

S-56

ENTRANCE EXAMINATION, 2018

QUESTION PAPER BOOKLET

Ph.D. (PHYSICS)

Marks: 50

Time: 2.00 hrs.

Hall Ticket No.:

I. Please enter your Hall Ticket Number on Page 1 of this question paper and on the OMR sheet without fail.

II. Read carefully the following instructions:

1. This Question paper has two parts: **PART - A** and **PART - B**.
2. **PART-A** consists of 20 objective type questions related to Research Methods.
3. **PART-B** consists of 20 objective type questions related to Physics.
4. All questions carry 2 marks each.
There is no negative marking
5. Answers are to be marked on the OMR answer sheet following the instructions provided there upon. An example is shown below

100.



6. Only Scientific Calculators are permitted. Mobile phone based calculators are not permitted. Logarithmic tables are not allowed.
7. Hand over the OMR answer sheet at the end of the examination to the Invigilator.

This book contains 16 pages

III. Values of physical constants:

$$c = 3 \times 10^8 \text{ m/s}; h = 6.63 \times 10^{-34} \text{ J.s}; k_B = 1.38 \times 10^{-23} \text{ J/K}$$

$$e = 1.6 \times 10^{-19} \text{ C}; \mu_0 = 4\pi \times 10^{-7} \text{ Henry/m}; \epsilon_0 = 8.85 \times 10^{-12} \text{ Farad/m,}$$

$$\mu_B = 9.274 \times 10^{-24} \text{ J/T}$$

S-56

This page is intentionally left blank.
Blank sheets are provided for rough work at the end of this booklet.

PART - A

1. A particle of mass M is moving along a wire, kept vertical to the ground. The ground is assumed to be rigid (i.e., $V(z) = \infty$ for $z < 0$). The minimum energy of this particle, calculated using uncertainty principle, is

- A. $\frac{3}{2}(\frac{1}{4})^{(1/3)} M^{(1/3)} (g\hbar)^{(2/3)}$
 B. $\frac{3}{2}(\frac{1}{4})^{(1/3)} M^{(2/3)} (g\hbar)^{(1/3)}$
 C. $\frac{3}{2}(\frac{1}{4})^{(1/3)} M^{(1/3)} (g\hbar)^{(1/3)}$
 D. $\frac{3}{2}(\frac{1}{4})^{(1/3)} M^{(1/2)} (g\hbar)^{(2/3)}$

2. For an inductor and capacitor connected in series, the equation describing the variation of charge is

$$L \frac{d^2Q}{dt^2} + \frac{Q}{C} = 0,$$

where L is the inductance, C is the capacitance and Q is the charge. An analogous equation can be written for a simple harmonic oscillator with position x , mass m and spring constant k . The mechanical analogs of L , C and Q respectively are

$L \quad C \quad Q$

- A. $m \quad k \quad x$
 B. $m \quad \frac{1}{k} \quad x$
 C. $k \quad x \quad m$
 D. $\frac{1}{k} \quad \frac{1}{m} \quad x$

3. Given

$$\frac{2 + 5 + 8 + \dots + n \text{ terms}}{7 + 11 + 15 + \dots + n \text{ terms}} = \frac{23}{35}$$

the value of n is

- A. 16
 B. 17
 C. 14
 D. 15

4. The next word in the following group of words is
 [Civic, Kayak, Level,]

- A. Valley
 B. Plump
 C. Racecar
 D. Ambigram

5. If $f(x) = \frac{\cos^2 x + \sin^4 x}{\sin^2 x + \cos^4 x}$, for $x \in R$, then $f(2018) =$
- 1
 - 2
 - 3
 - 4
6. Following forces act simultaneously on a particle at rest, which is placed at the origin of the coordinate system. $\vec{F}_1 = -4\hat{i} - 5\hat{j} + 5\hat{k}$, $\vec{F}_2 = 5\hat{i} + 8\hat{j} + 6\hat{k}$, $\vec{F}_3 = -3\hat{i} + 4\hat{j} - 7\hat{k}$ and $\vec{F}_4 = 2\hat{i} - 3\hat{j} - 2\hat{k}$. Then the particle will move
- in $x - y$ plane
 - in $x - z$ plane
 - in $y - z$ plane
 - along x -axis
7. A ball of mass m hangs from a support by a string of length l . The probability that the ball is displaced by a small height d , through collisions by the surrounding molecules at a temperature T , is
- $\exp(-mgd/kT)$
 - $\exp(-mgd^2/lkT)$
 - (d/l)
 - $\left[\exp\left(-\frac{mgd}{kT}\right) - 1 \right]^{-1}$
8. Under space-inversion and time-reversal transformations, the Poynting vector transforms as
- odd, even
 - even, odd
 - even, even
 - odd, odd
9. If $A = (100 \pm 3)$ and $B = (6 \pm 4)$ what is the error in $A + B$ and $A - B$?
- 7 and -7
 - 7 and -1
 - 5 and 5
 - 5 and -5
10. The actual height of a flagpole is 3m. At 8:00 am it's shadow is about 4m long, at 10:00 am it's shadow is about 2m and at 1:00 pm it's shadow is about 0.7m long. At 5:00 pm the length of it's shadow will be
- longer than 4m
 - longer than 2m but shorter than 4m
 - longer than 0.7m but shorter than 2m
 - about 2m

11. When drugs are given orally to a patient, the drug concentration 'c' in the blood stream is given by the formula,

$$c = k t e^{-at}$$

where 'k' is dependent on parameters such as the dose administered. 'a' depends on the absorption and elimination rates of the drug. If $k = 2$ and $a = 0.25$, where t is in seconds and c is in mg/ml, the time at which the maximum concentration is reached is given by the solution of the equation

- A. $2t e^{-0.25t} = 0$
 B. $2e^{-0.25t} - 2te^{-0.25t} = 0$
 C. $2e^{-0.25t} - 0.5te^{-0.25t} = 0$
 D. $2te^{-0.25t} = 2$
12. For fitting a curve to the given data points, using the least squares method
- A. all the data points have to pass through the curve
 B. at least two data points have to pass through the curve
 C. at least one data point has to pass through the curve
 D. the data points need not pass through the curve
13. Lloyd, Martin, Neil, Olsen, Peter, Quinn and Robin are sitting around a circle facing the centre. Quinn is second to the right of Robin and Peter is second to the right of Quinn. Olsen is sitting between Lloyd and Robin and Neil is not an immediate neighbour of Robin. Three of the following four are alike in a certain way, based on their seating positions, and so form a group. Which one does not belong to the group?
- A. Lloyd, Peter
 B. Robin, Martin
 C. Neil, Peter
 D. Olsen, Robin
14. The states $|1\rangle$, $|2\rangle$ and $|3\rangle$ are orthonormal. For what values of x , are the states given below mutually orthogonal?

$$|\psi_1\rangle = 5|1\rangle - 3|2\rangle + 2|3\rangle \quad \text{and} \quad |\psi_2\rangle = |1\rangle - 5|2\rangle + x|3\rangle$$

- A. 10
 B. 5
 C. -5
 D. -10
15. The 2^3S_1 state of Helium is said to be metastable because
- A. it cannot decay to the ground state through a dipole transition
 B. the triplet state is always a long lived state
 C. it is isolated from the other singlet states
 D. the $J = 1 \rightarrow 0$ transition is not dipole allowed

16. Given $f(n+1) = 2f(n) - 1$, for $n \geq 1$ and $f(1) = 3$, $f(n)$ is
- A. 2^n
 - B. $2^n + 1$
 - C. $2^{n+1} - 1$
 - D. $2^{n+1} - 3$
17. A nuclear bomb explodes 300 km above the surface of the moon. The sound of explosion on the moon will
- A. be heard before the flash of explosion is seen
 - B. be heard after the flash of explosion is seen
 - C. be heard at the same time as the explosion occurs
 - D. not be heard at all
18. Given $\Phi = X^2 - YZ$,
 $\vec{\nabla}\Phi$ at point (3,4,1) has the value
- A. $6\hat{i} - 4\hat{j} - \hat{k}$
 - B. $6\hat{i} - \hat{j} - 4\hat{k}$
 - C. $4\hat{i} - 6\hat{j} - \hat{k}$
 - D. $6\hat{i} - 3\hat{j} - 2\hat{k}$
19. If $\vec{A} = 2\hat{i} - 3\hat{j} + \hat{k}$ and $\vec{A} \cdot \vec{B} = 0 = \vec{A} \times \vec{B}$, then \vec{B} is
- A. perpendicular to \vec{A}
 - B. parallel to \vec{A}
 - C. $4\hat{i} + 3\hat{j} + \hat{k}$
 - D. 0
20. Pointing to a man's picture, a woman said "His brother's father is the only son of my grandfather". How is the woman related to the man in the picture?
- A. Daughter
 - B. Sister
 - C. Wife
 - D. Aunt

PART - B

21. The wave function describing a particle of mass m , confined to move in one dimension between $x = 0$ and $x = a$, is given by

$$\psi(x) = \alpha_1 \sin \frac{\pi x}{a} + \alpha_2 \sin \frac{3\pi x}{2a} + \beta_1 \cos \frac{3\pi x}{a} + \beta_2 \cos \frac{5\pi x}{2a}.$$

Which of the following statements is correct?

- A. $\alpha_1 + \alpha_2 = 0$
 B. $\alpha_1 + \beta_1 = 0$
 C. $\alpha_1 + \beta_2 = 0$
 D. $\alpha_2 + \beta_1 = 0$
22. If the potentials corresponding to a given charge and current distributions are given by

$$\phi(\vec{r}, t) = 0 \quad \vec{A}(\vec{r}, t) = -\frac{1}{4\pi\epsilon_0} \frac{qt}{r^2} \hat{r},$$

the electric and magnetic fields are

- A. $\vec{E} = 0, \vec{B} = 0$
 B. $\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \hat{r}, \vec{B} = 0$
 C. $\vec{E} = 0, \vec{B} = \frac{1}{4\pi\epsilon_0} \frac{qt}{r^3} \hat{r}$
 D. $\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \hat{r}, \vec{B} = \frac{1}{4\pi\epsilon_0} \frac{qt}{r^3} \hat{r}$
23. A rectangular wave guide, of infinite extent along z -axis, is placed with one corner at $x = 0$ and $y = 0$, and the diagonally opposite corner at $x = a$ and $y = a$. If only TM modes are excited in this wave guide, the cutoff frequency of the lowest non-vanishing mode is

- A. $\frac{C\pi}{a}$
 B. $\frac{C\pi}{2a}$
 C. $\sqrt{2} \left(\frac{C\pi}{a} \right)$
 D. $2 \left(\frac{C\pi}{a} \right)$

24. A system consists of N non-interacting particles with possible energy states 0 and ϵ . The total energy of the system is given by

- A. $\frac{N}{1 + e^{\beta\epsilon}}$
 B. $\frac{N\epsilon}{1 + e^{\beta\epsilon}}$
 C. $\frac{\epsilon}{1 + e^{\beta\epsilon}}$
 D. $\frac{N\epsilon}{(1 + e^{\beta\epsilon})^2}$

25. The residue of the function $\frac{z}{1 - \cos z}$ is

- A. $\text{res}|_{z=2\pi k} = \begin{cases} 2 & \text{if } k = 0 \\ 0 & \text{if } k \neq 0 \end{cases}$
 B. $\text{res}|_{z=2\pi k} = \begin{cases} 1/2 & \text{if } k = 0 \\ 0 & \text{if } k \neq 0 \end{cases}$
 C. $\text{res}|_{z=0} = 0$
 D. $\text{res}|_{z=0} = 2$

26. The inverse Laplace transform of the function

$$\tilde{f}(s) = \frac{s+3}{s(s+1)} \quad \text{for } s > 1 \quad \text{is}$$

- A. $3t - 2e^{-2t}$
 B. $3 - 2e^{-t}$
 C. $3 - 2e^{-2t}$
 D. $3t + 2e^{-t}$

27. The expectation value of the energy of a harmonic oscillator, calculated using a trial wave function

$$\psi_\lambda(x) = \frac{1}{x^2 + \lambda}, \quad \lambda > 0, \quad \text{is } \frac{\hbar^2}{4m\lambda} + \frac{m\omega^2\lambda}{2}.$$

The ground state energy of the harmonic oscillator, calculated using the variational method, is

- A. $\hbar\omega$
 B. $\frac{\hbar}{2}\omega$
 C. $\sqrt{2}\hbar\omega$
 D. $\frac{1}{\sqrt{2}}\hbar\omega$

28. The partition function for a free gas of N indistinguishable classical particles, each of mass m , in a very large container of volume V at a temperature T is given by

$$Z_N = \frac{1}{N!} \left(\frac{V}{\lambda_T^3} \right)^N, \quad \text{where } \lambda_T = (2\pi\hbar^2/mk_B T)^{1/2} \quad \text{is the thermal deBroglie wavelength.}$$

The partition function in the thermodynamic limit, with the number density n , can be written as

- A. $Z_N = \left(\frac{e}{n\lambda_T^3} \right)^N$
 B. $Z_N = \left(\frac{1}{n\lambda_T^3} \right)^N$
 C. $Z_N = N \left(\frac{e}{n\lambda_T^3} \right)$
 D. $Z_N = N \left(\frac{e}{n\lambda_T^3} \right)$

29. The system described by the Lagrangian

$$\mathcal{L} = \frac{1}{2}m\dot{r}^2 - \alpha e^{-\mu r^2}/r^2, \quad \text{where } \alpha \text{ and } \mu \text{ are constants,}$$

- A. is invariant under rotations in 3-dimensions only
- B. is invariant under translation in time only
- C. is invariant under rotations in 3-dimensions and translations in time
- D. is invariant under rotations in 3-dimensions, translations in space and translations in time

30. The Hamiltonian corresponding to the Lagrangian

$$L = \frac{m}{2} \frac{d\vec{r}}{dt} \cdot \frac{d\vec{r}}{dt} + m \frac{d\vec{r}}{dt} \cdot (\vec{\omega} \times \vec{r}) + \frac{m}{2} (\vec{\omega} \times \vec{r})^2, \quad \text{is}$$

- A. $\frac{\vec{p} \cdot \vec{p}}{2m} + \vec{p} \cdot (\vec{\omega} \times \vec{r})$
- B. $\frac{\vec{p} \cdot \vec{p}}{2m} + \vec{p} \cdot (\vec{r} \times \vec{\omega})$
- C. $\frac{\vec{p} \cdot \vec{p}}{2m} + \vec{p} \cdot (\vec{\omega} \times \vec{r}) - \frac{m}{2} (\vec{\omega} \times \vec{r})^2$
- D. $\frac{\vec{p} \cdot \vec{p}}{2m} + \vec{p} \cdot (\vec{\omega} \times \vec{r}) + \frac{m}{2} (\vec{\omega} \times \vec{r})^2$

31. The entropy of rolling a dice, assuming it to be a microcanonical ensemble, is

- A. $S = \frac{1}{6} k_B \ln 6$
- B. $S = 6 k_B \ln 6$
- C. $S = k_B \ln 6$
- D. $S = 6 k_B \ln \frac{1}{6}$

- 32. If the electric field propagating in a conducting medium along the z -direction is $\vec{E}_0 e^{K_{Im}x} e^{i(K_{Re}x - \omega t)}$ (where K_{Re} and K_{Im} are the real and imaginary parts of the wave vector), the corresponding magnetic field is

- A. $\vec{B} = -\frac{\vec{E}_0}{\omega} (K_{Re} - iK_{Im}) e^{K_{Im}x} e^{i(K_{Re}x - \omega t)} \hat{y}$
- B. $\vec{B} = \frac{\vec{E}_0}{\omega} (K_{Re} - iK_{Im}) e^{K_{Im}x} e^{i(K_{Re}x - \omega t)} \hat{y}$
- C. $\vec{B} = \frac{\vec{E}_0}{\omega} (K_{Re} + iK_{Im}) e^{K_{Im}x} e^{i(K_{Re}x - \omega t)} \hat{y}$
- D. $\vec{B} = -\vec{E}_0 \omega (K_{Re} - iK_{Im}) e^{K_{Im}x} e^{i(K_{Re}x - \omega t)} \hat{y}$

33. An incident pion of energy 10 GeV collides with a proton of rest mass 938 MeV. The energy in the center of mass frame after the collision (for very high energies) is

- A. 4.33 GeV
- B. 3.06 GeV
- C. 2.17 GeV
- D. 1.53 GeV

34. The particle w mediating the weak interaction has a mass $M = 80 \text{ GeV}/c^2$. In the framework of a Yukawa-like model, the range of the weak interaction is

- A. $24.7 \times 10^{-18} \text{ m}$
- B. $2.47 \times 10^{-18} \text{ m}$
- C. $24.7 \times 10^{-18} \text{ cm}$
- D. $2.47 \times 10^{-18} \text{ cm}$

35. An atomic spectral line corresponding to the transition $^1P_1 \rightarrow ^1S_0$ at 1850 \AA is split in a magnetic field of 1 T . The number of lines observed perpendicular to the field and the shift in frequency, respectively, are

- A. 3, 14MHz
- B. 2, 14MHz
- C. 2, 14GHz
- D. 3, 14GHz

36. The displacement of a damped harmonic oscillator as a function of time is given by

$$f(t) = \begin{cases} 0 & \text{for } t < 0 \\ e^{-t/\tau} \sin \omega_0 t & \text{for } t \geq 0 \end{cases}$$

The Fourier transform of $f(t)$ is

- A. $\frac{1}{2} \left[\frac{1}{(\omega + \omega_0 - i/\tau)} - \frac{1}{(\omega - \omega_0 - i/\tau)} \right]$
- B. $\left[\frac{1}{(\omega_0 - i/\tau)} - \frac{1}{(\omega_0 + i/\tau)} \right]$
- C. $\cos \omega_0 \tau$
- D. $\frac{1}{2} \left[\frac{1}{(\omega - \omega_0 + i/\tau)} - \frac{1}{(\omega + \omega_0 - i/\tau)} \right]$

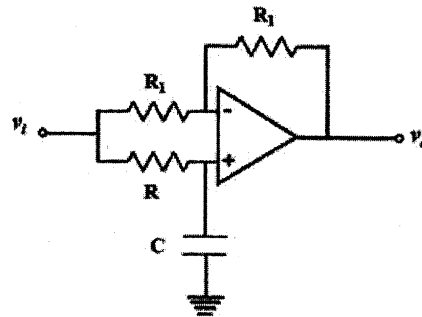
37. A laser beam of wavelength $\lambda = 1.15 \text{ microns}$ is used in a Michelson interferometer to obtain interference fringes. The fringes remained visible for a path length of 8 m . The lower limit on coherence time and spectral width, respectively, are

- A. $3.75 \times 10^7 \text{ s}$, $0.8 \times 10^{-13} \text{ m}$
- B. $3.75 \times 10^7 \text{ s}$, $1.6 \times 10^{-13} \text{ m}$
- C. $2.7 \times 10^{-8} \text{ s}$, $0.8 \times 10^{-13} \text{ m}$
- D. $2.7 \times 10^{-8} \text{ s}$, $1.6 \times 10^{-13} \text{ m}$

38. A crystal plane intercepts the crystal axes at $0.5a$, b and is parallel to the c -axis. The Miller indices of the plane are

- A. (120)
- B. (210)
- C. (012)
- D. (102)

39. The spectrum of the H -atom, derived using Dirac's theory, predicts the same energy for
- A. $n = 1, l = 0, j = 1/2$ and $n = 1, l = 0, j = -1/2$
 - B. $n = 3, l = 0, j = 1/2$ and $n = 3, l = 1, j = 1/2$
 - C. $n = 3, l = 1, j = 3/2$ and $n = 3, l = 1, j = 1/2$
 - D. $n = 3, l = 2, j = 3/2$ and $n = 2, l = 1, j = 3/2$
40. The voltage gain (A_v), for the op-amp circuit shown below, is



- A. $A_v = 1 + j\omega CR$
- B. $A_v = \frac{1}{1 + j\omega CR}$
- C. $A_v = \frac{1 + j\omega CR}{1 - j\omega CR}$
- D. $A_v = \frac{1 - j\omega CR}{1 + j\omega CR}$