ENTRANCE EXAMINATIONS – 2020

(Ph.D. Admissions - January 2021 Session)

Y-61

Ph.D. Applied Mathematics

Hall Ticket Number

Time: 2 hours
Max. Marks: 70

Part A: 35 Marks
Part B: 35 Marks

Instructions

1. Write your Hall Ticket Number on the OMR Answer Sheet given to you. Also write the Hall Ticket Number in the space provided above.
2. Answers are to be marked on the OMR sheet.
3. Please read the instructions carefully before marking your answers on the OMR answer sheet.
4. Hand over the OMR answer sheet at the end of the examination.
5. No additional sheets will be provided. Rough work can be done in the question paper itself/space provided at the end of the booklet.
6. Calculators are not allowed.
7. There are a total of 70 questions in **PART A** and **PART B** together.
8. Each correct answer carries 1 mark.
9. The appropriate answer should be coloured with either a blue or a black ball point or a sketch pen. DO NOT USE A PENCIL.
10. $\mathbb{R}$ denotes the set of real numbers, $\mathbb{C}$ the set of complex numbers, $\mathbb{Z}$ the set of integers, $\mathbb{Q}$ the set of rational numbers, and $\mathbb{N}$ the set of all natural numbers.
11. This booklet contains **15** pages including this page and excluding pages for the rough work. Please check that your paper has all the pages.

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Part-A

1. Let X be the universal set and $A, B \subseteq X$. Then the set $\bigcup\{C \subseteq X | A \cap C \subseteq B\}$ is equal to

   A. $A^c \cup B$

   B. $A \cup B$

   C. $A \cap B$

   D. $A \cup B^c$

2. The number of ways of selecting 3 boys and 4 girls from a group of 8 boys and 6 girls is

   A. 480

   B. 408

   C. 804

   D. 840

3. In a certain code language METHOD is written as NDUGPC and VECTOR is written as WDDSPQ. Then the word BOTTLE is written as

   A. APSUKF

   B. CNUSMD

   C. CMVSND

   D. ASPVKF

4. The number of ways of arranging the letters of the word EXAMINATION is

   A. $\frac{11!}{(2!)^2}$

   B. $\frac{11!}{2}$

   C. $\frac{11!}{(2!)^3}$

   D. $\frac{11!}{(2!)^{24}}$

5. Suppose the statements P, Q are true and R is false. Then the truth value of the statement $(P \to (Q \to R)) \Leftrightarrow ((P \vee Q)) \to R$

   A. is 0

   B. is 1

   C. some times 1 and some times 0

   D. can't be determined

6. Find the wrong number in the series: 5,10,17,26,36,50,65,82

A. 17
B. 26
C. 36
D. 50

7. From his house, Raju went 5 km to the north. Then he turned west and covered 4 km. Then he turned south and covered 3 km. Finally turning to east, he covered 2 km. In which direction is he from his house?

A. South-west
B. South-east
C. North-east
D. North-west

8. The size of the set $\{n \in \mathbb{Z} : 3n - 1 \text{ is divisible by } n + 8\}$ is

A. 6
B. 0
C. 2
D. ∞

9. The sum of all 4-digit positive integers formed using the digits $1, 2, 4, 5$ without repetition is

A. 99972
B. 79992
C. 99997
D. 97992

10. The number of integral ($x_i \in \mathbb{Z}$) solutions to the equation $x_1 + x_2 + x_3 + x_4 + x_5 = 50$ such that $x_1 \geq 0$, $x_2 \geq 1$, $x_3 \geq 0$, $x_4 \geq 3$, and $x_5 \geq 0$ is

A. $\binom{50}{4}$
B. $\binom{53}{4}$
C. $\binom{54}{4}$
D. $\binom{54}{3}$

11. The increasing order of the numbers $3^{32}, 2^{60}, 5^{24}, 7^{12}$ is

A. $7^{12}, 5^{24}, 3^{32}, 2^{60}$

B. $2^{60}, 7^{12}, 3^{32}, 5^{24}$

C. $7^{12}, 5^{24}, 3^{32}, 2^{60}$

D. $7^{12}, 3^{32}, 5^{24}, 2^{60}$

12. The number of ways to rearrange digits $1, 2, 3, 4, 5, 6, 7$ having at least one of the blocks 14, 235, or 76 is

A. $7! - 2 \times 5! + 3!$

B. $2 \times 6! - 2 \times 4! + 3!$

C. $7! + 2 \times 5! + 3!$

D. $2 \times 6! + 2 \times 4! - 3!$

13. The size of the set $\{3 \leq n \leq 1000 : \text{ sum of decimal digits of n is } 11\}$ is

A. $\binom{11}{3} - 3$

B. $\binom{13}{2} - 3$

C. $\binom{11}{3} - 9$

D. $\binom{13}{2} - 9$

14. There are 5 boys having distinct heights. The number of ways to arrange them in a line such that no three consecutive boys have heights in increasing order is

A. $5! - 50$

B. $5! - 70$

C. $5! - 3!$

D. $\frac{5!}{2} - 1$

15. Let m, n be positive integers. The number of functions $f : \{1, 2, 3 \ldots, m\} \to \{10, 20, 30, \ldots, 10n\}$ such that $f(i) \leq f(j)$ for every $i < j$ is

A. $\binom{m+n}{n-1}$

B. $\binom{m+n}{n}$

C. $\binom{m+n-1}{n-1}$

D. $\binom{m+n-1}{m-1}$

16. A person started from the point P and walked 5 km North, turned right and walked for 10 km and turned left and walked 5 km more and finally turned left and walked 10 km to reach the point Q. The distance between P and Q is

A. 5 km
B. 10 km
C. 15 km
D. 20 km

17. The next number in the sequence 7200,3600,1200,300,60,. . . , is

A. 10
B. 15
C. 20
D. 30

18. In a code language, if QUEEN is written as OVCFL, then the code of KING in that language is

A. MKOF
B. PHIK
C. FOKM
D. IJLH

19. In a code language, if THREAT is written as RHTTAE, then the code of PEARLY in that language is

A. YLRAEP
B. YLRPAE
C. AEPYLR
D. AEPRYL

20. In a code language, if MIGHT is written as GHMTI, then the code of EARTH in that language is

A. RTEHA
B. RTEAH
C. RTAEH
D. RETHA

21. 8 persons A, B, C, D, E, F, G and H are sitting in two rows opposite to each other. Each row has 4 persons. B and C are sitting in front of each other. C is between D and E. H is sitting immediate left of E. F is not in front of E. G and B are not neighbors. Then the person who is in front of A is

A. E

B. D

C. C

D. B

22. Two clocks are set correctly at 9 am on Monday. Both the clocks gain 3 min and 5 min respectively in an hour. What time will the second clock register, if the first clock gains 3 min in an hour shows the time as 27 min past 6 pm on the same day?

A. 6:27 pm

B. 6:45 pm

C. 6:25 pm

D. 6:50 pm

23. A, B, C, D, E, F and G are the names of two rivers, three canals and two valleys. B, G and D are not canals. E and F are not rivers. C is a canal and A is a valley. B, F and G are not valleys. Which are the two rivers?

A. A and D

B. B and D

C. B and G

D. A and G

24. Let $A = \{1, 2, 3\}$ and $R = \{(1, 2), (1, 3), (2, 3)\}$. Choose the correct alternative.

A. R is reflexive and transitive but not symmetric.

B. R is reflexive and symmetric but not transitive.

C. R is only transitive.

D. R is an equivalence relation.

25. Let A, B be two infinite subsets of $\mathbb{R}$. Choose the correct alternative.

A. $(A \setminus B) \cup (B \setminus A)$ can never be equal to A.

B. No bijection from $A \times B$ to A can exist.

C. If there is a bijection from $A \cup B$ to $A \cap B$, then $(A \cup B) \setminus (A \cap B)$ must be finite.

D. There is always a bijection from $\mathbb{R}$ to $A \times B$.

26. Pairs of sets are given in the table below. We say that the pair is 'matched' if the set in the left hand column is equal to the set in the right hand column. Choose the correct option.

1. $\{1,2\}$	I. $\{n \in \mathbb{Z} : \|n\| = 1\}$
2. $\{x \in \mathbb{R} : x^2 - 1 = 0\}$	II. $\{1,2,2\}$
3. $\{n \in \mathbb{N} : n$ is an even prime$\}$	III. $\{n \in \mathbb{N} : \|n\| = 2\}$

A. 1, I), (2, II), (3, III)

B. (1, I), (2, III), (3, II)

C. (1, III), (2, I), (3, II)

D. (1, II), (2, I), (3, III)

27. Let A, B, C, D be subsets of a set X. Which of the following is correct?

A. If $a \in A$ and $A \not\subseteq B$ then, $a \notin B$.

B. If $A \not\subseteq B$ and $B \not\subseteq C$ then, $A \not\subseteq C$.

C. If $A \subseteq B$ and $c \notin B$ then, $c \notin A$.

D. $A \cup B \subseteq C \cup D$ and $C \subseteq A$, then $B \subseteq D$

28. The sequence of symbols used in the place of circles to balance the equation 30 ◯ 28 ◯ 7 ◯ 5 ◯ 50 = 0 is ———

A. $\div, +, \times$ and $-$

B. $\times, +, \div$ and $-$

C. $-, +, \div$ and $\times$

D. $+, \div, \times$ and $-$

29. What is the next term in the following? MPN, JQJ, GRG, DSE, ———

A. ATC

B. BTC

C. ATD

D. BTD

30. What is the next term in the following? 15D3, 17G9, 20K27, 24P81, ———

A. 29V243

B. 30V243

C. 29V241

D. 29V143

31. In a row of children, Raj is eighth from the left and Ravi is fourth from the right. When Raj and Ravi exchange their positions, Raj will be fifteenth from the left. What will be the Ravi's position from the right?

A. Eighth
B. Eleventh
C. Fourth
D. Twelfth

32. Pointing to a man in a photograph, a woman said, "His brother's father is the only child of my grandfather". How is the woman related to the man in the photograph?

A. Mother
B. Sister
C. Aunt
D. Daughter

33. Arun runs 70 m East from his house, turns left and walks 90 m. Then turns right and walks 20 m to reach his office. In which direction is the office from his house?

A. Northeast
B. Southeast
C. East
D. North

34. Number of permutations of the word "SECUNDERABAD" is ——

A. 12!
B. $\frac{12!}{6}$
C. $\frac{12!}{8}$
D. $8 \times 12!$

35. The next number in the sequence, 25,21,36,32,47,43,..., is

A. 54
B. 55
C. 56
D. 58

TURN THE PAGE FOR PART B ⟶

Part-B

36. Which of the following is an integral domain?

A. $\frac{\mathbb{Z}[X]}{\langle X^2 \rangle}$

B. $\frac{\mathbb{Q}[X]}{\langle X^2-1 \rangle}$

C. $\frac{\mathbb{R}[X]}{\langle X^3+X+1 \rangle}$

D. $\frac{\mathbb{Q}[X,Y]}{\langle X,Y^2+1 \rangle}$

37. Let A_4 be the subgroup of even permutations of S_4, the permutation group on 4 letters. Which of the following is FALSE?

A. A_4 has a subgroup of order 3

B. A_4 not an abelian group

C. A_4 has a subgroup of order 6

D. A_4 is a normal subgroup of S_4

38. Let $n \geq 2$ be a natural number. Let A be a $n \times n$ nonzero, non-identity real matrix such that $A^2 = A$. Then,

A. A is invertible

B. $\text{rank}(A) = trace(A)$

C. $\dim \ker A = trace(A)$

D. $\text{rank}(A) = \dim \ker(A)$

39. Let G be a finite group of order n, and let $e \in G$ denote its identity. Suppose that for every divisor d of n there are at most d elements a of G such that $a^d = e$. Then G is

A. a cyclic group

B. can be a nonabelian group

C. $\{e\}$

D. an abelian group but may not be cyclic

40. Denote by μ_n a primitive nth root of unity. Consider the following field extensions of $\mathbb{Q}$. $K_1 = \mathbb{Q}(\mu_4)$, $K_2 = \mathbb{Q}(\sqrt{2} + \sqrt{3})$, $K_3 = \mathbb{Q}(\mu_7)$, and K_4 is splitting field of $X^4 - 2$. Put $n_i = [K_i : \mathbb{Q}]$. Then which order among the following is correct?

A. $n_1 < n_2 < n_3 < n_4$

B. $n_1 < n_2 = n_4 < n_3$

C. $n_1 = n_2 = n_4 < n_3$

D. $n_1 < n_2 < n_3 = n_4$

41. Let $A = (a_{ij})$ be an $n \times n$ matrix over $\mathbb{R}$ such that $a_{ij} = 1$ for every i, j. Then choose the correct statement(s) from the following:

(a) $\dim_{\mathbb{R}}(\ker A) = n - 1$

(b) A is diagonalizable

(c) The characteristic polynomial of A is $X^{n-1}(X - n) \in \mathbb{R}[X]$.

(d) A is not diagonalizable.

A. Only (a) is correct

B. Only (a), (b), (c) are correct

C. Only (c) is correct

D. Only (a), (c), (d) are correct

42. The center of $D_{2\times 8} = \langle r, s | r^8 = s^2 = 1, rs = sr^{-1} \rangle$ is

A. $\{1\}$, where 1 is the identity element.

B. $D_{2\times 8}$.

C. $\{1, r^4\}$.

D. $\{1, r, r^2, \ldots, r^7\}$.

43. If $f : \mathbb{R}^2 \to \mathbb{R}$ is $f(x, y) = xy$, then the differential $Df_{(a,b)}(x, y)$ of f at a point (a, b) is

A. 0

B. ab

C. $ax + by$

D. $bx + ay$

44. Which of the following can be the class equation of a group G of order 10?

A. $1 + 2 + 2 + 5$

B. $1 + 1 + 2 + 2 + 2 + 2$

C. $1 + 2 + 3 + 4$

D. $1 + 1 + 1 + 2 + 5$

45. Which of the following statements is/are correct?

I. An integral domain is a field.

II. A finite integral domain is a field.

III. A field is an integral domain.

IV. $\mathbb{Z}_3 \times \mathbb{Z}_3$ is a field with 9 elements.

A. II, III and IV

B. II and III

C. III and IV

D. I and IV

46. Match the entries in the two columns:

	Rings		Properties
I.	$\mathbb{Z} \oplus \mathbb{Z}$	*i.*	Field
II.	$\mathbb{Z}_p$, p a prime number	*ii.*	Non-commutative ring
III.	$M_2(\mathbb{Z})$,the ring of 2×2 matrices	*iii.*	Integral domain but not a field
IV.	$\mathbb{Z}$	*iv.*	Commutative ring but not an integral domain

A. (I,i), (II,ii), (III,iii), (IV,iv)

B. (I,iii), (II,iv), (III,i), (IV,ii)

C. I,iv), (II,i), (III, ii), (IV,iii)

D. (I,ii), (II,iii), (III,iv), (IV,i)

47. The sequence $(\frac{1}{n})$ belongs to the Banach space l^p if and only if

A. $1 < p < \infty$

B. $1 < p \leq \infty$

C. $1 \leq p < \infty$

D. $1 \leq p \leq \infty$

48. The topology in which $\mathbb{R}$ is T_1 but not T_2 is

A. discrete topology

B. indiscrete topology

C. lower limit topology

D. cofinite topology

49. Suppose G is a cyclic group of order 49. Then the number of elements of order 7 in G is

A. 7

B. 6

C. 4

D. 14

50. Which of the following is TRUE?

A. There exists a continuous surjective map $f : \mathbb{R} \to \mathbb{Q}$.

B. There exists a continuous injective map $f : \mathbb{R} \to \mathbb{Q}$.

C. There does not exist any continuous map $f : \mathbb{R} \to \mathbb{Q}$.

D. If $f : \mathbb{R} \to \mathbb{Q}$ is continuous, then f is a constant map.

51. Let X be a topological space and $\mathcal{F} = \{A \subset X : A \text{ is nonempty and connected}\}$. Consider the following two statements:
I. If $\mathcal{F} = \{\{x\} : x \in X\}$, then X is a discrete space.
II. If X is a discrete space, then $\mathcal{F} = \{\{x\} : x \in X\}$.
Then,

A. both I and II are true
B. I is true but not II
C. II is true but not I
D. neither I nor II is true

52. Which of the following is TRUE?

A. There exists a continuous map from $[0,1]$ onto $(0,1)$.
B. There exists a continuous map from $(0,1)$ onto $[0,1]$.
C. There exists a continuous map from $[0,1]$ onto $[0,1)$.
D. There exists a continuous map from $[0,1]$ onto $(0,1]$.

53. Let V, W be two normed linear spaces, and $L(V, W)$ be the space of all bounded linear maps from V to W, equipped with the operator norm. Then, which of the following is TRUE?

A. $L(V, W)$ is complete if V is complete.
B. $L(V, W)$ is complete if and only if both V, W are complete.
C. $L(V, W)$ is complete if W is complete.
D. $L(V, W)$ is complete if and only if at least one of V, W is finite dimensional.

54. Which of the following is TRUE for a finite commutative ring with unity?

A. An element is not a zero divisor if and only if it is a unit.
B. Every zero divisor is nilpotent.
C. If an element is not nilpotent then it must be a unit.
D. An element is a unit if and only it is not nilpotent.

55. Consider the four groups $G_1 = \mathbb{Z}_{21}, G_2 = \mathbb{Z}_{43}, G_3 = \mathbb{Z}_{16}, G_4 = \mathbb{Z}_{42}$. The correct arrangement of them in the increasing order of their number of subgroups is

A. G_2, G_1, G_3, G_4
B. G_3, G_1, G_4, G_2
C. G_3, G_4, G_2, G_1
D. G_1, G_2, G_4, G_3

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56. Let $f : \mathbb{R} \to \mathbb{R}$ be $f(x) = 2/(1+e^x)$. Then $f(\mathbb{R})$ is equal to:

A. $(-2, 2)$
B. $(0, 2)$
C. $(0, 1]$
D. $[1, 2)$

57. The operator norm of the orthogonal projection from $\mathbb{R}^2$ onto the line $y = x/2$ is

A. 1/2
B. 2
C. 1/4
D. 1

58. Let f, g be real polynomials of degree 8 and d respectively. A value of d for which the set $\{x \in \mathbb{R} : f(x) = g(x) - 5x^4\}$ becomes nonempty is:

A. 5
B. 7
C. 9
D. 4

59. Together with $(1, 1, 0)$ and $(2, 2, 2)$, which of the following vector will form a basis of $\mathbb{R}^3$?

A. $(3, 3, 3)$
B. $(0, 0, 3)$
C. $(-1, -1, 0)$
D. $(1, 2, 0)$

60. For the complex analytic function $z \mapsto e^{1/z}$, the singularity at 0 is:

A. an essential singularity
B. a removable singularity
C. a pole of order 1
D. a pole of order ≥ 2

61. How many units are there in the ring $\{a + bi : a, b \in \mathbb{Z}\}$ of Gaussian integers?

A. 4
B. 2
C. 8
D. infinitely many

62. Let p be an irreducible polynomial in $\mathbb{R}[x]$. Then,

A. $\deg(p)$ must be 1
B. $\deg(p)$ must be 0 or 1
C. $\deg(p)$ must be 1 or 2
D. $\deg(p)$ must be 2

63. Consider the following table containing various notions and results in Functional Analysis. Match the entries in the two columns.

1. Linear functionals on a Hilbert space	a. Bounded inverse theorem
2. Bijective linear maps between Banach spaces	b. Hahn - Banach theorem
3. Extension of bounded linear functional	c. Riesz Representation theorem

A. (1,b), (2,a), (3,c)
B. (1,c), (2,a), (3,b)
C. (1,c), (2,b), (3,a)
D. (1,a), (2,c), (3,b)

64. Consider the following table containing various notions and results in Topology. Match the entries in the two columns.

1. countable intersection of dense open sets	a. Tychonoff's theorem
2. product of compact sets	b. Arzela-Ascoli theorem
3. compact subsets of $\{f : [0,1] \to \mathbb{R} : f \text{ is continuous}\}$	c. Baire category theorem

A. (1,b), (2,a), (3,c)
B. (1,c), (2,b), (3,a)
C. (1,a), (2,c), (3,b)
D. (1,c), (2,a), (3,b)

65. Which of the following statements is true for the eigenvalue problem

$$\begin{cases} y'' + (x^2 - 1 - \lambda)y = 0, \ x \in (0,1), \\ y(0) = 0, \ y(1) = 0. \end{cases}$$

A. All the eigenvalues are negative
B. All the eigenvalues are positive
C. There is a positive eigenvalue and a negative eigenvalue
D. There is a purely imaginary eigenvalue

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66. The Cauchy problem $\begin{cases} y' = f(y), \ x \in \mathbb{R}, \\ y(0) = 0, \end{cases}$ has infinitely many solutions if $f(y) =$

A. $|y|^{\pi+e}$

B. $|y|^{\pi-e}$

C. $|y|$

D. $\sin|y|$

67. The point $(0,0)$ is a simple critical point of $\begin{cases} x' = 2x + 3y + \sqrt{x^2+y^2} f(x,y), \\ y' = 3x + 7y + \sqrt{x^2+y^2} g(x,y), \end{cases}$

if $\big(f(x,y), g(x,y)\big) =$

A. $\big(\cos(xy), \sin(xy)\big)$

B. $\big(1 - \cos(xy), \sin(xy)\big)$

C. $\big(2 - \cos(xy), 1 + \sin(xy)\big)$

D. $\big(\cos^2(xy), \sin^2(xy)\big)$

68. There exists no integral surface of $u_t + u_x = 0$, $(x,t) \in \mathbb{R}^2$, containing the curve $(s, s, f(s))$ (where s is a parameter), if $f(s) =$

A. 0

B. π

C. 10

D. $\sin s$

69. If u is a harmonic function on $\{(x,y) | x^2+y^2 < 2\}$, and $u(x,y) = 2021$ whenever $x^2+y^2 = 1$, then $u(0,0) =$

A. $2021/(2\pi)$

B. $2021/\pi$

C. 2021

D. 2021π

70. If u is a solution of $\begin{cases} u_t = u_{xx}, \ x \in \mathbb{R}, \ t > 0, \\ u(x,0) = e^{-x^2}, \ x \in \mathbb{R}, \end{cases}$ then $\lim_{t \to 0} u(1,t) =$

A. 1

B. $(e^{-1} + e)/2$

C. e

D. e^{-1}

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University of Hyderabad
Entrance Examinations – January, 2021

School :Mathematics and Statistics
Course/Subject : Ph.D in Mathematics/Applied Mathematics

Q.No.	Answer	Q.No.	Answer	Q.No	Answer
1	A	26	D	51	C
2	D	27	C	52	B
3	B	28	D	53	C
4	C	29	C	54	A
5	B	30	A	55	A
6	C	31	B	56	B
7	D	32	B	57	D
8	A	33	A	58	C
9	B	34	C	59	D
10	A	35	D	60	A
11	D	36	D	61	A
12	B	37	C	62	C
13	D	38	B	63	B
14	A	39	A	64	D
15	C	40	A	65	A
16	B	41	B	66	B
17	A	42	C	67	B
18	D	43	D	68	D
19	C	44	A	69	~~A~~ C
20	A	45	B	70	D
21	A	46	C		
22	B	47	B		
23	C	48	D		
24	C	49	B		
25	A	50	D		

27/1/2021

Signature
School/Department/Centre
25/1/2021