

ENTRANCE EXAMINATIONS – 2023
Ph.D. Mathematics/Applied Mathematics

Hall Ticket No.	
-----------------	--

Time : 2 hours
Max. Marks : 70

PART A: 35 Marks
PART B: 35 Marks

Instructions

1. Write your Hall Ticket Number on the OMR Answer Sheet given to you. Also write the Hall Ticket Number in the space provided above.
2. Answers are to be marked on the OMR sheet.
3. Please read the instructions carefully before marking your answers on the OMR answer sheet.
4. Hand over the OMR answer sheet at the end of the examination to the Invigilator.
5. No additional sheets will be provided. Rough work can be done in the question paper itself/space provided at the end of the booklet.
6. Calculators are not allowed.
7. There are a total of 70 questions in **PART A** and **PART B** together.
8. Each correct carries **1 mark**. In PART-A, there is **0.33** negative mark for each wrong answer. There is no negative mark in PART-B.
9. The appropriate answer(s) should be coloured with either a blue or black ball point or a sketch pen. **DO NOT USE A PENCIL.**
10. This book contains **15 pages** including this page and excluding pages for the rough work. Please check that your paper has all the pages.
11. \mathbb{R} denotes the set of real numbers, \mathbb{C} the set of complex numbers, \mathbb{Z} the set of integers, \mathbb{Q} the set of rational numbers, and \mathbb{N} the set of all natural numbers.

Part-A

1. What is the next pair in the following sequence?
5:7, 11:13, 17:19, _____
 - A. 21:23
 - B. 37:39
 - C. 29:31
 - D. 47:49
2. Consider the map $f : \mathbb{Z}_{\geq 0} \times \mathbb{Z} \rightarrow \mathbb{Z}$ defined by $f(m, n) = 2^m(2n + 1)$, then f is
 - A. one-one but not onto
 - B. both one-one and onto
 - C. onto but not one-one
 - D. is not well defined
3. The number of 5 letter words formed with the letters A, B and C such that each letter appears at least once is
 - A. 60
 - B. 90
 - C. 120
 - D. 150
4. The unit digit of 2^{4^8} is
 - A. 2
 - B. 4
 - C. 8
 - D. 6
5. Consider the relation " \leq " on the power set $\rho(\mathbb{N})$ of \mathbb{N} , defined as $A \leq B \Leftrightarrow A \cap B \neq \emptyset$ for $A, B \subseteq \mathbb{N}$. Then the relation " \leq " on $\rho(\mathbb{N})$ is
 - A. reflexive and symmetric but not transitive
 - B. symmetric but not reflexive
 - C. transitive but not symmetric
 - D. equivalence relation
6. For two subsets A, B of a non empty set X , $A\Delta B$ is defined as $A\Delta B = (A \cup B) \setminus (A \cap B)$. Another representation of $A\Delta B$ among the following is
 - A. $(A^c \cap B) \cup (A \cap B)$
 - B. $(A^c \cap B) \cup (A \cap B^c)$
 - C. $(A \cap B) \cup (A \cap B^c)$
 - D. $A \setminus B$

7. In a certain code language STATISTICS is written as TATSSIITSC and UNIVERSITY is written as VINUREISYT. Then in the same code language PHILOSOPHY is written as
- LIHPSOOPYH
 - LIPHSOPOYH
 - LIHPOSOPHY
 - LIHPSOPOYH
8. The number of 11 letter words formed with the letters of the word MATHEMATICS
- $\frac{11!}{2!}$
 - $\frac{11!}{2!2!}$
 - $\frac{11!}{2!2!2!}$
 - $11!$
9. A boy climbs up a 10 m high pole. He climbs up 3 m in first minute, but slips down 2 m in the second minute, again climbs up 3 m in the third minute and slips down 2 m in the fourth minute and so on. He reaches the top of the pole in
- 15 minutes
 - 17 minutes
 - 19 minutes
 - 20 minutes
10. Suppose that $a, b \in \mathbb{N} \cup \{0\}$. What is the number of ordered pairs (a, b) with $\max\{a, b\} = k^2$, where $k \in \mathbb{N}$?
- $2k^2 - 1$
 - $2k^2$
 - $2k^2 + 1$
 - k^2
11. The value of the integral $\int_0^{\pi/2} (\sin^5 x + \cos^5 x) dx$ is
- a negative real number
 - equal to $\pi/2$
 - greater than $\pi/2$
 - a positive number less than or equal to $\pi/2$
12. The number of equivalence relations on $\{a, b, c, d\}$ containing 8 elements are
- 4
 - 2
 - 8
 - 3

13. Given two positive integers a and b such that $\gcd(a, b) = p$ (prime). Consider the following statements:

S_1 : Possible values of $\gcd(a^3, b)$ are p , p^2 , and p^3 .

S_2 : Possible values of $\gcd(a^3, b^2)$ are p^2 , and p^3 .

Then

- A. both S_1 and S_2 are true
B. S_1 is true but S_2 is false
C. S_1 is false but S_2 is true
D. both S_1 and S_2 are false
14. The sum of squares of first n natural numbers with one of them missed is 51. What is the number that was missed?
- A. 1
B. 2
C. 3
D. 4
15. The average weight of a football team (11-players) reduces by 3 kg when two players with weight 70 kg and 73 kg are replaced by 2 youngster of equal weight. What is the weight of these young players?
- A. 55 kg
B. 56 kg
C. 58 kg
D. 60 kg
16. For any $n \in \mathbb{N}$, the value of $\gcd(n! + 1, (n + 1)! + 1)$ is
- A. $n + 1$
B. 1
C. n
D. $n - 1$
17. The number of bijective functions $f : \{a, b, c, d, e\} \rightarrow \{1, 2, 3, 4, 5\}$ such that $f(a) \neq 1$ and $f(b) \neq 2$, is
- A. 6
B. 13
C. 26
D. 78

18. Two plane mirrors facing each other are kept at 10° to each other. A point is located on the angle bisector. The number of images of the point is
- A. 35
 - B. 10
 - C. 5
 - D. 1
19. Let $f(x) = 6x^4 - 13x^3 + 9x^2 - 2x$ be a polynomial such that $f(\frac{u}{v}) = 0$ for some $u, v \in \mathbb{N}$. Consider the following statements:
- S_1 : 6 is a multiple of v .
- S_2 : $2 = uv$ for some $b \in \mathbb{Z}$.
- Then
- A. both S_1 and S_2 are true
 - B. S_1 is true but S_2 is false
 - C. S_1 is false but S_2 is true
 - D. both S_1 and S_2 are false
20. If the average of 10 numbers x_1, x_2, \dots, x_{10} is 11. Suppose each x_i is increased by i when i is even and each x_i is reduced by $i - 1$ when i is odd then the average of new integers is
- A. 11
 - B. 12
 - C. 13
 - D. 14
21. The number of diagonals in any octagon (8-sided polygon) is
- A. 19
 - B. 20
 - C. 21
 - D. 22
22. The value of $39 + 17 \times 8 \div 2 - 9\%$ of 600 is
- A. 53
 - B. 54
 - C. 55
 - D. 56
23. The number of positive integral solutions to the equation $x_1 + x_2 + x_3 + x_4 = 19$ is
- A. 814
 - B. 815
 - C. 816
 - D. 817

24. What comes in 209 and 210 positions of the following sequence?
 1, 2, 2, 1, 1, 1, 2, 2, 2, 2, 1, 1, 1, 1, 1, ...
- A. 1 and 2 respectively
 B. 2 and 1 respectively
 C. 1 and 1 respectively
 D. 2 and 2 respectively
25. On the sets \mathbb{N} and \mathbb{Z} , suppose \cup is written as \cap , \cap is written as \cup , \setminus is written as Δ and Δ is written as \setminus . Then the set $(\mathbb{N} \cup \mathbb{Z}) \Delta (\mathbb{Z} \setminus \mathbb{N}) = ?$
- A. \mathbb{N}
 B. \mathbb{Z}
 C. \emptyset
 D. $\mathbb{Z}_{\geq 0}$
26. The number of permutations of the letters of the word BENTONVILLE is equal to
- A. $11!$
 B. $\frac{11!}{2!2!2!}$
 C. $\frac{11!}{2!}$
 D. $\frac{11!}{2!2!}$
27. How many 5-digit numbers can be formed by using the digits 2 to 8 if repetition of digits is not allowed?
- A. 42
 B. 120
 C. 2520
 D. 5040
28. A committee of 4 persons is to be formed from a group of 4 men and 5 women. The committee should consist of 2 men and 2 women. In how many ways can this be done?
- A. 16
 B. 60
 C. 110
 D. 126
29. The coefficient of x^{20} in the expression $(x^2 - \frac{2}{x})^{22}$ is equal to
- A. $\binom{22}{8}$
 B. $4\binom{22}{20}$
 C. $2^8\binom{22}{8}$
 D. $2^{20}\binom{22}{2}$

30. Consider the following statements:

S_1 : If $a, b \in \mathbb{N}$ and $\gcd(a, b) = 1$ then $\gcd(a + b, a - b) = 1$.

S_2 : If $a, b \in \mathbb{N}$ and $\gcd(a, b) = 1$ then $\gcd(a + b, a - b) = 2$.

Then

- A. both S_1 and S_2 are true
 - B. S_1 is true but S_2 is false
 - C. S_1 is false but S_2 is true
 - D. both S_1 and S_2 are false
31. The cardinality of the set $\{n \in \mathbb{N} : 10 \leq n \leq 100 \text{ and } \gcd(n, 100) = 1\}$ is
- A. 40
 - B. 36
 - C. 37
 - D. 35
32. A subset S of \mathbb{Z} is said to have property (P) if $1 \in S$ and $a, b \in S \Leftrightarrow ab \in S$. Which of the following sets has property (P)?
- A. $S = \mathbb{Z} \setminus 8\mathbb{Z}$
 - B. $S = \mathbb{Z} \setminus 20\mathbb{Z}$
 - C. $S = \mathbb{Z} \setminus 30\mathbb{Z}$
 - D. $S = \mathbb{Z} \setminus \{0\}$.
33. Suppose that a jar filled with chocolates weighed 10 kg. After removing $3/4$ of the chocolates the weight of the jar along with the remaining chocolates is 6 kg. Then the weight of the total number of chocolates in the jar was
- A. $\frac{53}{60}$ kg
 - B. 5 kg
 - C. $\frac{71}{10}$ kg
 - D. $\frac{16}{3}$ kg
34. Which of the following is an injective function?
- A. $f: \mathbb{R} \rightarrow \mathbb{R}$ given by $f(x) = x - 1$
 - B. $f: \mathbb{R} \rightarrow \mathbb{R}$ given by $f(x) = x^2 - 1$
 - C. $f: \mathbb{R} \rightarrow \mathbb{R}$ given by $f(x) = |x|$
 - D. $f: \mathbb{R} \rightarrow \mathbb{R}$ given by $f(x) = x - x^3$
35. If $e^x (1 + 2 \cosh x) = 3$ then
- A. $x = 0$
 - B. $x = 1$
 - C. $x = 2$
 - D. $x = 3$

Part-B

36. Let $f(r, \theta) = u(r, \theta) + iv(r, \theta)$ be an entire function such that

$$\frac{\partial u}{\partial r} \frac{\partial v}{\partial \theta} - \frac{\partial v}{\partial r} \frac{\partial u}{\partial \theta} = 2r$$

in the whole complex plane. Then

- A. f has infinite number of zeros
 B. $f(z) = az + b$, $|a| = \sqrt{2}$
 C. f is constant
 D. f is periodic
37. The set of all real values of a such that the series $\sum_{n=1}^{\infty} (1 - \cos(\frac{1}{n}))^{\frac{a}{3}}$ converges is
- A. $[\frac{1}{2}, \frac{3}{2}]$
 B. $[\frac{1}{2}, \frac{3}{2})$
 C. $(\frac{3}{2}, \infty)$
 D. $[\frac{3}{2}, \infty)$
38. $\lim_{n \rightarrow \infty} (2^n + 3^n + 5^n + 7^n + 11^n + 13^n + 17^n + 19^n)^{\frac{1}{n}} =$
- A. 3
 B. 7
 C. 13
 D. 19
39. Consider the following statements:
- S_1 : The improper Riemann integral $\int_0^{\infty} \sin x^2 dx$ exists.
 S_2 : The improper Riemann integral $\int_0^{\infty} |\sin x^2| dx$ exists.
- Then
- A. both S_1 and S_2 are true
 B. S_1 is true but S_2 is false
 C. S_1 is false but S_2 is true
 D. both S_1 and S_2 are false
40. What is the number of positive integers less than or equal to 1000 which are not divisible by 2, 3 and 5?
- A. 458
 B. 345
 C. 321
 D. 266

41. Let $f : \mathbb{C} \rightarrow \mathbb{C}$ be an entire function. Consider the following statements:

S_1 : If $\lim_{z \rightarrow \infty} \frac{f(z)}{z} = 0$ then f is constant.

S_2 : If there exists an entire function g and a constant $C > 0$ such that $|f(z)| \leq C|g(z)|$ for all $z \in \mathbb{C}$ then f is a constant multiple of g .

Then

- A. both S_1 and S_2 are true
 - B. S_1 is true but S_2 is false
 - C. S_1 is false but S_2 is true
 - D. both S_1 and S_2 are false
42. For the system $x' = x - 10y$, $y' = 4x + 5y$, the origin is
- A. sink
 - B. spiral source
 - C. center
 - D. spiral sink
43. If ϕ_1 and ϕ_2 are solutions of $y'' + a_1(x)y' + a_2(x)y = 0$ with Wronskian $W(\phi_1, \phi_2)(x) = 1$, $\forall x \in \mathbb{R}$ then $a_1(x)$ is equal to
- A. 0
 - B. 1
 - C. 2
 - D. 3
44. The singular solution of the differential equation $y = px + p^2$ (where $p = y'$) obtained by using p -discriminant is
- A. $y = x^2$
 - B. $y = -x^2$
 - C. $y = \frac{-x^2}{4}$
 - D. $y = \frac{x^2}{4}$

45. Consider the following statements:

S_1 : If $f_1(x, y) = \frac{1}{2}\sqrt{y}$, $|x| \leq 1$, $0 \leq y \leq 1$, then f_1 is Lipschitz.

S_2 : If $f_2(x, y) = 3x^2|y|$, $|x| \leq 1$, $|y| \leq 1$, then f_2 is Lipschitz.

Then

- A. both S_1 and S_2 are true
- B. S_1 is true but S_2 is false
- C. S_1 is false but S_2 is true
- D. both S_1 and S_2 are false

46. Consider the following two statements:

S_1 : There exists a rearrangement of series $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$ with partial sum S'_n such that $\liminf_{n \rightarrow \infty} S'_n = -1$ and $\limsup_{n \rightarrow \infty} S'_n = 1$.

S_2 : If $\sum_{n=0}^{\infty} a_n = A$, $\sum_{n=0}^{\infty} b_n = B$ and $c_n = \sum_{k=0}^n a_k b_{n-k}$ then $\sum_{n=0}^{\infty} c_n = AB$.

Then

- A. both S_1 and S_2 are true
- B. S_1 is true but S_2 is false
- C. S_1 is false but S_2 is true
- D. both S_1 and S_2 are false

47. Let (X, d) be a metric space and $\mathcal{C}(X)$ be the set of all complex-valued continuous, bounded functions with domain X . Let Y be a metric space $\mathcal{C}(X)$ with supremum norm. Suppose (f_n) is a sequence in $\mathcal{C}(X)$. Consider the following statements:

S_1 : If (f_n) converges to a function f in Y then (f_n) converges uniformly on X .

S_2 : If (f_n) converges to a function f uniformly on X , then (f_n) converges to f in Y .

Then

- A. both S_1 and S_2 are true
- B. S_1 is true but S_2 is false
- C. S_1 is false but S_2 is true
- D. both S_1 and S_2 are false

48. Assume that $E \subset \mathbb{R}^n$ is an open set and $f : E \rightarrow \mathbb{R}^n$ is one-to-one, continuous and has finite partial derivatives on E . Then $f(E)$ is open in \mathbb{R}^n if

- A. f is differentiable on E
- B. f is continuously differentiable on E
- C. determinant of Jacobian matrix of f is nonzero on E
- D. determinant of Jacobian matrix of f is equal to 0 on E

49. Which of the following are vector spaces over a specified fields?

- (a) \mathbb{C} over \mathbb{Q}
- (b) $\{x \in \mathbb{R} : x > 0\}$ over \mathbb{R}
- (c) $\{(x, y) \in \mathbb{F}_2^2 : x^2 + y^2 = 0\}$ over \mathbb{F}_2 where \mathbb{F}_2 is the field with 2 elements
- (d) $\mathbb{Q}[X]$ over \mathbb{Q}

- A. Only (a) and (d)
- B. Only (a), (b) and (c)
- C. Only (a), (c) and (d)
- D. (a), (b), (c) and (d)

50. Let $n \geq 2$ and A be an $n \times n$ matrix over \mathbb{R} such that every entry of A is $1 \in \mathbb{R}$. Consider the following.

- (a) A has only two distinct eigenvalues.
- (b) A is invertible.
- (c) A is diagonalizable.

Then

- A. only (a) and (c) are correct
- B. all (a), (b) and (c) are correct
- C. only (a) is correct
- D. only (b) is correct

51. Let $0 \neq A \in M_2(\mathbb{R})$, and consider the map $\ell_A: M_2(\mathbb{R}) \rightarrow M_2(\mathbb{R})$ given by $B \mapsto AB$. Then

- A. ℓ_A is surjective
- B. ℓ_A is a ring homomorphism
- C. ℓ_A is an \mathbb{R} -vector space homomorphism
- D. ℓ_A is injective

52. We denote by I_n the $n \times n$ identity matrix, and we denote by $rk(A)$ the rank of a matrix A . Match the following.

- | | |
|--|---------------------|
| (1) $0 \neq A \in M_5(\mathbb{C})$ such that $A^2 = 0$ | I) $rk(A) = 1$ |
| (2) $A \in M_3(\mathbb{C})$ such that $A^3 = I_3$ | II) $rk(A) = 2$ |
| | III) $rk(A) \leq 2$ |
| | IV) $rk(A) = 3$ |

- A. (1, I) and (2, II)
- B. (1, II) and (2, IV)
- C. (1, IV) and (2, III)
- D. (1, III) and (2, IV)

53. Match the following.

- | | |
|---|--------------------------------|
| 1) A symmetric matrix over \mathbb{R} | I) is diagonalizable |
| 2) A symmetric matrix over \mathbb{C} | II) need not be diagonalizable |
| 3) A hermitian matrix over \mathbb{C} | III) is invertible |

- A. (1, I), (2, I), and (3, I)
- B. (1, I), (2, II) and (3, I)
- C. (1, III), (2, I), and (3, I)
- D. (1, III), (2, II), and (3, I)

54. Suppose G is a group having exactly 12 elements of order 5. Then the number of subgroups of order 5 is given by

- A. 3
- B. 6
- C. 11
- D. 12

55. Suppose $a \in G$ and $O(a^4) = 12$, then the possible orders of a are

- A. 48
- B. 24
- C. 12
- D. 6

56. If $a, b \in G$ such that $O(a) = 12$ and $O(b) = 22$, then

- A. $(a) \cap (b) = (e)$
- B. $(a) \cap (b) \neq (e)$ then $a^2 = b^2$
- C. $(a) \cap (b) \neq (e)$ then $a^6 = b^{11}$
- D. $(a) \cap (b) \neq (e)$

57. Consider the following statements:

S_1 : The number of onto group homomorphisms from \mathbb{Z}_{20} to \mathbb{Z}_{10} is 4.

S_2 : The number of onto group homomorphisms from \mathbb{Z}_{20} to \mathbb{Z}_8 is 4.

Then

- A. both S_1 and S_2 are true
- B. S_1 is true but S_2 is false
- C. S_1 is false but S_2 is true
- D. both S_1 and S_2 are false

58. Let p be a prime. For $n \in \mathbb{N}$, let C_{p^n} be the set of all p^n -th roots of unity. Let $G = \bigcup_{n=1}^{\infty} C_{p^n}$ and H is a proper subgroup of G . Consider the following statements

S_1 : H can be an infinite but not a cyclic group.

S_2 : $H = C_{p^n}$ for some $n \in \mathbb{N}$.

Then

- A. both S_1 and S_2 are true
- B. S_1 is true but S_2 is false
- C. S_1 is false but S_2 is true
- D. both S_1 and S_2 are false

59. Let $\alpha \in \mathbb{Q}$ such that $\alpha \neq \beta^2$ for any $\beta \in \mathbb{Q}$. Then $[\mathbb{Q}[\sqrt[4]{\alpha}] : \mathbb{Q}]$ is equal to

- A. 2
- B. 4
- C. 12
- D. 24

60. Let $k \subset L \subset \bar{k}$, k, L are fields and \bar{k} algebraic closure of k . Suppose the minimal polynomial of $\alpha \in L$ over F has a unique root in \bar{k} . Consider the following statements:

S_1 : $\alpha \in k$ if $\text{char}(k) = 0$.

S_2 : $\alpha \in k$ if $\text{char}(k) \neq 0$.

Then

- A. both S_1 and S_2 are true
- B. S_1 is true but S_2 is false
- C. S_1 is false but S_2 is true
- D. both S_1 and S_2 are false

61. Suppose $\{H_\lambda : \lambda \in \Lambda\}$ is the collection of all subgroups of order n , for some fixed $n \in \mathbb{N}$. Consider the following statements

S_1 : $\cap H_\lambda$ is a normal subgroups of G .

S_2 : $\cap H_\lambda$ is a normal subgroups of G only if n is a prime.

Then

- A. both S_1 and S_2 are true
- B. S_1 is true but S_2 is false
- C. S_1 is false but S_2 is true
- D. both S_1 and S_2 are false

62. Suppose k, n are two natural numbers such that $k \leq n$. Then consider the following statements:

S_1 : If k divides n then $(x^k - 1)$ divides $(x^n - 1)$.

S_2 : If $(x^k - 1)$ divides $(x^n - 1)$ then k divides n .

Then

- A. both S_1 and S_2 are true
- B. S_1 is true but S_2 is false
- C. S_1 is false but S_2 is true
- D. both S_1 and S_2 are false

63. Suppose G is group of order 154 and consider the following statements:

S_1 : The Sylow-11 subgroup is normal.

S_2 : If it has a subgroup of order 77, then it has to be normal.

Then

- A. both S_1 and S_2 are true
- B. S_1 is true but S_2 is false
- C. S_1 is false but S_2 is true
- D. both S_1 and S_2 are false

64. Let $\sigma = (12345)(678)(910) \in S_{10}$. If σ^m is a 5-cycle, then

- A. $m = 6l$, for all $l \in \mathbb{N}$
- B. $m = 5l$ for all $l \in \mathbb{N}$
- C. $m = 12$
- D. $m = 3l$ for all $l \in \mathbb{N}$

65. Match the following.

1) $A \in M_4(\mathbb{R})$ satisfying $x^2 + 1 = 0$ I) is similar to $\begin{pmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$

2) $\begin{pmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$

II) is not similar to a real diagonal matrix

III) is similar to a real diagonal matrix

- A. (1, I), (2, III)
- B. (1, I), (2, I)
- C. (1, III), (2, III)
- D. (1, II), (2, III)

66. If $\sum_{n=0}^{\infty} (-x^2)^n = \sum_{n=0}^{\infty} a_n(x-3)^n$ whenever $|x| < 1$ then $\lim_{n \rightarrow \infty} |a_n|^{\frac{1}{n}}$ is equal to

- A. $\sqrt{10}$
- B. $\sqrt{8}$
- C. $\sqrt{6}$
- D. $\sqrt{5}$

67. Consider the ring $R = \left\{ \frac{a}{s} \text{ such that } a, s \in \mathbb{Z} \text{ and } 2 \text{ does not divide } s \right\} \subset \mathbb{Q}$ then

- A. R is a field
- B. the ideal generated by $\frac{4}{1}$ is a maximal ideal of R
- C. there are two distinct ideals of R
- D. the ideal generated by $\frac{2}{1}$ is the only maximal ideal of R

68. Suppose that the characteristic polynomial of a square matrix A over a field F is

$$\chi_A(X) = (X - a_1)(X - a_2) \cdots (X - a_n) \in F[X], \quad \text{where } a_i \in F \text{ are not necessarily distinct.}$$

Then

- A. A is diagonalizable
 - B. A is triangulable
 - C. A is always invertible
 - D. A is always not invertible
69. Let \mathcal{P}_{n-1} be a vector space of polynomials in one variable over a field F of degree $n-1$. Suppose that $a_1, a_2, \dots, a_n \in F$ are pairwise distinct elements. Consider the following map

$$T: \mathcal{P}_{n-1} \rightarrow F^n \quad \text{given by } p \mapsto (p(a_1), p(a_2), \dots, p(a_n)).$$

Then T is

- A. only an injective map
 - B. only a surjective map
 - C. only a bijective map
 - D. an isomorphism of vector spaces
70. Suppose that $A, B \in M_n(\mathbb{R})$ having the same minimal and characteristic polynomials. Then
- A. A is similar to B only if $n = 1$ or $n = 2$ or $n = 3$
 - B. A is similar to B only if $n = 2$
 - C. A is similar to B only if $n = 3$
 - D. A is similar to B only if $n = 3$ or $n = 4$

C-4

Rough Work

10/7/2023

University of Hyderabad
Ph.D. Entrance Examinations - 2023

REVISED

School/Department/Centre
Course : Ph.D.

School of Mathematics and Statistics
Subject : Mathematics/Applied Mathematics

Q.No.	Answer	Q.No.	Answer	Q.No.	Answer
1	C	26	B	51	C
2	A	27	C	52	D
3	D	28	B	53	B
4	D	29	C	54	A
5	B	30	D	55	A
6	B	31	B	56	C
7	D	32	D	57	B
8	C	33	D	58	C
9	A	34	A	59	B
10	C	35	A	60	B
11	D	36	B	61	B
12	D	37	C	62	A
13	A	38	D	63	A
14	B	39	B	64	C
15	A	40	D	65	B
16	B	41	A	66	01 marks to all
17	D	42	B	67	C and/or D
18	A	43	A	68	B
19	A	44	C	69	D
20	B	45	C	70	A
21	B	46	B		
22	A	47	A		
23	C	48	C		
24	D	49	C or D		
25	A	50	A		

Note/Remarks :

Signature

School/Department/Centre

07/7/2023