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ENTRANCE EXAMINATIONS – 2021
Ph.D. Applied Mathematics

Hall Ticket No.	

Time : 2 hours
Max. Marks : 70

PART A: 35 Marks
PART B: 35 Marks

Instructions

1. Write your Hall Ticket Number on the OMR Answer Sheet given to you. Also write the Hall Ticket Number in the space provided above.

2. Answers are to be marked on the OMR sheet.

3. Please read the instructions carefully before marking your answers on the OMR answer sheet.

4. Hand over the OMR answer sheet at the end of the examination to the Invigilator.

5. No additional sheets will be provided. Rough work can be done in the question paper itself/space provided at the end of the booklet.

6. Calculators are not allowed.

7. There are a total of 70 questions in **PART A** and **PART B** together.

8. Each correct carries **1 mark**.

9. The appropriate answer(s) should be coloured with either a blue or black ball point or a sketch pen. DO NOT USE A PENCIL.

10. This book contains **16 pages** including this page and excluding two pages for the rough work. Please check that your paper has all the pages.

11. $\mathbb{R}$ denotes the set of real numbers, $\mathbb{C}$ the set of complex numbers, $\mathbb{Z}$ the set of integers, $\mathbb{Q}$ the set of rational numbers, and $\mathbb{N}$ the set of all natural numbers.

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Part-A

1. What will be the next entry in the following series?
   G25W5, I36U6, L49S7, __

   A. P64P8
   B. P64Q8
   C. O65Q8
   D. P65P8

2. If $x = \sqrt{0.5 + \sqrt{0.5 + \sqrt{0.5 + \cdots}}}$, then $x = ?$

   A. $(1 + \sqrt{2})/2$
   B. $(1 + \sqrt{3})/2$
   C. $(0.5 + \sqrt{3})$
   D. ∞

3. Fill in the blank in the following: 7, 16, 29, 46, 67, __

   A. 86
   B. 88
   C. 93
   D. 92

4. $14 \div 2 + 4 \times 2 - (3 + 2 \times 5) = ?$

   A. $-25/3$
   B. $-58/5$
   C. -10
   D. 2

5. Find the missing entry: BZ, HT, NN, __, ZB

   A. TH
   B. TU
   C. LF
   D. TI

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6. The number of permutations of the letters of the word 'APPLICATION' is ___

A. $11!$

B. $11! - (2! + 2! + 2!)$

C. $\dfrac{3 \times 11!}{4!}$

D. $\dfrac{11!}{6!}$

7. In how many ways can 5 blue, 3 orange and 2 black chairs be arranged in a row if the chairs of same color are indistinguishable?

A. 2530

B. 2520

C. 2500

D. 2422

8. Ajay started walking from the front door of his house and after walking 20 meters straight he turned his left and walked 50 meters. After this, he turned to his right and walked 80 meters. Find the distance between the end-point of his walk and the door of his house?

A. $50\sqrt{3}$

B. 50

C. $50\sqrt{5}$

D. 250

9. In a code language, if 'BAG' is coded as '379', 'RED' as '245' and 'DOOR' as '9663' then how will 'BOARD' be coded in this language?

A. 36725

B. 39725

C. 36425

D. 36745

10. Choose the correct alternative:

A. $f: \mathbb{R} \to \mathbb{R}$ defined by $f(x) = x^2 - 2$ is surjective.

B. $f: \mathbb{R} \to \mathbb{R}$ defined by $f(x) = 2|x| - x$ is surjective.

C. $f: \mathbb{R} \to \mathbb{R}$ defined by $f(x) = 2x - 2$ is not surjective.

D. $f: \mathbb{Z} \to \mathbb{Z}$ defined by $f(n) = 2n - 1$ is not surjective.

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11. Choose the correct statement:

A. There are $x, y \in \mathbb{R}$ with $\big||x| - |y|\big| > |x - y|$.
B. For every $x, y \in \mathbb{R}$, $|x| - |y| \leq |x - y|$.
C. For every $x, y, z \in \mathbb{R}$, $x + y > z$ implies that $x^2 + y^2 > z^2$.
D. There are $x, y \in \mathbb{R}$ with $|x + y| > |x| + |y|$.

12. For $a, b \in \mathbb{N}$, choose the correct alternative:

A. If $\gcd(a, b) = 1$ then, $\gcd(a + b, a - b)$ is 1 or 2.
B. If a divides b then, $\gcd(a, b) = b$.
C. It is possible to have $\gcd(a, b) = 1$ and $\gcd(a + b, ab) \neq 1$.
D. If $\gcd(a, b) = 1$ then, $\gcd(a + b, a^2 + b^2) = 1$.

13. On the collection of all lines in the Euclidean plane, the relation defined by "*is not perpendicular to*" is:

A. an equivalence relation.
B. reflexive and symmetric, but not transitive.
C. symmetric and transitive, but not reflexive.
D. reflexive and transitive, but not symmetric.

14. An example of a subset S of $\mathbb{Z}$ satisfying the property that $ab \in S$ whenever $a, b \in S$, is:

A. $S = \mathbb{Z} \setminus 6\mathbb{Z}$
B. $S = \mathbb{Z} \setminus 21\mathbb{Z}$
C. $S = \mathbb{Z} \setminus 3\mathbb{Z}$
D. $S = \mathbb{Z} \setminus 15\mathbb{Z}$

15. Which of the following is a binary operation $*$ on the specified set?

A. Set $\mathbb{N}$ and $*$ is defined by $x * y = x/y$.
B. Set $\mathbb{N} \cup \{0\}$ and $*$ is defined by $a * b = |a - b|$.
C. Set $\mathbb{Z}$ and $*$ is defined by $x * y = x^y$.
D. Set $\mathbb{Q}$ and $*$ is defined by $x * y = x/y$.

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16. Let A, B and C be nonempty subsets of a set X. Choose the correct alternative:

A. $A \cup B = A \cap B$ if and only if $A = B$.

B. If $A \cup B \subseteq A \cup C$, then $B \subseteq C$.

C. If $A \cap B \subseteq A \cap C$, then $B \subseteq C$.

D. The inclusion $B \subseteq C$ does not imply that $A \cap B \subseteq A \cap C$.

17. Let X be a set and $A, B \subseteq X$. Then the set $A^c \cup (A \cap B)$ is equal to

A. $A^c \cup B$

B. $A \cup B$

C. $A \cap B$

D. $A \cup B^c$

18. In a certain code language CONFERENCE is written as DNOEFQFMDD and WORKSHOP is written as XNSJTGPO. Then the word COLLOQUIUM is written as

A. DMKNPPVIVH

B. DNMKPPVHVL

C. DMNKPQPHVL

D. DNMKQPVIVG

19. The number of ways of arranging the letters of the word TELEPHONE is

A. $9! \times 3!$

B. $6! \times 3!$

C. $\frac{9!}{3!}$

D. $6! \times 3$

20. Find the wrong number in the series: 4,7,11,19,28,39,52

A. 7

B. 39

C. 19

D. 11

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21. Find the next word in the series: HBV, JER, LHN, __

A. NLJ

B. NJK

C. NJL

D. NKJ

22. If the first and second letters in the word UNIVERSITY were interchanged, also the third and the fourth letters, the fifth and the sixth letters and so on, then which letter would be the sixth letter counting from the right

A. E

B. I

C. R

D. V

23. In a certain code language 24685 is written as 33776. How is 36591 written in that code?

A. 44662

B. 46682

C. 44682

D. 45682

24. Determine the next number in the series: 4,5,7,11,19,35,__

A. 76

B. 67

C. 59

D. 55

25. In a code language, the n^{th} letter in the English alphabet is coded to the k^{th} letter, where $k := 3n + 2 (\mathrm{mod}\ 26)$. Then the number of letters that are invariant in this code is

A. 4

B. 3

C. 2

D. 1

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26. It takes eight hours for a 600 km journey, if 120 km is done by train and the rest by car. It takes 20 minutes more, if 200 km is done by train and the rest by car. The ratio of the speed of the train to that of the car is:

A. 5:6

B. 4:5

C. 3:4

D. 2:3

27. Five friends are sitting on a bench. A is to the left of B but on the right of C. D is to the right of B but on the left of E. Who are at the extremes?

A. A,B

B. A,D

C. B,D

D. C,E

28. A box contain RED, BLUE and GREEN coloured marbles. If all but 18 of the marbles are BLUE, all but 12 of the marbles are GREEN and all but 24 of the marbles are RED, then which of the following statement is true?

A. Box contains more RED marbles than BLUE or GREEN ones.

B. Box contains more BLUE marbles than RED or GREEN ones.

C. Box contains more GREEN marbles than RED or BLUE ones.

D. Box contains equal number of GREEN and BLUE marbles.

29. Vinod and Binod are two students among a group of n students. The number of ways to allocate n different rooms in a line to this group of students so that Vinod and Binod are not in adjacent rooms is

A. $(n-2)(n-1)!$

B. $2(n-1)!$

C. $(n-1)!$

D. $2(n-2)!$

30. A group of 8 boys are to be seated at a round table. If three boys Rohan, Mohan and Roshan (among them) are not allowed to seat next (adjacent) to each other, then number of possible seating arrangements is

A. $6 \times 6!$

B. $7!$

C. $7 \times 7!$

D. $8 \times 8!$

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31. The number of 10 length strings using six digits $1, 2, 3, 4, 5, 6$ such that number of 2's appearing is even is

A. $\frac{6^{10}-4^{10}}{2}$

B. $\frac{6^{10}+4^{10}}{2}$

C. $6^{10} - 4^{10}$

D. $6^{10} + 3^{10}$

32. The residue class of the integer $x = \sum_{n=1}^{29} \dfrac{31!}{n}$ is

A. $8 \pmod{17}$

B. $9 \pmod{17}$

C. $10 \pmod{17}$

D. $11 \pmod{17}$

33. The last two digits of $4^{3^{13}}$ are

A. 64

B. 36

C. 84

D. 56

34. The number of solutions to the equation $x_1 + x_2 + \cdots + x_5 = 20$ with $x_i \geq i$ for $i = 1, 2, \ldots, 5$ is

A. $\binom{15}{5}$

B. $\binom{10}{5}$

C. $\binom{9}{5}$

D. 1084

35. The sum of all 4-digit numbers formed using digits $1, 3, 4, 8$ without repetition is

A. 160656

B. 199990

C. 166056

D. 106656

TURN THE PAGE FOR PART B →

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Part-B

36. Match the following:

Rings		Properties	
I.	$\mathbb{R}[\epsilon]/(\epsilon^2)$ a quotient ring	a.	Field
II.	$\mathbb{Q}[\sqrt{2}]$	b.	Non-commutative ring
III.	$M_2(\mathbb{R})$,the ring of 2×2 matrices	c.	Integral domain but not a field
IV.	$\mathbb{R}[X]$ a polynomial ring	d.	Commutative but not an integral domain

A. (I,a), (II,b), (III,c), (IV,d)

B. (I,d), (II,a), (III,b), (IV,c)

C. (I,c), (II,d), (III,a), (IV,b)

D. (I,b), (II,c), (III,d), (IV,a)

37. Match the following:

Field extensions		Properties	
I.	$\mathbb{R}(X)$ over $\mathbb{R}$	a.	Simple
II.	$\mathbb{C}$ over $\mathbb{R}$	b.	Transcendental
III.	$\mathbb{Q}[\sqrt{p}\,\vert\,\text{all } p \text{ prime}]$ over $\mathbb{Q}$	c.	Algebraic but not simple
IV.	$\mathbb{F}_2(X,Y)$ over $\mathbb{F}_2(X^2,Y^2)$	d.	Infinite algebraic

A. (I,a), (II,b), (III,c), (IV,d)

B. (I,c), (II,d), (III,a), (IV,b)

C. (I,b), (II,a), (III,d), (IV,c)

D. (I,b), (II,c), (III,d), (IV,a)

38. Let $\mathbb{Z}_{56}^{\times} = \{\bar{i} | gcd(i,n) = 1\}$ be a multiplicative group. Match the following:

Elements		The order in $\mathbb{Z}_{56}^{\times}$	
I.	$\overline{9}$	a.	1
II.	$\overline{27}$	b.	2
III.	$\overline{5}$	c.	3
IV.	$\overline{1}$	d.	6

A. (I,a), (II,d), (III,b), (IV,c)

B. (I,b), (II,c), (III,a), (IV,d)

C. (I,d), (II,b), (III,c), (IV,a)

D. (I,c), (II,b), (III,d), (IV,a)

39. Match the following:

	Rings		Properties
I.	$\mathbb{Z}_{100}$	a.	has no non-zero nilpotents
II.	$\mathbb{Z}_{529}$	b.	has non-trivial idempotents
III.	$\mathbb{Z}_{91}$	c.	every zero divisor is nilpotent
IV.	$\mathbb{Z}_{380}$	d.	sum of two non-units is again a non-unit

A. (I,c), (II,d), (III,a), (IV,b)
B. (I,c), (II,a), (III,b), (IV,d)
C. (I,b), (II,d), (III,a), (IV,c)
D. (I,d), (II,c), (III,b), (IV,a)

40. Let $C[0,1]$ be the ring of all continuous real-valued functions on $[0,1]$, and let $I = \{f \in C[0,1] : f(\frac{1}{4}) = 0 = f(\frac{3}{4})\}$. Then,

A. I is not an ideal
B. I is an ideal but not a prime ideal
C. I is a prime ideal but not a maximal ideal
D. I is maximal ideal

41. Let G be a group with $|G| = 33$. Then the center $Z(G)$ of G is

A. $\{e\}$
B. $\mathbb{Z}_3$
C. $\mathbb{Z}_{11}$
D. G

42. If $f(z) = \dfrac{z^2}{(z^2+1)^2}$, the consider the following statements.

(I) f has a pole at $z = i$ with residue $\dfrac{-i}{4}$.

(II) f has a pole at $z = -i$ with residue $\dfrac{i}{4}$.

Then which of the following is true?

A. Only (I) is true.
B. Only (II) is true.
C. Both (I) and (II) are True.
D. Both (I) and (II) are False.

43. Consider the function $f(z) = z^4 - 5z + 1$. Then consider the following statements.
(I) The number of zeroes of f in $|z| < 1$ are 1.
(II) The number of zeroes of f in $1 < |z| < 2$ are 3.
Then which of the following is true?

A. Only (I) is true.
B. Only (II) is true.
C. Both (I) and (II) are True.
D. Both (I) and (II) are False.

44. Let $f(z) = 1 - \cos z$. Then consider the following statements .
(I) f has no zeroes.
(II) $z = (2k+1)\pi$, $(k = 0, \pm 1, \pm 2, \ldots)$ are zeroes of f of order 1.
(III) $z = 2k\pi$, $(k = 0, \pm 1, \pm 2, \ldots)$ are zeroes of f order 1.
(IV) $z = 2k\pi$, $(k = 0, \pm 1, \pm 2, \ldots)$ are zeroes of f order 2.
Then which of the following is true?

A. Only (I) is True.
B. Both (II) and (III) are True.
C. Only (III) is True.
D. Only (IV) is True.

45. Match the following functions with their Maclaurin series expansions.

(a) $\sin z$	(i) $\sum_{n=0}^{\infty} \frac{z^{2n+1}}{(2n+1)!}$, $\lvert z\rvert < \infty$.
(b) $\cos z$	(ii) $\sum_{n=0}^{\infty} (-1)^n \frac{z^{2n+1}}{(2n+1)!}$, $\lvert z\rvert < \infty$.
(c) $\sinh z$	(iii) $\sum_{n=0}^{\infty} (-1)^n \frac{z^{2n}}{(2n)!}$, $\lvert z\rvert < \infty$.
(d) $\cosh z$	(iv) $\sum_{n=0}^{\infty} \frac{z^{2n}}{(2n)!}$, $\lvert z\rvert < \infty$.

A. (a,i), (b,ii), (c,iii), (d,iv).
B. (a,ii), (b,iii), (c,i), (d,iv).
C. (a,iv), (b,ii), (c,iii), (d,i).
D. (a,iii), (b,i), (c,ii), (d,iv).

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46. The volume of the part of the solid sphere $x^2 + y^2 + (z+2)^2 \leq 9$, lying above xy-plane is equal to

A. $8\pi/3$
B. $6\pi/3$
C. $2\pi/3$
D. $\pi/3$

47. Match the equations with their types:

(i) $4\dfrac{\partial^2 u}{\partial x^2} - 4\dfrac{\partial^2 u}{\partial x \partial y} + 5\dfrac{\partial^2 u}{\partial y^2} = 0$	(a) elliptic type.
(ii) $x^2\dfrac{\partial^2 u}{\partial x^2} - y^2\dfrac{\partial^2 u}{\partial y^2} = 0$	(b) parabolic type.
(iii) $\dfrac{\partial u}{\partial t} - \dfrac{\partial^2 u}{\partial x^2} = 0$	(c) hyperbolic type.

A. (i,c), (ii,a), (iii,b).
B. (i,a), (ii,c), (iii,b).
C. (i,b), (ii,c), (iii,a).
D. (i,a), (ii,b), (iii,c).

48. Let $L^p[a,b]$ be the class of all p-integrable functions over $[a,b]$. Let $p \leq q$ be non-negative real numbers such that $\frac{1}{p} + \frac{1}{q} = 1$. If $f \in L^p[a,b]$ and $g \in L^q[a,b]$, then

A. $fg \in L^1[a,b]$
B. $fg \in L^p[a,b]$
C. $fg \in L^q[a,b]$
D. $fg \in L^{p+q}[a,b]$

49. Let $0 < p < 1$ and $f, g \in L^p[a,b]$ be such that $f \geq 0$ and $g \geq 0$. Then

A. $\|f+g\|_p \neq \|f\|_p + \|g\|_p$.
B. $\|f-g\|_p \leq \|f\|_p + \|g\|_p$.
C. $\|f+g\|_p \geq \|f\|_p + \|g\|_p$.
D. $\|f-g\|_p \leq \max\{\|f\|_p, \|g\|_p\}$.

50. The real linear space $C^1[0,1]$ of all continuously differentiable functions defined on $[0,1]$ equipped with the supremum norm given by $\|f\|_\infty = \sup\{|f(x)| : x \in [0,1]\}$ is

A. a complete normed space.
B. an incomplete normed space.
C. not a normed space.
D. a non-separable normed space.

51. Let $f(x) = \frac{1}{x}$ for $x \in (1, \infty)$. Then

A. $f \in L^2(1,\infty)$ and $\|f\|_2 = 1$.
B. $f \in L^2(1,\infty)$ and $\|f\|_2 < 1$.
C. $f \in L^2(1,\infty)$ and $\|f\|_2 > 1$.
D. $f \notin L^2(1,\infty)$ and $\|f\|_2 = \infty$.

52. The Wronskian $W(\cos x, \sin x, e^{-x}) =$?

A. 2
B. $2e^x$
C. $2e^{-x}$
D. $2e^{-2x}$

53. The function $f(x,y) = xy^2$ is Lipschitz continuous on ___

A. $|x| \leq 1$, $|y| \leq 1$
B. $|x| \leq 1$, $|y| < \infty$
C. $|x| < \infty$, $|y| \leq 1$
D. $|x| \leq 2$, $|y| < \infty$

54. The solution of the differential equation $y'' + 5y' + 6y = 0$ is:

A. continuously differentiable.
B. unbounded on $[0, \infty)$.
C. not differentiable.
D. not monotonic.

55. Let $\{f_n\}$ be a sequence of real-valued differentiable functions defined on $\mathbb{R}$. If $\{f_n\}$ converges to a function f uniformly on $\mathbb{R}$, then the sequence $\{f_n'\}$ ___

A. converges uniformly to f'.
B. converges pointwise to f', but may not be uniformly.
C. converges pointwise to some function, but may not be to f'.
D. may not be convergent.

56. Consider the set $\mathbb{R}$ with following topologies: τ_1 = cofinite topology, τ_2 = discrete topology, τ_3 = standard Euclidean topology and τ_4 = lower limit topology. Then

A. $\tau_4 \supset \tau_2 \supset \tau_3 \supset \tau_1$

B. $\tau_2 \supset \tau_3 \supset \tau_4 \supset \tau_1$

C. $\tau_2 \supset \tau_4 \supset \tau_1 \supset \tau_3$

D. $\tau_2 \supset \tau_4 \supset \tau_3 \supset \tau_1$

57. The number of Hausdorff topologies on a set with 5 elements is

A. 25

B. 5

C. 1

D. 15

58. Every ____ subset of the real line is a Borel set.

A. uncountable

B. bounded

C. connected

D. Lebesgue measurable

59. The set of points at which a given function $f : \mathbb{R} \to \mathbb{R}$ is continuous is always:

A. an F_σ set

B. a G_δ set

C. an open set

D. a closed set

60. Let G be a group of order pq, where p, q are primes with $p < q$. Then

A. G is cyclic.

B. G is abelian, but may not be cyclic.

C. G has a normal subgroup of order p, but G may not be abelian.

D. G has a normal subgroup of order q, but G may not be abelian.

61. Up to similarity, how many different 7×7 complex matrices are there whose minimal polynomial is $(x-1)^2(x-2)^3$?

A. 7
B. 5
C. 3
D. 2

62. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be the linear map given by $T(x, y, z) := (3x + y, y, 3z)$. Then T is

A. triangulable over $\mathbb{R}$, but not invertible.
B. invertible, but not triangulable over $\mathbb{R}$.
C. neither invertible nor diagonalizable over $\mathbb{R}$.
D. both invertible and diagonalizable over $\mathbb{R}$.

63. How many non-zero nilpotent elements are there in the ring $\mathbb{Z}_{2020}$?

A. 0
B. 1
C. 2
D. 3

64. In cylindrical co-ordinates (ρ, φ, z), $x = \rho\cos\varphi$, $y = \rho\sin\varphi$, $z = z$, Laplace's equation $\nabla^2 A = 0$ can be written as

A. $\dfrac{\partial^2 A}{\partial \rho^2} + \dfrac{1}{\rho}\dfrac{\partial A}{\partial \rho} + \dfrac{1}{\rho^2}\dfrac{\partial^2 A}{\partial \varphi^2} + \dfrac{\partial^2 A}{\partial z^2} = 0.$

B. $\dfrac{\partial^2 A}{\partial \rho^2} + \dfrac{1}{\rho^2}\dfrac{\partial A}{\partial \rho} + \dfrac{\partial^2 A}{\partial \varphi^2} + \dfrac{\partial^2 A}{\partial z^2} = 0.$

C. $\dfrac{\partial^2 A}{\partial \rho^2} + \dfrac{\partial^2 A}{\partial \varphi^2} + \dfrac{\partial^2 A}{\partial z^2} = 0.$

D. $\dfrac{\partial^2 A}{\partial \rho^2} - \dfrac{1}{\rho}\dfrac{\partial A}{\partial \rho} + \dfrac{1}{\rho^2}\dfrac{\partial^2 A}{\partial \varphi^2} + \dfrac{\partial^2 A}{\partial z^2} = 0.$

65. The equation $u_{xx} + 2xu_{xy} + u_{yy} = 0$ is

A. parabolic when $x = \pm 1$, and elliptic when $x = 0$.
B. parabolic when $x = \pm 1$, and hyperbolic when $x = 0$.
C. hyperbolic when $x > 1$, and parabolic when $x < -1$.
D. parabolic when $x > 1$, and hyperbolic when $x < -1$.

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66. For the linear system $\frac{dx}{dt} = ax + by$, $\frac{dy}{dt} = cx + dy$, where a, b, c, d are real constants, the critical point $(0,0)$ is a spiral if

A. $a = d \neq 0$, $b = c = 0$.
B. $a = d$, either $b = 0$ or $c = 0$.
C. $a = d$, b and c are of opposite signs.
D. $a = d$, b and c are of same signs.

67. Let $f(x, y)$ be an arbitrary function of x and y. The differential equation $yz\ \mathrm{d}x + 2xz\ \mathrm{d}y + f(x, y)\ \mathrm{d}z = 0$ is exact when $f(x, y)$ is equal to

A. $\phi(\xi)/y$ where $\xi = xy^2$ and ϕ is arbitrary function of ξ.
B. $\phi(\xi)/x$ where $\xi = xy^2$ and ϕ is arbitrary function of ξ.
C. $\phi(\xi)/y^2$ where $\xi = x^2y$ and ϕ is arbitrary function of ξ.
D. $\phi(\xi)/x^2$ where $\xi = x^2y$ and ϕ is arbitrary function of ξ.

68. The equation $x^2y''(x) + 3xy'(x) + y = 0$ subject to the conditions $\alpha y(1) + y'(2) = 1$ and $y'(1) + \beta y(2) = 2$ has

A. no solution if $\alpha = 1/4$ and $\beta = 2$.
B. no solution if $\alpha = 2$ and $\beta = 1/4$.
C. no solution if $\alpha = 4$ and $\beta = 1/2$.
D. no solution if $\alpha = 4$ and $\beta = 2$.

69. The system $\frac{dx}{dt} = y + 1 - \cos x$, $\frac{dy}{dt} = \sin(x + y)$.

A. has only one critical point which is stable.
B. has infinitely many critical points which are stable.
C. has finitely many critical points which are unstable.
D. has infinitely many critical points which are saddle points.

70. Let $f, g : \mathbb{R} \to \mathbb{R}$ be functions. Let $u(x, t)$ be the solution of the equation $\frac{\partial^2 u}{\partial x^2} + 2\frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 u}{\partial y^2} = 0$ subject to $u(x, 0) = f(x)$ and $u(0, y) = g(y)$. Then $u(x, y)$ is equal to

A. $(x - y)^{-1}\left(x^2 f(x - y) - yg(y - x)\right)$.
B. $(x - y)^{-1}\left(xf(x - y) - y^2 g(y - x)\right)$.
C. $(x - y)^{-1}\left(x^2 f(x - y) - y^2 g(y - x)\right)$.
D. $(x - y)^{-1}\left(xf(x - y) - yg(y - x)\right)$.

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University of Hyderabad
Entrance Examinations - 2021

School of Mathematics and Statistics
Course/Subject : Ph.D. in Applied Mathematics

Q.No.	Answer	Q.No.	Answer	Q.No.	Answer	Q.No.	Answer
1	B	26	C	51	A	76	
2	B	27	D	52	C	77	
3	D	28	C	53	A	78	
4	D	29	A	54	Cancelled	79	
5	A	30	A	55	D	80	
6	C	31	B	56	D	81	
7	B	32	B	57	C	82	
8	C	33	A	58	C	83	
9	A	34	C	59	B	84	
10	D	35	D	60	D	85	
11	B	36	B	61	B	86	
12	A	37	C	62	D	87	
13	B	38	D	63	B	88	
14	C	39	Cancelled	64	A	89	
15	B	40	B	65	A	90	
16	A	41	D	66	C	91	
17	A	42	C	67	A	92	
18	B	43	C	68	A	93	
19	C	44	D	69	D	94	
20	D	45	B	70	D	95	
21	D	46	A	71		96	
22	C	47	B	72		97	
23	D	48	A	73		98	
24	B	49	C	74		99	
25	C	50	B	75		100	

Note/Remarks : For Question Nos. 39 & 54 benefit will be given to all candidates.

Signature of the Head/Dean,
School/Department/Centre