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Entrance Examination: Ph.D. Applied Mathematics, 2022

Hall Ticket Number

Time: 2 hours
Max. Marks: 70

Part A: 35 Marks
Part B: 35 Marks

Instructions

1. **Write your Hall Ticket Number on the OMR Answer Sheet given to you. Also write the Hall Ticket Number in the space provided above.**
2. Answers are to be marked on the OMR sheet.
3. Please read the instructions carefully before marking your answers on the OMR answer sheet.
4. Hand over the OMR answer sheet at the end of the examination.
5. No additional sheets will be provided. Rough work can be done in the question paper itself/space provided at the end of the booklet.
6. Calculators are not allowed.
7. There are a total of 70 questions in **PART A** and **PART B** together.
8. Each correct answer carries 1 mark.
9. The appropriate answer should be coloured with either a blue or a black ball point or a sketch pen. DO NOT USE A PENCIL.
10. $\mathbb{R}$ denotes the set of real numbers, $\mathbb{C}$ the set of complex numbers, $\mathbb{Z}$ the set of integers, $\mathbb{Q}$ the set of rational numbers, and $\mathbb{N}$ the set of all natural numbers.
11. This booklet contains **14** pages including this page and excluding pages for the rough work. Please check that your paper has all the pages.

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Part-A

1. A boy and a girl are talking:
"I am a boy"- said the first child.
"I am a girl"- said the second child. At least one of them is lying. Then who is/are lying?

A. The first child.

B. The second child.

C. Both

D. No one

2. If $48 \rightarrow 24$, $11 \rightarrow 10$ and $39 \rightarrow 36$, then $26 \rightarrow ?$

A. 29

B. 23

C. 0

D. 12

3. A bike is moving at the speed of 80 km/h. The instantaneous velocity of the upper most points of its wheels is

A. 80 km/h forward.

B. 160 km/h forward.

C. 80 km/h backward.

D. 160 km/h backward.

4. What would be the duration of a year, if the distance between the earth and the sun gets tripled?

A. 1662 days.

B. 365 days.

C. 1095 days.

D. 1894 days.

5. If in a city 80% people like coffee and 70% people like bournvita, if x% people like exactly one type of beverage, then

A. $20\% \leq x \leq 60\%$.

B. $20\% \leq x \leq 40\%$.

C. $10\% \leq x \leq 50\%$.

D. $70\% \leq x \leq 80\%$.

6. Find the missing entry: AB, BD, CF, DH, ___, FL

A. DK

B. EJ

C. EI

D. EK

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7. $5 + 90\%\ of\ 80 - (\frac{10}{3} + 5 \times \frac{7}{3}) = ?$

A. $\frac{531}{9}$
B. 62
C. 61
D. $\frac{530}{9}$

8. What comes next in the following sequence
one, thousand, million, billion, _____

A. quadrillion
B. trillion
C. quintillion
D. septillion

9. The total number of ways in which 12 different beads can be arranged to form a necklace

A. $\frac{12!}{2}$
B. 12!
C. 11!
D. $\frac{11!}{2}$

10. In a code language if 56417 is written as 75609. How is 36353 written in the same code language?

A. 55555
B. 55545
C. 55535
D. 55565

11. Which of the following is **not** correct for the binary operation "$\circ$" defined on respective sets

A. $a \circ b = a \times b$ on $\mathbb{N}$
B. $a \circ b = ab^{-1}$ on $\mathbb{Q}$
C. $a \circ b = a + b - ab$ on $\mathbb{Z}$ has identity element in $\mathbb{Z}$
D. $a \circ b = a + b - ab$ on $\mathbb{Z}$ is commutative

12. What comes next in the sequence
6, 12, 20, 30, 42, __

A. 56
B. 57
C. 58
D. 59

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13. Find the next word in the series: ADBC, HKIJ, ORPQ, _____

A. WXYV

B. VYWX

C. VWYX

D. WYXV

14. Priya started walking from her hostel to her college. Intially she walked 300 meters to wards east then she walked 300 meters towards south, from that point she can see her college in east side in 100 meters. Then the shortest distance between priya's hostel and college is?

A. 400 meters

B. 700 meters

C. 500 meters

D. 600 meters

15. Let a and b are two positive integers such that $a + b + ab = 122$. Then the value of $(ab)^2$ is

A. 3600

B. 4900

C. 6400

D. not uniquely determined.

16. A man fills basket with candies in such a way that the number of candies added on each successive day is same as the number already present in the basket. This way the basket gets completely filled in 20 days. After how many days the basket was $(1/4)^{th}$ full?

A. 5

B. 18

C. 9

D. 12

17. Every time a ball falls to ground, it bounces back to one-third of the height it fell from. A ball is dropped from a height of 59049 cm. The maximum height from the ground to which it can rise after the ninth bounce is

A. 59.049 cm

B. 590.49 cm

C. 5.9049 cm

D. 3 cm

18. The remainder when $(1 + 10!)^2(1 + (10!)^2)^3$ is divided by 35 is

A. 5

B. 10

C. 1

D. 0

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19. Which of the following is the best approximation for $\sin(0.2)$?

A. $(0.2)^\circ$.

B. $\left(0.2 \times \dfrac{180}{\pi}\right)^\circ$.

C. $\left(0.2 \times \dfrac{90}{\pi}\right)^\circ$.

D. $\left(0.2 \times \dfrac{180}{\pi}\right)$.

20. If the radius of the earth were to shrink by 2% and if it's mass remains the same, then the acceleration due to gravity on the earth's surface

A. decreases by 4.11%.

B. decreases by 2.12%.

C. increases by 4.12%.

D. increases by 2.11%.

21. The unit digit of $1! + 2! + 3! + \ldots + 50! =$?

A. 1

B. 2

C. 3

D. 4

22. Suppose the letters of the word PLANT are shuffled to make 5 letter words. How many of these words begin with T and do not end with A

A. 15

B. 16

C. 17

D. 18

23. The number of digits in the number $2^{19} \times 3^3 \times 5^{21} \times 7^2$ when written in base 10 is

A. 23

B. 24

C. 25

D. 26

24. What comes next in the sequence 2, 9, 28, 65, 126, ...

A. 214

B. 215

C. 216

D. 217

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25. The value of the sum $(1-1^2+1^3)+(2-2^2+2^3)+(3-3^2+3^3)+\ldots+(10-10^2+10^3)=?$

A. 2695

B. 2696

C. 2697

D. 2698

26. Five people namely A, B, C, D, and E are sitting in a row. D and E are not in middle. A and C are adjacent to D. B is adjacent to A. Then who are sitting in 2^{nd} and 4^{th} positions?

A. B and A

B. B and D

C. C and D

D. D and E

27. Fill in the blank:
5, 11, 28, ___, 126, 296, 369

A. 53

B. 78

C. 80

D. 100

28. What will be the next entry in the following series?
P23BD03, M22FH06, I20JL10, D18NP15, ———

A. X14RT21

B. X15RT21

C. X14RQ21

D. X14RT22

29. If $3\times 4-2+15/x(3\times 5-8/4+3)=110/3$ then x is equal to

A. 3

B. 9/256

C. 12

D. 9

30. If $x=\iota^{\iota^{\iota^{\cdot^{\cdot^{\cdot}}}}}$, where $\iota=\sqrt{-1}$ then $x^{1/x}$ is

A. not defined

B. equal to ι

C. equal to $-\iota$

D. equal to ∞

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31. Ajay and his friends ordered a pizza. Ajay took $\frac{1}{5}$th part of pizza and the rest is equally divided among his friends. If the part of pizza Ajay got is 3 times as much as each of his friends then how many friends Ajay has?

A. 12

B. 9

C. 6

D. 15

32. Rohan is a grandfather of Raj, Jyotsna is the wife of Raj, Rani is the only kid of Rohan and Suhas is the husband of Rani. Then how is Jyotsna related with Suhas?

A. daughter

B. sister

C. daughter-in-law

D. sister-in-law

33. Let $\mathbb{Q}[X]$ be the polynomial ring in one variable over rationals. We say that $f \in \mathbb{Q}[X]$" is related to" $g \in \mathbb{Q}[X]$ if f divides g. Then "is related to" is

A. an equivalence relation

B. symmetric and transitive

C. reflexive and symmetric

D. reflexive and transitive

34. Which of the following set is uncountable?

A. $\mathbb{Q}$

B. $\mathbb{Q} \times \mathbb{Q}$

C. $\mathbb{Z} \times \mathbb{Z} \times \mathbb{Z}$

D. The set of all irrational numbers

35. Which of the following is a finite set?

A. The set of all integers that divide 2

B. $\{n \in \mathbb{Z} : 2 \text{ divides } n\}$

C. $\{2^n : n \in \mathbb{N}\}$

D. $\{2^{2n} : n \in \mathbb{N}\}$

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Part-B

36. The solutions of the system $x' = -14x + 20y$, $y' = -10x + 14y$ starting in a neighborhood of origin generate _____ orbits in the $xy-$plane.

A. unbounded

B. spiral and bounded

C. spiral and bounded

D. closed

37. Let y_1 and y_2 be the solutions of $y''+y = 0$ and $y''+9y = 0$, respectively. Further $y_1(0) = y_2(0)$, $y_1'(0) = y_2'(0)$, $y_1(1) = y_2(1)$, and $y_1'(1) = y_2'(1)$. Then $\int_0^1 y_1(x)y_2(x)dx =$ _____.

A. 0

B. 1

C. 9

D. 4

38. The differential equation $(1-x^2)y'' - xy' + y = 0$ is ——— equation.

A. Chebyshev

B. Legendre

C. Hermite

D. Bessel

39. Consider the Ordinary Differential Equation $x^2(x-1)^2y'' + (x-1)y' + y = 0$. Let $x_0 = 0$ and $x_1 = 1$. Then

A. x_0 is a regular singular point and x_1 is an irregular singular point

B. Both x_0 and x_1 are regular singular points

C. x_0 is an irregular singular point and x_1 is a regular singular point

D. Both x_0 and x_1 are irregular singular points

40. Let x_n be a sequence in a metric space (X, d). If S is the set of all sub-sequential limit points of x_n and if a is a limit point of S then ———

A. $\lim_{n \to \infty} x_n = a$

B. $a \in \overline{S} - S$, the boundary of S

C. $a \in S$

D. $\lim_{n \to \infty} d(x_n, a) > 0$

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41. Consider the sequence $\{f_n\}$ of functions defined by $f_n(x) = x^n n^2 (1-x)^{1/2}$, $0 \le x \le 1$. Then ——.

A. $\{f_n\}$ converges uniformly on interval $[0, 1]$.
B. $\{f_n\}$ is divergent.
C. $\lim\limits_{n \longrightarrow \infty} \int_0^1 f_n(x)dx = \int_0^1 \lim\limits_{n \longrightarrow \infty} f_n(x)dx$
D. $\{f_n\}$ converges pointwise but not uniformly on interval $[0, 1]$.

42. Suppose f maps an open set $E \subset \mathbb{R}^n$ into $\mathbb{R}^m$. Then f is differentiable on E if ——

A. the directional derivative $f'(\mathbf{x}; \mathbf{u})$ exists in all directions $\mathbf{u}$ for all $\mathbf{x} \in E$.
B. Jacobian matrix of f exists at all points $\mathbf{x} \in E$.
C. f is bounded on E.
D. all the partial derivatives $D_j f_i$ of component functions of f exist and are continuous on E.

43. The boundary value problem $y'' + y = 0$, satisfying $y(0) = y(\pi) = 0$ has —

A. a unique solution
B. no solution
C. only two solutions
D. infinitely many solutions

44. Let $a, b \in \mathbb{R}$. If all the roots of the characteristic polynomial of $y'' + ay' + by = 0$ have negative real parts, then

A. all solutions tend to 0 as x tends to ∞
B. all solutions tend to ∞ as x tends to ∞
C. there exists a solution which does not tend to 0 as x tends to ∞
D. all solutions tend to $-\infty$ as x tends to ∞

45. If one solution of $y'' - \frac{2}{x^2}y = 0$ is $\phi_1(x) = x^2$, $x > 0$ then a basis for its solutions is —

A. $\{x^2, x\}$
B. $\{x^2, x^{-1}\}$
C. $\{x^2, x^3\}$
D. $\{x^2, x^4\}$

46. The Bessel functions satisfy —

A. $J_0'(x) = J_1(x)$
B. $J_0'(x) = 2J_1(x)$
C. $J_0'(x) = -2J_1(x)$
D. $J_0'(x) = -J_1(x)$

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47. Which of the following are exact differential equations?

(I) $(x^2 + xy)dx + xydy = 0$.
(II) $(x + y)dy + (x - y)dx = 0$.
(III) $x^2y^3dx - x^3y^2dy = 0$.
(IV) $\cos(x)\cos^2(y)dx - \sin(x)\sin(2y)dy = 0$

A. (II) and (III)
B. (IV)
C. (I) and (II)
D. (III)

48. Which of the following statements are true?

(I) If ϕ_1, ϕ_2 are linearly independent functions on an interval I then they are linearly independent on any interval $J \subset I$.
(II) If ϕ_1, ϕ_2 are linearly independent solutions of linear ODE $y'' + ay' + by = 0$ on an interval I then they are linearly independent on any interval $J \subset I$.

A. (I) and (II)
B. neither (I) nor (II)
C. (I)
D. (II)

49. Which of the following subsets of $\mathbb{R}^2$ (with the standard topology) are not compact?

(I) a circle
(II) a parabola
(III) $\{(x, y) \in \mathbb{R}^2 | y = \sin(1/x), 0 < x \leq 1\}$
(IV) $\{(x, y) \in \mathbb{R}^2 | |x| + |y| = 4\}$

A. (II) and (III)
B. (I) and (IV)
C. (I) and (II)
D. (II) and (IV)

50. Identify the correct type of PDEs in first list from second list:

PDE	Type
(I) $u_{xx} - u_{yy} = 0$	(i) Parabolic
(II) $u_{xx} + u_y = 0$	(ii) Hyperbolic
(III) $u_{xx}+u_{yy} = 0$	(iii) Elliptic

A. (I)-(iii); (II)-(i); (III)-(ii)
B. (I)-(ii); (II)-(iii); (III)-(i)
C. (I)-(ii); (II)-(i); (III)-(iii)
D. (I)-(iii); (II)-(ii); (III)-(i)

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51. Which of the following are true?
(I) Every Cauchy sequence in a metric space is bounded.
(II) Every Cauchy sequence in a metric space is convergent.
(III) A Cauchy sequence is convergent if and only if it has a convergent subsequence.

A. (I) and (II)

B. (II) and (III)

C. (II)

D. (I) and (III)

52. Let $\sum_{n=-\infty}^{\infty} a_n z^n$ be the Laurent series expansion of the function $f(z) = \dfrac{1}{z^2 \tanh z}$ where $0 < |z| < \frac{\pi}{2}$. Then $\operatorname{Res} f(z) (a_1)^{-1}$ is

A. 11

B. 17

C. 15

D. 8

53. The radius of convergence of power series $\sum_{n=1}^{\infty} \phi(n^3) d(n^3) z^{3n+2}$ (where $\phi(n)$ is the Euler's totient function and $d(n)$ is the divisor function) is

A. ∞

B. 0

C. 1

D. 2

54. Let $f(z)$ be an entire function with $z = \infty$ a pole of order k. Then

A. f can have atmost k zeros.

B. f may have more than k zeros.

C. f is bounded.

D. f is periodic.

55. Let $f(x) = a3^x + b\cos \pi x + c|x|^3$ and $S = \{(a, b, c) \in \mathbb{R}^3 \mid f(x) \text{ has a root lying between } 0 \text{ and } 1\}$. Then

A. S is not a subspace of $\mathbb{R}^3(\mathbb{R})$.

B. S is a 1-dimensional subspace of $\mathbb{R}^3(\mathbb{R})$.

C. S is a 2-dimensional subspace of $\mathbb{R}^3(\mathbb{R})$.

D. S is a 3-dimensional subspace of $\mathbb{R}^3(\mathbb{R})$.

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56. The sum $1^{47} + 2^{47} + 3^{47} + 4^{47} + 5^{47} + 6^{47}$ is not divisible by

A. 3

B. 7

C. 11

D. 13

57. If N is a positive integer such that $N/2$ is a square, $N/3$ is a cube, $N/5$ is a fifth power, then the minimum possible value of $N/10^6$ lies in the interval

A. $[3 \times 10^7, 4 \times 10^7]$.

B. $[4 \times 10^7, 5 \times 10^7]$.

C. $[2 \times 10^6, 3 \times 10^6]$.

D. $[4 \times 10^6, 5 \times 10^6]$.

58. If the characteristic polynomial of a 100×100 matrix A over $\mathbb{R}$ is $(x-1)(x-2)\cdots(x-100)$ then the dimension of the vector space $V = \{B \in M_{100}(\mathbb{R}) : AB = BA\}$ is

A. 10^2

B. 10^3

C. 10^4

D. 10

59. Let $n > 1$ be a natural number. Suppose that A is a square matrix such that $A^n = 0 \in M_n(\mathbb{R})$. Choose the correct alternative.

A. A is a zero matrix

B. A is diagonalizable

C. The trace of A is zero

D. A is invertible

60. Choose the correct alternative.

A. $e^{\sqrt{3}} > 6$

B. $e^{\sqrt{3}} \in (5.7, 6]$

C. $e^{\sqrt{3}} \in [5.47, 5.7]$

D. $e^{\sqrt{3}} \in [5.41, 5.47)$

61. Choose the correct statement.

A. There are two non-isomorphic abelian groups of order 6

B. There is a unique, up to isomorphism, non-abelian group of order 6

C. There are two non-isomorphic non-abelian groups of order 6

D. Every group of order 6 is abelian.

62. Suppose H is a proper subgroup of $\mathbb{Z}$ containing $14, 30, 50$. Then $H =$

A. $50\mathbb{Z}$

B. $105\mathbb{Z}$

C. $2\mathbb{Z}$

D. $14\mathbb{Z}$

63. Suppose G is a group. Consider the following statements:
I: $\{x \in G \mid x^2 = e\}$ is a subgroup.
II: $\{x \in G \mid x^2 = e\}$ is a subgroup if G abelian.

A. Both I and II are true.

B. II is true but not I.

C. I and II both are false.

D. I is true but not II.

64. Let $G = \mathbb{Z}_{20}^*$ be the set of all units in $\mathbb{Z}_{20}$. Let $H = \{x \in G \mid x \equiv 1(mod\ k)\}$. Then H is a subgroup if

A. $k = 3$

B. $k = 14$

C. $k = 5$

D. $k = 7$

65. Suppose $o(G) = p^3$, p a prime and G is non-abelian. Then

A. $o(Z(G)) = p$

B. $o(Z(G)) = p^2$

C. $o(Z(G)) = 1$

D. $o(Z(G)) = p^3$

66. Suppose G is a finite abelian group of order n and for every divisor of n there exists exactly one subgroup of G. Then

A. G is cyclic for any n

B. G is cyclic only if n is a power of a prime

C. G is cyclic only if n is a prime

D. G is cyclic only if n is a product of two primes.

67. Suppose that G is a cyclic group of order 49. Then the number of elements of order 7 is

A. 6

B. 7

C. 48

D. 14

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68. Suppose that G is a p-group. Then,

A. For any proper subgroup H of G, H is a proper subgroup of $N_G(H)$.

B. For a proper subgroup H of G, H is proper subgroup of $N_G(H)$ only if H is normal.

C. There exists a proper subgroup H of G such that $H = N_G(H)$.

D. For a proper subgroup H of G, $N_G(H)$ is proper subgroup of G.

69. Suppose $f(x), g(x) \in \mathbb{Q}[X]$ are two irreducible polynomials over $\mathbb{Q}$. Let α, β be two roots of $f(x), g(x)$ such that $\deg_{\mathbb{Q}}\alpha < \deg_{\mathbb{Q}}\beta$. Consider the following statements.
I. if $f(x)$ is irreducible over $\mathbb{Q}[\beta]$, then $g(x)$ is irreducible over $\mathbb{Q}[\alpha]$.
II. if $g(x)$ is irreducible over $\mathbb{Q}[\alpha]$, then $f(x)$ is irreducible over $\mathbb{Q}[\beta]$.

Which of the following is true.

A. Both I and II are true.

B. I is true but not II.

C. II is true but not I.

D. I and II are not true.

70. Consider the following.

1) $\mathbb{Z}[X]/(2, X)$	I) is a field of characteristic 0
2) $\mathbb{Q}[X]/(X^2)$	II) is not an integral domain
3) $\mathbb{R}[X]/(X)$	III) is a field of characteristic 2
	IV) is a finite ring but not an integral domain.

Match the properties of rings.

A. (1, IV), (2, II), and (3, I)

B. (1, III), (2, II) and (3, III)

C. (1, III), (2, II), and (3, I)

D. (1, IV), (2, II), and (3, I)

University of Hyderabad
Ph.D. Entrance Examinations - 2022

School/Department/Centre : Mathematics and Statistics
Course : Ph.D. Subject : Applied Mathematics

Q.No.	Answer	Q.No.	Answer	Q.No.	Answer
1	C	26	B	51	D
2	D	27	A	52	C
3	B	28	A	53	C
4	D	29	D	54	A
5	C	30	B	55	B
6	B	31	A	56	D
7	B	32	B	57	A
8	B	33	D	58	A
9	D	34	D	59	C
10	B	35	A	60	C
11	B	36	D	61	B
12	A	37	A	62	C
13	B	38	A	63	B
14	C	39	C	64	C
15	C	40	C	65	A
16	B	41	D	66	A
17	D	42	D	67	A
18	C	43	D	68	A
19	B	44	A	69	A
20	C	45	B	70	C
21	C	46	D		
22	D	47	B		
23	B	48	D		
24	D	49	A		
25	A	50	C		

Note/Remarks :

Signature
School/Department/Centre 22/11/2022