I. Please enter your Hall Ticket Number on Page 1 of this question paper and on the OMR sheet without fail.

II. Read carefully the following instructions:

1. This Question paper has two Sections: Section A and Section B
2. Section A consists of 25 objective type questions of two marks each. 
   There is negative marking of 0.66 mark for every wrong answer.
   The marks obtained by the candidate in this Section will be used for resolving the tie cases.
3. Section B consists of 50 objective type questions of one mark each.
   There is no negative marking in this Section.
4. Answers are to be marked on the OMR answer sheet following the instructions provided there upon. An example is shown below

   \[ \begin{array}{ccc} 100 & \box{A} & \box{B} & \box{C} \\ \end{array} \]

5. Only Scientific Calculators are permitted. Mobile phone based calculators are not permitted. Logarithmic tables are not allowed.
6. Hand over the OMR answer sheet at the end of the examination to the Invigilator.

III. Values of physical constants:
   \[ c = 3 \times 10^8 \text{ m/s}; \quad h = 6.63 \times 10^{-34} \text{ J.s}; \quad k_B = 1.38 \times 10^{-23} \text{ J/K} \]
   \[ e = 1.6 \times 10^{-19} \text{ C}; \quad \mu_0 = 4\pi \times 10^{-7} \text{ Henry/m}; \quad \epsilon_0 = 8.85 \times 10^{-12} \text{ Farad/m} \]
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SECTION-A

1. The solution of the equation, \( \sin(x) + y' = 0 \), where \( y(0) = 1 \), is
   A. \( y(x) = \sqrt{2} \sin(x) + 1 \)
   B. \( y(x) = \sqrt{2} \cos(x) + 1 \)
   C. \( y(x) = \sqrt{2} \sin(x) - 1 \)
   D. \( y(x) = \sqrt{2} \cos(x) - 1 \)

2. The three vectors \( \vec{A} = 3\hat{i} + 2\hat{j} + 5\hat{k} \), \( \vec{B} = 7\hat{i} + 5\hat{j} + \alpha\hat{k} \) and \( \vec{C} = 4\hat{i} + 2\hat{j} + 9\hat{k} \) are coplanar when \( \alpha \) takes the value
   A. 8.5
   B. 12.5
   C. 10.5
   D. 6.5

3. The series \( \sum_{n=1}^{\infty} \frac{1}{n!(\log n)^p} \) is a
   A. convergent series for \( p > 1 \)
   B. divergent series for \( p > 1 \)
   C. convergent series for \( p = 1 \)
   D. divergent series for all values of \( p \)

4. The series expansion of the function \( f(t) = \sin^5 t \) is
   A. \( \frac{5}{8} \sin^2 t + \frac{5}{16} \sin^3 t + \frac{1}{16} \sin^5 t \)
   B. \( \frac{5}{8} \sin t - \frac{5}{16} \sin 3t + \frac{1}{16} \sin 5t \)
   C. \( \frac{5}{8} \sin t + \frac{5}{16} \sin 2t + \frac{5}{24} \sin 3t + \frac{5}{32} \sin 4t \)
   D. \( \frac{5}{8} \sin t + \frac{5}{16} \sin^2 t + \frac{5}{24} \sin^3 t + \frac{5}{32} \sin^4 t \)

5. Out of the given equations the only equation which is an exact differential is
   A. \((4x^3y^2 - 2xy)dx + (3x^3y^2 - x^2)dy = 0\)
   B. \((x^2 + y^2 + x)dx + xy dy = 0\)
   C. \((3e^{3x}y - 4x)dx + 6e^{3x}dy = 0\)
   D. \(\cos y dx + (\sin x - \sin y)dy = 0\)
6. The Fourier transform of the function \( f(t) = \begin{cases} e^{-\alpha t} & \text{for } t > 0 \\ 0 & \text{for } t < 0 \end{cases} \)

for \( \alpha > 0 \), is

A. \( \frac{1}{\alpha - i\omega} \)
B. \( \frac{1}{\alpha + i\omega} \)
C. \( \frac{1}{\alpha - i\omega} e^{-(\alpha + i\omega)t} \)
D. \( \frac{1}{\alpha + i\omega} e^{(\alpha + i\omega)t} \)

7. A body of mass ‘\( m \)’ is subjected to a resistive force of \( bv \) (where \( v \) is the velocity and \( b \) is a constant). There is no restoring force in the medium. The displacement ‘\( x \)’ as a function of time ‘\( t \)’, in terms of its initial velocity ‘\( v_0 \)’ and the coefficient \( \gamma = \frac{k}{m} \), is

A. \( -\gamma e^{-\gamma t} \)
B. \( -v_0 e^{-\gamma t} \)
C. \( \frac{-v_0}{\gamma} e^{-\gamma t} \)
D. \( \frac{-v_0}{\gamma} e^{-2\gamma t} \)

8. A rocket of mass \( M_0 \), takes off with a constant velocity \( v_0 \), in a uniform gravitational field. The rocket loses fuel mass, as it is propelled by the gas, which is ejected with a velocity ‘\( u \)’ relative to the rocket. At a later time \( t_f \), given that the mass of the rocket is \( M_f \), the velocity of the rocket \( v_f \) is equal to

A. \( u \ln \left( \frac{M_0}{M_f} \right) - gt_f \)
B. \( -u \ln \left( \frac{M_0}{M_f} \right) \)
C. \( u \ln \left( \frac{M_0}{M_f} \right) \)
D. \( u \ln \left( \frac{M_f}{M_0} \right) - gt_f \)

9. The rotational kinetic energy of a thin disk, of mass 20 kg and radius 70 cm, rotating like a merry-go-round at an angular speed of 120 radians/min, is

A. 5.9 J
B. 9.8 J
C. 19.6 J
D. 21.6 J
10. An uniform rope of mass $M$ and length $L$ is pivoted at one end and whirls with uniform angular velocity $\omega$. The tension in the rope, at a distance $r$ from the pivot, (neglecting gravity) is given by

A. $\frac{M\omega^2 r^2}{L}$
B. $\frac{M\omega^2}{L} (L^2 - r^2)$
C. $\frac{M\omega^2}{2L} r^2$
D. $\frac{M\omega^2}{2L} (L^2 - r^2)$

11. Three charges are placed at the corners of a square of side ‘a’ as shown in the figure. The work necessary to place another charge $-q$ in the fourth corner is

A. $-\frac{q^2}{4\pi\varepsilon_0 a\sqrt{2}}$
B. $\frac{q^2}{4\pi\varepsilon_0 a}$
C. $-\frac{q^2}{4\pi\varepsilon_0 a}$
D. $\frac{q^2}{4\pi\varepsilon_0 a\sqrt{2}}$

12. The capacitance of two coaxial metal cylinder tubes of radii $a$, $b$ and length $2L$ is

A. $\frac{4\pi\varepsilon_0}{L} \frac{ab}{(a-b)}$
B. $\frac{L}{4\pi\varepsilon_0} \left(\frac{1}{a} - \frac{1}{b}\right)$
C. $\frac{4\pi\varepsilon_0 L}{\ln(b/a)}$
D. $\frac{1}{4\pi\varepsilon_0} \ln(a/b)$
13. A monochromatic light of wavelength $\lambda$ is falling normally on a diffraction grating with period $d$. If the angle between the direction to the first and second order of the Fraunhofer maxima is $\Delta \theta$, the wavelength of the light is

A. $\frac{d \sin \Delta \theta}{\sqrt{5 - 4 \cos \Delta \theta}}$

B. $\frac{d \sin \Delta \theta}{\cos \Delta \theta}$

C. $d \sin \Delta \theta$

D. $\frac{d \sin^2 \Delta \theta}{\sqrt{5 - 4 \cos^2 \Delta \theta}}$

14. Consider two symmetrical thin lenses, one is converging with focal length $f_1$ and refractive index $n_1$ and the other is diverging with focal length $f_2$ and refractive index $n_2$. The radius of curvature of both the lenses is $R$. If the lenses are put together and submerged in water (refractive index $n_w$), the effective focal length of the system is

A. $\frac{2(n_1 - n_2)}{n_w R}$

B. $\frac{2n_w R}{(n_1 + n_2)}$

C. $\frac{2n_w}{R(n_1 + n_2)}$

D. $\frac{n_w R}{2(n_1 - n_2)}$

15. A gas is enclosed in a cylinder fitted with a piston which moves adiabatically, according to the equation $P^3 V^5 = \text{constant}$. The work done by the gas in adiabatic expansion from $(P_i, V_i) = (10^5 \text{ Pa}, 10^{-3} \text{ m}^3)$ to $(P_f, V_f) = (\frac{10^5}{32} \text{ Pa}, 8 \times 10^{-3} \text{ m}^3)$ is

A. 225 J

B. 700 J

C. 90 J

D. 112.5 J
16. A thermodynamic system obeys equations \( U = \frac{1}{2} PV \) and \( T^2 = \frac{AU^{3/2}}{V^{1/2}N^{1/2}} \) where \( A \) is positive. Here \( P, V, T, N \) and \( U \) are pressure, volume, temperature, number of particles and internal energy respectively. The entropy \( 'S' \) of the system can be expressed as

A. \( S = A^{-1/2}U^{1/4}V^{1/2}N^{1/4} + \text{constant} \)
B. \( S = 4A^{-1/2}U^{1/4}V^{1/2}N^{1/4} + \text{constant} \)
C. \( S = 125A^{-1/2}U^{1/4}V^{1/2}N^{1/4} + \text{constant} \)
D. \( S = 5A^{-1/2}U^{1/4}V^{1/2}N^{1/4} + \text{constant} \)

17. An LCR circuit, consisting of a resistance of 10 \( \Omega \), inductance of 1 H and a variable capacitance, is connected to a supply of 220 V, 50 Hz source. The capacitance required to achieve a series resonance condition is

A. 5 \( \mu F \)
B. 10 \( \mu F \)
C. 15 \( \mu F \)
D. 20 \( \mu F \)

18. In the given circuit, if the DC gain of the transistor changes from 100 to 200, the collector current will

A. increase by 2 times
B. decrease by 2 times
C. remains the same
D. saturate at 1.2 A

19. Two equal and opposite relativistic forces were applied along the \( y \)-axis at time \( t = 0 \) at spatial points \((0,0,0)\) and \((a,0,0)\) in a rest frame. In a frame moving with a velocity \( \vec{v} = vi \), the time-like separation of the two forces is,

A. \( \Delta t' = \frac{va}{c^2 \sqrt{1 - v^2/c^2}} \)
B. \( \Delta t' = -\frac{a}{c \sqrt{1 - v^2/c^2}} \)
C. \( \Delta t' = \frac{a}{c \sqrt{1 - v^2/c^2}} \)
D. \( \Delta t' = -\frac{va}{c^2 \sqrt{1 - v^2/c^2}} \)
20. A relativistic particle of rest mass \( m_0 \) is moving in free-space with a speed \( v \). If \( c \) is the speed of light in free-space, then the value of \( v \), at which it's (relativistic) kinetic energy is equal to it's rest mass energy \( (m_0c^2) \), is

A. \( v = \frac{1}{2}c \)
B. \( v = \frac{\sqrt{3}}{2}c \)
C. \( v = \frac{1}{4}c \)
D. \( v = \frac{\sqrt{3}}{4}c \)

21. An isolated and uncharged copper sphere of radius 1 cm is irradiated by an ultraviolet light of wavelength 200 nm. The work function for copper is 4.7 eV. The charge induced on the sphere, due to the photoelectric effect, is

A. 1.66 pC
B. 4.7 pC
C. 0.023 pC
D. 2.5 pC

22. Two pendulums, each of length 'l' and mass 'm' are coupled with a spring of natural frequency \( \omega_c \). The resultant frequency, of the out-of-phase vibration, is

A. \( \left( \frac{g}{l} - 2\omega_c^2 \right)^{1/2} \)
B. \( \left( \frac{g}{l} - \omega_c^2 \right)^{1/2} \)
C. \( \left( \frac{g}{l} - \frac{\omega_c^2}{2} \right)^{1/2} \)
D. \( \left( \frac{g}{l} + 2\omega_c^2 \right)^{1/2} \)

23. The Youngs modulus of a rod of mass \( m \), cross-sectional area \( a \) and length \( l_o \), fixed at one end is \( Y \). If a force is applied to stretch the rod, the period of small oscillation will be

A. \( 2\pi \sqrt{\frac{ml_o}{AY}} \)
B. \( \pi \sqrt{\frac{ml_o}{AY}} \)
C. \( 2\pi \sqrt{\frac{m}{AY}} \)
D. \( 2\pi \sqrt{\frac{ml_oY}{A}} \)
24. Consider a particle confined in a one dimensional box of length $a$. The wave function of the first excited state is given by

$$\psi(x) = \begin{cases} \sqrt{\frac{2}{a}} \sin\left(\frac{2\pi x}{a}\right), & \text{for } 0 < x < a \\ 0, & \text{elsewhere} \end{cases}$$

The average value of the position of the particle is

A. $a/4$
B. $a/16$
C. $a/8$
D. $a/2$

25. The average kinetic energy of a thermal neutron is given by $\frac{3}{2}k_B T$. The de Broglie wavelength at 0°C temperature is (mass of the neutron is $1.6 \times 10^{-24}$ g)

A. 1.52 Å
B. 1.85 Å
C. $\infty$
D. 1 Å
26. The following differential equation

\[ L \frac{d^2Q}{dt^2} + R \frac{dQ}{dt} + \frac{Q}{C} = 0 \]

is an equation of

A. order one, degree one
B. order two, degree two
C. order two, degree one
D. order one, degree two

27. The even component of the function \( f(t) = \begin{cases} \text{e}^t & t > 0 \\ 0 & t < 0 \end{cases} \) is given by

A. \( f_{even}(t) = \begin{cases} \frac{1}{2}e^{-t} & t > 0 \\ -\frac{1}{2}e^t & t < 0 \end{cases} \)
B. \( f_{even}(t) = \begin{cases} 2e^{-t} & t > 0 \\ -2e^t & t < 0 \end{cases} \)
C. \( f_{even}(t) = \begin{cases} \frac{1}{2}e^{-t} & t > 0 \\ \frac{1}{2}e^t & t < 0 \end{cases} \)
D. \( f_{even}(t) = \begin{cases} -2e^{-t} & t > 0 \\ 2e^t & t < 0 \end{cases} \)

28. The solution of the differential equation

\[ \frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 0 \]

is

A. \( e^x, e^{-2x} \)
B. \( e^x, e^{-x} \)
C. \( e^{-x}, e^{2x} \)
D. \( e^{2x}, e^{-2x} \)
29. Given that \( f(x) \) is an even periodic function of period \( 2L \), its Fourier series will be of the form

A. \( \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n + b_n) \sin \frac{n\pi x}{L} \)

B. \( \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \sin \frac{n\pi x}{L} \)

C. \( \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos \frac{n\pi x}{L} \)

D. \( \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \sin \frac{n\pi x}{L} + \sum_{n=1}^{\infty} b_n \cos \frac{n\pi x}{L} \)

30. The eigenvalues of the matrix

\[
A = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}
\]

are

A. \(-1, -2\)

B. \(1, 2\)

C. \(-1, 2\)

D. \(1, -2\)

31. The product of the two given series

\[
\left( x - \frac{1}{3!} x^3 + \frac{1}{5!} x^5 + \cdots \right) \left( 1 - \frac{1}{2!} x^2 + \frac{1}{4!} x^4 + \cdots \right)
\]

is

A. \(e^{2x}\)

B. \(\frac{1}{2}(\cos 2x)\)

C. \(e^{-2x}\)

D. \(\frac{1}{2}(\sin 2x)\)
32. The rank of the matrix

\[ A = \begin{bmatrix}
  1 & -1 & 1 & -1 \\
  -1 & 1 & -1 & 1 \\
  1 & -1 & 1 & -1 \\
  -1 & 1 & -1 & 1 
\end{bmatrix} \]

is

A. 4  
B. 3  
C. 2  
D. 1

33. A normalized Gaussian distribution with mean value \( \mu \) and standard deviation \( \sigma \) is given by

A. \( p(x) = \frac{1}{2\pi\sigma} e^{-\left(\frac{x-\mu}{\sigma}\right)^2} \)  
B. \( p(x) = \frac{1}{\sqrt{2\pi\sigma}} e^{-\left(\frac{x-\mu}{\sigma}\right)^2} \)  
C. \( p(x) = \frac{1}{\sqrt{2\pi\sigma}} e^{\left(\frac{x-\mu}{\sigma}\right)^2} \)  
D. \( p(x) = \frac{1}{2\pi\sigma} e^{-\left(\frac{\mu}{\sigma}\right)^2} \)

34. A flying space craft cannot be in the shape of a cylinder because

A. a cylinder is inherently unstable about its axial axis  
B. a cylinder is inherently unstable about its transverse axis  
C. a cylinder is only neutrally stable about its axial axis  
D. a cylinder is unstable along all the three axes

35. Consider a dumbell made up of two spheres, each of radius \( b \) and mass \( M \), separated by a thin rod. The distance between the centers of the spheres is \( 2l \). The dumbell rotates with an angular velocity \((\vec{\omega})\) given by \( \vec{\omega} = \omega_x \hat{j} + \omega_z \hat{k} \). The three components of the angular momentum will be related as,

A. \( L_x = L_y = L_z = 0 \)  
B. \( L_x = 0; \frac{L_y}{L_x} \neq \frac{\omega_y}{\omega_x} \)  
C. \( L_x = 0; \frac{L_y}{L_x} = \frac{\omega_y}{\omega_x} \)  
D. \( L_x \neq 0; L_y = L_z \)
36. A particle of mass \( m \) is moving from region I to region II. Region I and II are separated by a plane parallel to the \( y \) axis. In region I, the particle is moving under a constant potential \( u_1 \) and in region II under a constant potential \( u_2 \) \((u_1 \neq u_2)\) with velocities \( v_1 \) and \( v_2 \) respectively. If \( \theta_1 \) and \( \theta_2 \) are the angles between the normal to the plane of separation and \( v_1, v_2 \) are velocities in regions I and II respectively, then the ratio of \( \frac{v_2}{v_1} \) is given by,

\[
\frac{\sin \theta_1}{\sin \theta_2}
\]

B. \( \frac{\cos \theta_1}{\cos \theta_2} \)

C. \( \frac{\tan \theta_1}{\tan \theta_2} \)

D. \( \frac{1 - \cos \theta_1}{1 - \cos \theta_2} \)

37. A solid spherical object (of radius \( r \) and mass \( m \)) is rolling down (without slipping) with an acceleration \( a = \frac{3}{2}g \sin \theta \) along an inclined plane which makes an angle \( \theta \) with the horizontal line. Here \( g \) is the acceleration due to gravity and the object starts from rest from the upper end of the inclined plane of length \( L \), then the total (translational and rotational) kinetic energy of the object at the lower end of the inclined plane is

A. \( \frac{5}{2}mgL \sin \theta \)

B. \( \frac{3}{2}mgL \sin \theta \)

C. \( \frac{5}{2}mgL \sin \theta \)

D. \( mgL \sin \theta \)

38. If the speed of light is the escape velocity of a particle from the surface of a planet for which \( \frac{GM}{c^2} = 743 \) m, where \( G \) is the gravitational constant, \( M \) is the mass of the planet and \( c \) is the speed of light, then the radius of this planet is

A. 0.743 km

B. 1.486 km

C. 2.229 km

D. 6.000 km
39. A thick viscous liquid is flowing through a horizontal tube due to a constant pressure difference between the ends. If the radius of the tube is doubled and the length is reduced to half, then the volume rate of the liquid flow

A. increases by 32 times
B. increases by 8 times
C. decreases by 16 times
D. remains constant

40. If the earth is moving in a circular orbit of radius $1.5 \times 10^{11}$ m around the sun, then the mass of the sun is ($G=6.67\times10^{-11}$ $m^2/Kg$)

A. $2.67 \times 10^{35}$ Kg
B. $2.3 \times 10^{36}$ Kg
C. $1.9 \times 10^{36}$ Kg
D. $2.3 \times 10^{41}$ Kg

41. Two artificial satellites $S_1$ and $S_2$ of mass $m$ and $2m$ respectively, are orbiting the earth in elliptical orbits such that the period of $S_1$ is double that of $S_2$. If the semi-major axis of $S_1$ is half of $S_2$, then the ratio of the total energy between $S_1$ and $S_2$ is given by,

A. 1:1
B. 2:1
C. 4:1
D. 1:2

42. If a charge $Q$ moves counter clockwise with a speed $v$ around a circle of radius $R$, in a uniform magnetic field $B$ (pointing into the page), then the direction and magnitude of the magnetic force $F$, respectively is

A. $F$ points towards the centre of the circle, $F = QvB$
B. $F$ points away from the centre of the circle, $F = QvB$
C. $F$ points in the direction of $B$, $F = QvB$
D. $F$ and $B$ are opposite to each other and the net force is zero.

43. The magnetic field $B$, inside the solenoid depends on the

A. resistivity of the wire used for making the solenoid
B. number of turns per unit length of the solenoid
C. cross-sectional area of the solenoid
D. distance from the axis of the solenoid.
44. Ampère's law in integral form offers an efficient way of calculating the
   A. electric field inside a current carrying conductor
   B. electromagnetic field of an oscillating charge
   C. force on a charged particle moving in a uniform magnetic field
   D. magnetic field due to steady current flowing in a wire

45. Two infinite parallel plates carry equal but opposite uniform charge density ±σ. The electric field $\vec{E}$ in between and outside the plates respectively are
   A. $\frac{\sigma}{\epsilon_0}$, 0
   B. $\frac{\sigma}{2\epsilon_0}$, 0
   C. 0, $\frac{\sigma}{\epsilon_0}$
   D. 0, $\frac{\sigma}{2\epsilon_0}$

46. A hollow metal sphere having a potential of +400 V with respect to the ground contains a charge of $5 \times 10^{-9}$ C. The electric potential at the center of the sphere is
   A. 0 V
   B. 800 V
   C. 400 V
   D. -400 V

47. The criteria for total internal reflection of the sound wave at the air-water interface is
   A. sound wave must travel from water to air with incidence angle above the critical angle.
   B. sound wave must travel from air to water with incidence angle above the critical angle
   C. sound wave must travel from water to air with incidence angle below the critical angle
   D. sound wave must travel from air to water with incidence angle below the critical angle
48. An electrical field,

\[ \vec{E} = E_0 \cos(kz - \omega t)\hat{x} - E_0 \sin(kz - \omega t)\hat{y} \]

where \( E_0 \) is the amplitude, \( \omega \) angular frequency, and \( k \) the propagation constant, is

A. right circularly polarized
B. linearly polarized
C. left circularly polarized
D. right elliptically polarized

49. Which of the following will promote a liquid to flow in streamline motion through narrow tubes?

A. low viscosity and low density
B. low viscosity and high density
C. high viscosity and low density
D. high viscosity and high density

50. Consider two coherent plane light waves of wavelength \( \lambda \) separated by a distance \( d \) falling normally on a screen (with a divergence angle \( \theta \ll 1 \text{ rad} \)), at a distance \( l \) from the source. If the amplitude of the waves are equal, then the separation between the neighboring maxima, on the screen, is

A. \( \lambda \theta \)
B. \( \lambda / \theta \)
C. \( \lambda \theta / dl \)
D. \( \lambda^2 / \theta d \)

51. In a two-beam interferometer, an orange colour mercury line, composed of two wavelengths \( \lambda_1 \) and \( \lambda_2 \) is used. The lowest order of interference corresponding to the least contrast in the fringe pattern is

A. \( \frac{\lambda_1}{2(\lambda_2 - \lambda_1)} \)
B. \( \frac{(\lambda_1 + \lambda_2)}{(\lambda_1 - \lambda_2)} \)
C. \( \frac{(\lambda_1 - \lambda_2)}{(\lambda_1 + \lambda_2)} \)
D. \( \frac{(\lambda_1 + \lambda_2)}{2\lambda_1} \)
52. Consider a natural light of intensity $I_o$ falling on a system of three identical in-line polarizers (the maximum transmission coefficient of each polarizer is $T$), with the principal direction of the middle polarizer forming an angle $\phi$ with the other two polarizers. The intensity of the output light, after passing through the system is

A. $\frac{1}{2}I_oT^2\cos^2\phi$
B. $\frac{1}{4}I_oT^3\cos^4\phi$
C. $I_oT^2\cos^2\phi$
D. $\frac{1}{4}I_oT^3\cos^2\phi$

53. A light source is moving in free-space away from a stationary observer with velocity $\vec{v}$. Let $c$ be the speed of light in the free-space. If the time period of the signal received by the observer is $T$, then the corresponding wavelength $\lambda$ is

A. $\lambda = cT$
B. $\lambda = |\vec{v}|T$
C. $\lambda = (c + |\vec{v}|)T$
D. $\lambda = (c - |\vec{v}|)T$

54. Two relativistic particles are moving in free-space with $\vec{v}_1 = v\hat{i}$ and $\vec{v}_2 = -v\hat{i}$ along the $x$-axis and negative $x$-axis respectively. Let $c$ be the speed of light in free-space. The speed of one particle with respect to the other is

A. 0
B. $2v$
C. $\frac{2v}{1 - \frac{v^2}{c^2}}$
D. $\frac{2v}{1 + \frac{v^2}{c^2}}$

55. The required speed of a clock to run in free-space at a rate of one-third of that at rest is

A. $1/3$ times the speed of light in free-space
B. $1/9$ times the speed of light in free-space
C. $2\sqrt{2}/3$ times the speed of light in free-space
D. $8/9$ times the speed of light in free-space
56. The energy required to completely remove an electron from the \( n = 2 \) (where \( n \) is the principle quantum number) state of a hydrogen atom is

A. 1.51 eV  
B. 12.0 eV  
C. 13.6 eV  
D. -13.6 eV

57. Which one of the following statement is true?

A. Nuclear force is always an attractive force.  
B. Nuclear force is always a repulsive force.  
C. Nuclear force is both attractive and repulsive depending on the distance between the particles.  
D. Nuclear force is both attractive and repulsive depending on the spin of the particles.

58. If 10 moles of a certain radioactive material decays into 1.25 moles in 75 days, then the half life of the material is

A. 37.5 days  
B. 25 days  
C. 50 days  
D. 23.44 days

59. The Legendre transform of the function \( y = \frac{x^2}{4} \) in terms of \( P = \frac{dy}{dx} \) is

A. \( \psi = -P^2 \)  
B. \( \psi = -4P^2 \)  
C. \( \psi = P^2 \)  
D. \( \psi = \frac{P^2}{4} \)

60. According to the third law of thermodynamics, entropy of a perfect crystal at absolute zero temperature is

A. always negative  
B. always positive  
C. equal to zero  
D. equal to one
61. If $T_1$ and $T_2$ are temperatures (expressed in °K) of the two reservoirs of a Carnot engine, with $T_1 > T_2$, then the efficiency of the engine $\eta$ obeys

A. $\eta \geq \frac{T_1 - T_2}{T_1}$
B. $\eta \leq \frac{T_1 - T_2}{T_2}$
C. $\eta \leq \frac{T_1 - T_2}{T_1}$
D. $\eta \geq \frac{T_1 - T_2}{T_2}$

62. An isolated system with an equilibrium distribution of particles at a constant energy $E$, pressure $P$ and volume $V$ is known as a

A. Micro-canonical ensemble
B. Canonical ensemble
C. Grand canonical ensemble
D. Macro canonical ensemble

63. In an analog to digital converter (ADC), if the number of digital output bit increases for the same analog input, then the

A. accuracy of the ADC increases
B. range of the ADC increases
C. conversion speed of the ADC increases
D. power consumption of the ADC decreases

64. An npn transistor with DC gain of 100 is configured as common emitter amplifier. A base supply of 10 V forward biases the emitter diode through a 220 kΩ resistor. A collector supply of 10 V reverse biases the collector diode through 1 kΩ resistor. During the circuit operation, if the base resistor is open, then the collector-emitter voltage is

A. 0.7 V
B. 5.5 V
C. 9.3 V
D. 10 V
65. A fullwave bridge rectifier circuit is connected to 220 V, 50 Hz a.c. input supply. If one of the diodes in the bridge is open, then the ripple frequency at the load output is

A. 0 Hz  
B. 50 Hz  
C. 100 Hz  
D. 150 Hz

66. In the circuit given below, the input root mean square voltage \((V_{in})\) is 220 V and the diode is an ideal diode. If the primary to secondary winding turn ratio of the transformer is 5:1, then the d.c. load voltage is

![Circuit Diagram]

A. 67.9 V  
B. 48 V  
C. 21.6 V  
D. 15.3 V

67. If \(N\) number of resistors, each of value \(R\), are connected in series, the resultant resistance is \(X\). When the same resistors are connected in parallel, the total resistance is

A. \(\frac{X}{N^2}\)  
B. \(\frac{X}{2N}\)  
C. \(N^2X\)  
D. \(\frac{X}{RN^2}\)

68. When a zener diode is used under forward biased condition, it will behave as a

A. voltage regulator  
B. resistor  
C. PN junction diode  
D. capacitor

69. The octal equivalent of decimal number \((49.5)_{10}\) is

A. 1110001.1  
B. 16.4  
C. 61.4  
D. 41.3
70. For a horizontal streamline motion of an incompressible fluid, which of the following is correct?

A. The sum of its static and dynamic pressures remains constant.
B. The sum of its potential energy and kinetic energy remains constant.
C. The sum of its pressure energy and potential energy remains constant.
D. The kinetic energy remains constant.

71. Which of the following represents an eigenfunction

A. \( f(x) \)
B. \( f(x) \)
C. \( f(x) \)
D. \( f(x) \)

72. Consider a particle of mass \( m \) bound in a square well of length \( L \) and depth of potential \( V_0 \). The minimum kinetic energy required to cross the potential barrier is

A. \( \frac{\hbar^2 m}{L^2} \)
B. \( \frac{\hbar^2}{mL^2} \)
C. \( \frac{\hbar^2}{8mL^2} \)
D. \( \frac{\hbar^2 m}{8L^2} \)

73. In a harmonic oscillator, the ground state energy is 2.4 eV. If the oscillator undergoes a transition from its fourth excited state to the second, by emitting a photon, the energy of the emitted photon is

A. 4.8 eV
B. 14.4 eV
C. 7.2 eV
D. 9.6 eV
74. Which one of the following is the weakest bond?
   A. covalent bond
   B. ionic bond
   C. metallic bond
   D. Vander waals bond

75. Which form of the material exhibits highly anisotropic properties?
   A. Polycrystalline
   B. Single crystalline
   C. Amorphous
   D. Liquid
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Note/Remarks: * For question 56, all students will get benefit.

Signature
School/Department/Centre