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ENTRANCE EXAMINATIONS – 2020

M.Sc. Mathematics/Applied Mathematics

Hall Ticket No.	

Time : 2 hours
Max. Marks : 100

PART A: 50 Marks
PART B: 50 Marks

Instructions

1. Write your Hall Ticket Number on the OMR Answer Sheet given to you. Also write the Hall Ticket Number in the space provided above.
2. Answers are to be marked on the OMR sheet.
3. Please read the instructions carefully before marking your answers on the OMR answer sheet.
4. Hand over the OMR answer sheet at the end of the examination to the Invigilator.
5. No additional sheets will be provided. Rough work can be done in the question paper itself/space provided at the end of the booklet.
6. Calculators are not allowed.
7. There are a total of 50 questions in **PART A** and **PART B** together.
8. Each correct answer carries **2 marks and each wrong answer carries -0.66 mark. Each question has only one correct option.**
9. The appropriate answer(s) should be coloured with either a blue or black ball point or a sketch pen. DO NOT USE A PENCIL.
10. This book contains **14 pages** including this page and excluding pages for the rough work. Please check that your paper has all the pages.
11. The question paper can be taken away by the candidate at the end of the examination.
12. **Notations** : $\mathbb{R}$ denotes the set of real numbers, $\mathbb{C}$ the set of complex numbers, $\mathbb{Q}$ the set of rational numbers, $\mathbb{Z}$ the set of integers and $\mathbb{N}$ the set of natural numbers $\{1, 2, 3, \ldots\}$, and $\varnothing$ the empty set. For a set A, A^c denotes its complement. For a ring R and a positive integer n, $M_n(R)$ denotes the set of all $n \times n$ matrices with entries from R.

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PART - A

1. Let S_3 be a permutation group on three symbols. Then group $S_3 \times \mathbb{Z}/2\mathbb{Z}$ is isomorphic to

   (A) The dihedral group with 12 elements, D_{12}.

   (B) The alternating group A_4 .

   (C) The cyclic group of order $\mathbb{Z}/12\mathbb{Z}$.

   (D) $\mathbb{Z}/2\mathbb{Z} \times \mathbb{Z}/2\mathbb{Z} \times \mathbb{Z}/3\mathbb{Z}$.

2. Suppose $f : [a, b] \longrightarrow \mathbb{R}$, is a continuous map. Which of the following is true.

   (A) if $\int_a^b f(x)dx = 0$, then $f = 0$.

   (B) if $| \int_a^b f(x)dx |= 0$, then $f = 0$.

   (C) if there exists $c \in (a, b)$ such that $f(c) \neq 0$, then $\int_a^b f(x)dx \neq 0$.

   (D) if $\int_a^b f^2(x)dx = 0$, then $f = 0$.

3. If A is an orthogonal matrix of odd order then

   (A) $A - I$ or $A + I$ is necessarily singular.

   (B) A is always singular.

   (C) zero is an eigenvalue of A.

   (D) $A - I$ is always non-singular.

4. Let $A = \{(-1)^n(1\frac{1}{n}) \mid n \in \mathbb{N}\}$. The limit points of A are

   (A) 1.

   (B) -1.

   (C) $-1, 0, 1$.

   (D) $-1, 1$.

5. Suppose $f : \mathbb{R} \longrightarrow \mathbb{R}$ and $a \in \mathbb{R}$. Suppose there exists $m > 0$ and $r \in \mathbb{N}$ such that,$|f(x) - f(a)| < m|x - a|^r$. Choose the correct answer.

   (A) f is differentiable at a, for any r.

   (B) f is differentiable at a, if $m > 1$.

   (C) f is differentiable at a, if $r \geq 2$.

   (D) f is differentiable at a, if $m = 1$.

6. The orthogonal trajectories of a family of circles passing through the points $(0, 2)$ and $(0, -2)$ are circles with

   (A) centres at $(0, c/2)$ and radius $\sqrt{c^2 - 16}/2, |c| > 4$.

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(B) centres at $(0, -c/2)$ and radius $\sqrt{c^2 - 16}/2, |c| > 4$.

(C) centres at $(0, 0)$ and radius $\sqrt{c^2 - 16}/2, |c| > 4$.

(D) centres at $(0, c/2)$ and radius $c/2$.

7. Let $\mathbb{R}$ with the Euclidean (or standard) metric. Let $f : \mathbb{R} \to \mathbb{R}$ be given by

$$f(x) = \begin{cases} x^2, & \text{if } x \in \mathbb{Q}, \\ x - 2, & \text{if } x \in \mathbb{R} \setminus \mathbb{Q}. \end{cases}$$

Then

(A) f is continuous on $\mathbb{Q} \cap (0, \infty)$.

(B) f is continuous only at 1 and -2.

(C) f is not continuous at every point.

(D) f is continuous on $\mathbb{R}$.

8. The least distance of the point $(10, 7)$ from the circle $x^2 + y^2 - 4x - 2y - 20 = 0$ is

(A) 10.

(B) 15.

(C) 5.

(D) 0.

9. The equation of the sphere having the circle $x^2+y^2+z^2+10y-4z-8 = 0, x+y+z-3 = 0$ as a great circle is

(A) $x^2 + y^2 + z^2 - 4x + 6y - 8z + 4 = 0$.

(B) $x^2 + y^2 + z^2 + 4x - 6y - 8z + 4 = 0$.

(C) $x^2 + y^2 + z^2 - 4x - 6y - 8z - 4 = 0$.

(D) $x^2 + y^2 + z^2 = 16$.

10. Which of the following is conditionally convergent ?

(A) $\sum_{n=1}^{\infty} \frac{1}{n}$.

(B) $\sum_{n=1}^{\infty} \frac{1}{n^2}$.

(C) $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$.

(D) $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$.

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11. Let $S = \{1, 2, 4, 6, 7, 8\}$. The number of 3-digit integers formed using the digits from S such that the integer is either an even number or else an odd number but without repetition of digits is

(A) 184.

(B) 186.

(C) 212.

(D) 108.

12. The necessary and sufficient condition for the differential equation $M(x, y)dx + N(x, y)dy = 0$ to be exact is

(A) $\frac{\partial M}{\partial y} = -\frac{\partial N}{\partial x}$.

(B) $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$.

(C) $\frac{\partial M}{\partial x} = \frac{\partial N}{\partial y}$.

(D) $\frac{\partial M}{\partial x} = -\frac{\partial N}{\partial y}$

13. A sequence $\{x_n\}$ of real numbers satisfies $|x_{n+2} - x_{n+1}| \leq c|x_{n+1} - x_n|$, for all $n \in \mathbb{N}$ and for some $0 < c < 1$. Then $\{x_n\}$ is called

(A) cauchy.

(B) convergent.

(C) contractive.

(D) divergent.

14. If $\tilde{\mathbf{A}}$ be vector valued function on $\mathbb{R}^3$, then which of the following is always true?

(A) $curlcurl\tilde{\mathbf{A}} = \mathbf{grad}(\mathbf{div}\tilde{\mathbf{A}}) - \nabla^2\tilde{\mathbf{A}}$.

(B) $curlcurl\tilde{\mathbf{A}} = \mathbf{0}$.

(C) $grad(div\tilde{\mathbf{A}}) = \mathbf{0}$.

(D) $curlcurl\tilde{\mathbf{A}} = -\nabla^2\tilde{\mathbf{A}}$.

15. The volume generated by revolving the region in the first quadrant bounded by $y = x^3$ and $y = 4x$ about the x-axis is

(A) $\frac{512\pi}{21}$.

(B) $\frac{128\pi}{21}$.

(C) $\frac{256\pi}{21}$.

(D) $\frac{1024\pi}{21}$.

16. How many group homomorphisms are there from $\mathbb{Z}_5$ to $\mathbb{Z}_{10}$?

(A) 1.

(B) 2.

(C) 5.

(D) 10.

17. Let $V = span\{\mathbf{x_1}, \mathbf{x_2}\}$; where $\mathbf{x_1} = \begin{bmatrix} 3 \\ 6 \\ 0 \end{bmatrix}$ and $\mathbf{x_2} = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$. Then orthogonal basis $\{\mathbf{v_1}, \mathbf{v_2}\}$ for V is given by

(A) $\mathbf{v_1} = \begin{bmatrix} 3 \\ 6 \\ 0 \end{bmatrix}$; $\mathbf{v_2} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$.

(B) $\mathbf{v_1} = \begin{bmatrix} 3 \\ 6 \\ 0 \end{bmatrix}$; $\mathbf{v_2} = \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}$.

(C) $\mathbf{v_1} = \begin{bmatrix} 3 \\ 0 \\ 6 \end{bmatrix}$; $\mathbf{v_2} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$.

(D) $\mathbf{v_1} = \begin{bmatrix} 3 \\ 0 \\ 6 \end{bmatrix}$; $\mathbf{v_2} = \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}$.

18. Let $f : [-1, 1] \to \mathbb{R}$ be differentiable at $x = 0$, then $\lim_{n\to\infty} \left[\sum_{k=1}^{N} f\left(\frac{k}{n}\right) - Nf(0) \right]$

(A) is equal to 0.

(B) is equal to $N(N+1)/2$.

(C) is equal to $N/2$.

(D) does not exist.

19. Let $f : [0, 1] \to \mathbb{R}$ be defined as $f(x) = \begin{cases} x^2, & x \in [0,1] \cap \mathbb{Q}, \\ x^3, & x \in [0,1] \setminus \mathbb{Q}. \end{cases}$ Let $U(f)$ and $L(f)$ denote the upper and lower Riemann integrals of f. Then

(A) $0 < U(f) - L(f) < \frac{1}{20}$.

(B) $\frac{1}{10} < U(f) - L(f) < 1$.

(C) $\frac{1}{20} < U(f) - L(f) < \frac{1}{10}$.

(D) $1 < U(f) - L(f) < 2$.

20. Eigenvalues of idempotent matrices can only be

(A) 0 and 1.

(B) 0.

(C) 1.

(D) 0, 1 and 2.

21. Let V, W be two finite dimensional vector spaces over $\mathbb{R}$. Suppose $T : V \to W$ be any function. Which of the following statements is/are true?

I. If T is a linear map, then $T(0) = 0$.

II. If $T : (V, +) \to (W, +)$ is a group homomorphism, then T is a linear map.

III. If $T : (V, +) \to (W, +)$ is a group homomorphism, then T need not be a linear map.

IV. Both I and II are true, only when $V = W$.

(A) I and II .

(B) I and III.

(C) IV.

(D) I, III and IV.

22. Let $A = \{a + b\sqrt{2} : a, b \in \mathbb{Z}\}$. Consider the following statements.

S_1: There exists a non constant sequence in $A \cap [0, 1]$,

S_2: There exists a non constant Cauchy sequence in A.

Then

(A) both S_1 and S_2 are true.

(B) S_1 is true and S_2 is false.

(C) S_1 is false and S_2 is true.

(D) both S_1 and S_2 are false.

23. Suppose $a_n > 0$, $\forall n \in \mathbb{N}$ and $\sum_{n=1}^{\infty} a_n$ converges. Consider the following statements:

S_1 : The series $\sum_{n=1}^{\infty} \frac{e^{a_n}}{n^{\frac{3}{2}}}$ converges

S_2 : The series $\sum_{n=1}^{\infty} \frac{|\sin a_n|}{a_n}$ converges

Then

(A) both S_1 and S_2 are true.

(B) S_1 is true and S_2 is false.

(C) S_1 is false and S_2 is true.

(D) both S_1 and S_2 are false.

24. Let $A = \begin{bmatrix} 1 & -1 & 1 \\ 1 & 1 & 1 \\ 2 & 3 & \alpha \end{bmatrix}$ and $b = \begin{bmatrix} 1 \\ 3 \\ \beta \end{bmatrix}$. Consider

S_1 : The equations $Ax = b$ has a unique solution when $\alpha = 3$ and $\beta = 2$.

S_2 : The equations $Ax = b$ has an infinite number solutions when $\alpha = 2$ and $\beta = 2$.

Then

(A) S_1 and S_2 are true.

(B) S_1 and S_2 are false.

(C) S_1 is true but S_2 is false.

(D) S_1 is false but S_2 is true.

25. Let $f(x, y) = \log\left(\cos^2 e^{x^2}\right) + \sin(x + y)$. Consider

S_1 : $\frac{\partial^2 f}{\partial x \partial y} = -1$ at $x = 0$ and $y = \pi/2$.

S_2 : $\frac{\partial^2 f}{\partial x^2} = 4\tan(e)$ at $x = 0$ and $y = 0$.

Then

(A) S_1 and S_2 are true.

(B) S_1 and S_2 are false.

(C) S_1 is true but S_2 is false.

(D) S_1 is false but S_2 is true.

PART - B

26. Consider $f(x) = (\sin x)^{\sin x}, x \in (0, \pi)$.
(I) $f(x)$ has a minimum at $x = \pi/2$.
(II) $f(x)$ has a maximum at $x = \pi/2$.
(III) $f(x)$ has a maximum at $x = \sin^{-1}(\frac{1}{e})$.

Then

(A) Only (I) is True.

(B) Only (II) is True.

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(C) Both (I) and (II) are True.

(D) Both (I) and (III) are False.

27. Which of the following are TRUE?
(1) Every bounded sequence in **R** has a convergent subsequence.
(2) Every sequence in **R** has a monotone subsequence.
(3) Every subsequence of an unbounded sequence in **R** is unbounded.
(4) The sum of two unbounded sequences in **R** is again an unbounded sequence.

(A) (1) and (4) .

(B) (1) and (2).

(C) (2) and (3).

(D) (2) and (4) .

28. Let H, K be two subgroups of a finite group G. Which of the following statements is/are correct?

I. HK is always a subgroup of G.

II. If H is normal and K is not normal subgroup, then only HK is a subgroup of G.

III. HK is need not be a subgroup of G.

IV. HK is a subgroup of G, if H is a normal subgroup of G.

(A) I and II.

(B) III and IV.

(C) II and III.

(D) II and IV.

29. Which of the following statements is/are true or False ?

A. In a metric space, a ball can contain another ball of strictly bigger radius.

B. Let (X, d) be a metric space and let A, B and C be three subsets of X. Define

$$d(A, B) = \inf_{(a,b)\in A\times B} d(a, b).$$

Then, $d(A, C) \leq d(A, B) + d(B, C)$.

(A) Both **A** and **B** are true.

(B) Both **A** and **B** are false.

(C) **A** is false and **B** is true.

(D) **A** is true and **B** is false.

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30. Consider the following statements:

I. An abelian group is cyclic.

II. A group of a prime order is cyclic.

III. A cyclic group is abelian.

IV. $\mathbb{Z}_2 \times \mathbb{Z}_2$ is cyclic.

Which of the following statements is/are correct:

(A) II, III and IV.

(B) II and III.

(C) II and IV.

(D) I and IV.

31. An ordering of three of the numbers $0, 1, 2, 3, 4, 5$ matching with the sequence 'determinant, trace, rank' for the matrix $\begin{bmatrix} 2 & 1 & 3 \\ 0 & 1 & 1 \\ -2 & 0 & -2 \end{bmatrix}$ is

(A) 2,5,3.

(B) 4,1,3.

(C) 0,2,1.

(D) 0,1,2 .

32. Let $[x]$ denote the greatest integer less than or equal to $x \in \mathbb{R}$. Then match the following:

(1) $f(x) = [x - 1]$, $x \in \mathbb{R}$	(i) continuous but not differentiable at $x = 1$
(2) $g(x) = \|x - 1\|^3$, $x \in \mathbb{R}$	(ii) discontinuous at $x = 1$
(3) $h(x) = \|x - 1\| + \sin^2(\|x - 1\|)$, $x \in \mathbb{R}$	(iii) differentiable at $x = 1$

(A) (1)-(iii), (2)-(i), (3)-(ii) .

(B) (1)-(i), (2)-(ii), (3)-(iii) .

(C) (1)-(ii), (2)-(iii), (3)-(i) .

(D) (1)-(ii), (2)-(i), (3)-(iii) .

33. Match the following set/equations to the corresponding geometric objects in the xyz-space:

(1) $\{(1,2,t) : t \in \mathbb{R}\}$	(i) plane parallel to the xy-plane
(2) $x = 0,\ z = 5$	(ii) line parallel to the z-axis
(3) $z = 5$	(iii) line parallel to the y-axis
(4) $x = 5$	(iv) plane orthogonal to the xy-plane

(A) (1)-(ii), (2)-(i), (3)-(iii), (4)-(iv).

(B) (1)-(ii), (2)-(iii), (3)-(i), (4)-(iv) .

(C) (1)-(iv), (2)-(iii), (3)-(ii), (4)-(i).

(D) (1)-(iii), (2)-(iv), (3)-(ii), (4)-(i)

34. Given that there are real constants a, b, c, and d such that the identity $\lambda x^2 + 2xy + y^2 = (ax + by)^2 + (cx + dy)^2$ holds for all x and y in $\mathbb{R}$. Then match the following

(i.) λ lies in	(a.) $[1, \infty)$ if $c \geq 0$
(ii.) b and d lie in	(b.) $[0, \infty)$
(iii.) a lies in	(c.) $[-1, 1]$

(A) (i.)-(b.),(ii.)-(c.), (iii.)-(a.).

(B) (i.)-(a.),(ii.)-(c.), (iii.)-(b.).

(C) (i.)-(c.),(ii.)-(b.), (iii.)-(a.).

(D) (i.)-(b.),(ii.)-(a.), (iii.)-(c.).

35. Match the following

(i.) $u = x^2 \cos 2y,$ $v = x^2 \sin 2y$	(a.) $J\left(\frac{u,v}{x,y}\right)=0$ at $x = 1,\ y = 1$.
(ii.) $u = x\sqrt{1-y^2} + y\sqrt{1-x^2},$ $v = \sin^{-1} x + \sin^{-1} y$	(b.) $J\left(\frac{u,v}{x,y}\right) > 0$ at $x = 1,\ y = 1$.
(iii.) $u = \cosh x \cos y,$ $v = \sinh x \sin y$	(c.) $J\left(\frac{u,v}{x,y}\right)=4$ at $x = 1,\ y = 1$.

(A) (i.)-(c.),(ii.)-(a.), (iii.)-(b.).

(B) (i.)-(a.),(ii.)-(c.), (iii.)-(b.).

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(C) (i.)-(c.),(ii.)-(b.), (iii.)-(a.).

(D) (i.)-(b.),(ii.)-(a.), (iii.)-(c.).

36. Match the following statements.

(a) For a real Hermitian matrix	(i) zero is always an eigenvalue.
(b) For a real skew-Hermitian matrix	(ii) the eigenvalues are zero or purely imaginary.
(c) For a unitary matrix	(iii) the eigenvalues have unit modulus.
(d) For a singular matrix	(iv) the eigenvalues are real.

Then

(A) (a)-(iii), (b)-(ii), (c)-(i), (d)-(iv).

(B) (a)-(iv), (b)-(ii), (c)-(iii), (d)-(i).

(C) (a)-(iv), (b)-(i), (c)-(iii), (d)-(ii).

(D) (a)-(i), (b)-(ii), (c)-(iv), (d)-(iii).

37. Match the following differential equations with their integrating factors.

(a) $(2x^2 + y)dx + (x^2y - x)dy = 0$	(i) x^2y.
(b) $(3y + 4xy^2)dx + (2x + 3x^2y)dy = 0$.	(ii) $\frac{1}{x^3y^3}$.
(c) $(xy - 1)dx + (x^2 - xy)dy = 0$.	(iii) $\frac{1}{x}$.
(d) $xdy + ydx + 3x^3y^4dy = 0$.	(iv) $\frac{1}{x^2}$.

Then

(A) (a)-(i), (b)-(ii), (c)-(iii), (d)-(iv).

(B) (a)-(iii), (b)-(ii), (c)-(i), (d)-(iv).

(C) (a)-(iv), (b)-(i), (c)-(iii), (d) - (ii).

(D) (a)-(iii), (b)-(ii), (c)-(i), (d)-(iv).

38. Let f be a function from $\mathbb{R}^2$ to $\mathbb{R}$. Then match the following

(i.) $f(x,y) = \dfrac{x^3+y^3}{3x^2+4xy}$	(a.) is not a homogeneous function.
(ii.) $f(x,y) = \sin^{-1}(x+y)$	(b.) is a homogeneous function of degree 0.
(iii.) $f(x,y) = \ln\left(\dfrac{x^3+y^3}{3x^2y+4xy^2}\right)$	(c.) is a homogeneous function of degree 1.

(A) (i.)-(c.),(ii.)-(a.), (iii.)-(b.).

(B) (i.)-(a.),(ii.)-(c.), (iii.)-(b.).

(C) (i.)-(c.),(ii.)-(b.), (iii.)-(a.).

(D) (i.)-(b.),(ii.)-(a.), (iii.)-(c.).

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39. Match the following functions with their Laplace transforms.

(a) $e^{2x}\sin 3x$	(i) $\frac{s-2}{(s-2)^2+9}$.
(b) $e^{2x}\cos 3x$	(ii) $\frac{3}{(s-2)^2+9}$.
(c) $e^{2x}\sinh 3x$	(iii) $\frac{(s-2)}{(s-2)^2-9}$.
(d) $e^{2x}\cosh 3x$	(iv) $\frac{3}{(s-2)^2-9}$.

Then

(A) (a)-(i), (b)-(ii), (c)-(iii), (d)-(iv).

(B) (a)-(iii), (b)-(ii), (c)-(i), (d)-(iv).

(C) (a)-(ii), (b)-(i), (c)-(iv), (d) - (iii).

(D) (a)-(iii), (b)-(ii), (c)-(i), (d)-(iv).

40. In this question, S_4 is the symmetric group on four letters, and A_3 is the alternating group. Let $D_8 = \langle r, s : r^4 = s^2 = 1, srs = r^{-1}\rangle$ be the dihedral group with 8 elements.

1. A_3	I. 2
2. S_4	II. 3
3. $D_8 \times \mathbb{Z}/2\mathbb{Z}$	III. 1.

We say that the pair is 'matched' if the *minimal* number of generators of a group listed on the left hand column equals to a number in the right hand column. Which of the pairs are matched?

(A) (I,1); (II,3); (III,2).

(B) (I,2); (II,1); (III,3).

(C) (I,3); (II,2); (III,1) .

(D) (I,2); (II,3); (III,1) .

41. If $f(x) = \int_x^{x^2} e^{-t^2}dt$ then

(A) $f'(0) \leq f'(1) \leq f'(2)$.

(B) $f'(0) \leq f'(2) \leq f'(1)$.

(C) $f'(1) \leq f'(0) \leq f'(2)$.

(D) $f'(2) \leq f'(1) \leq f'(0)$.

42. Let $y = \phi(x)$ be a solution to $x^2y''(x) - 2y(x) = 0$, $y(1) = 1$ and $y(2) = 1$, then which of the following is true ?

(A) $\phi(1/2) \leq \phi(3/2) \leq \phi(5/2)$.

(B) $\phi(3/2) \leq \phi(1/2) \leq \phi(5/2)$.

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(C) $\phi(1/2) \leq \phi(5/2) \leq \phi(3/2)$.

(D) $\phi(3/2) \leq \phi(5/2) \leq \phi(1/2)$.

43. Let 2 and 3 be the eigenvalues of a 2×2 matrix A. Then arrange the following matrices as per the descending values of their respective traces.
$P : A^{-1}$.
$Q : A^2$.
$R : (A - I)$.
$S : (A + I)$.

(A) S, R, Q, P.

(B) Q, S, R, P.

(C) Q, S, P, R.

(D) S, Q, P, R.

44. If $\vec{A}, \vec{B}, \vec{C}, \vec{D}$ represent the vectors $(z_1 - z_0)$, $(z_2 - z_0)$, $(z_3 - z_0)$ and $(z_4 - z_0)$, where $z_0 = 1 + i, z_1 = -3 + 2i, z_2 = 2 + 3i, z_3 = 1 + 5i, z_4 = 1 + 3i$, then arrange them in the descending order of the length of the vectors.

(A) $|\vec{C}|, |\vec{A}|, |\vec{B}|, |\vec{D}|$.

(B) $|\vec{A}|, |\vec{B}|, |\vec{C}|, |\vec{D}|$.

(C) $|\vec{A}|, |\vec{C}|, |\vec{B}|, |\vec{D}|$.

(D) $|\vec{C}|, |\vec{A}|, |\vec{D}|, |\vec{B}|$.

45. If $A(n)$ denote the area of the region $\{(x, y) \in \mathbb{R}^2 : |x|^n + |y|^n \leq 1\}$, $n \in \mathbb{N}$ then the arrangement of $A(2), A(3), A(4), A(5)$ in the ascending order is

(A) $A(4)$, $A(3)$, $A(2)$, $A(5)$.

(B) $A(5)$, $A(3)$, $A(4)$, $A(2)$.

(C) $A(2)$, $A(3)$, $A(4)$, $A(5)$.

(D) $A(2)$, $A(3)$, $A(5)$, $A(4)$.

46. Let y_k, $1 \leq k \leq 4$ be the solution of $y'' + 10^k y = 0$. The arrangement of y_k's in the decreasing order of their number of zeros in $[-10, 10]$ is

(A) y_1, y_3, y_2, y_4.

(B) y_1, y_2, y_3, y_4.

(C) y_4, y_2, y_1, y_3.

(D) y_4, y_3, y_2, y_1.

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47. Arrange the following subgroups of $\mathbb{Z}_{100}$ in decreasing order of their size (i.e. cardinality). : $H_1 =< 15 >, H_2 =< 6 >, H_3 =< 24 >, H_4 =< 30 >$.

(A) H_2, H_3, H_1, H_4.

(B) H_1, H_2, H_3, H_4.

(C) H_2, H_3, H_1, H_4.

(D) H_4, H_1, H_3, H_2.

48. Consider the following spheres :
$S_1 : x^2 + y^2 + z^2 - 2x - 2y - 4z = 20$.
$S_2 : x^2 + y^2 + z^2 + 2x - 2y - 6z = 14$.
$S_3 : x^2 + y^2 + z^2 = 4y - 2z$.
$S_4 : x^2 + y^2 + z^2 + 10x - 6y + 6z + 34 = 0$.
Arrange the spheres as per the decreasing order of their radii.

(A) S_2, S_1, S_3, S_4.

(B) S_1, S_2, S_3, S_4.

(C) S_1, S_2, S_4, S_3.

(D) S_2, S_1, S_4, S_3.

49. Let $X = \{1, 2, 3\}$. Arrange the following sets in an increasing order of cardinalities:
$S_1 =$ The set of all symmetric relations on X.
$S_2 =$ The set of all bijections of X.
$S_3 =$ The set of equivalence relations on X.
$S_4 =$ The set of all relations on X.
$S_5 =$ The set of all functions from X to X.
$S_6 =$ The set of all reflexive relations on X.

(A) $|S_3| \leq |S_2| \leq |S_5| \leq |S_1| \leq |S_6| \leq |S_4|$.

(B) $|S_2| \leq |S_1| \leq |S_3| \leq |S_5| \leq |S_6| \leq |S_4|$.

(C) $|S_5| \leq |S_1| \leq |S_3| \leq |S_6| \leq |S_4| \leq |S_2|$.

(D) $|S_2| \leq |S_5| \leq |S_3| \leq |S_1| \leq |S_6| \leq |S_4|$.

50. Consider the general system $AX = \mathbf{0}$ of n equations in n unknowns. Arrange the following statements in the order
I. $\det A \neq 0$
II. The system has unique solution
III. Solution is $A^{-1}X$
IV. Rank(A)=Rank([A B])

(A) III,, II, IV, I.

(B) II, III, IV, I..

(C) I , IV, II, III.

(D) III, II, I, IV .