Entrance Examination: Ph.D. Mathematics/Applied Mathematics, 2017

Hall Ticket Number

Time: 2 hours
Max. Marks: 80

Part A:40 Marks
Part B:40 Marks

Instructions

1. Write your Hall Ticket Number in the OMR Answer Sheet given to you. Also write the Hall Ticket Number in the space provided above.
2. Answers are to be marked on the OMR sheet.
3. Please read the instructions carefully before marking your answers on the OMR answer sheet.
4. Hand over the OMR answer sheet at the end of the examination.
5. No additional sheets will be provided. Rough work can be done in the question paper itself/space provided at the end of the booklet.
6. Calculators are not allowed.
7. There are a total of 40 questions in **PART A** and **PART B** together.
8. Each correct answer carries 2 marks.
9. The appropriate answer(s) should be coloured with either a blue or a black ball point or a sketch pen. DO NOT USE A PENCIL.
10. $\mathbb{R}$ denotes the set of real numbers, $\mathbb{C}$ the set of complex numbers, $\mathbb{Z}$ the set of integers, $\mathbb{Q}$ the set of rational numbers and $\mathbb{N}$ the set of all natural numbers.
11. This book contains **9** pages including this page and excluding pages for the rough work. Please check that your paper has all the pages.

Part-A

1. Suppose the statements P and Q are false and the statements R and S are true. Then the truth value of the statement $(S \to P) \iff \sim (Q \vee R)$

   (A) is **True**.

   (B) is **False**.

   (C) is sometimes **True** and sometimes **False**.

   (D) cannot be determined.

2. The negation of the statement "There are apples that cost Rs 25/- or more" is

   (A) No apple costs less than Rs. 25/-.

   (B) Some apples cost less than Rs. 25/-.

   (C) There is an apple that costs Rs. 20/-.

   (D) All apples cost less than Rs. 25/-.

3. Consider the statements:

   - All bats are cats.
   - No cat is a rat.
   - All rats are tigers.
   - Some tigers are fighters.

   Which of the following conclusions are **true**?

   S_1 : Some tigers are cats.

   S_2 : At least some fighters being rats is a possibility.

   S_3 : At least some cats are definitely bats.

   S_4 : None of the fighters is a rat.

   (A) Both S_1 and S_2 are true.

   (B) Both S_3 and S_4 are true.

   (C) Both S_2 and S_3 are true.

   (D) Both S_1 and S_4 are true.

4. Let $X = \{(x, y) : 0 \le x \le 4,\ 0 \le y \le 4\}$. For $i, j \in \mathbb{N} \cup \{0\}$, define $A_{i,j} = \{(x, y) : i \le x \le i+2,\ j \le y \le j+2\}$. The number of elements in the set $\{(i, j) : A_{i,j} \subset X\}$ is

   (A) 16.

   (B) 12.

   (C) 9.

   (D) 4.

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5. Let $X = \{1, 2, \cdots, 9\}$, $Y = \{1, 2, 3\}$ and $Z = \{1, 2, \cdots, 6\}$.
If $F = \{A \subseteq X : Y \subset A \text{ and } A \not\subseteq Z\}$, then the number of elements in F is equal to

(A) 64.
(B) 60.
(C) 56.
(D) 52.

6. Let $P(A)$ denote the power set of A. Let ϕ be the empty set. Then the number of elements in $P(P(P(\phi)))$ is

(A) 0.
(B) 1.
(C) 2.
(D) 4.

7. In a hexagon ABCDEF, each vertex is joined with the remaining vertices with line segments. The number of line segments which are inside the hexagon is

(A) 8.
(B) 9.
(C) 12.
(D) 15.

8. Let A, B, C be the vertices of a triangular field of sides 10m, 10m and 15m. Assume that at each vertex a cow is tied with a rope of length 4m. The area that these three cows can graze together, in square meters, is

(A) 14π.
(B) 12π.
(C) 10π.
(D) 8π.

9. Raghu is at a point A. He walks 3 km to the North and then he turns to his left. He walks 4 km in this direction. He turns left again and walks 6 km. If he wishes to reach point A again, then the direction he should be walking and the distance he will have to cover are

(A) South-East direction and 5 km.
(B) North-East direction and 5 km.
(C) South-East direction and 4 km.
(D) North-East direction and 4 km.

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10. The missing term in the following figure is

C 5	F 3	I 15
L 1	?	R 4
U 5	X 12	A 60

(A) B 5.

(B) O 5.

(C) B 4.

(D) O 4.

Instructions for Questions 11 and 12: Read the following information carefully and answer the following questions.

Six people A, B, C, D, E and F are sitting on the ground at the vertices of a regular hexagon. A is not adjacent to B or C; D is not adjacent to C or E; B and C are adjacent; F is in the middle of D and C.

11. Which of the following is not a possible neighbour pair?

(A) A and F.

(B) D and F.

(C) B and E.

(D) C and F.

12. Which of the following is in the right sequence for the above data?

(A) A, F, B.

(B) F, A, E.

(C) B, C, F.

(D) D, A, B.

13. In a certain code language MATH is written as MBTI and ABEL is written as ACEM. Then the possible word for JUNE in that code language is

(A) JWKP

(B) JVNF

(C) JPNQ

(D) BAST

14. In a certain code language FOOD is written as HQQF and RAVI is written as TCXK. Then the possible word for JOIN in that code language is

(A) LQKP

(B) NQPK

(C) JPKQ

(D) BAST

15. In a certain code language WONDER is written as VPMEDS. Then the possible word for MASTER in that code language is

(A) LBRWDW.

(B) LCRVDV.

(C) LBSVDV.

(D) LBRUDS.

Instructions for Questions 16–20: Read the following information carefully and answer the following questions.

In a certain code language:
"politics is not money" is written as "sa ri ga ma";
"media driven state politics" is written as "pa da ni sa";
"money controls many things" is written as "ok ri oo ha";
"media controls state politics" is written as "ok pa sa da".

16. What may be the possible code for "money controls politics" in the code language?

(A) ni pa sa.

(B) ri ok sa.

(C) ri pa ni.

(D) ga pa ma.

17. What may be the possible code for "media controls many things" in the code language?

(A) ok da oo ha

(B) ri ok oo ha

(C) ok ri oo ni

(D) ok da pa sa

18. What may be the possible code for "state driven politics" in the code language?

(A) ga sa da

(B) pa ni sa

(C) ga ma ni

(D) pa ni ga

19. What may be the possible code for "media is powerful" in the given code language?

(A) xi ga da.

(B) xi ma sa.

(C) xi da ri.

(D) xi ni ha.

20. What may be the possible code for "state and controls" in the code language?

(A) pa zz ok.

(B) sa da zz.

(C) oo ri zz.

(D) ha ga pa.

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Part-B

21. Let $P(x) = x(x-1)(x-2)(x-3)$. Consider the statements

S_1 : P' has 3 real roots.

S_2 : P'' has 2 real roots.

Then

(A) both S_1 and S_2 are true.
(B) S_1 is true but S_2 is false.
(C) S_2 is true but S_1 is false
(D) both S_1 and S_2 are false.

22. Let $f_n(x) = \dfrac{x^n}{1+x^2}$, for $n \in \mathbb{N}$. Then which of the following statements is **true**?

(A) f_n converges uniformly on $[0,1]$.
(B) f_n converges uniformly on $[-1, 1-\epsilon]$ for $\epsilon \in (0,1)$.
(C) $\sum f_n$ converges uniformly on $[0,1)$.
(D) $\sum f_n$ converges uniformly on $[-1+\epsilon, 1-\epsilon]$ for $\epsilon \in (0,1)$.

23. Let $C[0,1] = \{f : [0,1] \to \mathbb{R} : f \text{ is continuous}\}$. Consider $C[0,1]$ with the norm $\|f\|_\infty = \sup\limits_{x\in[0,1]} |f(x)|$. If $\{f_n\}$ is a bounded sequence in $(C[0,1], \|\cdot\|_\infty)$, then $\{f_n\}$ has a convergent subsequence with respect to $\|\cdot\|_\infty$ provided

(A) each f_n is a polynomial.
(B) $f_1 \geq f_2 \geq \cdots \geq 0$.
(C) $f_1 \leq f_2 \leq f_3 \leq \cdots$.
(D) $\{f_n : n \in \mathbb{N}\}$ is equicontinuous.

24. Let D be the unit open disc. Which of the following statements need not imply that an analytic function f is a constant function?

(A) $f : D \to \mathbb{C}$, Range$(f) = \{a + ib : b = 3a,\ a \in \mathbb{R}\}$.
(B) $f : \mathbb{C} \to \mathbb{C}$, $|f(z)| \geq 5$, $\forall\, z \in \mathbb{C}$.
(C) $f : \mathbb{C} \to \mathbb{C}$, $f(\{z : Re(z) = 0\})$ is bounded.
(D) $f : D \to \mathbb{C}$, $f(\frac{1}{n}) = 0$, $\forall\, n \in \mathbb{N}$.

25. The nature of the singular points of $f(z) = \dfrac{z+1}{(z^2+1)(z^4-1)}$ is

(A) a pole of order 1 and 2 poles of order 2 each.
(B) a pole of order 2 and 2 poles of order 1 each.
(C) 3 poles of order 1.
(D) 4 poles of order 1.

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26. Consider the following statements.

S_1 : $f : \mathbb{R} \to \mathbb{R}$ is a measurable function if and only if $f^{-1}\big((-\infty, r)\big)$ is a measurable set for every $r \in \mathbb{Q}$.

S_2 : $f : \mathbb{R} \to \mathbb{R}$ is a measurable function if and only if $f^{-1}(U)$ is a measurable set for every open set U in $\mathbb{R}$.

Then

(A) both S_1 and S_2 are true.
(B) S_1 is true but S_2 is false.
(C) S_2 is true but S_1 is false
(D) both S_1 and S_2 are false.

27. Pick up a true statement from the following.

(A) If $f : \mathbb{R} \to \mathbb{R}$ is Lebesgue integrable then f^2 is Lebesgue integrable.
(B) If $f : \mathbb{R} \to \mathbb{R}$ is Lebesgue integrable then $f^{1/2}$ is Lebesgue integrable.
(C) If $f : \mathbb{R} \to \mathbb{R}$ is Lebesgue integrable then $|f|$ is Lebesgue integrable.
(D) If $f : \mathbb{R} \to \mathbb{R}$ is Lebesgue integrable then f is Riemann integrable in every compact interval.

28. Let X be any topological space, and Y a Hausdorff topological space. Let $f, g : X \to Y$ be continuous maps. Consider the following two sets:

$$P = \{(y, y) : y \in Y\} \subseteq Y \times Y,$$
$$Q = \{x \in X : f(x) = g(x)\} \subseteq X.$$

Then pick up a statement which is always **true** from the following.

(A) P is open in $Y \times Y$ and Q is open in X.
(B) P is open in $Y \times Y$ and Q is closed in X.
(C) P is closed in $Y \times Y$ and Q is open in X.
(D) P is closed in $Y \times Y$ and Q is closed in X.

29. Let X' denote the dual of the normed linear space X. Consider the following statements.

S_1 : $(\ell^1)' = \ell^\infty$.
S_2 : $(\ell^\infty)' = \ell^1$.

Then

(A) both S_1 and S_2 are true.
(B) S_1 is true but S_2 is false.
(C) S_2 is true but S_1 is false
(D) both S_1 and S_2 are false.

30. Let X be a Hilbert space and $T : X \to X$ be a bounded linear operator satisfying $T^*T = I$. Then T is

(A) a normal operator.
(B) a self-adjoint operator.
(C) an isometry.
(D) a compact operator.

31. Consider the following statements

S_1 : Every complex $n \times n$ matrix of finite order (i.e, $A^m = I_n$ for some m) is diagonalizable.
S_2 : Every real and symmetric matrix is diagonalizable.

Then

(A) both S_1 and S_2 are true.
(B) S_1 is true but S_2 is false.
(C) S_2 is true but S_1 is false
(D) both S_1 and S_2 are false.

32. Let A be a complex matrix of order 5×5 with minimal polynomial $(X-1)^2(X+1)$. Then the number of possible Jordan canonical forms for the matrix A is

(A) 1.
(B) 2.
(C) 3.
(D) 4.

33. The number of distinct 3-cycles in S_6 is

(A) 120.
(B) 40.
(C) 3!.
(D) 6!.

34. The degree of the field extension $\mathbb{Q}\left(\sqrt{2}, \sqrt{3}, \sqrt{6}\right)$ over $\mathbb{Q}$ is

(A) 2.
(B) 4.
(C) 6.
(D) 8.

35. Consider the ring $R = (\mathbb{Z}/12\mathbb{Z}) \times (\mathbb{Z}/49\mathbb{Z})$ with pointwise addition and multiplication. Then the number of units in R is

(A) 21.
(B) 42.
(C) 84.
(D) 168.

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36. The group generated by two elements, a and b with relations $a^4 = b^4 = e$, and $a^3b = ab^3 = ba$ is isomorphic to

(A) $\mathbb{Z}/8\mathbb{Z}$.

(B) S_3.

(C) the group of quaternions.

(D) $\mathbb{Q}/\mathbb{Z}$.

37. Consider the regular Sturm-Liouville eigenvalue problem $(py')' + (q + \lambda s)y = 0$, $y(0) = 0$, $y(1) = 0$ where $p > 0$, $s > 0$. Which of the following statements is **true**?

(A) There exists a sequence λ_n of eigenvalues such that $\lambda_n \to 0$ as $n \to \infty$.

(B) There exists a sequence λ_n of eigenvalues such that $\lambda_n \to \infty$ as $n \to \infty$.

(C) It is possible to have a purely imaginary eigenvalue.

(D) Eigenfunctions are always of the form $\sin(\alpha x)$ for some $\alpha \in \mathbb{R}$.

38. Consider the system $\begin{cases} \dfrac{dx}{dt} = 2x + 3y, \\ \dfrac{dy}{dt} = 5x + 6y. \end{cases}$

Then

(A) there are no critical points for the given system.

(B) there exists a critical point which is stable and there exists a critical point which is unstable.

(C) all critical points are stable.

(D) all critical points are unstable.

39. The solution of the problem $\begin{cases} u_t(x,t) = u_{xx}(x,t), \ 0 < x < \pi, \\ u(x,0) = \sin(x), \ 0 < x < \pi, \\ u(0,t) = u(\pi,t) = 0, \ t \geq 0. \end{cases}$

is

(A) $u(x,t) = e^{-t}\sin(2x) - e^{-2t}\sin(x)$.

(B) $u(x,t) = e^{-t}\sin(2x) + e^{-2t}\sin(x)$.

(C) $u(x,t) = e^{-t}\sin(x)$.

(D) $u(x,t) = e^{t}\sin(x)$.

40. Pick up the partial differential equation which is parabolic from the following

(A) $u_{xx} + u_{yy} = x^2$.

(B) $u_t - u_{xx} + u^2 = 0$.

(C) $u_{tx} + u^2 = 0$.

(D) $u_{tt} - u_{xx} + u^2 = 0$.

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