## C-7

## ENTRANCE EXAMINATION, 2023

## QUESTION PAPER BOOKLET

## Ph.D. (PHYSICS)

## Marks: 70

Time: 2.00 hrs.
Hall Ticket No.: $\square$

1. Please enter your Hall Ticket Number on Page 1 of this question paper and on the OMR sheet without fail.
2. Read carefully the following instructions:
(a) This Question paper has two parts: Part - A and Part - B.
(b) Part - A consists of 20 multiple-choice questions related to Research methods.
(c) Part - B consists of 20 multiple-choice questions related to Physics.
(d) All questions carry 1.75 marks each.
(e) There is negative marking of 0.50 marks for every wrong answer. The marks obtained by a candidate in Part-A will be used for resolving tie cases.
(f) Answers are to be marked on the OMR answer sheet following the instructions provided there upon. An example is shown below

(g) Only non-scientific, non-programmable calculators are permitted. Mobile phone based calculators are not permitted. Logarithmic tables are not allowed.
(h) No additional sheets will be provided. Rough work can be done in the question paper itself/space provided at the end of the booklet.
(i) Hand over the OMR answer sheet at the end of the examination to the invigilator.

$$
\text { This booklet contains } 18 \text { pages }
$$

3. Values of physical constants:
$c=3 \times 10^{8} \mathrm{~m} / \mathrm{s} ; h=6.63 \times 10^{-34} \mathrm{~J} . \mathrm{s} ; k_{B}=1.38 \times 10^{-23} \mathrm{~J} / \mathrm{K}$
$e=1.6 \times 10^{-19} \mathrm{C} ; \mu_{\circ}=4 \pi \times 10^{-7}$ Henry $/ \mathrm{m} ; \epsilon_{\circ}=8.85 \times 10^{-12} \mathrm{Farad} / \mathrm{m}$

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Blank sheets are provided for rough work at the end of this booklet

## Part - A

1. A jar contains buttons of four different colors, yellow, green, red and blue. Given that the number of yellow buttons is twice the number of green buttons and the number of red buttons is twice the number of yellow buttons, and the number of blue buttons is twice the number of red buttons. The probability of picking a blue button from the jar is
A. $8 / 15$
B. $4 / 15$
C. $2 / 15$
D. $1 / 15$
2. The visible region of sunlight scattered from the road can be polarized at certain angles, leading to glare that can temporarily blind a person. Motor vehicle drivers wear polaroid glasses to cut-off this glare. To effectively cut-off this glare, the axis of the polaroid glass should be oriented
A. parallel to the surface of the road.
B. $45^{\circ}$ to the surface of the road.
C. $60^{\circ}$ to the surface of the road.
D. perpendicular to the surface of the road.
3. A log-log plot displays two-dimensional data using logarithmic scale on both axes. A straight line graph on a log-log scale will correspond to which of the following distribution on the linear scale
A. Power law distribution
B. Sinusoidal distribution
C. Poisson distribution
D. Gaussian distribution
4. For the sale of an item, its price is reduced by $x \%$ from the regular price. After the sale, if the price is increased by the same percentage, the item does not return to its original price. The graph of increased price versus $x$ will resemble a
A. straight line
B. parabola
C. hyperbola
D. circle
5. The minimum distance between the curves of $y=e^{x}$ and $y=\ln (x)$ is
A. $1 / 2$
B. 1
C. $\sqrt{2}$
D. $e$
6. Consider an ant at one corner of a cube of side length " $a$ ". The shortest path the ant must travel to reach the far corner of the cube is
A. $5 a$
B. $\sqrt{2} a$
C. $a / \sqrt{2}$
D. $\sqrt{5} a$
7. A $10 \times 10 \times 10$ cube is constructed from thousand unit cubes. The number of unit cubes that have at least one face on the surface of the larger cube is
A. 729
B. 512
C. 488
D. 271
8. Consider an experiment in which gamma rays emitted by a radioactive source are counted. There are 5212 counts in a time interval of 1 sec . The distribution of counts is random in time and follows Poisson probability function. The relative error in the count is
A. $14 \%$
B. $1.4 \%$
C. $0.14 \%$
D. $0.014 \%$
9. The series $1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\ldots$. converges to
A. $1 / 2$
B. $\pi / 4$
C. $\ln (3 / 2)$
D. $\ln (2)$
10. The Fourier transform $F(\omega)=\int_{-\infty}^{+\infty} d t e^{-i \omega t} f(t)$ of a real function $f(t)$ necessarily satisfies which one of the following :
A. $F(\omega)=F(-\omega)$
B. $F(\omega)=F^{*}(-\omega)$
C. $F(\omega)=-F^{*}(\omega)$
D. $F(\omega)=F^{*}(\omega)$
11. The value of the integral $\int_{-1}^{+1} d x H_{3}(x) P_{2}(x)$ (where, $H_{n}(x)$ and $P_{n}(x)$ denote Hermite and Legendre polynomials, respectively) is
A. $\sqrt{2 \pi} \times 3$ !
B. 0
C. $4 / 6$
D. 1
12. A given matrix $M$ has unit determinant (i.e., $|M|=+1$ ) and $M=e^{i \alpha N}$, where $\alpha>0$. The most appropriate statement about the matrix $N$ is :
A. Traceless and anti-symmetric
B. Traceless and symmetric
C. Traceless
D. Anti-symmetric
13. If $f(x)$ is a symmetric function, then the value of $\int_{-3 a}^{+3 a} f(x) \delta\left(x^{2}-a^{2}\right) d x$ is
A. 0
B. $f\left(a^{2}\right)$
C. $\frac{f(a)}{2}$
D. $\frac{f(a)}{|a|}$
14. If the determinant of a $4 \times 4$ matrix $A$ is 5 , the determinant of $\left(2 A^{2}\right)^{-1}$ is
A. $1 / 25$
B. 50
C. $1 / 50$
D. 25
15. Which one of the following quantity is invariant under Lorentz transformation?
A. Charge
B. Charge density
C. Current
D. Electric field
16. The radius of convergence of Taylor series expansion of the function $\frac{1}{\cosh (x)}$ around $x=0$, is
A. $\infty$
B. $\pi$
C. $\frac{\pi}{2}$
D. 1
17. The eigenfunction of the differential operator $\left[x+\frac{d}{d x}\right]$, corresponding to eigenvalue $\lambda$, is of the form
A. $e^{-i \lambda x+x^{2} / 2}$
B. $e^{\lambda x-x^{2} / 2}$
C. $e^{i \lambda x-x^{2} / 2}$
D. $e^{-i\left(\lambda x+x^{2} / 2\right)}$
18. The rotation group in two-dimension can be represented as $R(\theta)=\left(\begin{array}{cc}\cos \theta & -\sin \theta \\ \sin \theta & \cos \theta\end{array}\right)$.

If one takes two consecutive rotations, i.e., $\theta_{1}$ followed by $\theta_{2}$, which of the following relation is true :
A. $R\left(\theta_{1}+\theta_{2}\right)=R\left(\theta_{1}\right) R\left(\theta_{2}\right)$
B. $R\left(\theta_{1}+\theta_{2}\right)=R\left(\theta_{1}\right)+R\left(\theta_{2}\right)$
C. $R\left(\theta_{1}+\theta_{2}\right)=R\left(\theta_{1}\right)-R\left(\theta_{2}\right)$
D. $R\left(\theta_{1}+\theta_{2}\right)=\left[R\left(\theta_{1}\right), R\left(\theta_{2}\right)\right]$
19. At very low temperatures $T \ll T_{F}$ (where, $T_{F}$ is the Fermi temperature), the specific heat of an ideal Fermi gas is proportional to
A. $T$
B. $T^{3}$
C. $T^{2}$
D. $1 / T$
20. The given phase-space plot is for a system undergoing which one of the following kind of motion:

A. Damped harmonic oscillation
B. Forced harmonic oscillation
C. Chaotic oscillation
D. Simple harmonic oscillation

## Part - B

21. A total of " $n$ " identical particles are arranged in a 2 -dimensional plane, such that each particle is attached only to its nearest neighbours. The particles are attached by identical springs and allowed to execute small oscillations about their equilibrium positions. The total number of allowed normal modes of oscillation for $n \gg 1$ are
A. $n$
B. $2 n$
C. $4 n$
D. $8 n$
22. The transformation $(q, p) \rightarrow(Q, P)$ is canonical. Given, $Q=\alpha \sqrt{q} e^{t} \cos (p)$ and $P=\beta \sqrt{2 q} e^{-t} \sin (p)$, the values of $\alpha, \beta$ should satisfy,
A. $\alpha \beta=1$
B. $\alpha=\beta=\sqrt{2}$
C. $\alpha \beta=\sqrt{2}$
D. $\alpha \beta=-\sqrt{2}$
23. For a charge distribution with charge density $\rho(r)=\frac{q k}{r^{2}} e^{-k r}$, (where $k>0$ ) the electric field $\vec{E}(\vec{r})$ for $r>0$ is
A. $\frac{q}{\varepsilon_{0} r^{2}}\left[1+e^{-k r}\right] \hat{r}$
B. $\frac{q}{\varepsilon_{0} r^{2}}\left[1-e^{-k r}\right] \hat{r}$
C. $\frac{q}{4 \pi \varepsilon_{0} r^{2}}\left[1-e^{-k r}\right] \hat{r}$
D. $\frac{q k^{2}}{\varepsilon_{0}}\left[1-e^{-k r}\right] \hat{r}$
24. Consider an interface on the XY plane, separating two dielectric media of permittivities respectively $\varepsilon_{1}=2 \varepsilon_{0}$ and $\varepsilon_{2}=3 \varepsilon_{0}$. If the electric field at the interface in medium 1 is $\vec{E}_{1}=2 \hat{x}+3 \hat{y}+5 \hat{z}$, the electric field $\vec{E}_{2}$ at interface in medium 2 is,
A. $2 \hat{x}+3 \hat{y}+\frac{10}{3} \hat{z}$
B. $2 \hat{x}+3 \hat{y}+5 \hat{z}$
C. $\frac{1}{2} \hat{x}+\frac{1}{3} \hat{y}+\frac{1}{5} \hat{z}$
D. $2 \hat{x}+3 \hat{y}$
25. The electric field of an electromagnetic wave is $\vec{E}=A \sin (\omega t-k z) \hat{x}$. If this wave has to satisfy the wave equation, the value of $k$ should be
A. $\omega \sqrt{\mu \varepsilon}$
B. $\omega^{2} \mu \varepsilon$
C. $\omega / \sqrt{\mu \varepsilon}$
D. $\omega(\mu \varepsilon)^{2}$
26. Consider a classical ideal gas in a container of volume $V$ with $N$ particles ( $N \gg 1$ ). If this system has $\Omega$ accessible microstates, the number of accessible microstates of the same classical ideal gas confined to a volume 3 V is
A. $3 \Omega$
B. $3 N \Omega$
C. $3^{N} \Omega$
D. $\frac{\Omega}{3}$
27. Consider a four-level system where the excited state is three-fold degenerate. The difference in energy between the excited state and the ground state is $\Delta$. This four-level system is in thermal equilibrium at temperature $T$. If at $T=T_{0}$, the probability of finding the system in the ground state and the probability of finding the system in any one of the three excited states are equal, then $T_{0}$ is
A. $\frac{\Delta}{k_{B} \log _{e} 3}$
B. $\frac{\Delta}{3 k_{B}}$
C. $\frac{\Delta}{9 k_{B}}$
D. $\frac{\Delta}{k_{B} e^{3}}$
28. The value of the integral, $I=\int \frac{i \cos (i z \pi)}{z^{2}+1} d z$ over a closed contour $|z+i|=3 / 2$ (in the anticlockwise sense) is
A. $\frac{\pi}{2}$
B. $-\frac{\pi}{2}$
C. $\pi$
D. $-\pi$
29. The ground state wave function of one-dimensional harmonic oscillator is given as $\psi(x)=A e^{-\alpha x^{2} / 2}$, where $A$ is a normalization constant and $\alpha>0$. The ground state energy shift due to the perturbation $V(x)=\lambda x^{4}$ (where $\lambda$ is a constant) is
A. $\frac{3}{4} \frac{\lambda}{\alpha^{2}}$
B. $\frac{3}{2} \lambda \alpha^{2}$
C. $\frac{3}{2} \frac{\lambda}{\alpha}$
D. $\lambda \sqrt{\frac{\pi}{\alpha}}$
30. The Hamiltonian of an axially symmetric rotator is given as

$$
H=\frac{L_{x}^{2}+L_{y}^{2}}{2 I_{1}}+\frac{L_{z}^{2}}{2 I_{3}}
$$

The energy spectrum of the system in the angular momentum state $|l, m\rangle$ is
A. $\frac{\hbar^{2}}{2}\left[\frac{l(l+1)}{I_{1}}+m^{2}\left(\frac{1}{I_{3}}-\frac{1}{I_{1}}\right)\right]$
B. $\frac{\hbar^{2}}{2}\left[\frac{l(l+1)+m^{2}}{I_{1}}+\frac{m^{2}}{I_{3}}\right]$
C. $\frac{\hbar^{2}}{2} \frac{m^{2}}{I_{3}}$
D. $\frac{\hbar^{2}}{2}\left(\frac{l(l+1)}{I_{1}}+\frac{m^{2}}{I_{3}^{2}}\right)$
31. A measurement is made on an electron in the spin state given by

$$
\psi=\frac{1}{\sqrt{5}}\binom{i}{2} .
$$

The standard deviation for the measurement of the $x$-component of spin angular momentum is,
A. $\frac{\hbar}{2}$
B. $\frac{\hbar}{\sqrt{10}}$
C. $\sqrt{\frac{3}{20}} \hbar$
D. 0
32. If $\vec{\pi}=(\vec{p}+e \vec{A})$ is the kinetic momentum for an electron in a magnetic field $(\vec{B}=\nabla \times \vec{A})$, which of the following identity is quantum mechanically correct.
A. $\hat{\vec{\pi}} \times \hat{\vec{\pi}}=0$
B. $\hat{\vec{\pi}} \times \hat{\vec{\pi}}=i e \hbar \vec{B}$
C. $\hat{\vec{\pi}} \times \hat{\vec{\pi}}=-i e \hbar \vec{B}$
D. $\hat{\vec{\pi}} \times \hat{\vec{\pi}}=i e \hbar[(\vec{B}+\vec{E}) / c] \hat{\sigma}_{3}$
33. The pupil diameter of a human eye is 2 mm . The approximate resolving ability of the eye at 600 nm wavelength, of an object located at 20 m and 70 m distance respectively is
A. 3.2 mm and 12.8 mm
B. 7.3 mm and 25.6 mm
C. 10.9 mm and 36.4 mm
D. 14.6 mm and 51.2 mm
34. At a given center of mass energy $E_{\mathrm{CM}}$, the ratio of the cross sections for the reactions (a) $p+n \rightarrow d+\pi^{0}$ and (b) $p+p \rightarrow d+\pi^{+}$is,
A. $\sigma(a): \sigma(b)=1: 2$
B. $\sigma(a): \sigma(b)=2: 1$
C. $\sigma(a): \sigma(b)=1: 3$
D. $\sigma(a): \sigma(b)=2: 3$
35. The radius of the Fermi sphere of a free electron in a monovalent metal with fcc structure in which the volume of the unit cell is $a^{3}$ will be,
A. $\frac{1}{a}$
B. $\left(\frac{3 \pi^{2}}{a^{3}}\right)^{1 / 3}$
C. $\left(\frac{12 \pi^{2}}{a^{3}}\right)^{1 / 3}$
D. $\left(\frac{\pi^{2}}{a^{3}}\right)^{1 / 3}$
36. The ratio of the number of family of directions with $<110>$ orientation to another family of directions with $<011\rangle$ orientation in a tetragonal system is
A. $1: 1$
B. $1: 2$
C. $1: 3$
D. $1: 4$
37. The ratio of vibrational frequencies of the optical branch to acoustic branch at the zone boundary for a one-dimensional diatomic chain of atoms having masses $M$ and $m$ (with $M>m$ ) alternatively arranged is, (nearest neighbour distance is " $a$ ")
A. $\sqrt{M}: \sqrt{m}$
B. $\sqrt{m}: \sqrt{M}$
C. $M: \sqrt{\cos (k a)}$
D. $\sqrt{\cos (k a)}: \dot{m}$
38. A $\Sigma^{0}$ particle at rest decays to a $\Lambda^{0}$ particle through $\Sigma^{0} \rightarrow \Lambda^{0}+\gamma$. The energy of the outgoing photon in terms of the particle masses is
A. $\left(\frac{m_{\Sigma}^{2}-m_{\Lambda}^{2}}{2 m_{\Sigma}}\right) c^{2}$
B. $\left(\frac{m_{\Sigma}^{2}+m_{\Lambda}^{2}}{2 m_{\Sigma}}\right) c^{2}$
C. $\left(\frac{m_{\Sigma}^{2}+m_{\Lambda}^{2}}{2 m_{\Lambda}}\right) c^{2}$
D. $\left(\frac{m_{\Sigma}^{2}-m_{\Lambda}^{2}}{2 m_{\Lambda}}\right) c^{2}$
39. The Boolean expression for the given Karnaugh-map (K-map) is

| YZ |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| W | 00 | 01 | 11 | 10 |
| 00 | 1 | 1 | 0 | 1 |
| 01 | 1 | 1 | 0 | 1 |
| 11 | 1 | 1 | 0 | 1 |
| 10 | 1 | 1 | 0 | 0 |

A. $\bar{Y}+W Z+X \bar{Z}$
B. $\bar{Y}+\bar{W} Z+\bar{X} Z$
C. $Y+W Z+X Z$
D. $\bar{Y}+\bar{W} \bar{Z}+X \bar{Z}$
40. The open-loop gain of the given op-amp is 25,000 with $Z_{\text {in }}=4 M \Omega$ and $Z_{\text {out }}=50 \Omega$. The input and output impedance of the closed loop (cl) circuit is

A. $Z_{\text {in }(c l)}=1.0 k \Omega, Z_{\text {out }}=\infty$
B. $Z_{\text {in }(c l)}=\infty, Z_{\text {out }}=1.92 \Omega$
C. $Z_{\text {in }(c l)}=1.0 \mathrm{k} \Omega, Z_{\text {out }}=1.92 \Omega$
D. $Z_{\text {in }(c l)}=\infty, Z_{\text {out }}=0$
FOR ROUGH WORK ONLY

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Entrance Examination-2023
School of Physics
Ph.D. (Physics) (Code: B-4)

| Question No | Answer | Question No | Answer |
| :---: | :---: | :---: | :---: |
| 1 | A | 21 | B |
| 2 | D | 22 | C |
| 3 | A | 23 | B |
| 4 | B | 24 | A |
| 5 | C | 25 | A |
| 6 | D | 26 | C |
| 7 | C | 27 | A |
| 8 | B | 28 | C |
| 9 | D | 29 | A |
| 10 | B | 30 | A |
| 11 | B | 31 | A |
| 12 | C | 32 | C |
| 13 | D | 33 | B |
| 14 | C | 34 | A |
| 15 | A | 35 | C |
| 16 | C | 36 | B |
| 17 | B | 37 | A |
| 18 | A | 38 | A |
| 19 | A | 39 | D |
| 20 | A | 40 | C |

