ENTRANCE EXAMINATIONS – 2018

(Ph.D. Admissions - January 2019 Session)

Ph.D. Mathematics/Applied Mathematics

Hall Ticket No.	

Time : 2 hours
Max. Marks : 80

PART A: 40 MARKS
PART B: 40 MARKS

Instructions

1. Write your Hall Ticket Number on the OMR Answer Sheet given to you. Also write the Hall Number in the space provided above.

2. Answers are to be marked on the OMR sheet.

3. Pleas read the instructions carefully before marking your answers on the OMR answer sheet.

4. Hand over the OMR answer sheet at the end of the examination to the Invigilator.

5. No additional sheets will be provided. Rough work can be done in the question paper itself/space provided at the end of the booklet.

6. Calculators are not allowed.

7. There are a total of 40 questions in **Part - A** and **Part - B** together.

8. Each correct answer carries 2 marks and there is **negative marking** of **0.66** for each wrong answer.

9. The appropriate answer(s) should be coloured with either a blue or black ball point or a sketch pen. DO NOT USE A PENCIL.

10. $\mathbb{R}$ denotes the set of real numbers, $\mathbb{C}$ the set of complex numbers, $\mathbb{Z}$ the set of integers, $\mathbb{Q}$ the set of rational numbers, and $\mathbb{N}$ the set of all natural numbers.

11. This book contains **9 pages** including this page and excluding pages for the rough work. Please check that your paper has all the pages.

S-53

Part-A

1. Let $*$ be a non-empty relation on a non-empty collection of non-empty sets defined by $A * B$ if and only if $A \cap B = \emptyset$. Then which one of the following is TRUE?

   (A) $*$ is reflexive and transitive.

   (B) $*$ is symmetric and not transitive.

   (C) $*$ is an equivalence relation

   (D) $*$ is not reflexive and not symmetric.

2. In a job hiring committee, half the number of experts voted for Mr. A and one third voted for Mr. B. Two members voted for both and four of them did not vote for either. How many experts were there in all?

   (A) 6

   (B) 12

   (C) 18

   (D) 24

3. Let X, Y and Z be three sets. Which one of the following is FALSE?

   (A) $Z \setminus (Y \cup X) = (Z \setminus Y) \setminus X$.

   (B) $X \setminus (Y \cup Z) = (X \setminus Y) \setminus (Y \cup Z)$.

   (C) $Y \setminus (Z \cup X) = (Y \setminus X) \setminus Z$.

   (D) $X \setminus (Y \cap Z) = (X \setminus Y) \setminus (Y \cap Z)$.

4. Let X, Y and Z be three sets and $f : X \to Y$ and $g : Y \to Z$ be maps. Let $h = g \circ f : X \to Z$. Suppose g is surjective. Then which one of the following is FALSE?

   (A) If f is not surjective, then h is not necessarily surjective.

   (B) If f is not surjective, then h can still be surjective.

   (C) If f is surjective, then h is surjective.

   (D) If f is not injective, then h can still be injective.

5. Let S be a countable infinite set. If $S = \bigcup_{i=1}^{\infty} S_i$ is such that S_i's are pairwise disjoint, then which one of the following is TRUE?

   (A) At least one of the sets S_i has to be finite.

   (B) At least one of the sets S_i has to be infinite.

   (C) All the sets S_i can be infinite.

   (D) Union of any two S_i's has to be infinite.

S-53

6. If in a certain code language 'BEAUTIFUL' is coded as '573041208' and 'BUTTER' is coded as '504479', then how will 'FUTURE' be coded in that language?

   (A) 204097

   (B) 201497

   (C) 704092

   (D) 204079

7. If in a code language, 'COULD' is written as 'BNTKC' and 'MARGIN' is written as 'LZQFHM', how will 'MOULDING' be written in that code?:

   (A) CHMFINTK

   (B) LNKTCHMF

   (C) NITKHCMF

   (D) LNTKCHMF

8. If 'MILD' is coded as 'NKOH', then 'GATE' should be coded as ___.

   (A) IBVJ

   (B) HCWI

   (C) KDXK

   (D) ICWA

9. If the animals which can walk are called swimmers, animals who crawl are called flying, those living in water are called snakes and those which fly in the sky are called hunters, then what will a lizard be called?

   (A) Swimmers.

   (B) Snakes.

   (C) Flying.

   (D) Hunters.

10. In a certain code language '735924' is written as '826833'. How is '357918' written in that code?

    (A) 448807

    (B) 884472

    (C) 466825

    (D) 448827

S-53

11. Consider the following statements as facts. Fact 1: All dogs like to run; Fact 2: Some dogs like to swim; Fact 3: Some dogs are white in colour. Then which one of the following statements must also be a fact?

(A) Dogs who like to swim are white in colour.

(B) Dogs who like to run are white in colour.

(C) Dogs who like to swim also like to run.

(D) Dogs who like to run also like to swim.

12. A sequence of numbers looks like 70, 71, 76, _, 81, 86, 70, 91, 96. Which number should fill the blank?

(A) 70

(B) 71

(C) 75

(D) 80

13. If $24 * 13 = 2$, $27 * 43 = 2$, and $53 * 11 = 6$, then what is $51 * 11$?

(A) 4

(B) 5

(C) 6

(D) 7

14. Let $\mathcal{A}$ be a nonempty collection of sets. Consider the statement "*if* $x \in \cup_{A \in \mathcal{A}} A$, *then* $x \in A$ *for at least one* $A \in \mathcal{A}$". The contrapositive of this statement is:

(A) if $x \in A$ for at least one $A \in \mathcal{A}$ then $x \in \cup_{A \in \mathcal{A}} A$.

(B) if $x \notin \cup_{A \in \mathcal{A}} A$ then $x \notin A$ for at least one $A \in \mathcal{A}$.

(C) if $x \notin A$ for all $A \in \mathcal{A}$ then $x \notin \cup_{A \in \mathcal{A}} A$.

(D) if $x \notin A$ for at least one $A \in \mathcal{A}$ then $x \notin \cup_{A \in \mathcal{A}} A$.

15. The question paper of an examination has 4 questions with 4 choices for each answer. In order to get two identical answer scripts, how many candidates should necessarily appear for the examination?

(A) 2

(B) 4

(C) 4^4

(D) $4^4 + 1$

16. Consider the sequence : 3 8 4 8 8 2 6 8 5 4 8 1 8 8 8 3 2 8 2. How many 8's are present in this sequence, which are divisible by both its preceding and following numbers?

(A) 7
(B) 6
(C) 5
(D) 4

17. From a group of 7 men and 6 women, five persons are to be selected to form a committee so that at least 3 men are there in the committee. In how many ways can it be done?

(A) 624
(B) 702
(C) 756
(D) 812

18. There are 30 people in a group. If all shake hands with one another, how many handshakes are possible?

(A) 30!
(B) 29!
(C) 870
(D) 435

19. In a cricket championship, there are 21 matches. If each team plays exactly one match with every other team, the number of teams is:

(A) 7
(B) 8
(C) 9
(D) 10

20. A box contains 4 red, 3 white, and 2 blue balls. Three balls are drawn at random. The number of ways of selecting the balls of different colours is:

(A) 12
(B) 24
(C) 48
(D) 168

Part-B

21. Let A be a 3×3 real symmetric matrix with $Det(A) = 1$. Let $\lambda_1, \lambda_2, \lambda_3$ be the eigenvalues of A. Then always

    (A) $\lambda_1 = \lambda_2 = \lambda_3 = 1$.

    (B) $\lambda_i = 1$ for some $i \in \{1, 2, 3\}$.

    (C) $\lambda_1\lambda_2\lambda_3 = 1$.

    (D) $\lambda_1\lambda_2\lambda_3 > 1$.

22. Let A be a 3×4 matrix and B be a 4×3 matrix. Then always

    (A) AB is singular.

    (B) BA is singular.

    (C) AB is non-singular.

    (D) BA is non-singular.

23. Let A be a finite commutative ring with 1. Then,

    (A) A is a field.

    (B) There is $a \in A$ which is neither a unit nor a zero divisor.

    (C) Every element of A is a unit or a zero divisor.

    (D) There is A with $|A| = 12$ and any $a \in A \setminus \{0\}$ is not a zero divisor.

24. Let P be a monic polynomial in $\mathbb{Z}[X]$. Then,

    (A) All roots of P belong to $\mathbb{Z}$.

    (B) All roots of P belong to $(\mathbb{R} \setminus \mathbb{Q}) \cup \mathbb{Z}$.

    (C) All roots of P belong to $(\mathbb{C} \setminus \mathbb{Q}) \cup \mathbb{Z}$.

    (D) All roots of P can belong to $\mathbb{Q} \setminus \mathbb{Z}$.

25. If G is a group of order 33, then the center of G is isomorphic to:

    (A) $\{e\}$

    (B) $\mathbb{Z}_3$

    (C) $\mathbb{Z}_{11}$

    (D) G

26. Let $f_n : [0,1] \to \mathbb{R}$ be $f_n(x) = 1 - x^n$ for $n \in \mathbb{N}$. Then,

(A) (f_n) converges uniformly on $[0,1]$.

(B) (f_n) converges uniformly on $[0,1/2]$ but not on $[0,1]$.

(C) (f_n) converges uniformly on $[1/2,1]$ but not on $[0,1]$.

(D) (f_n) converges uniformly on $[0,1] \cap \mathbb{Q}$ but not on $[0,1]$.

27. Let $f, g : \mathbb{R} \to \mathbb{R}$ be differentiable. Then which of the following is FALSE?

(A) $x \mapsto f(x) - g(-x)$ is differentiable on $\mathbb{R}$.

(B) $x \mapsto f(x)g(x)$ is differentiable on $\mathbb{R}$.

(C) $x \mapsto f(g(x))$ is differentiable on $\mathbb{R}$.

(D) $x \mapsto \max\{f(x), g(x)\}$ is differentiable on $\mathbb{R}$.

28. Let $f : \mathbb{R} \to \mathbb{R}$ be defined as $f(0) = 1$, $f(x) = 0$ if $x \in \mathbb{R} \setminus \mathbb{Q}$, and $f(p/q) = 1/q$ if $p \in \mathbb{Z} \setminus \{0\}$, $q \in \mathbb{N}$, and p, q are relatively prime. Then,

(A) $\{x \in \mathbb{R} : f \text{ is continuous at } x\} = \mathbb{Z} \setminus \{0\}$.

(B) $\{x \in \mathbb{R} : f \text{ is continuous at } x\} = \mathbb{R} \setminus \mathbb{Z}$.

(C) $\{x \in \mathbb{R} : f \text{ is continuous at } x\} = \mathbb{Q} \setminus \{0\}$.

(D) $\{x \in \mathbb{R} : f \text{ is continuous at } x\} = \mathbb{R} \setminus \mathbb{Q}$.

29. Let μ be the Lebesgue measure on $\mathbb{R}$, and $Y \subset \mathbb{R}$ be a (Lebesgue) measurable set. Then which of the following is TRUE?

(A) If $\mu(Y) > 0$, then Y contains a non-measurable subset.

(B) If Y is closed in $\mathbb{R}$ and $\mu(Y) < \infty$, then Y is bounded.

(C) If Y is open in $\mathbb{R}$ and $\mu(Y) < \infty$, then $\overline{Y}$ is compact.

(D) If Y is compact and uncountable, then $\mu(Y) > 0$.

30. Consider $[0,1]$ with the Lebesgue measure. Which of the following is FALSE?

(A) If $f : [0,1] \to \mathbb{R}$ is Riemann integrable, then f is Lebesgue integrable.

(B) If $f : [0,1] \to \mathbb{R}$ is measurable and $f \geq 0$, then f is Lebesgue integrable.

(C) If $f : [0,1] \to \mathbb{R}$ is Riemann integrable, then f is measurable.

(D) If $f : [0,1] \to \mathbb{R}$ is Lebesgue integrable, then $|f|$ is Lebesgue integrable.

31. Let X be a compact Hausdorff topological space. Then,

(A) X is metrizable and separable.

(B) X is metrizable but may not be separable.

(C) X is normal but may not be metrizable.

(D) X is separable but may not be normal.

32. Consider $\mathbb{R}^2$ with the Euclidean metric. Then which of the following is FALSE?

(A) Every connected subset of $\mathbb{R}^2$ is path connected.

(B) Every bounded subset of $\mathbb{R}^2$ is totally bounded.

(C) Every uncountable subset of $\mathbb{R}^2$ has a limit point in $\mathbb{R}^2$.

(D) Every subset of $\mathbb{R}^2$ is second countable with respect to the subspace topology.

33. Choose the FALSE statement:

(A) If $f : \mathbb{C} \setminus \{0\} \to \mathbb{C}$ is analytic and bounded, then f is constant.

(B) If $f : \mathbb{C} \to \mathbb{C}$ is analytic and non-constant, then f is surjective.

(C) If $f : \mathbb{C} \to \mathbb{C}$ is analytic and $f(\mathbb{C}) \subset \mathbb{R}$, then f is constant.

(D) If $f : \mathbb{C} \setminus \{0\} \to \mathbb{C}$ is analytic and non-constant, then f is an open map.

34. Let $f : \mathbb{C} \to \mathbb{C}$ be analytic, and suppose f is NOT a polynomial. Define $g : \mathbb{C}\setminus\{0\} \to \mathbb{C}$ as $g(z) = f(1/z)$. Then,

(A) 0 is a removable singularity of g.

(B) 0 is a pole of g of order 1.

(C) 0 is a pole of g of order ≥ 2.

(D) 0 is an essential singularity of g.

35. Consider $l^2 := \{(a_n) : a_n \in \mathbb{R}$ for every $n \in \mathbb{N}$ and $\sum_{n=1}^{\infty} |a_n|^2 < \infty\}$ with the norm $\|(a_n)\| = (\sum_{n=1}^{\infty} |a_n|^2)^{1/2}$. Also, let $X = \{(a_n) \in l^2 : a_n \in \mathbb{Q}$ for every $n \in \mathbb{N}\}$, and $Y = \{(a_n) \in l^2 : a_n = 0$ for all but finitely many $n \in \mathbb{N}\}$. Then,

(A) Both X and Y are dense in l^2.

(B) X is dense in l^2 but Y is not dense in l^2.

(C) Y is dense in l^2 but X is not dense in l^2.

(D) Neither X nor Y is dense in l^2.

36. Let X be a Banach space over $\mathbb{C}$, and let X' denote the dual space of X. Which of the following is FALSE?

(A) If $A \subset X$ is bounded, then $f(A)$ is bounded for every $f \in X'$.

(B) If $A \subset X$ is such that $f(A)$ is bounded for every $f \in X'$, then A is bounded.

(C) If (x_n) converges in X, then $(f(x_n))$ converges in $\mathbb{C}$ for every $f \in X'$.

(D) If (x_n) is a sequence in X such that $(f(x_n))$ converges in $\mathbb{C}$ for every $f \in X'$, then (x_n) converges in X.

37. Let $\overline{F}(x, y, z) = (y, -x, x^2y^2z^2)$ and S be the surface $x^2 + y^2 + 3z^2 = 1$, $z \leq 0$. If $\widehat{n}$ is the unit outward normal to S, then $\iint_S curl(\overline{F}) \cdot \widehat{n} dS =$ __.

(A) -2π

(B) 2π

(C) 1

(D) 0

38. The transformation $\xi = y - \dfrac{x^2}{2}, \quad \eta = x$

reduces the equation $u_{xx} + 2xu_{xy} + x^2u_{yy} = 0$ to

(A) $u_{\eta\eta} = u_\xi$

(B) $u_{\eta\eta} + u_\xi = 0$

(C) $u_{\eta\eta} + 2\eta u_{\eta\xi} + \eta^2 u_{\xi\xi} = 0$

(D) $u_{\eta\eta} + u_{\xi\xi} = 0$

39. The equation $u_{xx} + (2x + 3)u_{xy} + 6xu_{yy} = 0$ is

(A) hyperbolic for all values of x.

(B) elliptic for all values of x.

(C) parabolic for $x = 3/2$ and hyperbolic for $x \neq 3/2$.

(D) parabolic for $x = -3/2$ and hyperbolic for $x \neq -3/2$.

40. For the system of equations $\dfrac{dx}{dt} = 8x - y^2, \quad \dfrac{dy}{dt} = -6y + 6x^2$

(A) the critical points are $(0, 0)$ and $(2, 4)$, and $(0, 0)$ is an unstable spiral point.

(B) the critical points are $(0, 0)$ and $(2, 4)$, and $(0, 0)$ is an unstable saddle point.

(C) $(0, 0)$ is the only critical point.

(D) there are no critical points.