T - 6

Entrance Examination : M.Sc. Mathematics, 2019

Hall Ticket Number [ ] [ ] [ ] [ ] [ ] [ ] [ ] [ ]

Time : 2 hours
Max. Marks : 100

Part A : 25 marks
Part B : 75 marks

Instructions

1. Write your Hall Ticket Number in the OMR Answer Sheet given to you. Also write the Hall Ticket Number in the space provided above.
2. Answers are to be marked on the OMR answer sheet.
3. Please read the instructions carefully before marking your answers on the OMR answer sheet.
4. Hand over the OMR answer sheet at the end of the examination.
5. The question paper can be taken by the candidate at the end of the examination.
6. No additional sheets will be provided. Rough work can be done in the question paper itself/space provided at the end of the booklet.
7. Calculators are not allowed.
8. There are a total of 50 questions in **Part A** and **Part B** together.
9. **There is a negative marking in Part A. Each correct answer carries 1 mark and each wrong answer carries -0.33 mark. Each question in Part A has only one correct option.**
10. **There is no negative marking in Part B. Each correct answer carries 3 marks. In Part B some questions haves more than one correct option. All the correct options have to be marked in OMR sheet other wise zero marks will be credited.**
11. The appropriate answer(s) should be coloured with either a blue or a black ball point or a sketch pen. DO NOT USE A PENCIL.
12. This booklet contains TWELVE (12) pages including this page and excluding pages for rough work. Please check that your paper has all the pages.
13. **Notations** : $\mathbb{R}$ denotes the set of real numbers, $\mathbb{C}$ the set of complex numbers, $\mathbb{Q}$ the set of rational numbers, $\mathbb{Z}$ the set of integers and $\mathbb{N}$ the set of natural numbers $\{1, 2, 3, \ldots\}$, and $\varnothing$ the empty set. For a set A, A^c denotes its complement. For a ring R and a positive integer n, $M_n(R)$ denotes the set of all $n \times n$ matrices with entries from R.

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Part A

1. The contrapositive of the statement "If it rains then you take an umbrella" is

   (A) if it does not rain, then you do not take an umbrella.

   (B) if you do not take an umbrella, then it does not rain.

   (C) if you take an umbrella, then it rains.

   (D) none of the above.

2. Consider the statements- **P** : G is a cyclic group, **Q** : $|G|$ is prime. Which one of the following is TRUE?

   (A) **P** implies **Q** and **Q** does not imply **P**.

   (B) **Q** implies **P** and **P** does not imply **Q**.

   (C) **P** and **Q** are equivalent.

   (D) None of the above.

3. Let A be a finite set and $f : \mathcal{P}(A) \longrightarrow \mathcal{P}(A)$ be a set map. Then $f(B) = B$ for some $B \subseteq A$, if

   (A) $|A|$ is even.

   (B) $|A|$ is odd.

   (C) f is surjective and for any $C, D \in \mathcal{P}(A)$, $C \subseteq D$ implies $f(C) \subseteq f(D)$.

   (D) f is injective.

4. Let X, Y and Z be three sets and $f : X \longrightarrow Y$ and $g : Y \longrightarrow Z$ be set maps. Let $h = g \circ f : X \longrightarrow Z$. Suppose g is surjective. Then which one of the following is FALSE?

   (A) If f is not surjective, then h need not be surjective.

   (B) If f is not surjective, then h can still be surjective.

   (C) If f is not injective, then h is not injective.

   (D) If f is not injective, then h can still be injective.

5. The highest power of 3 that divides 1000! is

   (A) 498.

   (B) 499.

   (C) 500.

   (D) 501.

6. The remainder of 2^{2025} when divided by 13 is

   (A) 2.

   (B) 3.

   (C) 4.

   (D) 5.

7. The total number of positive divisors of 7777 is

   (A) 8.

   (B) 7.

   (C) 10.

   (D) 11.

8. Which one of the following is TRUE?

   (A) $1 + \frac{1}{2} + \frac{1}{2^2} + \cdots + \frac{1}{2^{n-1}} < 2 - \frac{1}{n+1}$, for all $n \in \mathbb{N}$.

   (B) $1 + \frac{1}{2} + \frac{1}{2^2} + \cdots + \frac{1}{2^{n+1}} > 2 + \frac{1}{n-1}$, for some $n \in \mathbb{N}$.

   (C) $1 + \frac{1}{2} + \frac{1}{2^2} + \cdots + \frac{1}{2^n} < 2$, for all $n \in \mathbb{N}$.

   (D) None of the above.

9. Let $n \geq 3$ be an integer. Which one of the following is TRUE?

   (A) $\displaystyle\sum_{\substack{0 \leq i \leq n \\ i \text{ is even}}} \binom{n}{i} = \sum_{\substack{0 \leq j \leq n \\ j \text{ is odd}}} (-1)^{\frac{j+1}{2}} \binom{n}{j}$.

   (B) $\displaystyle\sum_{\substack{0 \leq i \leq n \\ i \text{ is even}}} \binom{n}{i} = \sum_{\substack{0 \leq j \leq n \\ j \text{ is odd}}} \binom{n}{j}$.

   (C) $\displaystyle\sum_{\substack{0 \leq i \leq n \\ i \text{ is even}}} (-1)^{\frac{i}{2}} \binom{n}{i} = \sum_{\substack{0 \leq j \leq n \\ j \text{ is odd}}} \binom{n}{j}$.

   (D) None of the above.

10. In a job hiring committee, half the number of experts voted for Mr. A and one fourth voted for Mr. B. Two members voted for both and four of them did not vote for either. How many experts were there in all?

    (A) 6.

    (B) 8.

    (C) 10.

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(D) 12.

11. For an integer $n \geq 1$, the total number of reflexive relations on a set of n elements is

(A) 2^{n^2-n}.

(B) $n^2 - n$.

(C) 2^n.

(D) n.

12. There are five files numbered $1, 2, 3, 4, 5$, which are kept in a pile such that file number 1 is below file number 5, file number 3 is above file number 4, file number 2 is below file number 1, file number 4 is above file number 5. Then which file is in the bottom?

(A) 4.

(B) 3.

(C) 2.

(D) 5.

13. There are 2 identical RED and 4 identical BLUE balls. These balls need to be packed into 8 different boxes, so that each box will contain at most one ball. The number of ways of doing this is

(A) 420.

(B) 840.

(C) 105.

(D) 210.

14. Let $m \geq 2,\ n \geq 2$ be integers. Then $\binom{m+n}{2}$ equals to

(A) $\binom{m}{2}\binom{n}{2} + mn$.

(B) $\binom{m}{2} + \binom{n}{2} - mn$.

(C) $\binom{m}{2} + \binom{n}{2} + mn$.

(D) none of the above.

15. The number of rearrangements of any 6 letters from the collection $\{A, B, B, C, C, C, C\}$ is

(A) 90.

(B) 120.

(C) 105.

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(D) 100.

16. Let X be a subset of $\mathbb{R}$. Which one of the following is FALSE?

    (A) $\mathbb{R} \setminus X$ is bounded implies X is uncountable.

    (B) $\mathbb{R} \setminus X$ is countable implies X is unbounded.

    (C) $\mathbb{R} \setminus X$ is uncountable need not imply X is countable.

    (D) $\mathbb{R} \setminus X$ is unbounded implies X is bounded.

17. The function $f(x) = \sin(1/x)$ for $x \in (0, 1]$, is an example of

    (A) an unbounded function.

    (B) a non-differentiable function.

    (C) a function with simple discontinuity.

    (D) a function which does not have limit at 0.

18. Let C be the circle $x^2 + y^2 - 2x - 4y = 20$. Let T_1 and T_2 be the tangent lines of C at $(1, 7)$ and $(4, -2)$ respectively. Then the intersection point of T_1 and T_2 is

    (A) (16, 5).

    (B) .(16, 7).

    (C) (16, 9).

    (D) (16, 1).

19. Let C be a circle with center at $(3, -1)$. If the line $2x - 5y + 18 = 0$ makes a chord of length 6 in C, then the equation of C is

    (A) $(x - 3)^2 + (y + 1)^2 - 38 = 0$.

    (B) $(x - 3)^2 + (y + 1)^2 - 39 = 0$.

    (C) $(x - 3)^2 + (y + 1)^2 - 29 = 0$.

    (D) $(x - 3)^2 + (y + 1)^2 - 38^2 = 0$.

20. If the line $x \cos\theta + y \sin\theta - 17 = 0$ touches the circle $x^2 + y^2 = 289$, then

    (A) $\theta \in (\pi/2, 2\pi/3)$.

    (B) $\theta \in [0, \pi)$.

    (C) $\theta \in \mathbb{R}$.

    (D) none of the above.

21. The radius of the circle formed by the intersection of $x^2 + y^2 + z^2 = 1$ and $x + y + z = 1$ is

    (A) $\frac{\sqrt{2}}{\sqrt{3}}$.

    (B) $\frac{2}{3}$.

    (C) $\frac{\sqrt{2}}{9}$.

    (D) 3.

22. Let A and B be two events with probabilities $P(A) = 0.7$, and $P(A \cap B) = 0.5$. Which one of the following is TRUE?

    (A) $P(B)$ could be 0.45.

    (B) $P(B)$ could be 0.84.

    (C) $P(B)$ is 0.9.

    (D) None of the above.

23. In a class of 40 students, the average marks of first five students is 52, of next five students is 82, of next twenty students is 64 and of the rest ten students is 74. The average marks of the entire class is

    (A) 68.25.

    (B) 67.25.

    (C) 68.5.

    (D) 68.

24. From the pack of 52 cards, two cards are drawn at random. The probability that one is an ace and the other is a king is

    (A) $\frac{2}{13}$.

    (B) $\frac{1}{169}$.

    (C) $\frac{16}{169}$.

    (D) $\frac{8}{663}$.

25. Suppose $A = \{x_n \in \mathbb{R} \mid n \in \mathbb{Z}\}$ is an infinite set. If no point of $\mathbb{R}$ is a limit point of A, then

    (A) A is open.

    (B) A is closed.

    (C) A is both open and closed.

    (D) none of the above.

Part B

26. Let V, W be two vector spaces over $\mathbb{R}$ and $T : V \longrightarrow W$ be a non-zero linear transformation. Let A be a non-empty subset of V. Then, which of the following statements are TRUE?

    (A) If $Span(A) = V$ then $Span(T(A)) = W$.

    (B) If $Span(T(A)) = W$ then $Span(A) = V$.

    (C) If A is linearly independent then $T(A)$ is linearly independent.

    (D) If $T(A)$ is linearly independent then A is linearly independent.

27. Let $A \in M_5(\mathbb{R})$ be a non-zero, non-identity and idempotent matrix. Which of the following statements are TRUE?

    (A) A is invertible.

    (B) 1 is an eigenvalue of A.

    (C) 0 is an eigenvalue of A.

    (D) 0 and 1 are the only eigenvalues of A.

28. Let $n \geq 3$, $A \in M_n(\mathbb{R})$ and $W = \{X \in \mathbb{R}^n \mid XA = 0\}$. Which of the following statements are TRUE?

    (A) If $W = \{0\}$, then $XA = (1, 2, 3, 0, \cdots, 0)$ has at most two solutions.

    (B) A is invertible if and only if $W = \{0\}$.

    (C) $\dim(W) = rank(A)$.

    (D) $\dim(W) = n - rank(A)$.

29. Let $A \in M_n(\mathbb{C})$ such that it has n distinct non-zero eigenvalues. If $AB = BA$, then which of the following statements are TRUE?

    (A) B has n distinct eigenvalues.

    (B) A and B have the same characteristic polynomial.

    (C) B is invertible.

    (D) None of the above.

30. Suppose $T : V \longrightarrow V$ is a linear transformation such that $T^2 = T$. Which of the following statements are TRUE?

    (A) $\ker(T) \subseteq \text{Image}(T)$.

    (B) $\ker(T) \supseteq \text{Image}(T)$.

    (C) $\ker(T) \cap \text{Image}(T) = \{0\}$.

    (D) $\ker(T) \cap \text{Image}(T) \neq \{0\}$.

31. Let $A \in M_4(\mathbb{R})$ be a matrix having eigenvalues $-2,\ -1,\ 1,\ 2$. If $B = A^4 - 5A^2 + 5I$, then which of the following statements are TRUE?

(A) $det(A + B) = 0$.

(B) $det(B) = 1$.

(C) $tr(A - B) = 0$.

(D) $tr(A + B) = 4$.

32. Let a, b be two real numbers. Which of the following are TRUE?

(A) If $a - \epsilon < b$ for every $\epsilon > 0$, then $a \leq b$.

(B) If $a < b + \epsilon$ for every $\epsilon > 0$, then $a < b$.

(C) If $a - \epsilon \leq b$ for some $\epsilon > 0$, then $a \leq b$.

(D) If $a + \epsilon \leq b$ for some $\epsilon > 0$, then $a < b$.

33. Let $(x_n)_{n \in \mathbb{N}}$ be a convergent sequence of real numbers, not converging to 0. Which of the following are TRUE?

(A) $x_n \neq 0$ for every $n \in \mathbb{N}$.

(B) $x_n \neq 0$ for infinitely many $n \in \mathbb{N}$.

(C) $\inf_{n \in \mathbb{N}} |x_n| > 0$.

(D) $\liminf_{n \to \infty} |x_n| > 0$.

34. Let $(x_n)_{n \in \mathbb{N}}$ be a sequence of real numbers converging to 0. For $n \geq 1$, define $y_n := \max\{x_1, \ldots, x_n\}$ and $z_n := \min\{x_1, \ldots, x_n\}$. Then which of the following are TRUE?

(A) $(y_n)_{n \in \mathbb{N}}$ converges to 0.

(B) $(z_n)_{n \in \mathbb{N}}$ converges to 0.

(C) $(x_n - z_n)_{n \in \mathbb{N}}$ is a convergent sequence.

(D) $(y_n + z_n)_{n \in \mathbb{N}}$ is a convergent sequence.

35. Let $(x_n)_{n \in \mathbb{N}}$ be a convergent sequence of real numbers. Denote $X = \{x_n : n \in \mathbb{N}\}$ and let $f : X \longrightarrow X$ be a set map. Then $(f(x_n))_{n \in \mathbb{N}}$ converges

(A) if f is injective.

(B) if f is surjectice.

(C) if f is bijective.

(D) only if f is a constant map.

36. Which of the following series are convergent?

(A) $\sum_{n=1}^{\infty} \sqrt{n^{3/2}-1} - \sqrt{n^{3/2}}$.

(B) $\sum_{n=1}^{\infty} \frac{e^n}{1+ne^n}$.

(C) $\sum_{n=1}^{\infty} \frac{\left(\cos(\frac{n\pi}{8})\right)^n}{n}$.

(D) None of the above.

37. Let $f : [-1,1] \longrightarrow \mathbb{R}$ be a function. Then f is differentiable at 0 if

(A) $\lim_{h\to 0} \frac{f(h)-f(-h)}{2h}$ exists in $\mathbb{R}$.

(B) $\lim_{h\to 0} \frac{f(3h^2)-f(0)}{h^2}$ exists in $\mathbb{R}$.

(C) $\lim_{h\to 0} \frac{f(5h^2)-f(0)}{h}$ exists in $\mathbb{R}$.

(D) $\lim_{h\to 0} \frac{f(7h^3)-f(0)}{h^3}$ exists in $\mathbb{R}$.

38. Which of the following functions are NOT integrable?

(A) $f(x) = \begin{cases} 1 & \text{if } x \in D, \\ 2 & \text{if } x \notin D, \end{cases}$

where D is a dense subset of $[-1,1]$ such that $D \neq \mathbb{Q}\cap[-1,1]$ and $D \neq \mathbb{Q}^c\cap[-1,1]$.

(B) $f(x) = \begin{cases} 1 & \text{if } x \in S, \\ 2 & \text{if } x \notin S, \end{cases}$

where S is the set of all roots of the polynomial $x^{14} + 2x^{13} + 2x^{12} + \cdots + 15$ in $[-1,1]$.

(C) $f(x) = \lim_{n\to\infty} \frac{\cos(n\pi x)}{n}$ for $x \in \mathbb{R}$.

(D) $f(x) = e^{-x^2}$ for $x \in \mathbb{R}$.

39. Let G be a finite group and $a, b \in G$. Suppose the order of a is m and of b is n, with $m \neq n$. Which of the following statemetns are FALSE?

(A) If G is abelian, then order of ab is mn.

(B) If $\gcd(m,n) = 1$ and $ab = ba$, then order of ab is mn.

(C) If $\gcd(m,n) = 1$, then order of ab is mn.

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(D) All of the above are false.

40. Let G be a group having no non-trivial proper subgroup. Then

(A) G is finite.

(B) G is cyclic.

(C) G has prime order.

(D) G is abelian.

41. Consider the system of congruence relations $x \equiv 1 \pmod 3$, $x \equiv 2 \pmod 4$, $x \equiv 3 \pmod 5$. The system has

(A) no simultaneous integer solution.

(B) infinitely many simultaneous integer solutions.

(C) only one simultaneous integer solution modulo 60.

(D) more than one simultaneous integer solutions modulo 60.

42. Let G be a finite group and H be a subgroup of index 5. Which of the following statements are TRUE?

(A) For all $x \in G$, $x^5 = 1$.

(B) For all $x \in H$, $x^5 = 1$.

(C) For all $x \in G$, $x^5 \in H$.

(D) None of the above.

43. The ring $\mathbb{Z}_{100}$ has

(A) unique maximal ideal.

(B) exactly three maximal ideals.

(C) exactly two maximal ideals.

(D) exactly four maximal ideals.

44. Let p be a prime, n be a positive integer and C_{p^n} be the set of all p^n-th roots of unity. Then $G = \cup_{n=1}^{\infty} C_{p^n}$ is

(A) an infinite abelian group but not cyclic.

(B) an infinite cyclic group.

(C) a finite cyclic group.

(D) none of the above.

45. Suppose R is a commutative ring with unity. If the sum of any two non-units in R is again a non-unit in R, then characteristic of R is

(A) zero.

(B) either zero or a prime.

(C) either zero or a power of a prime.

(D) none of the above.

46. Let $f : \mathbb{Z}_{12} \longrightarrow \mathbb{Z}_{20},\ g : \mathbb{Z}_{20} \longrightarrow \mathbb{Z}_{12}$ be defined as $f(x) = 10x,\ g(x) = 9x$. Then

(A) $f,\ g$ both are ring homomorphisms.

(B) $f,\ g$ both are group homomorphisms.

(C) f is a ring homomorphism but g is not a ring homomorphism.

(D) g is a ring homomorphism but f is not a ring homomorphism.

47. In a class of 50 students, 20 students appeared for Algebra test, 10 students appeared for Analysis test, 10 students appeared for Topology test. And 4 students have appeared for all the three tests. Then, the number of students who did not appear for any of the three tests is

(A) at least 18.

(B) at most 17.

(C) equal to 17.

(D) none of the above.

48. The equation $y' = \dfrac{2y}{x}$, $x > 0$, $y(0) = 0$ has

(A) exactly one solution.

(B) has no solution.

(C) infinitely many solutions.

(D) none of the above.

49. The general solution of $x'' - 2x' + x = e^t$ is

(A) $x = (c_1 + c_2t)e^t + \dfrac{1}{4}e^{-t}$.

(B) $x = (c_1 + c_2t)e^{-t} + \dfrac{1}{4}e^{-t}$.

(C) $x = (c_1 + c_2t)e^{-t} - \dfrac{1}{4}e^{-t}$.

(D) $x = (c_1 + c_2t)e^t + \dfrac{1}{4}e^t$.

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50. Solutions of $y' + y = xy^4$ are

(A) $y^{-3} = x + \frac{1}{3} + ce^{3x}$.

(B) $y^3 = x + \frac{1}{3} + ce^{3x}$.

(C) $y^{-3} = x + \frac{1}{3} + ce^{-3x}$.

(D) $y^3 = x + \frac{1}{3} + ce^{-3x}$.

Revised Key

University of Hyderabad
Entrance Examinations - 2019

School/Department/Centre : School of Mathematics and Statistics

Course/Subject : M.Sc. Mathematics/Applied Mathematics

Q.No.	Answer	Q.No.	Answer
1	B	26	D
2	B	27	B C D
3	C	28	A B D
4	D	29	D
5	A	30	C
6	D	31	A B D
7	A	32	A D
8	C	33	B D
9	B	34	C D
10	B	35	A C
11	A	36	D
12	C	37	D
13	A	38	A
14	C	39	A C D
15	C	40	A B C D
16	D	41	B C
17	D	42	D
18	B	43	C
19	A	44	A
20	C	45	C
21	A	46	B D
22	A	47	A
23	B	48	C
24	D	49	XXXX
25	B	50	A

Note/Remarks : 1. Questions numbered 26 to 50 have multiple correct options as indicated above. 2. As there is a typo, Q. No. 49 Benefit will be given to all.

06/06/2019

M. S. 06/06/2019

Signature
School/Department/Centre

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