Entrance Examination – 2016 : M.Sc. Mathematics

Hall Ticket Number

Time: 2 hours
Max. Marks: 100

Part A:25 Marks
Part B:75 Marks

Instructions

1. Write your Hall Ticket Number on the OMR Answer Sheet given to you. Also write the Hall Ticket Number in the space provided above.
2. Answers are to be marked on the OMR sheet.
3. Please read the instructions carefully before marking your answers on the OMR answer sheet.
4. Hand over the OMR answer sheet at the end of the examination.
5. The question paper can be taken by the candidate at the end of the examination.
6. No additional sheets will be provided. Rough work can be done in the question paper itself/space provided at the end of the booklet.
7. Calculators are not allowed.
8. There are a total of 50 questions in **PART A** and **PART B** together.
9. **There is a negative marking in PART A. Each correct answer carries 1 mark and each wrong answer carries -0.33 mark. Each question in PART A has only one correct option.**
10. **There is no negative marking in PART B. Each correct answer carries 3 marks. In PART B some questions have more than one correct option. All the correct options have to be marked in OMR sheet otherwise zero marks will be credited.**
11. The appropriate answer(s) should be coloured with either a blue or a black ball point or a sketch pen. DO NOT USE A PENCIL.
12. $\mathbb{R}$ denotes the set of real numbers, $\mathbb{C}$ the set of complex numbers, $\mathbb{Z}$ the set of integers, $\mathbb{Q}$ the set of rational numbers and $\mathbb{N}$ the set of all natural numbers.
13. This book contains 10 pages including this page and excluding pages for the rough work. Please check that your paper has all the pages.

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PART-A

1. Which of the following is an uncountable subset of $\mathbb{R}^2$?

   (a) $\{(x,y) \in \mathbb{R}^2 : x \in \mathbb{Q} \text{ or } x+y \in \mathbb{Q}\}$.
   (b) $\{(x,y) \in \mathbb{R}^2 : x \in \mathbb{Q} \text{ and } y \in \mathbb{Q}\}$.
   (c) $\{(x,y) \in \mathbb{R}^2 : x \in \mathbb{Q} \text{ or } y \in \mathbb{Q}\}$.
   (d) $\{(x,y) \in \mathbb{R}^2 : x \in \mathbb{Q} \text{ or } y^2 \in \mathbb{Q}\}$.

2. Which of the following is an unbounded subset of $\mathbb{R}^2$?

   (a) $\{(x,y) \in \mathbb{R}^2 : x^2+y^2 \leq 1\}$.
   (b) $\{(x,y) \in \mathbb{R}^2 : x+y \leq 1\}$.
   (c) $\{(x,y) \in \mathbb{R}^2 : |x|+|y| \leq 1\}$.
   (d) $\{(x,y) \in \mathbb{R}^2 : |x|+y^2 \leq 1\}$.

3. Let $f : \mathbb{R} \to \mathbb{R}$ be a twice differentiable function. Then which of the following is true?

   (a) If $f(0) = 0 = f''(0)$ then $f'(0) = 0$.
   (b) f is a polynomial.
   (c) f' is continuous.
   (d) If $f''(x) > 0$ for all x in $\mathbb{R}$ then $f(x) > 0$ for all x in $\mathbb{R}$.

4. The non-zero values for x_0 and x_1 such that the sequence defined by the recurrence relation $x_{n+2} = 2x_n$, is convergent are

   (a) $x_0 = 1$ and $x_2 = 1$.
   (b) $x_0 = 1/2$ and $x_1 = 1/4$.
   (c) $x_0 = 1/10$ and $x_1 = 1/20$.
   (d) none of the above.

5. The set of all values of a for which the series $\displaystyle\sum_{n=1}^{\infty} \frac{a^n}{n!}$ converges is

   (a) $(0,\infty)$. (b) $(-\infty,0]$.
   (c) $(-\infty,\infty)$. (d) $(-1,1)$.

6. Consider $f(x) = \begin{cases} |x|, & \text{if } -1 \leq x \leq 1, \\ x^2, & \text{otherwise.} \end{cases}$ Then

   (a) f is not continuous at 0.
   (b) f is not continuous at 1.
   (c) f is not continuous at -1.
   (d) f is continuous at all points.

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7. Consider the statement S : "Not all students in this class are tall". The statement S means

   (a) All students in this class are short.
   (b) All short students are in this class.
   (c) At least one student in this class is not tall .
   (d) No short student is in this class.

8. A subset S of $\mathbb{N}$ is infinite if and only if

   (a) S is not bounded below.
   (b) S is not bounded above.
   (c) $\exists n_0 \in \mathbb{N}$ such that $\forall n \geq n_0$, $n \in S$.
   (d) $\forall a \in S$, $\exists x \in \mathbb{N}$ such that $x < a$.

9. In a cyclic group of order 35, the number of elements of order 35 is

   (a) 1. (b) 4. (c) 6. (d) 24.

10. Let A be an $n \times n$ matrix. Consider the following statements.

    S_1 : If the rank of A is equal to n then the rank of the adjoint matrix of A is also equal to n.

    S_2 : If the rank of A is equal to $n-2$ then the rank of the adjoint matrix of A is also equal to $n-2$.

    Pick up a true statement from the following.

    (a) S_1 and S_2 are true. (b.) S_1 is true but not S_2.
    (c) S_2 is true but not S_1. (d.) Neither S_1 nor S_2 is true.

11. The locus of a point P which is at the same distance from two planes $x+y+z=1$, $z=0$ is

    (a) an unbounded set.
    (b) a sphere.
    (c) a pair of parallel planes.
    (d) a pair of intersecting planes.

12. The value of the integral $\oint_C (x^3+x)dx + (1+y^2+y^3)dy$, where $C = \{(x(t), y(t)) \,/\, x(t) = 2+3\cos t,\ y(t) = 5+4\sin t,\ 0 \leq t < 2\pi\}$, is

    (a) 0. (b) π. (c) 10π. (d) 12π.

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13. Let V be the region which is common to the solid sphere $x^2+y^2+z^2 \leq 1$ and the solid cylinder $x^2+y^2 \leq 0.5$. Let ∂V be the boundary of V and $\hat{n}$ be the unit outward normal drawn at the boundary. Let $\vec{F} = (y^2+z^2)\hat{i} + (z^2 - 2x^2)\hat{j} + (x^2+2y^2)\hat{k}$. Then the value of

$$\iint_{\partial V} \vec{F} \cdot \hat{n} dS \text{ is equal to}$$

(a) 0. (b) 1. (c) -1. (d) π.

14. Let $R([a,b])$ be the set of all Riemann integrable functions on $[a,b]$. Consider the following statements.

S_1 : $f \in R([a,b])$ whenever there exist $g, h \in R([a,b])$ such that $g \leq f \leq h$.

S_2 : $f \in R([a,b])$ whenever there exist two continuous functions g, h on $[a,b]$ such that $g \leq f \leq h$.

Which of the following statements is true?

(a) S_1 and S_2 are true. (b) S_1 is true but not S_2.
(c) S_2 is true but not S_1. (d) Neither S_1 nor S_2 is true.

15. The shortest distance from the sphere $x^2+y^2+z^2-2x-4y-6z+11=0$ to the plane $x+y+z=3$ is equal to

(a) $\sqrt{3}$. (b) $2\sqrt{3}$. (c) $3\sqrt{3}$. (d) $4\sqrt{3}$.

16. A solution of $(x^2y^2+y^4+2x)dx + 2y(x^3+xy^2+1)dy = 0$ is

(a) $x^2 + \log|x^2-y^2| =$ constant.
(b) $x^2y + \log|x^2-y^2| =$ constant.
(c) $x^2y + \log(x^2+y^2) =$ constant.
(d) $xy^2 + \log(x^2+y^2) =$ constant.

17. Let $f, g : \mathbb{R} \to \mathbb{R}$, $f(x) = x^2$, $g(x) = \sin x$. Which of the following statements is true?

(a) f and g are uniformly continuous.
(b) f and g are not uniformly continuous.
(c) f is uniformly continuous but g is not.
(d) f is not uniformly continuous but g is uniformly continuous.

18. If A and B are square matrices satisfying $AB = BA$, $\det(A) = 1$ and $\det(B) = 0$ then $\det(A^3B^2 + A^2B^3)$ is equal to

(a) -1. (b) 0. (c) 1. (d) 2

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19. Which of the following subsets are subspaces of $\mathbb{R}^3$?

(a) $\{(x, y, z) \in \mathbb{R}^3 \,/\, 5x - y + z = 0\}$
(b) $\{(x, y, z) \in \mathbb{R}^3 \,/\, 5x - y + z = -1\}$
(c) $\{(x, y, z) \in \mathbb{R}^3 \,/\, x, y, z \text{ are rationals}\}$
(d) $\{(x, y, z) \in \mathbb{R}^3 \,/\, x^2 + y^2 + z^2 = 1\}$

20. A palindrome is a word which reads the same backward or forward (eg. MADAM, ANNA). The number of palindromes of length 11 (eleven) can be formed from an alphabet of K letters is equal to

(a) K^6. (b) K^5. (c) $\binom{K}{6}$. (d) $\binom{K}{5}$.

21. The center of the ring of 2×2 matrices over $\mathbb{R}$ is

(a) $\left\{\begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix} / \, a, b \in \mathbb{R}\right\}$. (b) $\left\{\begin{pmatrix} a & 0 \\ 0 & a \end{pmatrix} / \, a \in \mathbb{R}\right\}$.
(c) $\left\{\begin{pmatrix} 0 & b \\ a & 0 \end{pmatrix} / \, a, b \in \mathbb{R}\right\}$. (d) $\left\{\begin{pmatrix} 0 & a \\ a & 0 \end{pmatrix} / \, a \in \mathbb{R}\right\}$.

22. The set of units of the Gaussian ring $\{a + ib \,/\, a, b \in \mathbb{Z}\}$ is

(a) $\{\pm 1, \pm i\}$. (b) $\mathbb{Z} \cup i\mathbb{Z}$.
(c) $\{a + ib \,/\, a, b \in \{\pm 1, 0\}\}$. (d) $\mathbb{Z}$.

23. Group of automorphisms of $(\mathbb{Z} / 10\mathbb{Z}, +)$ is isomorphic to

(a) $(\mathbb{Z} / 2\mathbb{Z}) \times (\mathbb{Z} / 2\mathbb{Z})$. (b) $\mathbb{Z} / 2\mathbb{Z}$.
(c) $\mathbb{Z} / 4\mathbb{Z}$. (d) $\mathbb{Z} / 10\mathbb{Z}$.

24. The system of equations $6x_1 - 2x_2 + 2\alpha x_3 = 1$ and $3x_1 - x_2 + x_3 = 5$ has no solution if α is equal to

(a) -5. (b) -1. (c) 1. (d) 5.

25. Suppose every collection of three distinct numbers from 1, 2, ...,9 is equally likely to be selected, then

(a) the probability that the sum of the selected numbers is even is more than the probability that the sum of the selected numbers is odd.
(b) the probability that the sum of the selected numbers is even is equal to the probability that the sum of the selected numbers is odd.
(c) the probability that the product of the selected numbers is even is less than $1/2$.
(d) the probability that the product of the selected numbers is odd is equal to $2/3$.

PART-B

26. Which of the following are true?

(a) There is a surjective function $f : \mathbb{N} \to \mathbb{N} \times \mathbb{N}$.

(b) There is an injective function $f : \mathbb{N} \times \mathbb{N} \times \mathbb{N} \to \mathbb{N}$.

(c) There is a bijective function $f : \mathbb{N} \times \mathbb{N} \to \mathbb{N} \times \mathbb{N} \times \mathbb{N}$.

(d) There is a bijective function $f : \{0,1\} \times \mathbb{N} \to \mathbb{N} \times \mathbb{N}$.

27. Let $f : \mathbb{R} \to \mathbb{R}$ be a differentiable function such that $f(1) = f(0) + 1$. Then which of the following are true?

(a) f' is constant.

(b) $f(2) = f(1) + 1$.

(c) $f'(x) = 1$ for some x in $(0,1)$.

(d) $|f'(x)| \leq 1$ for all x in $(0,1)$.

28. Let $\mathbb{P}(\mathbb{N}) = \{\text{All subsets of } \mathbb{N}\}$. Then which of the following are equivalence relations on $\mathbb{P}(\mathbb{N})$?

(a) $A \sim B$ if and only if $|A| = |B|$.

(b) $A \sim B$ if and only if $A \cup B = B$.

(c) $A \sim B$ if and only if $A \cup B = \mathbb{N}$.

(d) $A \sim B$ if and only if $A \cap B \neq \phi$.

29. Let $f(x) = x^3 + ax^2 + bx + c$ where a, b, c are real numbers. Suppose $c < 0$, $a + b + c > -1$ and $a - b + c > 1$. Then which of the following are true?

(a) All roots of $f(x)$ are real. .

(b) $f(x)$ has one real root and two complex roots.

(c) $f(x)$ has two roots in $(-1,1)$.

(d) $f(x)$ has at least one negative root.

30. Suppose $T : \mathbb{R}^3 \to \mathbb{R}^3$ is the map $T((x_1, x_2, x_3)) = (2x_1, x_2, 2x_1)$. Then which of the following is true?

(a) T has only two distinct eigenvalues.

(b) $\ker(T) \neq \{(0,0,0)\}$.

(c) T has three distinct eigenvalues.

(d) Range of T is isomorphic to $\mathbb{R}^2$.

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31. Consider the following statements.

$S_1 : \sum_{n=3}^{\infty} \frac{1}{(\log\log n)^{\log n}}$ is a convergent series.

$S_2 : \sum_{n=3}^{\infty} \frac{1}{n^{\log\log\log n}}$ is a convergent series.

Which of the following statements are true?

(a) S_1 and S_2 are true.
(b) S_1 is true but not S_2.
(c) S_2 is true but not S_1.
(d) Neither S_1 nor S_2 is true.

32. The functions $f(z)$ and $g(z)$ are such that the function

$$\vec{F} = \left(2x + yf(z)\right)\hat{\imath} + \left(2y + xf(z)\right)\hat{\jmath} + xyg(z)\hat{k}$$

can be written as a gradient of some scalar function.
Pick up the possible choices for f and g from the following.

(a) $f(z) = z^3$ and $g(z) = 3z^2$.
(b) $f(z) = 0$ and $g(z) = 0$.
(c) $f(z) = 1$ and $g(z) = 0$.
(d) $f = 1$ and $g = z$.

33. If β is the radius of the circle of intersection of the sphere $x^2 + y^2 + z^2 - 2x - 4y - 6z + \alpha = 0$ and the plane $x + y + z = 1$, then a relation between α and β is

(a) $3\alpha + 3\beta^2 = 17$.
(b) $3\alpha^2 - 3\beta^2 = 17$.
(c) $3\alpha^2 - 3\beta^2 = 67$.
(d) $3\alpha + 3\beta^2 = 67$.

34. Which of the following sequences converge to e?

(a) $\left(1 + \frac{1}{2n}\right)^n$.
(b) $\left(2 + \frac{1}{2!} + \frac{1}{3!} + \cdots + \frac{1}{n!}\right)$.
(c) $\left(1 + \frac{1}{n}\right)^n$.
(d) $\left(\frac{2n+1}{2n-2}\right)^n$.

35. For every pair of continuous functions $f, g : \mathbb{R} \to \mathbb{R}$, which of the following statements are "always" true?

(a) If $f(x) = g(x)$, $\forall x \in \mathbb{Q}$, then $f(x) = g(x)$, $\forall x \in \mathbb{R}$.
(b) $\{x \in \mathbb{R} \,/\, f(x) = g(x)\}$ is an open subset of $\mathbb{R}$.
(c) The product of f and g is continuous.
(d) If $h(x) = \begin{cases} \dfrac{f(x)}{g(x)}, & \text{for } g(x) \neq 0, \\ 0, & \text{otherwise,} \end{cases}$ then h is continuous.

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36. Let A and B be two $n \times n$ matrices such that $\text{rank}(A) = n$, $\text{rank}(B) = n - 1$. Then which of the following are true?

(a) $\det(A^3) = 0$.

(b) $\det(B) = 0$

(c) $\text{rank}(AB) = n - 1$.

(d) $\text{rank}(BA) = n - 1$.

37. Which of the following statements are true?

(a) Every finite group of even order contains at least one element of order 2.

(b) If every subgroup of a group is normal then the group is abelian.

(c) If G is an abelian group of odd order, then $x \to x^2$ is an automorphism of G.

(d) If the elements a, b in a group have finite order then the element ab is also of finite order.

38. For $A \subset \mathbb{R}$, define $\chi_A(x) = \begin{cases} 1 & \text{for } x \in A, \\ 0 & \text{for } x \notin A. \end{cases}$ Then χ_A is Riemann integrable over $[-1, 1]$ if

(a) $A = \mathbb{Q} \cap [-1, 1]$.

(b) $A = [-1, 1] \setminus \mathbb{Q}$.

(c) $A = \left\{1, \frac{1}{2}, \frac{1}{3}, \cdots\right\}$.

(d) $A = \{\pm 10^{-n} \,/\, n \in \mathbb{N}\}$.

39. A solution of $(D^2 + 1)^3 y = \sin x$ is

(a) $\sum_{n=1}^{3} \left(c_n x^{n-1} \sin x + d_n x^{n-1} \cos x\right)$ for some $c_n, d_n \in \mathbb{R}$, $1 \leq n \leq 3$.

(b) $\sum_{n=1}^{3} (c_n \sin^n x + d_n \cos^n x)$ for some $c_n, d_n \in \mathbb{R}$, $1 \leq n \leq 3$.

(c) $\sum_{n=1}^{3} (c_n \sin nx + d_n \cos nx)$ for some $c_n, d_n \in \mathbb{R}$, $1 \leq n \leq 3$.

(d) none of the above.

40. Let $\sum_{n=0}^{\infty} a_n$ be a divergent series of positive terms. Then it follows that

(a) $\sum_{n=0}^{\infty} a_n^2$ is also divergent.

(b) the sequence (a_n) does not converge to 0.

(c) the sequence (a_n) is not bounded.

(d) $\sum_{n=0}^{\infty} \sqrt{a_n}$ is also divergent.

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41. Let (a_n) be a sequence where all rational numbers are terms (and all terms are rational). Then

(a) no subsequence of (a_n) converges.

(b) there are uncountably many convergent subsequences of (a_n).

(c) every limit point of (a_n) is a rational number.

(d) no limit point of (a_n) is a rational number.

42. Let $y_1(x) = \sum_{n=1}^{3} c_n\phi_n(x)$ and $y_2(x) = \sum_{n=1}^{3} d_n\psi_n(x)$ be complementary solutions to $P(D)y = 0$ and $Q(D)y = 0$ respectively; where $P(D)y = (D^3+a_1D^2+a_2D+a_3)y$ and $Q(D)y = (D^3+b_1D^2+b_2D+b_3)y$. Then the general solution of $P(D)Q(D)y = 0$ is equal to

(a) $y = \sum_{n=1}^{3}(c_n\phi_n + d_n\psi_n)$.

(b) $y = \left(\sum_{n=1}^{3} c_n\phi_n\right)\left(\sum_{n=1}^{3} d_n\psi_n\right)$.

(c) $y = \sum_{n=1}^{3}(c_n\phi_n^2 + d_n\psi_n^2)$.

(d) None of these.

43. The number of elements in S_5 whose order is 2 is

(a) 10. (b) 12. (c) 25. (d) 40.

44. Consider the following statements

S_1 : There is no polynomial $P(x)$ with integer coefficients such that $P(5) = 5$ and $P(9) = 7$.

S_2 : If α and β are two odd integers then $\alpha^2 + \beta^2$ is not a perfect square.

Which of the following statements are true?

(a) S_1 and S_2 are true.

(b) S_1 is true but not S_2.

(c) S_2 is true but not S_1.

(d) Neither S_1 nor S_2 is true.

45. Let A be an $n \times n$ matrix. If $A^m = 0$ for some integer m then which of the following statements are true?

(a) If A is nilpotent then $\det(I + A) = 1$.

(b) If A is nilpotent then $A^n = 0$

(c) If every eigenvalue of A is 0, then A is nilpotent.

(d) If A is nilpotent then every eigenvalue of A is 0.

46. The number of automorphisms of $\mathbb{Z}[\sqrt{2}]$ is

(a) 1. (b) 2. (c) 4. (d) infinity.

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47. The kernel of a ring homomorphism from $\mathbb{R}[X]$ to $\mathbb{C}$ defined by $f(X) \mapsto f(3+2i)$ is

(a) $\langle X^2 - 6X + 13 \rangle$.
(b) $\langle X^2 + 6X + 5 \rangle$.
(c) $\mathbb{R}[X]$.
(d) $\{0\}$.

48. Pick up prime elements of the ring of Gaussian integers $G = \{x + iy \,/\, x, y \in \mathbb{Z}\}$ from the following

(a) 2.
(b) 3.
(c) 7.
(d) 13.

49. A subset S of $\mathbb{N}$ is said to be thick if among any 2016 consecutive positive integers, at least one should belong to S. Which of these subsets are thick?

(a) The set of the geometric progression $\{2, 2^2, 2^3, \cdots\}$.
(b) The set of the arithmetic progression $\{1000, 2000, 3000, \cdots\}$.
(c) $\{n \in \mathbb{N} \,/\, n > 2016\}$.
(d) The set of all composite numbers.

50. Three students are selected at random from a class of 10 students among which 4 students know C programming of whom 2 students are experts. If every such selection is equally likely, then the probability of selecting three students such that at least two of them know C programming with at least one out of the two selected being an expert in C programming is

(a) less than 1/4.
(b) greater than 1/4 but less than 1/2.
(c) greater than 1/2 but less than 3/4.
(d) greater than 3/4. .

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