \) cm ; \( m = +3.50 \)  
D. \( s' = +21.3 \) cm ; \( m = -2.33 \)

63. A glass plane-convex lens has its flat side toward the object. The convex side has a radius of curvature of 30.0 cm. The index of refraction of the glass for violet light \( (\lambda = 400 \) nm) is 1.537, and that for red light \( (\lambda = 700 \) nm) is 1.517. The colour purple is a mixture of red and violet. If a purple object is placed 80.0 cm from this lens, where are the red \( (S'_r) \) and violet \( (S'_v) \) images formed?

A. \( S'_r = -211 \) cm ; \( S'_v = -185 \) cm  
B. \( S'_r = 185 \) cm ; \( S'_v = 211 \) cm  
C. \( S'_r = 211 \) cm ; \( S'_v = 185 \) cm  
D. \( S'_r = 15.96 \) cm ; \( S'_v = 13.88 \) cm
64. What is the equivalent capacitance between A and B?

A. 2 \( \mu \text{F} \)  
B. 3 \( \mu \text{F} \)  
C. 4 \( \mu \text{F} \)  
D. 6 \( \mu \text{F} \)  

65. The equivalent resistance \( R_{eq} \) between the two points A and B is given by

A. \( R_{eq} = \frac{6}{5} R \)  
B. \( R_{eq} = \frac{5}{6} R \)  
C. \( R_{eq} = 3R \)  
D. \( R_{eq} = \frac{1}{3} R \)  

66. A 5 \( \mu \text{F} \) capacitor is charged by a 12 V battery through a resistance of 10\(^4\)\(\Omega\). The total amount of heat dissipated by the resistance during the charging process is equal to

A. 36 mJ  
B. 3.6 mJ  
C. 360 mJ  
D. 0.36 mJ
67. A parallel circular plate capacitor with radius 10 cm is getting charged by an increasing voltage. The displacement current density \( J_D \) is 0.5 \( \mu A/m^2 \). The magnetic field at a distance of 5 cm from the axis (point \( A \)) of the capacitor plate is approximately

A. \( 1.6 \times 10^{-14} \) T
B. \( 0.5 \times 10^{-14} \) T
C. \( 10 \times 10^{-14} \) T
D. 0 T

68. The electric potential in the \( xy \)-plane in a certain region of space is given by \( V(x, y) = C(6x^2y - 2y^3) \), where \( x \) and \( y \) are in meters and \( V \) is in volts and \( C \) is a constant. What is the magnitude of the \( y \)-component of the electric field at the point (-1,2)?

A. 0 V/m
B. 18C V/m
C. 24C V/m
D. 30C V/m

69. A circular ring of radius \( a \) has a line charge density \( \rho = \rho_o \sec^2 \theta \), where \( \theta \) is measured with respect to a fixed radius. The potential at the centre is equal to

A. \( \rho_o/a \)
B. \( \pi \rho_o/a \)
C. \( \pi \rho_o \)
D. 0

70. A particle of mass \( M \) at rest decays into two particles of masses \( m_1 \) and \( m_2 \) having nonzero velocities. The ratio of de-Broglie wavelengths \( \lambda_1/\lambda_2 \) of the particles is

A. \( m_1/m_2 \)
B. \( m_2/m_1 \)
C. 1
D. \( \sqrt{m_1/m_2} \)
71. For large principle quantum number \((n)\), the spacing between two neighbouring energy levels of a hydrogen atom is proportional to

A. \(1/n\)
B. \(1/n^2\)
C. \(1/n^3\)
D. \((1 + n)/n^2\)

72. A particle is confined in a one-dimensional box of length \(l\). Its ground state wave function is given by

\[
\psi(x) = \sqrt{\frac{2}{l^3}} \sin\left(\frac{\pi x}{l}\right), \text{ for } 0 < x < l
\]
\[
= 0 \quad , \text{ for } 0 > x > l
\]

The expectation value of the momentum operator will respect to the above wave function is equal to

A. 1
B. 0.5
C. \(l/2\)
D. 0

73. The current \(I_D\) flowing through the silicon diode shown below is given in the first approximation by

![Diode Circuit Diagram]

A. 0 mA
B. 0.33 mA
C. 0.38 mA
D. 0.43 mA
74. The current flowing through the zener diode shown in the figure below (with the Zener breakdown voltage equal to 6 V) is

A. 0 mA
B. 7 mA
C. 8.3 mA
D. 13 mA

75. The half life of thorium is $1.4 \times 10^{10}$ years. What is the time required for 10% of a sample of thorium to disintegrate?

A. $2.13 \times 10^9$ years
B. $2.13 \times 10^7$ years
C. $2.13 \times 10^6$ years
D. $2.13 \times 10^5$ years