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Entrance Examination : M.Sc. Mathematics, 2010

Hall Ticket Number

Time : 2 hours
Max. Marks. 75

Part A : 25 marks
Part B : 50 marks

Instructions

1. Write your Hall Ticket Number in the OMR Answer Sheet given to you. Also write the Hall Ticket Number in the space provided above.
2. There is negative marking. Do not gamble.
3. Answers are to be marked on the OMR answer sheet following the instructions provided there upon.
4. Hand over the question paper booklet and the OMR answer sheet at the end of the examination.
5. No additional sheets will be provided. Rough work can be done in the question paper itself/space provided at the end of the booklet.
6. Calculators are not allowed.
7. There are a total of 50 questions in Part A and Part B together.
8. The appropriate answer should be coloured in either a blue or black ball point or sketch pen. DO NOT USE A PENCIL.

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Part-A

Each question carries 1 mark. 0:33 marks will be deducted for each wrong answer. There will be no penalty if the question is left unanswered.

The set of real numbers is denoted by $\mathbb{R}$, the set of complex numbers by $\mathbb{C}$, the set of rational numbers by $\mathbb{Q}$, the set of integers by $\mathbb{Z}$, and the set of natural numbers by $\mathbb{N}$.

1. Let $f(x) = \cos|x|$ and $g(x) = \sin|x|$ then

   (A.) both f and g are even functions.

   (B.) both f and g are odd functions.

   (C.) f is an even function and g is an odd function.

   (D.) f is an odd function and g is an even function.

2. The sequence $\left\{(-1)^n\left(1+\frac{1}{n}\right)\right\}$ is

   (A.) bounded below but not bounded above.

   (B.) bounded above but not bounded below.

   (C.) bounded.

   (D.) not bounded.

3. If $f(x) = \begin{cases} \exp(x) - 1 - x, & x \neq 0, \\ 0, & x = 0, \end{cases}$ then $f'(0)$ is

   (A.) 0.

   (B.) 1.

   (C.) 1/2

   (D.) none of these.

4. Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ be a matrix with integer entries such that $b \neq 0$. If $A^2 + A + I_2 = 0$ then

   (A.) $a^2 - a - bc = 1$.

   (B.) $a^2 - a - bd = 1$.

   (C.) $a^2 + a + bc = -1$.

   (D.) $a^2 + a - bc = -1$.

5. The number of points at which the function $f(x) = |(|x| - 3)\sin(\pi x)| + |(x^2 - 1)(x^3 - 27)|$ takes zero value is

   (A.) 1.

   (B.) 2.

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(C.) 3.

(D.) 4.

6. Let $f(x) = \begin{cases} 2x, & \text{if } x \text{ is irrational,} \\ x+3, & \text{if } x \text{ is rational,} \end{cases}$ be a function defined from $\mathbb{R}$ to $\mathbb{R}$. Then the discontinuities of f are

(A.) all rational numbers.

(B.) all irrational numbers.

(C.) $\mathbb{R} \setminus \{2\}$.

(D.) $\mathbb{R} \setminus \{3\}$.

7. Consider the system of equations $AX = 0$, $BX = 0$ where A and B are $n \times n$ matrices and X is a $n \times 1$ matrix. Which of the following statements are true.

(i) $\det(A) = \det(B)$ implies that the two systems have the same solutions.

(ii) The two systems have the same solutions implies $\det(A) = \det(B)$.

(iii) $\det(A) = 0 \neq \det(B)$ implies that the two systems can have different solutions.

(A.) All are true.

(B.) (i) is true.

(C.) (iii) is true.

(D.) (i) and (ii) are true.

8. $\displaystyle\int \frac{(x+1)\exp(x)}{\cos^2(x\exp(x))} dx$ is equal to

(A.) $-\cot(x\exp(x)) + C$.

(B.) $\tan(x\exp(x)) + C$.

(C.) $\log(\sec(x\exp(x))) + C$.

(D.) $\csc(x\exp(x)) + C$.

9. If $f(x) = x^3 - 2x^2$ in $(0, 5)$ then the value of c to satisfy the Mean Value theorem is

(A.) 2.

(B.) 3.

(C.) 4.

(D.) None of these.

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10. A random variable X takes the values -1, 0 and 1 with probabilities $1/3$ each. Then the mean value of X is

(A.) 0.

(B.) 1.

(C.) 0.5

(D.) 0.25.

11. Two numbers are drawn without replacement from $1, 2, \cdots, 10$. The probability that their sum is an even number strictly lies in

(A.) $(0, 1/3]$.

(B.) $(1/3, 1/2]$.

(C.) $(1/2, 3/4]$.

(D.) $(3/4, 1]$.

12. $\lim_{x \to -1} \dfrac{\sqrt{2x+3}-1}{\sqrt{5+x}-2}$ is equal to

(A.) 4.

(B.) 3.

(C.) 2.

(D.) None of these.

13. For $X, Y \subset \mathbb{R}$, define $X+Y = \{x + y \,/\, x \in X, y \in Y\}$. An example where $X + Y \neq \mathbb{R}$ is

(A.) $X = \mathbb{Q}$, $Y = \mathbb{R} \setminus \mathbb{Q}$.

(B.) $X = \mathbb{Z}$, $Y = [1/2, 1/2]$.

(C.) $X = (-\infty, 100]$, $Y = \{p \in \mathbb{N} \,/\, p \text{ is prime}\}$.

(D.) $X = (-\infty, 100]$, $Y = \mathbb{Z}$.

14. Let $f : [0, 5] \to \mathbb{R}$ be continuous function with a maximum at $x = 2$ then

(A.) the derivative of f at 2 may not exist.

(B.) the derivative of f at 2 must not exist and be nonzero.

(C.) the derivative of f at 2 must not exist and be zero.

(D.) the derivative of f at 2 can not exist.

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15. The perimeter of the Cardiod $r = a(1+\cos\theta)$ is

(A.) $2a$.

(B.) $4a$.

(C.) $8a$.

(D.) none of these.

16. If $P(x) = x^3 + 7x^2 + 6x + 5$ then

(A.) P has no real root.

(B.) P has three real roots.

(C.) P has exactly one negative real root.

(D.) P has exactly two complex roots.

17. The number of diagonal 3×3 complex matrices A such that $A^3 = I$ is

(A.) 1.

(B.) 3.

(C.) 9.

(D.) 27.

18. The number of subgroups of order 4 in a cyclic group of order 12 is

(A.) 0.

(B.) 1.

(C.) 2.

(D.) 3.

19. Let G be an abelian group and let $f(x) = x^2$ be an automorphism of G if G is

(A.) finite.

(B.) finite cyclic.

(C.) prime order.

(D.) prime order ≥ 7.

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20. The series $\sum_{n=1}^{\infty}\left(\frac{1}{\sqrt{n}}-\frac{1}{\sqrt{n+1}}\right)$

(A.) converges to 1.

(B.) converges to 1/2.

(C.) converges to 3/4.

(D.) does not converge.

21. The sequence $\left\{1+\sum_{j=1}^{n}\frac{(-1)^j}{2j+1}\right\}$ is

(A.) unbounded and divergent.

(B.) bounded and divergent.

(C.) unbounded and convergent.

(D.) bounded and convergent.

22. The function $f:\mathbb{R}\to\mathbb{R}$ defined by $f(x)=\begin{cases}\frac{1-x}{|1-x|}, & |x|<1,\\ x^2, & |x|\geq 1\end{cases}$ is

(A.) continuous at all points.

(B.) not continuous at $x=\pm 1$.

(C.) differentiable at all points.

(D.) none of the these.

23. Let $T:\mathbb{R}^3\to\mathbb{R}^3$ be the linear transformation defined by $T((x_1,x_2,x_3))=(x_1+x_2,x_2+x_3,x_3+x_1)$. Then an eigenvalue for T is

(A.) 0.

(B.) 1.

(C.) 2.

(D.) 3.

24. The solutions of $x^2y''+xy'+4y=0$, $x\neq 0$ are

(A.) $\cos(\log x)$, and $\sin(\log x)$.

(B.) $\cos(\log x)$, and $\sin(\log x^2)$.

(C.) $\cos(\log x^2)$, and $\sin(\log x)$.

(D.) $\cos(2\log x)$, and $\sin(2\log x)$.

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25. The series $\sum_{n=1}^{\infty} \frac{x^{2n}}{n}$

(A.) converges in $(-1, 1)$.
(B.) converges in $[-1, 1]$.
(C.) converges in $[-1, 1)$.
(D.) converges in $(-1, 1]$.

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Part-B

Each question carries 2 marks. 0.66 marks will be deducted for a wrong answer. There will be no penalty if a question is unanswered.

26. The integrating factor of the differential equation $(y^2 - x^2y)dx + x^3dy = 0$ is

    (A.) $(xy)^{-1}$.

    (B.) $(xy)^{-2}$.

    (C.) xy.

    (D.) x^3y^3.

27. An example of an infinite group in which every element has finite order is

    (A.) non singular 2×2 matrices with integer entries.

    (B.) $(\mathbb{Q}/\mathbb{Z}, +)$.

    (C.) the invertible elements in $\mathbb{Z}$ under multiplication.

    (D.) the Quarternion group.

28. The value of the determinant $\begin{vmatrix} 1^2 & 2^2 & 3^2 & 4^2 \\ 2^2 & 3^2 & 4^2 & 5^2 \\ 3^2 & 4^2 & 5^2 & 6^2 \\ 4^2 & 5^2 & 6^2 & 7^2 \end{vmatrix}$ is

    (A.) 0.

    (B.) 1.

    (C.) 2.

    (D.) none of these.

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29. Three girls G_1, G_2, G_3 and 3 boys B_1, B_2, B_3 are made to sit in a row randomly. The probability that at least two girls are next to each other is

(A.) 0.

(B.) 1/10.

(C.) 1/20.

(D.) 9/10.

30. 3 red balls (all alike), 4 blue balls (all alike) and 3 green balls (all alike) are arranged in a row. Then the probability that all 3 red balls are together is

(A.) 1/15.

(B.) 1/10!.

(C.) 8!/10!.

(D.) 3/10!.

31. The equation $|x-1|+|x|+|x+1| = x+2$, $x \in \mathbb{R}$ has

(A.) no solution.

(B.) only one solution.

(C.) only two solutions.

(D.) infinitely many solutions.

32. A natural number 'n' is said to be "petty" if all its prime divisors are $< \sqrt{n}$. A natural number is square free if the square of a prime can not divide it. Then

(A.) Every square free number is petty.

(B.) All even numbers are petty.

(C.) There exists an infinite numbers which are pretty.

(D.) Square of a prime number is petty.

33. For the sequence $\left\{\sqrt{n} + \frac{(-1)^n}{\sqrt{n}}\right\}$ of real numbers

(A.) the greatest lower bound and least upper bound exist.

(B.) the greatest lower bound exists but not least upper bound.

(C.) the least upper bound exists but not the greatest lower bound.

(D.) neither the greatest lower bound nor the least upper bound exist.

34. Let $f : \mathbb{R} \to \mathbb{R}$ be a differentiable function and consider the following

(i) $|f(x) - f(y)| \leq 1$, $\forall x, y \in \mathbb{R}$ with $|x - y| \leq 1$.

(ii) $|f'(x)| \leq 1, \forall x \in \mathbb{R}$.

Then we have

(A.) (i) implies (ii) but (ii) does not imply (i).

(B.) (ii) implies (i) but (i) does not imply (ii).

(C.) (i) implies (ii) and (ii) implies (i).

(D.) (i) does not imply (ii) and (ii) does not imply (i).

35. Let $U = \{(a,b,c,d) \,/\, a+b = c+d\}$, $V = \{(a,b,c,d) \,/\, a = b,\ c = d\}$ be subspaces of $\mathbb{R}^4$. Then the dimensions U and V are

(A.) 1 and 2 respectively.

(B.) 2 and 3 respectively.

(C.) 3 and 2 respectively.

(D.) 3 and 4 respectively.

36. Let $f : [0,1] \to \mathbb{R}$ be continuous function and define $g : [0,1] \to \mathbb{R}$ as $g(x) = (f(x))^2$. Then

(A.) $\int_0^1 f dx = 0 \Rightarrow \int_0^1 g dx = 0$.

(B.) $\int_0^1 g dx = 0 \Rightarrow \int_0^1 f dx = 0$.

(C.) $\int_0^1 g dx = \left(\int_0^1 f dx\right)^2$.

(D.) $\int_0^1 f dx \leq \int_0^1 g dx$.

37. Let X be a set, $\{A_\alpha \,/\, \alpha \in I\}$ be a collection of subsets of X and $f : X \to X$ be a function. Then we have $f\left(\bigcap_{\alpha\in I} A_\alpha\right) = \bigcap_{\alpha \in I} f(A_\alpha)$ if

(A.) X is finite.

(B.) I is finite.

(C.) f is one-one.

(D.) f is onto.

38. The value of the integral $\int_0^1 \log\left(\sqrt{1+x} + \sqrt{1-x}\right) dx$ is

(A.) $\log\sqrt{2} - 1$.

(B.) $1 - \log\sqrt{2}$.

(C.) $\log\sqrt{2} + 1/2 + \pi/4$.

(D.) $\log\sqrt{2} - 1/2 + \pi/4$.

39. The derivative of the function $y = \sin^{-1}\left(\sqrt{\frac{x-1}{x+1}}\right) + \sec^{-1}\left(\sqrt{\frac{x+1}{x-1}}\right)$ is

(A.) -1.
(B.) 0.
(C.) 1.
(D.) none of these.

40. Let $T : \mathbb{R}^4 \to \mathbb{R}^3$ be the linear transformation defined by $T((x_1, x_2, x_3, x_4)) = c(x_1 - x_2, x_2 - x_3, x_3 - x_4)$. Then, which of the following statements are true?

(i) $\dim\mathrm{Ker}(T) = 1$ if $c \neq 0$.
(ii) $\dim\mathrm{Ker}(T) = 4$ if $c = 0$.
(iii) $\dim\mathrm{Ker}(T) = 1$ if T is onto.

(A.) (i) and (ii).
(B.) (ii) alone.
(C.) (ii) and (iii).
(D.) (i), (ii), (iii).

41. Let S_1 and S_2 be two series defined for $x \in (-1, 1)$ as $S_1 = \sum_{n=0}^{\infty}(\sin n)x^n$ and $S_2 = \sum_{n=0}^{\infty}(\sin n + \cos n)x^n$ then

(A.) S_1 and S_2 are convergent.
(B.) S_1 and S_2 are bounded but are not convergent.
(C.) S_1 is convergent.S_1 but S_2 is only bounded.
(D.) S_1 and S_2 are divergent.

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42. If $P = \begin{bmatrix} 3 & -3 & 3 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$ then P is invertible and P^{-1} is equal to

(A.) $(P^2 + P + I)/3$.
(B.) $(P^2 + P - I)/3$.
(C.) $(P^2 - P + I)/3$.
(D.) $(P^2 - P - I)/3$.

43. Let $\{x_n\}$, $\{y_n\}$ be two convergent real sequences and let $z_n = \max\{x_n, y_n\}$ for each $n \in \mathbb{N}$. Then

(A.) $\{z_n\}$ is convergent.
(B.) $\{z_n\}$ is bounded but may not be convergent.
(C.) $\{z_n\}$ may not be convergent but $\{z_n\}$ has a convergent subsequence.
(D.) $\{z_n\}$ is convergent if and only if $\exists n_0 \in \mathbb{N} \ni x_n = y_n \forall n \geq n_0$.

44. The solution of the differential equation $y' - y = xy^5$ is

(A.) $y = (-x + c\exp(-4x) + 1/4)^4$.
(B.) $y = (-x + c\exp(-4x) + 1/4)^{-4}$.
(C.) $y = (-x + c\exp(-4x) + 1/4)^{-1/4}$.
(D.) $y = (-x + c\exp(-4x) + 1/4)^{1/4}$.

45. Let $f : \mathbb{R} \to \mathbb{R}$ be a continuous function such that $f(n) = n$, $\forall n \in \mathbb{Z}$. Then

(A.) f is identity.
(B.) $|f(x)| \leq x$, $\forall x \in \mathbb{R}$.
(C.) $f(x) > 0$, $\forall x \in (0, \infty)$.
(D.) none of these.

46. Let $\{u, v, w\}$ be a linearly independent set in the vector space $\mathbb{R}^3$ and let $X = \text{span}\{u, v + w\}$ and $Y = \text{span}\{w, u + v\}$. Then the the dimension of $X \bigcap Y$ is

(A.) 0.
(B.) 1.
(C.) 2.
(D.) can not be found from the information.

47. Let $f(x) = x|x|$ and $g(x) = \sin|x|$ then

(A.) both f and g are differentiable functions.
(B.) f is differentiable function but g is not.
(C.) g is differentiable function but f is not.

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(D.) both f and g are not differentiable functions.

48. Let $u = x + ct$, $v = x - ct$ and $z = \log u + \sin v^2$ then $\dfrac{\partial^2 z}{\partial t^2} - c^2 \dfrac{\partial^2 z}{\partial x^2}$ is equal to

(A.) $-c$.
(B.) -1.
(C.) $-2c$.
(D.) 0.

49. If α, β and γ are the roots of the equation $15x^3 + 7x - 11 = 0$ then the value of $\alpha^3 + \beta^3 + \gamma^3$ is

(A.) 3/5.
(B.) 7/5.
(C.) 9/5.
(D.) 11/5.

50. Area of the region enclosed by the curves $y = x^2 - x - 2$ and $y = 0$ is

(A.) 7/2.
(B.) $-7/2$.
(C.) 9/2.
(D.) $-9/2$.