

4-62

ENTRANCE EXAMINATIONS – 2020
(Ph.D. Admissions - January 2021 Session)

Ph.D. Statistics

Hall Ticket No.	
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Time : 2 hours
Max. Marks : 70

PART A: 35 Marks
PART B: 35 Marks

Instructions

1. Write your Hall Ticket Number on the OMR Answer Sheet given to you. Also write the Hall Ticket Number in the space provided above.
2. Answers are to be marked on the OMR sheet.
3. Hand over the OMR answer sheet at the end of the examination to the Invigilator.
4. No additional sheets will be provided. Rough work can be done in the question paper itself/space provided at the end of the booklet.
5. Calculators are not allowed.
6. There are a total of 70 questions in **PART-A** and **PART-B** together of **one mark** each.
7. The appropriate answer(s) should be coloured with either a blue or black ball point or a sketch pen. **DO NOT USE A PENCIL.**
8. This book contains **12 pages** including this page and excluding pages for the rough work. Please check that your paper has all the pages.
9. Given below are the meanings of some symbols that may have appeared in the question paper:
R-The set of all real numbers, $E(X)$ -Expected value of the random variable X ; $V(X)$ -Variance of the random variable X ; $Cov(X, Y)$ -Covariance of the random variables X and Y ; $\rho_{X, Y}$ -correlation coefficient between X and Y ; iid-independent and identically distributed; pdf-probability density function; $B(n, p)$ and $N(\mu, \sigma^2)$ - respectively the Binomial and the Normal distributions with the said parameters; $Rank(A)$ and $det(B)$ -respectively the rank and the determinant of the matrices A and B .

Part-A

1. Two squares are selected at random from a chess board with equal probabilities, what is the probability that they have a common side?
 (A) $1/18$. (B) $1/36$. (C) $7/12$. (D) $7/24$.

2. If $A = [-3, 7]$, $B = (-1, 9)$ then $A \Delta B$ is equal to
 (A) union of two disjoint open intervals.
 (B) an open interval.
 (C) a closed interval.
 (D) none of the above is correct.

3. $X \sim N(65, 10)$, arrange the following events in increasing order of their probabilities:

$$E_1 = \{X > 75\} \quad E_2 = \{X \leq 58\} \quad E_3 = \{X > 90\} \quad E_4 = \{X \leq 65\}$$

- (A) E_1, E_2, E_3, E_4 . (B) E_3, E_1, E_2, E_4 .
 (C) E_3, E_1, E_4, E_2 . (D) E_2, E_1, E_3, E_4 .
4. Let X be a random variable such that $P(Y = 1) = p = 1 - P(Y = -1)$. Find $a(a \neq 1)$ such that $E(a^Y) = 1$
 (A) p . (B) $\frac{1-p}{p}$. (C) $\frac{p}{1+p}$. (D) p^2 .

5. Let $X \sim N_p(\mu, \Sigma)$. The moment generating function of X is

- (A) $M_X(t) = \exp(t'\mu + \frac{1}{2}t'\Sigma t)$.
 (B) $M_X(t) = \exp(t'\mu - \frac{1}{2}t'\Sigma t)$.
 (C) $M_X(t) = \exp(\frac{1}{2}t'\mu + t'\Sigma t)$.
 (D) $M_X(t) = \exp(\frac{1}{2}t'\mu + t'\Sigma t)$.

6. Let $X \sim N_n(\mu, \Sigma)$. Then $X'A_1X$ and $X'A_2X$ are independent if and only if
 (A) $A_1A_2 = 0$. (B) $A_1 + A_2 = 0$. (C) $A_1 = 0$. (D) $A_2 = 0$.

7. An accident occurs at a point X that is uniformly distributed on a road of length L . At the time of the accident, an ambulance is at a location Y that is also uniformly distributed on the road. Assuming that X and Y are independent, find the expected distance between the ambulance and the point of the accident.
 (A) $\frac{L}{3}$. (B) $\frac{L}{2}$. (C) $\frac{L}{4}$. (D) $\frac{2L}{3}$.

8. If X and Y are independent binomial random variables with identical parameters n and p , then $E[X|X + Y = m]$ is
 (A) $\frac{m}{2}$. (B) $\frac{m+1}{2}$. (C) $\frac{n+1}{2}$. (D) $\frac{n}{2}$.

9. The Central Limit Theorem states that
- (A) if n is large, and if the population is normal, then the sampling distribution of the sample mean can be approximated closely by a normal curve.
 - (B) if n is large, and if the population is normal, then the variance of the sample mean must be small.
 - (C) if n is large, then the sampling distribution of the sample mean can be approximated closely by a normal curve.
 - (D) if n is large then the distribution of the sample can be approximated closely by a normal curve.
10. In order for the Poisson probabilities to give good approximate values for the binomial probabilities, we must have the condition(s) that:
- (A) the population size is large relative to the sample size.
 - (B) the sample size is large.
 - (C) the probability, p , is small and the sample size is large.
 - (D) the probability, p , is close to .5 and the sample size is large.
11. If $\begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix} A \begin{bmatrix} 0 & 2 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then the matrix A is
- (A) $\begin{bmatrix} 3 & -4 \\ 3/4 & -1 \end{bmatrix}$.
 - (B) $\begin{bmatrix} -13/4 & 3/2 \\ 5/4 & -1/2 \end{bmatrix}$.
 - (C) $\begin{bmatrix} -17/4 & 3/4 \\ -7/4 & -1/4 \end{bmatrix}$.
 - (D) $\begin{bmatrix} 5/4 & 11/4 \\ 3 & -9/4 \end{bmatrix}$.
12. Suppose $N(t)$ is a Poisson process and $s < t$. Then the probability $P[N(s) = k | N(t) = n]$ will be
- (A) $e^{-s/t} \frac{(s/t)^k}{k!}$.
 - (B) $\binom{n}{k} \left(\frac{t}{s}\right)^k \left(1 - \frac{t}{s}\right)^{n-k}$.
 - (C) $\binom{n}{k} \left(\frac{s}{t}\right)^k \left(1 - \frac{s}{t}\right)^{n-k}$.
 - (D) $\left(\frac{s}{t}\right)^k \left(1 - \frac{s}{t}\right)^{n-k}$.
13. Which of the following is NOT true of the confidence level of a confidence interval?
- (A) The confidence level gives us the success rate of the procedure used to construct the confidence interval.
 - (B) The confidence level is often expressed as an area $1 - \alpha$, where α is the complement of the confidence level.
 - (C) The confidence level is also called the degree of confidence.
 - (D) There is a $1 - \alpha$ chance, where α is the complement of the confidence level, that the true value of the parameter will fall in the confidence interval produced by the sample.

14. Suppose we are interested in studying the factors that affect GPA of college students. Fifty colleges are selected at random and we collect the GPA of one male and one female student from each college. We also classify each college as public or private. Which of the following procedures is most appropriate to conduct first.
- (A) An independent 2 sampled t test on the GPA of male and female students.
 - (B) A paired t test on the GPA of male and female students.
 - (C) ANOVA on the four sets of GPAs: male students at private colleges, female students at private colleges, male students at public colleges, female students at public colleges.
 - (D) Two-Way ANOVA using the college status as one factor and gender as the second factor.
15. In a hypothesis test, a p -value is
- (A) a probability of observing a parameter more extreme than the one observed under the assumption that the null hypothesis is true.
 - (B) a probability of observing a parameter more extreme than the one observed under the assumption that the null hypothesis is false.
 - (C) a probability of observing a statistic at least as extreme as the one observed under the assumption that the null hypothesis is true.
 - (D) a probability of observing a statistic at least as extreme as the one observed under the assumption that the null hypothesis is false.
16. Let X be a set with 30 elements. Let A, B, C be subsets of X with 10 elements each such that $A \cap B \cap C$ has 4 elements. Suppose $A \cap B$ has 5 elements, $B \cap C$ has 6 elements, and $C \cap A$ has 7 elements, how many elements does $A \cup B \cup C$ have?
- (A) 16. (B) 14. (C) 15. (D) 30.
17. A statistic is said to be biased if it
- (A) has exactly the same value as the parameter.
 - (B) systematically underestimates or overestimates the parameter.
 - (C) is determined from a trimmed sample.
 - (D) leads to an erroneous conclusion about the sample.
18. The point $(3, 4)$ in the xy -plane is reflected w.r.t the x -axis and then rotated through 90 degrees in the clockwise direction in the plane about the origin. The final position of the point is
- (A) $(3, -4)$. (B) $(4, -3)$.
 (C) $(-3, -4)$. (D) $(-4, -3)$.
19. The complement of 4 heads in the toss of 4 coins is
- (A) All tails. (B) Exactly one tail.
 (C) Three heads. (D) At least one tail.

20. To determine if there are outliers in a least squares regression models data set, we could construct a Boxplot of the
- (A) response variables. (B) predictor variables.
(C) lurking variables. (D) residuals.
21. A club has a membership of 600 and operates a golf course and 12 tennis courts. The club would like to know how many members use each facility. A survey indicates that 61% regularly use the golf course, 45% regularly use the tennis courts, and 3% use neither of these facilities regularly. What Percentage of the 600 use at least one of the golf or tennis facilities?
- (A) 97%. (B) 3%. (C) 103%. (D) 9%.
22. What is the next number in the sequence 1, 5, 14, 30, 55,.....?
- (A) 71. (B) 81. (C) 91. (D) 101.
23. Identify which, if any, of the following is true:
- (A) If events A and B are independent then they are also disjoint.
(B) If events A and B are independent then $P(A \cap B) < P(A)P(B)$.
(C) If events A and B are independent then they cannot occur simultaneously.
(D) None of the above three statements are true.
24. The percentage of measurements that are above the 39th percentile is
- (A) 71%. (B) 39%. (C) 61%. (D) Cannot be determined.
25. If $I = \int_0^1 e^x dx$, then which of the following is true?
- (A) $I < 1$. (B) $1 < I < 2$. (C) $2 < I < e$. (D) $I > e$.
26. The difference between the squares of two consecutive odd numbers is 56, the product of the two numbers is
- (A) 105. (B) 143. (C) 195. (D) 255.
27. In an exam 3 marks are given for a correct answer and -1 for a wrong answer and no marks are awarded if a candidate does not attempt a question. A candidate attempted all 50 questions and scored 98 marks. How many questions did this candidate answer incorrectly?
- (A) 11. (B) 13. (C) 15. (D) 17.
28. Consider the two sets $A = \{a_1, a_2, a_3, a_4, a_5\}$ and $B = \{b_1, b_2, b_3, b_4, b_5\}$. How many one-to-one function f exist from A to B ?
- (A) 20. (B) 60. (C) 100. (D) 120.
29. Let $v = (1, 1)$ and $w = (1, -1) \in R^2$. Then a vector $u = (a, b) \in R^2$ is in the linear span of v and w
- (A) only when $a = b$. (B) for exactly one value of (a, b) .
(C) always. (D) for at most finitely many values of (a, b) .

30. The value of the following determinant is:

$$\begin{vmatrix} a+b & c+d & e & 1 \\ b+c & d+a & f & 1 \\ c+d & a+b & g & 1 \\ d+a & b+c & h & 1 \end{vmatrix}$$

- (A) 0.
- (B) $(a+b)(c+d)+e+f+g+h$.
- (C) 1.
- (D) $(a+b+c+d)(e+f+g+h)$.

31. Given the following least squares prediction equation, $\hat{y} = -173 + 74x$, we estimate y to _____ by _____ with each 1-unit increase in x

- (A) decrease; 74.
- (B) decrease; 173.
- (C) increase; 74.
- (D) increase; 173.

32. Maximum number of positive roots of $x^6 + 9x^5 + 2x^3 - x^2 - 2$ is

- (A) 0.
- (B) 1.
- (C) 3.
- (D) 5.

33. If x, y are nonzero real numbers, then $x^2 + xy + y^2$ is

- (A) always positive.
- (B) always negative.
- (C) zero.
- (D) sometimes positive, sometimes negative.

34. Manish gets test marks of 71, 76, 80, and 86. He gets 90 marks on his final exam. If the tests each count for 10% and the final exam counts for 60% of the composite marks, then rounded to one decimal place, his final composite marks is:

- (A) 85.3 .
- (B) -71.2 .
- (C) 241.8 .
- (D) 80.6 .

35. Which measure of central tendency may not exist for all numeric data sets?

- (A) Mean.
- (B) Median.
- (C) Mode.
- (D) Variance.

Part-B

36. Suppose X_1, X_2, X_3, X_4 are non-negative independent random variables, where X_2, X_3, X_4 are *iid* Poisson random variables with mean 3, and $X_1 + X_2 + X_3 + X_4$ also has Poisson distribution with mean 11. Then X_1 follows a
- (A) Binomial distribution. (B) Poisson distribution.
 (C) Hypergeometric distribution. (D) Discrete Uniform distribution.

37. Let $((0, 1], \mathcal{B}_{(0,1]}, P)$ be a σ -probability space where $\mathcal{B}_{(0,1]}$ is the usual (Borel) σ -algebra of subsets of $(0, 1]$ and P is a probability measure on $\mathcal{B}_{(0,1]}$ that satisfies $P((a, b]) = b - a, \forall 0 \leq a \leq b \leq 1$. Let $\{X_n, n = 1, 2, \dots\}$ be a sequence of random variables defined on this probability measure space as follows: $X_n(\omega) = \begin{cases} (-1)^n & \omega \in (0, 1/2] \\ (-1)^{n+1} & \omega \in (1/2, 1] \end{cases}$. Consider the following statements
- 1 $E(X_n) = 0, \forall n = 1, 2, \dots$
 - 2 $X_n \rightarrow 0$ a.s.(P).
 - 3 X_1, X_2, \dots are identically distributed.
 - 4 X_1, X_2, \dots are not identically distributed.

The correct statements are:

- (A) only 1, 2 and 4. (B) only 2 and 4.
 (C) only 1 and 3. (D) only 1, 3, and 4.
38. Let A_n denote the event that the n th and $(n + 1)$ th tosses of a coin show heads and let B_n denote the event that the $(2n - 1)$ th and the $(2n)$ th tosses of this coin show heads. Then
- (A) $\limsup_{n \rightarrow \infty} B_n \subseteq \limsup_{n \rightarrow \infty} A_n$.
 (B) $\liminf_{n \rightarrow \infty} A_n \subseteq \limsup_{n \rightarrow \infty} A_n \subseteq \limsup_{n \rightarrow \infty} B_n$.
 (C) $\liminf_{n \rightarrow \infty} B_n^c \subseteq \liminf_{n \rightarrow \infty} A_n^c$.
 (D) None of the above is correct.

39. X_1, X_2, \dots are *iid* random variables distributed uniformly over $\{1, 2, \dots, 10\}$, that is $Pr(X_m = i) = 1/10, i = 1, 2, \dots, 10$ and for all $m = 1, 2, \dots$. Let $S_n = X_1 + \dots + X_n$, read the following statements and identify the correct ones.
- 1 $Pr(S_n > S_{n+1}) = 1$ for every $n = 1, 2, \dots$
 - 2 The sequence $\{\frac{S_n}{n}\}$ converges with probability 1 to $E(X_i)$.
 - 3 $\lim_{n \rightarrow \infty} Pr(\frac{S_n}{\sqrt{n}} > 5.5\sqrt{n}) = 1/2$
 - 4 $\lim_{n \rightarrow \infty} Pr(\frac{S_n}{\sqrt{n}} > 5.5\sqrt{n}) = 1$

The correct statements are:

- (A) only 2 and 3. (B) only 1, 2 and 3.
 (C) only 1 and 3. (D) only 1, 2, and 4.

40. The second moment of a random variable X defined on the σ -probability space (Ω, \mathcal{B}, P) is 36. Then
 (A) $|E(X)| \leq E(|X|) \leq 6$. (B) $|E(X)| \geq E(|X|) \geq 6$.
 (C) $6 \leq |E(X)|$. (D) $|E(X)| \leq 6 \leq E(|X|)$.

41. Random variables X_1 and X_2 are *iid* and their characteristic functions are real valued $\phi(t)$, $t \in \mathbb{R}$. The characteristic function of $X_1 - \frac{X_2}{2}$ is
 (A) $\frac{\phi(t)}{2}$. (B) $\phi(t/2)^2$. (C) $\phi(t)$. (D) $\phi(t)\phi(t/2)$.

42. Suppose X_1, \dots, X_n is a random sample from the $U(-\theta, \theta)$, $\theta > 0$ population, and let $X_{(1)}, \dots, X_{(n)}$ be the corresponding order statistics. Then which of the following is true?
 (A) $X_{(n)}$ is the Maximum Likelihood estimator for θ .
 (B) $X_{(n)}$ is a minimal sufficient statistic for θ .
 (C) $-X_{(1)}$ is a sufficient statistic for θ .
 (D) None of the above is correct.

43. The one step Transition probability matrix of an Homogeneous Markov chain with state space $S = \{1, 2, 3, 4\}$ is

$$\begin{pmatrix} 1/2 & 1/2 & 0 & 0 \\ 1/3 & 1/3 & 0 & 1/3 \\ 0 & 1/3 & 1/3 & 1/3 \\ 0 & 2/3 & 0 & 1/3 \end{pmatrix}$$

- (A) This Markov chain is irreducible.
 (B) All the states of this Markov Chain are positive recurrent.
 (C) There is only one recurrent state in this Markov chain.
 (D) This Markov chain has only one transient(non-recurrent) state.

44. For a non-negative integer valued random variable N

- (A) $\sum_{i=0}^{\infty} iP(N > i) = \frac{1}{2}(E(N^2) - E(N))$.
 (B) $\sum_{i=0}^{\infty} iP(N > i) = \frac{1}{2}(E(N^2) + E(N))$.
 (C) $\sum_{i=0}^{\infty} iP(N > i) = \frac{1}{2}E(N)$.
 (D) $\sum_{i=0}^{\infty} iP(N < i) = \frac{1}{2}(E(N^2) - E(N))$.

45. If Z is standard normal random variable, then $Cov(Z, Z^2)$ equals
 (A) $\frac{1}{3}$. (B) 1. (C) 0. (D) $\frac{1}{2}$.

46. Let X, Y, Z be independent and uniformly distributed over $(0, 1)$. Then $P(X \geq YZ)$ equals
 (A) $\frac{1}{2}$. (B) $\frac{1}{4}$. (C) $\frac{1}{3}$. (D) $\frac{3}{4}$.
47. Let X_1, X_2, \dots, X_n be a random sample from a Poisson distribution with parameter λ . Then the Cramer-Rao lower bound for the unbiased estimator of λ is
 (A) $\frac{\lambda}{n}$. (B) $\frac{\lambda}{n-1}$. (C) $\frac{\lambda^2}{n}$. (D) $\frac{\lambda}{\sqrt{n}}$.
48. Suppose X follows the exponential distribution with parameter 1. Define Y to be the integer part of $X + 1$, that is

$$Y = i + 1 \quad \text{if and only if} \quad i \leq X < i + 1, \quad i = 0, 1, 2, \dots$$
 Then distribution of Y
 (A) Exponential with parameter 1. (B) Geometric with parameter $p = 1 - e^{-1}$.
 (C) Geometric with parameter $p = e^{-1}$. (D) Uniform $(0, 1)$.
49. Let X_1, X_2, \dots, X_n be a random sample from $U(-\theta, \theta)$, $\theta > 0$ and $X_{(1)} = \min(X_1, X_2, \dots, X_n)$ and $X_{(n)} = \max(X_1, X_2, \dots, X_n)$. Then $T = (X_{(1)}, X_{(n)})$ is
 (A) sufficient but not complete. (B) both sufficient and complete.
 (C) complete but not sufficient. (D) neither sufficient not complete.
50. Let X be an exponential random variable with parameter $\lambda = 1$. Then $P(X > 10 | X > 5)$ is
 (A) e^{-5} . (B) $1 - e^{-5}$. (C) e^{-10} . (D) $1 - e^{-10}$.
51. Let $S = \begin{pmatrix} 100 & 60 \\ 60 & 100 \end{pmatrix}$ be a sample covariance matrix. Then the percentage of variance explained by the first principal component is
 (A) 70%. (B) 74%. (C) 80%. (D) 88%.
52. If X and Y are independent exponential random variables with parameter β , then the distribution of $\frac{X}{X+Y}$ is
 (A) Exponential with parameter β . (B) Exponential with parameter 2β .
 (C) Uniform $(0, 1)$. (D) Uniform $(0, \beta)$.
53. Suppose the random variable X follows the exponential distribution with mean 1. Then $E(3X^{13})$ is
 (A) $3 \cdot 13!$. (B) 3. (C) $13!$. (D) 1.
54. Suppose (X_1, \dots, X_n) follows a multinomial distribution with m trials and cell probabilities p_1, p_2, \dots, p_n . Then $\text{Cov}(X_i, X_j)$ is equal to
 (A) $mp_i p_j$. (B) $p_i p_j$. (C) $np_i p_j$. (D) $-mp_i p_j$.

55. Let X_1, X_2 and X_3 be independent random variables with $X_k, k = 1, 2, 3$ having probability density function $f_k(x) = k\theta e^{-k\theta x}, 0 < x < \infty, \theta > 0$. Then a sufficient statistic for θ is
 (A) $X_1 + X_2 + X_3$. (B) $X_1 + 2X_2 + 3X_3$.
 (C) $X_1X_2X_3$. (D) $3X_1 + 2X_2 + X_3$.
56. In a Markov chain with state space $\{0, 1, 2\}$ and one step transition matrix given by $P = \begin{bmatrix} 1/4 & 3/4 & 0 \\ 1/3 & 1/3 & 1/3 \\ 0 & 1/4 & 3/4 \end{bmatrix}$, the value of $p_{01}^{(2)}$ will be:
 (A) $3/4$. (B) $9/16$. (C) $3/16$. (D) $7/16$.
57. The 95% asymptotic confidence interval for θ of the Poisson distribution is given by
 (A) $\bar{x} \pm 2.58\sqrt{\frac{\bar{x}}{n}}$. (B) $\bar{x} \pm 1.96\sqrt{\frac{\bar{x}}{n}}$.
 (C) $\bar{x} \pm 1.96\sqrt{\frac{\bar{x}}{\bar{x}}}$. (D) None of these.
58. In stratified sampling with population size equal to 1000, the population is divided into two strata with sizes $N_1 = 600$ and $N_2 = 400$ respectively. Under Neyman allocation if $S_1 = 24$ and the sample sizes from two strata are in the ration $n_1 : n_2 :: 2 : 1$ then the value of S_2 is
 (A) 16. (B) 36. (C) 18. (D) 24.
59. The arrivals of customers to a mall are in accordance with a Poisson process at the rate of 100 per minute. An arriving customer to the mall goes to the clothes section with probability $5/8$, to the food section with probability $2/8$ and to the electronics section with probability $1/8$. The expected number of customers that will visit the electronics section in a 12 hour period is
 (A) 9000. (B) 12000. (C) 6000. (D) 7200.
60. Which statement is NOT CORRECT?
 (A) The sample standard deviation measures variability of our sample values.
 (B) A larger sample will give answers that vary less from the true value than smaller samples (assuming both are properly chosen).
 (C) The standard error measures how much our estimate may vary if a new sample of the same size is chosen using the same sampling method.
 (D) A large sample size always gives unbiased estimators regardless of how the sample is chosen.
61. The statement "If there is sufficient evidence to reject a null hypothesis at the 10% significance level, then there is sufficient evidence to reject it at the 5% significance level" is:
 Please select the best answer of those provided below.
 (A) Always True.
 (B) Never True.
 (C) Sometimes True; the p-value for the statistical test needs to be provided for a conclusion.
 (D) Not Enough Information; this would depend on the type of statistical test used.

62. An experiment was conducted in which $n = 15$ subjects were timed in the 400 meter run on Monday, fed a carbohydrate heavy diet for the week, and timed in the 400 meter run on Friday. To determine if there is a difference in average performance the appropriate hypothesis testing procedure would be
- (A) a two sample F-test. (B) matched pairs t-test.
 (C) 2 sample z test. (D) none of these is appropriate.
63. You conduct a hypothesis test and you observe values for the sample mean and sample standard deviation when $n = 25$ that do not lead to the rejection of null hypothesis. You calculate a p-value of 0.0667. What will happen to the p-value if you observe the same sample mean and standard deviation for a sample of size greater than 25?
- (A) Increase. (B) Decrease. (C) Remain same. (D) Cannot be said exactly.
64. Let x_1, x_2, \dots, x_n be a random sample from a $N(0, \theta)$ distribution where the variance θ is unknown. The UMP test for $H_0 : \theta = \theta_0 (> 0)$ against $H_1 : \theta > \theta_0$ is of the form:
- (A) $\sum_1^n x_i^2 \leq C$ where C is some constant.
 (B) $\sum_1^n x_i \geq C$ where C is some constant.
 (C) $\sum_1^n x_i \leq C$ where C is some constant.
 (D) $\sum_1^n x_i^2 \geq C$ where C is some constant.
65. For a linear regression model, how does a confidence interval differ from a prediction interval?
- (A) Confidence intervals are used to measure the accuracy of the mean response of all the individuals in the population, while a prediction interval is used to measure the accuracy of a single individuals predicted value.
 (B) Confidence intervals are used to measure the accuracy of a single individuals predicted value, while a prediction interval is used to measure the accuracy of the mean response of all the individuals in the population.
 (C) Confidence intervals are constructed about the predicted values of y while prediction intervals are constructed about a particular value of x .
 (D) Confidence intervals are constructed about the predicted values of x while prediction intervals are constructed about a particular value of y .
66. To establish causation, a statistician must
- (A) take surveys in multiple locations. (B) use a sample size of at least $n=30$.
 (C) use regression analysis. (D) none of the above are correct.
67. In a BIBD with t treatments in b blocks of k plots each and r replicates, which one of the following is not true?
- (A) $rt = bk$. (B) $b \geq t$. (C) $r > k$. (D) $b \leq (r + t - k)$.
68. In a χ^2 test of independence, with m rows and n columns in the contingency table, the number of degrees of freedom associated with the test statistic is
- (A) $mn-1$. (B) $mn+1$. (C) $mn-m-n+1$. (D) $mn-m-n-1$.

69. X_1, \dots, X_n are i.i.d. random variables with absolutely continuous distribution function $F(x; \theta)$, then $-\sum_{i=1}^n \log F(X_i; \theta)$ has
- (A) Normal distribution. (B) Beta distribution.
(C) Gamma distribution. (D) Weibull distribution.
70. For a positive integer valued random variable X , $P(X > k) = \left(\frac{3}{4}\right)^k, k = 1, 2, \dots$, the expected value of X
- (A) is 1. (B) is 2. (C) is 3. (D) does not exist.

KEY - Ph.D (Statistics) - January 2021

Part-A

1-A	2-D	3-B	4-B	5-A
6-A	7-A	8-A	9-C	10-C
11-B	12-C	13-D	14-D	15-C
16-A	17-B	18-C	19-D	20-D
21-A	22-C	23-D	24-C	25-B
26-C	27-B	28-D	29-C	30-A
31-C	32-B	33-D	34-A	35-C

Part-B

36-B	37-C	38-A	39-B	40-A
41-D	42-D	43-D	44-A	45-C
46-D	47-A	48-B	49-A	50-A
51-D	52-B	53-A	54-D	55-B
56-D	57-B	58-C	59-A	60-D
61-C	62-B	63-B	64-D	65-A
66-D	67-B	68-C	69-C	70-C

