Entrance Examination: M.Sc. Mathematics, 2018

Hall Ticket Number [| | | | | | |]

Time: 2 hours
Max. Marks: 100

Part A:25 Marks
Part B:75 Marks

Instructions

1. Write your Hall Ticket Number in the OMR Answer Sheet given to you. Also write the Hall Ticket Number in the space provided above.
2. Answers are to be marked on the OMR sheet.
3. Please read the instructions carefully before marking your answers on the OMR answer sheet.
4. Hand over the OMR answer sheet at the end of the examination.
5. The question paper can be taken by the candidate at the end of the examination.
6. No additional sheets will be provided. Rough work can be done in the question paper itself/space provided at the end of the booklet.
7. Calculators are not allowed.
8. There are a total of 50 questions in **PART A** and **PART B** together.
9. **There is a negative marking in PART A. Each correct answer carries 1 mark and each wrong answer carries -0.333 mark. Each question in PART A has only one correct option.**
10. **There is no negative marking in PART B. Each correct answer carries 3 marks. In PART B some questions have more than one correct option. All the correct options have to be marked in OMR sheet, otherwise zero marks will be credited.**
11. The appropriate answer(s) should be coloured with either a blue or a black ball point or a sketch pen. DO NOT USE A PENCIL.
12. $\mathbb{R}$ denotes the set of real numbers, $\mathbb{C}$ the set of complex numbers, $\mathbb{Z}$ the set of integers, $\mathbb{Q}$ the set of rational numbers and $\mathbb{N}$ the set of all natural numbers.
13. This book contains 11 pages including this page and excluding pages for the rough work. Please check that your paper has all the pages.

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Part-A

1. Let $X \subset \mathbb{N}$ be a nonempty finite set and $Y \subset \mathbb{N}$ be an infinite set. Define $A = \{x - y : x \in X \text{ and } y \in Y\}$. Then

   (A) $\inf(A) = -\infty$ and $\sup(A) = \infty$.

   (B) $\inf(A) = -\infty$ and $\sup(A) < \infty$.

   (C) $\inf(A) > -\infty$ and $\sup(A) = \infty$.

   (D) $\inf(A) > -\infty$ and $\sup(A) < \infty$.

2. Let X be a nonempty set and $A, B \subset X$. If $(A \cup B) \setminus (A \cap B)$ is a finite set, then

   (A) Both A and B are finite sets.

   (B) Atleast one of A, B is a finite set.

   (C) X is a finite set.

   (D) None of the above.

3. Let $f, g : \mathbb{R} \to \mathbb{R}$ be a polynomials with deg $(f) = 5$ and deg $(g) = 7$. Let $A = \{x \in \mathbb{R} : f(x) = g(x)\}$. Then

   (A) $|A| \leq 5$.

   (B) $|A| \leq 7$.

   (C) A is finite, but $|A|$ can be arbitrarily large.

   (D) A can be finite.

4. Let X be a set with $2m + 1$ elements, where $m \geq 4$. The number of subsets of X having at least $m + 1$ elements is

   (A) 2^{2m+1}.

   (B) 2^{2m-1}.

   (C) 2^{2m}.

   (D) $2^m + 1$.

5. A box contain RED, BLUE and GREEN coloured balls. If all but 18 of the balls are BLUE, all but 12 of the balls are GREEN and all but 24 balls are RED, then which of the following is true.

   (A) Box contains more RED balls than BLUE or GREEN ones.

   (B) Box contains more BLUE balls than RED or GREEN ones.

   (C) Box contains more GREEN balls than RED or BLUE ones.

   (D) none of the above.

6. Vinod and Binod are two students among a group of n students. The number of ways to allocate n different rooms in a line to this group of students so that Vinod and Binod are not in adjacent rooms is

   (A) $(n-2)(n-1)!$.

   (B) $2(n-1)!$.

   (C) $(n-1)!$.

   (D) $2(n-2)!$.

Q-6

7. Let G be a group and $a, b \in G$ such that $o(a) = 6$, $o(b) = 2$ and $a^3b = ba$. Then $o(ab)$ is

   (A) 6.

   (B) 8.

   (C) 12.

   (D) 2.

8. Suppose G is a group and $x \in G$ such that order of x is $\geq |G|/2$. Then

   (A) G is cyclic group.

   (B) If G is abelian, then G is cyclic.

   (C) If G is finite, then G is cyclic.

   (D) none of the above.

9. Let $A\Delta B$ denote the symmetric difference of two sets: $A\Delta B := (A \setminus B) \cup (B \setminus A)$. Which of the follwing is true?

   (A) $A \cap (B\Delta A) = \emptyset$ always holds true.

   (B) $(A \cup (A\Delta B)) \cap B \neq B$ always holds true.

   (C) $A\Delta B = (A \cup B) \cap (A^c \cap B^c)$ always holds true.

   (D) None of the above is true.

10. Let A be a 5x5 real matrix. Suppose 0 is one of eigenvalues of A. Which of the following statement is true?

    (A) System $AX = 0$ has unique solution.

    (B) System $AX = C$ ha unique solution for any C.

    (C) $AX = 0$ has a non-trivial solution.

    (D) none of the above.

11. The order of (123)(145) in the permutation group S_5 is

    (A) 6.

    (B) 3.

    (C) 5.

    (D) 9.

12. The number of subgroups of order 2 in the permutation group S_3 is

    (A) 1.

    (B) 3.

    (C) 2.

    (D) 12.

Q-6

13. Which one the following subsets is a subspace of the vector space $\mathbb{R}^3$ over the field $\mathbb{R}$?

(A) $\{(u, v, w) \in \mathbb{R}^3 \mid 2u + 3v + 4w = 0\}$.
(B) $\{(u, v, w) \in \mathbb{R}^3 \mid 2u + 3v + 4w = 1\}$.
(C) $\{(u, v, w) \in \mathbb{R}^3 \mid u > 0, v > 0, w < 0\}$.
(D) $\{(u, v, w) \in \mathbb{R}^3 \mid u, v, w \text{ are rational}\}$.

14. The dimension of the vector space $\mathbb{Q}[\sqrt{2}]$ over the field $\mathbb{Q}$ is

(A) 1.
(B) ∞.
(C) 4.
(D) 2.

15. Let $V = \{(x_1, x_2, x_3, x_4) \in \mathbb{R}^4 \mid 2x_1 + \pi x_2 = 0, x_2 - x_3 - x_4 = 0\}$. A basis for V

(A) $\{(1, 0, 0, 0), (0, 1, 0, 0), (0, 0, 1, 0), (0, 0, 0, 1)\}$.
(B) $\{(1, -\frac{2}{\pi}, 0, -\frac{2}{\pi}), (0, 0, 1, -1)\}$.
(C) $\{(\pi, -2, 0, -2), (2\pi, -4, 0, -4)\}$.
(D) $\{(0, 0, 1, -1), (\frac{1}{2}, -\frac{1}{\pi}, 0, 0)\}$.

16. In a survey, it is found that 80 students know English, 60 know French, 50 know German, 30 know English and French, 20 know French and German, 15 know English and German and 10 students know all the three languages. How many students know English only?

(A) 45.
(B) 55.
(C) 20.
(D) 80.

17. If one of the roots of the quadratic equation $10x^2 - 29x + (k + 4) = 0$ is the reciprocal of the other root, then the value of k is

(A) 4
(B) 29
(C) 6
(D) -3

18 The number of subsets from 100 distinct objects containing an odd number of objects is

(A) $2^{100} - 1$.
(B) 2^{99}.
(C) 2^{50}.
(D) $2^{50} - 1$.

19. The proportions of those arrangements of the numerals $1, 2, \ldots, 9$ in which all the multiples of 3 appears consecutively is

    (A) 1/2.
    (B) 1/6.
    (C) 1/12.
    (D) 1/36.

20. The rate of convergence of Newton-Raphson method is

    (A) 2.
    (B) 3.
    (C) 1.
    (D) 1.618.

21. The Laplace transform of $t^{1/2}$ is

    (A) $\sqrt{\pi}/2s^{3/2}$
    (B) $\sqrt{\pi}/s^{3/2}$
    (C) $\sqrt{\pi}/2s^{1/2}$
    (D) $\sqrt{\pi}/s^{1/2}$

22. Let f and g be differentiable functions such that $f'(x) = 2g(x)$ and $g'(x) = -f(x)$ and let $T(x) = f(x)^2 - g(x)^2$. Then $T'(x)$ is equal to

    (A) $T(x)$.
    (B) 0.
    (C) $2f(x)g(x)$.
    (D) $6f(x)g(x)$

23. If G is an abelian group, then the number of conjugacy classes equal to

    (A) $0(G)$.
    (B) 1.
    (C) $0(G) - 0(Z(G)$.
    (D) 2.

24. Let p be a prime and $G = \{z \in \mathbb{C} \mid z \text{ is an } n^{th}\text{-root of } 1, \text{ for some } n \in \mathbb{N}\}$. then,

    (A) an infinite abelian group but not cyclic.
    (B) an infinite cyclic group.
    (C) it is a finite cyclic group.
    (D) None of these.

25. Let G be a finite group and H is a subgroup of G of index 2. Then

    (A) H is normal and $g^2 \in H$ for any $g \in H$.
    (B) H is normal and $g^2 = e$.
    (C) H is need not be normal.
    (D) None of the aboove.

Q-6

Part-B

26. Let (x_n) be a sequence of positive reais converging to 1. Which of the following sequences converge to 1?

    (A) $(\sqrt{x_n})$.

    (B) (x_n^3).

    (C) $(x_n^{1/n})$

    (D) $(2^{x_n} - 1)$.

27. Let $f : \mathbb{R} \to \mathbb{R}$ be $f(x) = 3x^4 - 2x^5$. Then which of the following are true?

    (A) f is surjective.

    (B) f is injective.

    (C) f is uniformly continuous on $\mathbb{R}$.

    (D) f is Lipschitz continuous on $[-100, 100]$.

28. For $k, n \in \mathbb{N}$, let $T_{k,n}$ be a linear map from $\mathbb{R}^k \to \mathbb{R}^n$. Which of the following are true ?

    (A) $T_{6,7} \circ T_{7,6} : \mathbb{R}^7 \to \mathbb{R}^7$ can never be injective.

    (B) $T_{6,7} \circ T_{7,6} : \mathbb{R}^7 \to \mathbb{R}^7$ can never be surjective.

    (C) $T_{7,6} \circ T_{6,7} : \mathbb{R}^7 \to \mathbb{R}^7$ can never be injective.

    (D) $T_{7,6} \circ T_{6,7} : \mathbb{R}^7 \to \mathbb{R}^7$ can never be surjective.

29. The greatest common divisor (gcd) of $5n + 3$ and $7n + 4$, for all $n \in \mathbb{N}$ is

    (A) 1.

    (B) 5.

    (C) n.

    (D) 2.

30. How many three digit numbers are divisible by 6?

    (A) 142.

    (B) 150.

    (C) 148.

    (D) 166.

31. The number of diagonals in an n-gon is

    (A) $\frac{n(n-1)}{2}$.

    (B) $n(n-3)$.

    (C) $n(n-1)$.

    (D) $\frac{n(n-3)}{2}$.

Q-6

32. Let $i = \sqrt{-1}$ in $\mathbb{C}$. Then i^i is

(A) 1.

(B) 0.

(C) $e^{-\frac{\pi}{2}}$.

(D) $e^{-i\frac{\pi}{2}}$.

33. Let G be an abelian group with the identity e. Which one of the following statements are true.

(A) $H = \{x \in G \,|\, \text{order of } x \text{ is odd}\}$ is a subgroup of G.

(B) $H = \{x \in G \,|\, \text{order of } x \text{ is even}\} \cup \{e\}$ is a subgroup of G.

(C) Every subgroup of G is normal.

(D) G is cyclic.

34. Let p be a prime number. Consider the group

$$\mathrm{SL}(2, \mathbb{Z}_p) = \left\{ \left(\begin{array}{cc} a & b \\ c & d \end{array} \right) \middle| \, a, b, c, d \in \mathbb{Z}_p \text{ and } ad - bc = 1 \right\}$$

under the matrix multiplication. Then the order of $A = \left(\begin{array}{cc} 1 & 1 \\ 0 & 1 \end{array} \right) \in \mathrm{SL}(2, \mathbb{Z}_p)$ is

(A) ∞.

(B) p.

(C) 1.

(D) $p - 1$.

35. In the ring $\mathbb{Z}_8[X]$, the element $4X^2 + 6X + 3$ is

(A) a nilpotent.

(B) a unit.

(C) an idempotent.

(D) a zero-divisor.

36. Suppose that $\phi : \mathbb{Z}_{20} \to \mathbb{Z}_{20}$ is an automorphism such that $\phi(5) = 5$. Then the number of possibilities for $\phi(1)$ is

(A) 4.

(B) 1.

(C) 5.

(D) 20.

37. Let X and Y be subspaces of finite dimensional vector space V. Let $X + Y = \{x + y \,|\, x \in X, y \in Y\}$. The dimension of the subspace $X + Y$ is always equal to

(A) $\dim(X + Y) = \dim(X) + \dim(Y) - \dim(X \cap Y)$.

(B) $\dim(X + Y) = \dim(X) + \dim(Y)$.

(C) $\dim(X + Y) = \max\{\dim(X), \dim(Y)\}$.

(D) $\dim(X + Y) = \dim(X) + \dim(Y) + \dim(X \cap Y)$.

8-6

38. A number is selected from $\{1, 2, \ldots 100\}$ if every outcome is equally likely, the probability that the selected number is a prime number given that it is odd is

(A) 1/4.

(B) 12/25.

(C) 1/3.

(D) 14/25

39. The nullity of $n \times n$ matrix all of whose terms are 1 is

(A) 1.

(B) n.

(C) $n - 1$.

(D) $n - 2$.

40. The proportion of 2×2 nonsingular matrices whose terms are either 0 or 1 is

(A) 1/2.

(B) 1/4.

(C) 1/8.

(D) 1/16.

41. The value of $\lim_{n \to \infty} \frac{1}{n}(1 + \frac{1}{2} + \cdots + \frac{1}{n})$ is

(A) 0.

(B) 1.

(C) ∞.

(D) e.

42. Suppose G is a finite group and H is a subgroup of G. If $[G : H] = 2$, then which of the following statements are true?

(A) If $x \in H$ and $y \notin H$, then $xy \in H$.

(B) If $x \notin H$ and $y \notin H$, then $xy^{-1} \in H$.

(C) If $x \notin H$ and $y \notin H$, then $xy \in H$.

(D) both (b) and (c) are true.

43. Let S_3 be the permutation group on $\{1, 2, 3\}$. Then there exists a non-trivial group homomorphism $f : S_3 \longrightarrow S_3$ such that

(A) Kernel $f = \{(12), e\}$.

(B) Kernel $f = \{(123), (132), e\}$.

(C) Kernel f can contains $(123), (12)$.

(D) None of the above.

Q-6

44. Let $T : V \longrightarrow W$ be a linear transformation, then

(A) if $v_1, v_2, \cdots v_n$ are linear independent then $T(v_1), T(v_2) \cdots, T(v_n)$ are linear independent.

(B) if $T(v_1), T(v_2) \cdots, T(v_n)$ are linear independent, then $v_1, v_2, \cdots v_n$ are linear independent.

(C) $T(v_1), T(v_2), \cdots, T(v_n)$ are linear independent if T is onto.

(D) None of these.

45. The solution of $y' + y = xy^4$ is given by

(A) $y^{-3} = x + \frac{1}{3} + ce^{3x}$.

(B) $y^3 = x + \frac{1}{3} + ce^{3x}$.

(C) $y^{-3} = x + \frac{1}{3} + ce^{-3x}$.

(D) $y^3 = x + \frac{1}{3} + ce^{-3x}$.

46. The function $f(x) = |x|^p,\ 0 < p < 1$ is

(A) not Lipschitz at $x = 0$.

(B) Lipschitz at $x = 0$ with Lipschitz constant 1.

(C) Lipschitz at $x = 0$.

(D) Differentiable at $x = 0$.

47. The initial value problem $\frac{dx}{dt} = x^{3/2}(t),\ x(0) = 0$ has

(A) unique solution.

(B) two solutions.

(C) infinitely many solutions.

(D) none of these.

48. If u is a function of x, y and z satisfies the partial differential equation

$$(y - z)\frac{\partial u}{\partial x} + (z - x)\frac{\partial u}{\partial y} + (x - y)\frac{\partial u}{\partial x} = 0.$$

Then the general solution is of the form

(A) $u = f(x + y + z, x^2 + y^2 + z^2)$.

(B) $u = f(xy + yz + zx, x^2 + y^2 + z^2)$

(C) $u = f(xyz, x^2 + y^2 + z^2)$

(D) $u = f(x + y + z, x^2y^2z^2)$

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49. The complete integral of the equation

$$(p+q)(z-xp-yq)=1,$$

where $p=\frac{\partial z}{\partial x}, q=\frac{\partial z}{\partial y}$ is

(A) $z=ax+by+\frac{1}{a-b}$.

(B) $z=ax+by+\frac{1}{a+b}$.

(C) $z=ax-by+\frac{1}{a+b}$.

(D) none of these.

50. The orthogonal trajectories of the family of rectangular hyperbolas $y=c_1/x$ is

(A) $y^2-x^2=c$.

(B) $y^2+x^2=c$.

(C) $x^2y^2=c$.

(D) $\frac{x^2}{y^2}=c$

Q-6