Entrance Examinations - 2021
M.Sc. Statistics

Hall Ticket Number | | | | | | | |

Time : 2 hours
Max. Marks. 100

Part A : 25 marks
Part B : 75 marks

Instructions

1. Write your Hall Ticket Number on the OMR Answer Sheet given to you. Also write the Hall Ticket Number in the space provided above.
2. Answers are to be marked on the OMR answer sheet.
3. Please read the instructions carefully before marking your answers on the OMR answer sheet.
4. Hand over the OMR answer sheet after the examination.
5. There are plain sheets in the booklet for rough work, no additional sheets will be provided.
6. There are a total of 50 questions in Part A and Part B together.
7. **Each question in Part – A has only one correct option and there is negative marking of 0.33.**
8. **There is no negative marking in Part - B. Some questions have more than one correct option. All the correct options have to be marked in the OMR answer sheet, otherwise zero marks will be credited.**
9. The appropriate answer(s) should be coloured with either a blue or a black ball point or a sketch pen. DO NOT USE A PENCIL.
10. **The maximum marks for this examination is 100, 25 for Part-A and 75 for Part-B, there will be NO INTERVIEW.**
11. Given below are the meanings of some symbols that may have appeared in the question paper:
$\mathbb{R}$-The set of all real numbers, $E(X)$-Expected value of the random variable X, $V(X)$-Variance of the random variable X, $Cov.(X,Y)$-Covariance of the random variables X and Y, $\rho_{X,Y}$ denotes the correlation coefficient between X and Y, *iid*-independent and identically distributed, *pdf*-probability density function, $B(n,p)$, $N(\mu,\sigma^2)$ and $U((a,b))$ denote respectively,the Binomial, the Normal and the Uniform distributions with the said parameters. $Rank(\mathbf{A})$ means rank of the matrix $\mathbf{A}$. Members of $\mathbb{R}^n$ are column vectors $\underset{\sim}{x}$ and $\underset{\sim}{0}$ is the column vector of zeros or the zero vector.
12. This book contains **12** pages including this page and excluding pages for rough work. Please check that your paper has all the pages.

2-7

Part - A

Find the correct answer and mark it on the OMR sheet. Each correct answer gets 1 (one) mark and wrong answer gets -0.33 marks

1. Random Experiments are

   I. experiments which when repeated under the same conditions result in the same outcome always.

   II. experiments whose outcome can be different when repeated.

   III. experiments whose possible outcomes are not known.

   IV. experiments all of whose possible outcomes are known.

   The correct statements are

   (A) only I and II. **(B)** only III and IV. **(C)** only II and IV. **(D)** only I,III and IV.

2. Of 20 balls how many balls should be each of two colours so that the number of different arrangements in a row is the most?

   (A) 11 of one colour and 9 of the other.

   (B) 15 of one colour and 5 of the other.

   (C) 12 of one colour and 8 of the other.

   (D) 10 of each colour.

3. In any collection of 10 people born in the year 2000, the probability that all of them have different birthdays is

   (A) nearly 0. **(B)** very close 1/3. **(C)** very close to 2/3. **(D)** very close to 1.

4. The probability of at least one of the events A and B occurring is 0.8 while the probability that at lest one of them does occur is 0.7, the probability of exactly one of them occurring

   (A) is 0.7. **(B)** is 0.5. **(C)** is 0.4.
   (D) can not be determined from the information given.

5. Exactly one marble in a bag of 100 marbles of equal sizes is red. If two marbles are to be removed one after the other, the probabilities P_1 and P_2 of the first drawn marble and the second drawn marble being red respectively are

   (A) $P_1 = \frac{1}{100}$, $P_2 = \frac{1}{99}$. **(B)** $P_1 = \frac{1}{100}$, $P_2 = 0$.
   (C) $P_1 = \frac{1}{100}$, $P_2 = \frac{1}{100}$. **(D)** $P_1 = \frac{1}{100}$, $P_2 = \frac{1}{98}$.

2-7

6. Which of the random variables described below is a Binomial random variable?

   (A) The number of red balls in 5 draws of one ball each without replacement from a bag containing 10 red and 10 blue balls.

   (B) The number of problems solved correctly out of 10.

   (C) The toss number at which the $5^{th.}$ heads shows up upon tossing of a fair coin.

   (D) The number of red balls in 5 draws of one ball each with replacement from a bag containing 10 red and 15 blue balls.

7. m numbers are drawn from the set $\{1, 2, \ldots, 2m\}$ without replacement, let X_1 and X_2 denote the mean of the m numbers drawn and the mean of the m numbers not drawn.

   (A) $X_1 = X_2$ always. **(B)** $E(X_1) = E(X_2)$ always.
   (C) $E(X_1) > E(X_2)$ always. **(D)** $V(X_1) < V(X_2)$ always.

8. Given below is the frequency distribution of marks of 50 students of a class

Marks	5	10	20
frequency	20	20	10

   Identify the correct option

   (A) The marks are symmetric about the median marks.

   (B) The median and mean marks are equal, but the data are not symmetric.

   (C) The data are positively skewed and the mean marks are more than the median.

   (D) The median is 15.

9. A blindfolded person randomly picks up 15 soaps from a super market that contains 6 brands of soap. The number of soaps of Brand number 1 in the 15 soaps picked up is a

   (A) Binomial random variable.

   (B) Negative Binomial random variable.

   (C) Multinomial random variable.

   (D) Hypergeometric random variable.

10. The number of e-mails that come into my Spam folder in a day is a Poisson random variable with mean 2. If the number of spam mails on each day is observed for say 200 days one is likely to find

    (A)2 spam mail on more days than 2. **(B)**5 spam mails on more days than 4.
    (C) 6 spam mails on more days than 5. **(D)** 4 spam mails on more days than 3.

11. Let X_1 denote the number of tosses of a fair coin till the first heads shows up and let X_2 denote the number of tosses of the same coin till the next heads shows up. The random variable $X_1 + X_2$

(A) has binomial distribution with parameters $2, 1/2$.

(B) has negative binomial distribution with parameters $2, 1/2$.

(C) has geometric distribution with parameter $1/2$.

(D) has hypergeometric distribution.

12. To the 20 numbers $a_1, \ldots, a_{20}$ add a positive quantity c to the 10 smaller numbers and subtract c from the rest. Let the new set of numbers so obtained be $\{b_1, \ldots, b_{20}\}$

(A) The mean and variance of the new set are the same as those for the original set.

(B) The variance of the new set is more than that of the original set.

(C) The variance of the new set is less than that of the original set.

(D) The variance may be different, but the mean remains the same.

13. You have 200 observations of the $B(6, 2/3)$ random variable, the most frequently occurring number is likely to be

(A) 2. (B) 3. (C) 4. (D) 5.

14. There are M_1 red and M_2 blue balls in a bag, let X denote the number of blue balls drawn in a draw of n balls from this bag, assume that $n < \min.(M_1, M_2)$ and let Y denote the number of red balls left in the bag after the draw, the correlation $\rho_{X,Y}$ between X and Y is

(A)1. (B) $\frac{M_1}{M_1+M_2}$. (C) 0. (D) -1.

15. $X \sim N(0, 2)$, $E(X^{11})$ is equal to

(A) 1. (B) 0. (C) 2. (D) 11.

16. $X_1 \sim U((-3, 3))$ and $X_2 \sim N(0, 1)$, identify the correct statement.

(A)$Pr(|X_1| > 2) > Pr(|X_2| > 2)$. (B)$E(X_1) < E(X_2)$.
(C) $Pr(|X_1| < 3) < Pr(|X_2| < 3)$. (D)$V(X_1) = V(X_2)$.

17. What should the sample size be so that the length of the 95% confidence interval for the mean of a normal random variable whose variance is 100 is at the most 10? For $Z \sim N(0,1)$, $Pr(Z \leq 1.645) = 0.95, Pr(Z \leq 1.96) = 0.975$.

    (A) could be 10.

    (B) should be at least 100.

    (C) should be at least 16.

    (D) should not be more than 15.

18. The 97.5% confidence interval for the mean μ based on a sample of size 25 is $(55, 70)$, this means that

    (A) μ is certainly between 55 and 70.

    (B) the hypothesis $H_0 : 62$ will be rejected at .025 level of significance.

    (C) the hypothesis $H_0 : 64$ will be accepted at .025 level of significance.

    (D) μ is more than 70 with probability 0.0125.

19. The critical regions of an Hypothesis testing problems at α_1 and α_2 levels of significance are C_1 and C_2 respectively, and, $C_1 \subset C_2$, so

    (A) $\alpha_1 = \alpha_2$. **(B)** $\alpha_1 > \alpha_2$. **(C)** $\alpha_1 < \alpha_2$. **(D)** can not say which is more.

20. Let M_1 denote the number of different simple random samples of size 4 from a population of 20 distinct units and let M_2 denote the number of stratified samples of size 4 with proportional allocation from the same populations which is stratified into 3 strata of sizes 10, 5 and 5. M_1/M_2 is

    (A) less than 1. **(B)** in $[1, 2.5]$. **(C)** in $(2.5, 4]$. **(D)** more than 4.

21. The household incomes of two countries C_1 and C_2 that have about the same number of households are normally distributed with mean and standard deviation Rs.30000&15000 and Rs.40000, &5000 respectively, it means that

    (A) more than half of the households in C_1 have more than the average income of C_2.

    (B) less than half of the households in C_2 have more than the average income of C_1.

    (C) more households in C_1 have incomes greater than Rs.50000 than in C_2.

    (D) more households in C_1 have incomes less than or equal to Rs.60000 than in C_2.

22. If an hypothesis H_0 regarding a population is true, the probability of drawing the sample that is got is 0.03, we should

(A) reject H_0 at 0.05 level of significance.

(B) accept H_0 at 0.04 level of significance.

(C) reject H_0 at 0.02 level of significance.

(D) reject H_0 at .01 level of significance.

23. $\mathbf{A}$ is a real $n \times n$ matrix and $\mathbf{A}\underset{\sim}{\mathbf{x}} = \mathbf{A}\underset{\sim}{\mathbf{y}}$ where neither of the distinct vectors $\underset{\sim}{\mathbf{x}}$ and $\underset{\sim}{\mathbf{y}}$ is $\underset{\sim}{\mathbf{0}}$, this means

I. $\mathbf{A}$ is singular.

II. $\mathbf{A}$ has to be the zero matrix.

III. All the rows of $\mathbf{A}$ are linearly independent.

IV. Some columns of $\mathbf{A}$ are linear combinations of other columns.

The correct statements are

(A) only II. (B) only I and IV. (C) only III. (D) only III and IV.

24. Let $\underset{\sim}{\mathbf{a}}_1, \underset{\sim}{\mathbf{a}}_2, \underset{\sim}{\mathbf{a}}_3, \underset{\sim}{\mathbf{a}}_4$ and $\underset{\sim}{\mathbf{a}}_5$ be the 5 linearly independent columns of a real 5×5 matrix $\mathbf{A}$. $\mathbf{B}$ is the 5×5 matrix whose columns are $\underset{\sim}{\mathbf{a}}_1, \underset{\sim}{\mathbf{a}}_1 + \underset{\sim}{\mathbf{a}}_2, \underset{\sim}{\mathbf{a}}_1 + \underset{\sim}{\mathbf{a}}_2 + \underset{\sim}{\mathbf{a}}_3, \underset{\sim}{\mathbf{a}}_1 + \underset{\sim}{\mathbf{a}}_2 + \underset{\sim}{\mathbf{a}}_3 + \underset{\sim}{\mathbf{a}}_4, \underset{\sim}{\mathbf{a}}_1 + \underset{\sim}{\mathbf{a}}_2 + \underset{\sim}{\mathbf{a}}_3 + \underset{\sim}{\mathbf{a}}_4 + \underset{\sim}{\mathbf{a}}_5$, the Rank of $\mathbf{B}$ is

(A)5. (B) 4 (C) 3 (D) 2.

25. If I have read at least one book in shelf A, not read at least one book in shelf B, and, not read any book in shelf C, then it means that

(A) I have not read any book in any of the 3 shelves.

(B) I have read some books in all the 3 shelves.

(C) I have read some books from both of the shelves A and B.

(D) I may have read just one book from shelf A and no book from shelves B and C.

Part - B

- **Questions (26)-(37) have more than one correct option.**
- **For the answer to be right all the correct options have to be marked on the OMR sheet.**
- **No credit will be given for partially correct answers.**
- **Questions (38)-(50) have only one correct option.**
- **Find the correct answers and mark them on the OMR sheet. Correct answers (marked in OMR sheet) to a question get 3 marks and zero otherwise.**

26. The probabilities $P(A_1), P(A_2)$ and $P(A_3)$ of 3 events A_1, A_2 are all in the interval $(0, 1)$.

(A) If A_1 and A_2 are independent, then $P(A_1^c|A_2^c) = P(A_1^c)$.

(B) If A_1 and A_2 are independent, then they are also conditionally independent given A_3, that is $P(A_1 \cap A_2|A_3) = P(A_1|A_3)P(A_2|A_3)$.

(C) If A_2 confirms A_1, that is if $P(A_1|A_2) > P(A_1)$, then A_1^c confirms A_2^c.

(D) If A_1, A_2, A_3 are pairwise independent, then $A_1 \cap A_2$ and A_3 are independent.

27. Select a point D in accordance with the uniform distribution on the base BC of an equilateral triangle ABC whose lengths are a units

(A) The probability that the area of ABD is at most $(1/6)^{th}$ of the area of ABC is $1/3$.

(B) The expected area of ABD is half the area of ABC.

(C) The expected area of ADC is more than half of the area of ABC.

(D) The probability that the area of ABD is more than $(3/4)^{th}$ the area of ABC is $1/4$.

28. For two randomly selected numbers without replacing from $1, \ldots, 10$, the probability

(A) that their sum is even is the same as the probability that their absolute difference is even.

(B) that their sum is even the same as the probability that their absolute difference is odd.

(C) that their product is even is more than the probability that their product is odd.

(D) their product is odd is the same as the probability that their absolute difference is odd.

29. Upon rolling a 6 faced die for which probability of 6 is p_6, the first, second and third 6 showed up in the 8^{th}, 16^{th} and the 20^{th} throws. So

(A) 8, 8 and 4 are three independent observations of the $G(p_6)$ random variable.

(B) 20 is an observation of the Negative Binomial random variable with parameters 3 and p_6 that is $NB(3, p_6)$.

(C) we have not got an observation of the $NB(4, p_6)$ random variable.

(D) we have not got an observation of the $B(3, p_6)$ random variable.

30. X is a random variable whose distribution is symmetric about 10, so

(A) The expected value of X is 10.

(B) The probability of the event $X > 0$ is more than $1/2$, that is $Pr(X > 0) > 1/2$.

(C) $Pr(X \leq 16) > Pr(X \geq 5)$.

(D) The random variables $Y = X - 10$ and $-Y = 10 - X$ are identically distributed.

31. $X_i \sim U((-1, 1))$, $i = 1, 2$ and are independent

(A) $X_1 + X_2$ and $X_1 - X_2$ are identically distributed.

(B) The correlation coefficient between $X_1 + X_2$ and $X_1 - X_2$ is 0.

(C) $X_1 + X_2$ and $X_1 - X_2$ are independent random variables.

(D) $X_1 + X_2$ and $X_1 - X_2$ are not independently distributed.

32. The marks of 10 students in two exams E_1 and E_2 are as follows:

E_1	55	55	55	55	55	55	55	55	55	80
E_2	34	41	47	52	55	55	62	67	72	80

(A) The variance in the marks in E_2 is more than the variance in the marks in E_1.

(B) 80 marks in E_1 is a superior performance to 80 marks in E_2.

(C) 80 marks in E_1 and 80 marks in E_2 are equally good.

(D) The average marks in the two exams are the same.

33. From a bag containing 10 red and 5 blue balls, keep removing one ball at a time till all the 5 blue balls have been removed. Let X denote the draw number in which the last blue ball got removed. Identify the correct statements.

(A) X has negative Binomial distributions with parameters 5 and 1/3.

(B) The expected value of X is less than 5.

(C) The expected value of X is more than 7.

(D) The most likely value of X is 15.

34. The probability density function of a random variable X is $f_X(x) = \begin{cases} \frac{1}{2}(1+x) & -1 \leq x \leq 0 \\ \frac{1}{2} - \frac{x}{6} & 0 \leq x \leq 3 \\ 0 & e.w \end{cases}$ .Identify the correct statements

(A) The distribution of this random variable is skewed to the right.

(B) The median of X is more than its expected value.

(C) The median and the expected value are equal.

(D) $Pr(-1 < X \leq -1/2) > Pr(5/2 < X < 3)$.

35. To estimate the average milk consumption in households of a locality the 1000 households are divided into 3 strata of 200, 500 and 300 households. Identify the correct statements regarding a stratified sample of 100 units from this stratified population

(A) The stratification should have been done by asking a child from each household to select a number from 1, 2 and 3 and then placing that household in the stratum number selected.

(B) The stratification could have been done based on size of the house.

(C) A sample allocation of 20, 50 and 30 units respectively is the sample allocation that is proportional to the strata sizes.

(D) Every household has the same probability of being included in the stratified sample with allocation proportional to stratum size.

36. For a random variable X, to test the hypothesis $H_0 : f_X(x) = \begin{cases} 2x & 0 < x < 1 \\ 0 & o.w \end{cases}$ versus the alternate hypothesis $H_1 : f_X(x) = \begin{cases} 6x(1-x) & 0 < x < 1 \\ 0 & o.w \end{cases}$ based on one observation the critical region is the interval $(0, 0.2)$. Identify the correct statements

(A) If the observed value is 0.1, we should accept H_1.

(B) The level of significance for this test is less than 0.05.

(C) The power of this test is more than 0.72.

(D) Accept H_0 if the observed value is 0.8.

37. $X_1, \ldots, X_n$ is a random sample from the $N(\mu, \mu^2)$ population.

(A) The sample mean $\bar{X}$ is an unbiased estimator for μ.

(B) $\bar{X}$ is a sufficient statistic for μ.

(C) There is no single sufficient statistic for μ.

(D) $\frac{1}{n}\sum_{i=1}^{n} X_i^2$ is an unbiased estimator for μ^2.

38. The average age and variance of 3 children of a family are 9 years and 6 respectively, the sum of the squares of their ages after three years is

(A) 350. (B) 450. (C) 400 (D) 475.

39. Take out two balls from a bag that has 6 red and 8 blue balls,

i. if they are of different colours, put them them back.

ii. if both are blue, put two red balls in the bag, but keep the drawn blue balls out.

iii. if both the balls drawn are red, put back two blue balls in the bag, but keep the drawn red balls out.

the expected number of red balls after this process is closest to

(A) 8. (B) 7. (C)6. (D) 5.

40. The variance of a finite random variable X is 25, identify the correct statement

(A) $E(X^2)$ could be 20. (B) $E(X^4)$ could be 750.
(C) $(E(X^2))^2$ could be 500. (D) $V(X^2)$ can not be 0.

41. Of the $N(>1)$ balls in a bag exactly one ball is red, to estimate N keep removing a ball from this bag till the red ball is drawn, suppose this occurred in the $15^{th.}$ draw, an unbiased estimate for N

(A) is 15. (B) is 30. (C) is 29. (D) can not be determined from the given data.

42. X_1 and X_2 are independently and identically distributed random variables with the following distribution $Pr(X_i = m) = 1/11, \quad m = -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5$ for $i = 1, 2$. The expected value of $Y = \frac{X_1}{X_1+X_2}$

(A) is 0. (B) is 1/2. (C) is 5.
(D) does not exist.

43. A coin for which the probability of heads showing up upon tossing is 2/3 is to be first tossed 10 times and then 15 times, given that a total number heads showed up is m,the number of heads showing up in the first 10 tosses is a

(A) Binomial random variable with parameters $10, 2/3$.

(B) Bernoulli random variable.

(C) Discrete uniform random variable.

(D) Hypergeometric random variable.

44. Given that the second heads showed up in the $10^{th.}$ toss of a coin, the probability that the first heads showed up in the $5^{th.}$ toss

(A) is 1/5. (B) is 1/9. (C) is 1/10. (D) can not be determined from the given data.

45. X_1 and X_2 are independent Poisson random variables with parameters 1 and 2 respectively, $E(X_1|X_1 + X_2 = 6)$ is equal to

(A) 6. (B) 4. (C)2. (D) 1.

46. The sum of squares of 4 distinct positive numbers is 36, their product

(A) is less than 81.

(B) is at least 81 but less than 86.

(C) is at least 87 but less than 100

(D) could be 112.

47. From a bag that contains 10 blue and M red balls, 6 balls were taken out and 4 of them were red. Which of the alternatives is the most likely value of M?

(A) 13. (B) 12 . (C) 11. (D) 10.

48. The means of at least 98% of all samples of size n should be within 10 from the population mean of a random variable whose variance 150, what can you say about the sample size n?

(A) n should be 50. (B) n can be 75. (C) n can be 85. (D) n should be 90.

49. Let p be the probability of heads(H) showing up upon tossing a coin if H, T, H, H, T, H are the outcomes upon tossing the said coin 6 times, an unbiased estimate for p^2 is

(A) 4/5. (B) 4/9. (C) 4/15. (D) 2/5.

50. The value of the sum $\sum_{j=0}^{n}\sum_{k=1}^{j}(-1)^j\binom{n}{j}\binom{j}{k}$ is

(A)1. (B)0. (C) 2^n. (D) 3^n.

University of Hyderabad
Entrance Examinations - 2021

School of Mathematics and Statistics
Course/Subject : M.Sc in Statistics

Q.No.	Answer	Q.No.	Answer	Q.No.	Answer	Q.No.	Answer
1	C	26	A, C	51		76	
2	D	27	B,D	52		77	
3	D	28	A,C	53		78	
4	Cancelled	29	A,B,C,D	54		79	
5	C	30	B,C,D	55		80	
6	D	31	A,B,D	56		81	
7	B	32	A,B	57		82	
8	B	33	C,D	58		83	
9	D	34	A,D	59		84	
10	Cancelled	35	B,C,D	60		85	
11	B	36	A,B,D	61		86	
12	D	37	A,C	62		87	
13	C	38	B	63		88	
14	A	39	C	64		89	
15	B	40	B	65		90	
16	A	41	C	66		91	
17	C	42	D	67		92	
18	C	43	D	68		93	
19	C	44	B	69		94	
20	D	45	C	70		95	
21	C	46	A	71		96	
22	A	47	A	72		97	
23	B	48	B or C	73		98	
24	A	49	D	74		99	
25	D	50	Cancelled	75		100	

Note: For Question Nos. 4, 10 and 50 benefit will be given to all candidates.
For Question no. 48 the correct answer is 'B' or 'C' and B & C.

Signature of the Head/Dean
School/Department/Centre