ENTRANCE EXAMINATION, FEBRUARY 2013

QUESTION PAPER BOOKLET

M. Tech. (Computational Techniques)

Marks: 75

Time: 2.00 hrs. Hall Ticket No.:

I. Please enter you **Hall Ticket Number on Page 1 of this question paper and on the OMR sheet** without fail.

II. Read carefully the following instructions:

1. This Question paper has two Sections: **Section A** and **Section B**
2. **Section A** consists of 25 objective type questions of one mark each. **There is negative marking of 0.33 mark for every wrong answer.** The marks obtained by the candidate in this Section will be used for resolving the tie cases.
3. **Section B** consists of 50 objective type questions of one mark each. **There is no negative marking in this Section.**
4. Answers are to be marked on the OMR answer sheet following the instructions provided there upon. An example is shown below

   100. (A) ● (C) (D)

5. Only Scientific Calculators are permitted. Mobile phone based calculators are not permitted. Logarithmic tables are not allowed.
6. Hand over the OMR answer sheet at the end of the examination to the Invigilator.

This book contains **22 pages**

III. Values of physical constants:
$c = 3 \times 10^8$ m/s; $h = 6.63 \times 10^{-34}$ J.s; $k_B = 1.38 \times 10^{-23}$ J/K
$e = 1.6 \times 10^{-19}$ C; $\mu_o = 4\pi \times 10^{-7}$ Henry/m; $\epsilon_o = 8.85 \times 10^{-12}$ Farad/m

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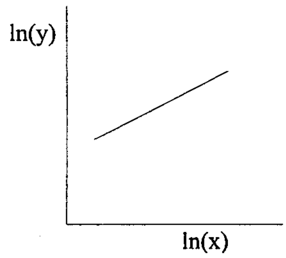
SECTION - A

1. Which of the following equations is best represented by the graph below



A. $xy = a$, where a is a constant

B. $y = mx + c$, where m and c are constants

C. $y = \exp(x) + a$, where a is a constant

D. $y = ax^m$, where a and m are constants

2. The equation of the plane tangent to a sphere $x^2 + y^2 + z^2 = 1$ at the point $\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$ is given by

A. $x + y + z = 0$

B. $x + y + z = \sqrt{3}$

C. $x + y + z = 3\sqrt{3}$

D. $2x + 2y + 2z = 1$

3. The set of all $n \times n$ complex matrices form a vector space under matrix addition. Which of the following is not a subspace?

A. The set of $n \times n$ complex matrices with all the diagonal elements equal to zero

B. The set of $n \times n$ complex matrices with the elements in the first column equal to zero

C. The set of $n \times n$ complex matrices with the elements in the first row equal to zero

D. The set of all $n \times n$ complex matrices with the determinant equal to zero

4. The eigenvalues of the matrix $\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ are given by

A. 0, 1, 1

B. $0, \pm\sqrt{2}$

C. $1, \pm\sqrt{2}$

D. 0, 0 , 2

5. $y = \sin x/x$ is a solution of the differential equation

A. $xy' + y = \cos x$

B. $xy' + y = \sin x$

C. $y' + xy = \cos x$

D. $y' + xy = \sin x$

6. If $\sin(z^2/4)$ is expanded in powers of z, the series will converge for

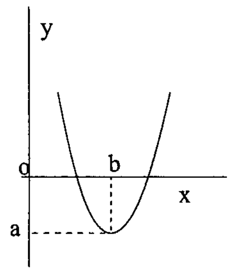
A. all values of z

B. $|z| < 2$

C. $|z| < 1$

D. $|z| < \frac{1}{2}$

7. The equation of the parabola shown in the figure is



A. $y = x^2 + a + b$

B. $y = (x - b)^2 + a$

C. $y = (x - b)^2 - a$

D. $y = (x + b)^2 - a$

8. The function $z^{-2}\sin(1/z)$ has a singular point at $z = 0$. The nature of this singularity is

A. pole of order 2

B. essential singularity

C. pole of order 2 and an essential singularity

D. pole of order 2 and a branch point

9. z_1 and z_2 are complex numbers. The modulus $|z_1 + z_2|$ is

A. $\geq |z_1| + |z_2|$

B. $\leq |z_1| + |z_2|$

C. $= |z_1| - |z_2|$

D. $= |z_1| \cdot |z_2|$

10. The value of the integral,

$$\int_{-\infty}^{+\infty} \delta(ax)\, f(x) dx,$$

where $\delta(t)$ is Dirac delta function, is given by

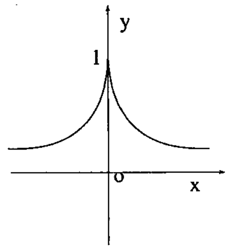
A. $\frac{1}{a} f(0)$

B. $af(0)$

C. $\frac{1}{|a|} f(0)$

D. $|a| f(0)$

11. The graph shown corresponds to the equation



A. $y = e^{x^2}$
B. $y = e^{-x}$
C. $y = e^{|x|}$
D. $y = e^{-|x|}$

12. Suppose that a variable Q only takes on five possible values: the value "1" with probability 0.25, the value "2" with probability 0.25, the value "3" with probability 0.05, the value "4" with probability 0.30, and the value "5" with probability 0.20. What is the probability that Q is an odd number?

A. 0.45
B. 0.55
C. 0.0025
D. 0.50

13. The number of root(s) to the equation $\tan x = x$ is

A. one
B. two
C. zero
D. infinite

14. In the Kepler problem the conserved quantites are

A. Energy only
B. Energy and angular momentum only
C. Energy and velocity
D. Energy, velocity and angular momentum

15. For a scattering experiment of a particle by a central potential what happens as the impact parameter decreases ?

A. scattering angle decreases
B. scattering angle increases
C. scattering angle is unchanged
D. scattering angle may increase or decrease

16. Consider a system described by the Hamiltonian $H(p_i, q_i, t)$. If $A(p_i, q_i, t)$ and $B(p_i, q_i, t)$ are two conserved quantities defined on the phase space of this system, which of the following statements is correct?

   A. $\{A, B\}$ and AB are conserved

   B. $\{A, B\}$ is conserved but not AB

   C. $\{A, B\}$ is not conserved but AB is

   D. neither $\{A, B\}$ nor AB is conserved

   where $\{A, B\}$ is the Poisson bracket of A and B.

17. A particle with spin 1 has orbital angular momentum $l = 2$. The allowed values of total angular momentum are

   A. $J = 3$ only

   B. $J = \pm\, 3$ only

   C. $J = 3,2,1$

   D. $J = \pm 3,\ \pm 2,\ \pm 1,\ 0$

18. Consider a particle moving under a SHO whose Hamiltonian is $H_o = \dfrac{p^2}{2m} + \dfrac{1}{2}m\omega^2 x^2$ (one-dimensional motion). If there is perturbation to the Hamiltonian by a term $V = \lambda x$, the change in the ground state energy, to first order in λ, is

   A. zero

   B. λ

   C. $-\lambda$

   D. $\dfrac{\lambda}{4}$

19. 'Zero point energy' is a result of

   A. Energy conservation

   B. Relativistic effects

   C. Uncertainty principle

   D. Pauli's exclusion principle

20. The Second law of thermodynamics says (qualitatively) that

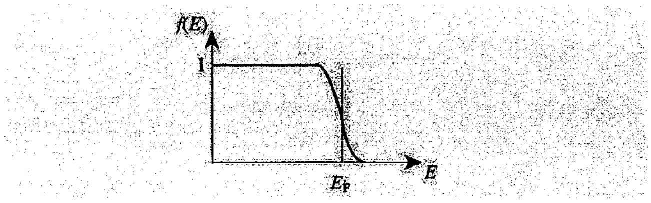
   A. Energy is always conserved

   B. Energy is always greater than entropy

   C. Entropy increases as disorder increases

   D. Hot water added to cold water produces warm water

21. The plot below represents



A. Bose-Einstein distribution at non-zero temperature
B. Bose-Einstein distribution at zero temperature
C. Fermi-Dirac distribution at non-zero temperature
D. Fermi-Dirac distribution at zero temperature

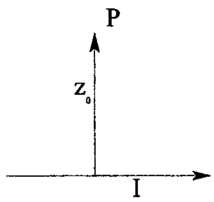
22. In a canonical ensemble, if P_r represents the probability that a system has energy ϵ_r, then the entropy S is given by (here k is the Botlzmann's constant)

A. $k \sum_r P_r^2 \ln P_r$
B. $-k \sum_r P_r^2 \ln P_r$
C. $k \sum_r P_r \ln P_r$
D. $-k \sum_r P_r \ln P_r$

23. The energy of a uniformly charged spherical shell of total charge q and radius R is proportional to

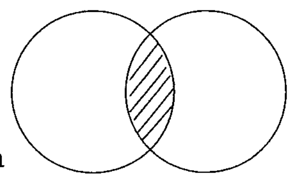
A. q/R
B. $q^2/R^{3/2}$
C. q/R^2
D. q^2/R

24. The magnetic field at P due to an infinite straight wire carrying steady current I is



A. $\mu_o I/2\pi Z_o$ pointing out of the page
B. $\mu_o I/2\pi Z_o$ pointing into the page
C. $\mu_o I/2\pi Z_o$ along Z_o
D. $\mu_o I/2\pi Z_o$ parallel to the wire

25. Consider two oppositely charged overlapping spheres with same radius and with charge densities $+\rho$ and $-\rho$. Which **one** of the following statements is **true** ?



A. Field is nowhere uniform
B. Field is everywhere uniform
C. Field in the region of overlap (shaded) is uniform
D. Field in the region of nonoverlap (unshaded) is uniform

SECTION - B

26. The curve parametrically given by $x = 1+2\sec(t)$, $y = -1+\tan(t)$, $t \in (-\pi/2, \pi/2)$ represents

A. an ellipse

B. a hyperbola

C. the right hand branch of a hyperbola

D. the left hand branch of hyperbola

27. If C is a unit circle in the complex z-plane with origin as the centre and traversed in an anti-clockwise direction, then the value of the integral $\int_C \frac{dz}{(z^2 - 1/4)}$ is

A. 2π

B. 0

C. $\pi/4$

D. π

28. Urn 1 contains 4 red and 6 green balls while urn 2 contains 6 red and 3 green balls. A ball is selected at random from urn 1 and transferred to urn 2. Then a ball is selected at random from urn 2. The probability that the ball from urn 2 is green is

A. 0

B. 9/25

C. 1/25

D. 6/25

29. Which of the following equations has damped oscillatory solution?

A. $\frac{d^2y}{dt^2} - 4\frac{dy}{dt} + y = 0$

B. $\frac{d^2y}{dt^2} - 3\frac{dy}{dt} + 2y = 0$

C. $\frac{d^2y}{dt^2} - 10\frac{dy}{dt} + 25y = 0$

D. $\frac{d^2y}{dt^2} + 5\frac{dy}{dt} + 25y = 0$

30. The value of the integral $\int_c \frac{3z^2+2}{(z-4)(z^2+9)}dz$,
where c is the circle $|3z-2|=1$ oriented in anticlockwise direction is

A. zero

B. $2\pi i$

C. $4\pi i$

D. $6\pi i$

31. The initial estimate of the root of the equation $f(x)=0$, found by using Newton-Raphson method, is $x_0=3$, $f(3)=5$. The angle, the tangent to the function $f(x)$ makes at $x=3$, is 57 ° with respect to the x-axis. The next estimate of the root, x_1 most nearly is

A. -3.2470

B. -0.2470

C. 3.2470

D. 6.2470

32. Which of the following is not a basis for the complex vector space C^3 consisting of all complex 3-tuples (ξ_1, ξ_2, ξ_3)

A. (1,0,0), (1,1,0), (1,1,1)

B. (1,0,0), (1,i,0), (1,1,i)

C. (1,0,0), (1,1,-1), (1,-1,1)

D. (1,0,0), (1,1,-1), (1,-1,0)

33. Which of the following matrices is non-invertible?

A. $\begin{bmatrix} 1 & -1 & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 1 \end{bmatrix}$

B. $\begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$

C. $\begin{bmatrix} 1 & -1 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 1 \end{bmatrix}$

D. $\begin{bmatrix} 1 & -1 & 0 \\ 1 & 0 & -1 \\ 0 & -1 & 1 \end{bmatrix}$

34. Vectors $\vec{r}_1 = (0,1,0)$ and $\vec{r}_2 = (1,2,3)$ can be combined to find scalar and vector products, $\vec{r}_1 \cdot \vec{r}_2$ and $\vec{r}_1 \times \vec{r}_2$ respectively. Which of the following statements is **true**?

   A. $(\vec{r}_1 \times \vec{r}_2)^2 = 0$

   B. $(\vec{r}_1 \cdot \vec{r}_2)^2 = 2$

   C. $(\vec{r}_1 \times \vec{r}_2)^2 = 10$

   D. $(\vec{r}_1 \cdot \vec{r}_2)^2 = 0$

35. The fourier transform of a gaussian function, $f(x) = e^{-x^2}$, in k-space is

   A. a gaussian in $1/k$, like e^{-1/k^2}

   B. a powerlaw in k, like $1/k^2$

   C. a sinusoidal function like $\sin k$ or $\cos k$

   D. a gaussian in k like e^{-k^2}

36. The probability of coins falling 'heads or tails' is $= \frac{1}{2}$. The probability of a cubic dice falling with side 1, or 2, or 3,$\cdots$, or 6 facing upwards, is $= \frac{1}{6}$. The probability when tossing a coin *and* rolling a dice of having the result coin-heads **and** dice = 4 is given by

   A. $\frac{1}{2} + \frac{1}{4} = \frac{3}{4}$

   B. $\frac{1}{2} + \frac{1}{6} = \frac{2}{3}$

   C. $\frac{1}{2} \times \frac{1}{6} = \frac{1}{12}$

   D. $\frac{1}{2} \times \frac{1}{4} = \frac{1}{8}$

37. Which of the following series does not diverge for $x > 1$

   A. $1 - x + x^2 + x^3 + \cdots$

   B. $1 - \frac{x^2}{2} + \frac{x^3}{3} + \cdots$

   C. $1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots$

   D. $1 + x^3 + x^6 + x^9 + \cdots$

38. $\vec{F}(\vec{r}) \equiv \vec{\nabla} \times \vec{G}(\vec{r})$ is the curl of a 3D vector field $\vec{G}(r)$, with $\vec{r} = (x,y,z)$. For $G(\vec{r}) = (0,x,0)$ the curl $\vec{F}(\vec{r})$ is:

   A. (0,0,0)

   B. (1,0,0)

   C. (0,1,0)

   D. (0,0,1)

39. The limit of the sequence

$$\sqrt{2};\ \sqrt{2\sqrt{2}}\ ;\ \sqrt{2\sqrt{2\sqrt{2}}}\ ;\ \sqrt{2\sqrt{2\sqrt{2\sqrt{2}}}}\ ;\ \cdots$$

is

A. 1
B. 2
C. $2\sqrt{2}$
D. ∞

40. If $M'_{ij} = \alpha M_{ij}$, $(i,j = 1,2,\cdots,n)$, then $\sqrt{|M'|}$ is proportional to

A. $\alpha^{1/2}$
B. α
C. $\alpha^{n/2}$
D. α^{n}

41. The principal value of $\ln(-2-2i)$ (where ln denotes the natural logarithm) is:

A. $\ln 2$
B. $\ln 2 + \frac{\pi}{2}$
C. $\ln\sqrt{5} + \frac{5}{4}\pi i$
D. $\ln\sqrt{8} - \frac{3\pi i}{4}$

42. The matrix M which diagonalizes the matrix $H = \begin{pmatrix} 3 & i \\ -i & 3 \end{pmatrix}$ is given by

A. $\begin{pmatrix} 1 & i \\ -i & 1 \end{pmatrix}$
B. $\begin{pmatrix} 1/\sqrt{2} & i/\sqrt{2} \\ -i/\sqrt{2} & 1/\sqrt{2} \end{pmatrix}$
C. $\begin{pmatrix} i/\sqrt{2} & -i/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix}$
D. $\begin{pmatrix} i/\sqrt{2} & 1/\sqrt{2} \\ -1/\sqrt{2} & i/\sqrt{2} \end{pmatrix}$

43. The Laplace transform $\mathcal{L}(s)$ of the function $f(t) = \sin^2 t$ is

A. $\dfrac{2}{s(s^2+4)}$

B. $\dfrac{1}{s^2+4}$

C. $2\sin(s)\cos(s)$

D. $\dfrac{2}{s^2+4} + \dfrac{1}{s}$

44. A box contains 10 screws needed in a certain order for assembling a product. Out of these, 6 are alike right handed screws, and the remaining 4 are alike left-landed screws. Now, if the 6 right-handed screws are needed first and the 4 left-handed screws are needed later, the probability that random drawing yields the screws in the desired order is:

A. (10-4)!(10-6)!

B. $\dfrac{6!4!}{10!}$

C. $\dfrac{6!}{10!} + \dfrac{4!}{10!}$

D. $\dfrac{10!}{6!4!}$

45. The function $\cos z + \cot z$ has poles at $z = n\pi$ where

A. $n = 0$ only

B. n is an odd integer only

C. n is an even integer only

D. n is any integer

46. The set $\{0, 1\}$ forms a group under addition, modulo 2. The inverses of elements 0 and 1 are respectively

A. 0, 1

B. 1, 0

C. 0, 0

D. 1, 1

47. The function $f(\theta) = \cos(3\theta^2)$, with θ real, is

A. a periodic function of θ with period $2\pi/3$

B. a periodic function of θ with period $(2\pi)^2/3$

C. a periodic function of θ with period $12\pi^2$

D. not a periodic function of θ

48. The error function is defined as

$$erf(x) = \frac{2}{\sqrt{\pi}} \int_o^x e^{-t^2} dt.$$

Which of the following statements is correct?

A. $erf(x)$ is an even function

B. $erf(x)$ is an odd function

C. $erf(0) = 1$

D. $erf(\infty) = 1$

49. Consider three operators X, Y, Z on a three dimensional complex vector space defined by

$$X(a, b, c) = (a + b, b + c, c + a + 1)$$
$$Y(a, b, c) = (a + b, b + c, c + a)$$
$$Z(a, b, c) = (|a + b|, |b + c|, |c + a|)$$

A. all the three operators X, Y, Z are linear operators

B. only X is a linear operator

C. only Y is a linear operator

D. only Z is a linear operator

50. The function given by

$$f(x) = \begin{cases} ax + 1, & x \leq \pi/2 \\ \sin x + b, & x > \pi/2 \end{cases}$$

is continuous if

A. $a = \sin b$

B. $a = \frac{2b}{\pi}$

C. $b = \frac{\pi a}{2}$

D. $b = \sin a$

51. Roots of an equation $f(x) = (x + \cos x)(x - \cos x)$ can be found as an intersection of two graphs. If one of the graphs is $f_1(x) = x^2$, then the other graph is

A. $\cos x$

B. $-2 \cos x$

C. $\cos^2 x$

D. $- \cos^2 x$

52. The transformation $(q,p) \longrightarrow (Q,P)$ is given to be canonical where $Q = \alpha\sqrt{q}\,e^t \cos p$, $P = \beta\sqrt{2q}\,e^{-t}\sin p$. Then α and β should satisfy

A. $\alpha\beta = 1$

B. $\alpha = \beta = \sqrt{2}$

C. $\alpha\beta = \sqrt{2}$

D. $\alpha\beta = -\sqrt{2}$

53. The Hamiltonian corresponding to the Lagrangian

$$L = \frac{m}{2}\frac{d\vec{r}}{dt} \cdot \frac{d\vec{r}}{dt} + m\frac{d\vec{r}}{dt} \cdot (\vec{\omega} \times \vec{r}) + \frac{m}{2}(\vec{\omega} \times \vec{r})^2$$

is

A. $\dfrac{\vec{p}\cdot\vec{p}}{2m} + \vec{p}\cdot(\vec{\omega}\times\vec{r}),$

B. $\dfrac{\vec{p}\cdot\vec{p}}{2m} + \vec{p}\cdot(\vec{\omega}\times\vec{r}) + \dfrac{m}{2}(\vec{\omega}\times\vec{r})^2,$

C. $\dfrac{\vec{p}\cdot\vec{p}}{2m} + \vec{p}\cdot(\vec{\omega}\times\vec{r}) - \dfrac{m}{2}(\vec{\omega}\times\vec{r})^2,$

D. $\dfrac{\vec{p}\cdot\vec{p}}{2m} + \vec{p}\cdot(\vec{r}\times\vec{\omega}),$

54. For a particle of mass m, moving under the influence of a central potential, the Poisson bracket $\{L_1, \{L_2, L_3\}\}$ is
(here L_1, L_2, and L_3 are the components of the angular momentum)

A. zero

B. $\{L_2, \{L_3, L_1\}\}$

C. $\{L_3, \{L_1, L_2\}\}$

D. $\{L_2, \{L_1, L_3\}\}$

55. A particle of mass m moves in a central force field defined by $\vec{F} = -\dfrac{k\vec{r}}{r^4}$. If E is the total energy supplied to the particle, then its speed is given by

A. $[kr^2 + 2E/m]^2$

B. $[kr^2 + 2Em]^{1/2}$

C. $\left[\dfrac{k}{mr^2} + \dfrac{2E}{m}\right]^{1/2}$

D. $\left(\dfrac{km}{r^2} + \dfrac{2E}{m}\right)^2$

56. The mutual potential energy V of two particles depends on their spatial separation according to

$$V = \frac{a}{r^2} - \frac{b}{r} \qquad ; \qquad a > 0,\ b > 0\ .$$

For what separation, are the particles in static equilibrium?

A. a/b

B. $a/2b$

C. a^2/b

D. $2a/b$

57. If the Hamiltonian for a system in two dimensions is $H = \frac{p_x^2 + p_y^2}{2m} - \frac{k}{r}$. The Poisson bracket $\{p_x, H\}$ is

A. zero

B. p_x/m

C. $-k/r^2$

D. $-kx/r^3$

58. A particle of mass m has uncertainty a in its position. Its kinetic energy must be equal to or greater than

A. $\hbar^2/(2ma)$

B. $\hbar^2/(8ma^2)$

C. $\hbar^2/(2ma^2)$

D. $\hbar^2 a^2/2m$

59. Given that $S_z = \frac{1}{2}\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ and the spin state of a particle being $\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$, subsequent measurement of spin along z-direction would yield the spin value as

A. $\frac{1}{2}$

B. $-\frac{1}{2}$

C. zero

D. half of the measurements yield $\frac{1}{2}$ and the other half yield $-\frac{1}{2}$

60. Consider a state in which the coordinate x has zero uncertainty. Then a possible form of the wave function is

A. $\psi(x) = e^{-\alpha|x|}$
B. $\psi(x) = \delta(x - x_o)$
C. $\psi(x) = e^{-x^2/\sigma^2}$
D. $\psi(x) = e^{ikx}$

61. On the Hilbert space of square integrable functions $\psi(x)$, $\int_{-\infty}^{\infty} dx|\psi(x)|^2 < \infty$, the action of the parity operator P is defined as follows

$$P\psi(x) = \psi(-x)$$

Which of the following statements concerning P is ***not*** correct?

A. P is a hermitian operator
B. P has eigenvalues +1 and -1
C. P is a unitary operator
D. P is a projection operator

62. If $|n>$, with $n = 0, 1, 2, \cdots$ denote the eigenstates of the Hamiltonian H of a one dimensional quantum mechanical harmonic oscillator corresponding to the eigenvalues $(n + \frac{1}{2})\hbar\omega$, then the expectation value of H in the state

$$|\psi>= 1|1> +2|2> -i|3>$$

is

A. $15\hbar\omega$
B. $\frac{15}{6}\hbar\omega$
C. $7\hbar\omega$
D. $\frac{7}{6}\hbar\omega$

63. The density operator corresponding to the state $\frac{1}{\sqrt{2}}[|0> +i|1>]$ is

A. $\frac{1}{2}[|0><0| + |1><1|]$
B. $\frac{1}{2}[|0><0| - |1><1|]$
C. $\frac{1}{2}[|0><0| - i|0><1| + i|1><0| + |1><1|]$
D. $\frac{1}{2}[|0><0| + i|0><1| + i|1><0| - |1><1|]$

64. The ratio of the population of two levels 1 and 2, with the energy E_1 and $E_2 (E_1 < E_2)$ of a classical system in contact with a heat bath at temperature T is given by (where $\epsilon = E_1 - E_2$)

    A. ϵ/kT

    B. $\exp(-\epsilon/kT)$

    C. $\log(\epsilon/kT)$

    D. $\exp(+\epsilon/kT)$

65. Consider N independent particles, each of which can be in two states, with energies 0 and ϵ. The partition function of this system is

    A. $Z = (1 + e^{-\beta\epsilon})^N$

    B. $Z = N(1 + e^{-\beta\epsilon})$

    C. $Z = N\epsilon e^{-\beta\epsilon}$

    D. $Z = \dfrac{e^{-\beta\epsilon}}{[1 + e^{-\beta\epsilon}]}$

66. The entropy S of a system at fixed volume V depends on absolute temperature T as $S = -aT^3 - bT^2 + S_o$ where a, b, S_o are constants. The specific heat C_v is

    A. $-(aT^2 + bT) + \dfrac{S_o}{T}$

    B. $3aT^2 + 2bT$

    C. $-6aT - 2b$

    D. $3aT^3 + 2bT^2$

67. A quantum mechanical system has 3 energy levels, with energies $E_o, 2E_o, 3E_o$ and respective degeneracies 1,2 and 3. The partition function for this system is $\left(\beta = \dfrac{1}{k_B T}\right)$

    A. $e^{-\beta E_o} + e^{-2\beta E_o} + e^{-3\beta E_o}$

    B. $e^{-\beta E_o} + 2e^{-2\beta E_o} + 3e^{-3\beta E_o}$

    C. $\dfrac{e^{-\beta E_o} + 2e^{-\beta E_o} + 3e^{-3\beta E_o}}{6e^{-\beta E_o}}$

    D. $\dfrac{e^{-\beta E_o} + 2e^{-2\beta E_o} + 3e^{-3\beta E_o}}{6e^{-6\beta E_o}}$

68. The probability distribution of points in phase space representing a microcanonical ensemble as a function of energy, is expressible as one of the following

    A. uniform distribution
    B. Gaussian
    C. Lorentzian
    D. Dirac's δ-function

69. Given that ρ is the number density of the particles and λ is the de-Broglie wavelength, the quantum statistics reduces to classical case under the following conditions:

    A. $\rho\lambda^3 \ll 1$
    B. $\rho\lambda^3 = 1$
    C. $\rho\lambda^3 \gg 1$
    D. $\rho = 0$

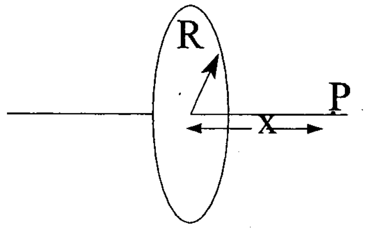
70. Which one of the following vector potentials corresponds to a constant magnetic field in the *positive* z direction

    A. $(A_x, A_y, A_z) = \left(-\frac{y}{2}, \frac{x}{2}, 0\right)$
    B. $(A_x, A_y, A_z) = \left(\frac{y}{2}, -\frac{x}{2}, 0\right)$
    C. $(A_x, A_y, A_z) = (y, x, 0)$
    D. $(A_x, A_y, A_z) = (0, 0, z)$

71. A point charge q is placed outside, at a distance d from the centre, of a grounded conducting sphere of radius r. The magnitude q' and the position d' of the image charge are

    A. $q' = -\frac{a}{d^2}q,\ d' = \frac{a}{d}$
    B. $q' = -\frac{a}{d}q,\ d' = \frac{a^2}{d}$
    C. $q' = -\frac{a^2}{d}q,\ d' = \frac{a}{d}$
    D. $q' = -\frac{a}{d}q,\ d' = \frac{a}{d^2}$

72. A thin, nonconducting ring of radius R, as shown below, has a charge Q uniformly spread out on it. The electric potential at a point P, which is located on the axis of symmetry at a distance x from the centre of the ring, is given by



A. $\dfrac{Q}{4\pi\epsilon_o x}$

B. $\dfrac{Qx}{4\pi\epsilon_o(R^2+x^2)}$

C. $\dfrac{Q}{4\pi\epsilon_o\sqrt{R^2+x^2}}$

D. $\dfrac{Q}{4\pi\epsilon_o(R^2+x^2)^{3/2}}$

73. A rectangular waveguide has a cross-sectional dimension of 3 cm $\times 1$ cm. It is filled with a material of dielectric constant 4. What is its cutoff frequency for the dominant mode?

A. 5 GHz.

B. 1.25 GHz.

C. 15 GHz.

D. 2.5 GHz.

74. A double slit interference arrangement is illuminated with green light of wavelength 546 nm. The two slits are 0.12 mm apart and the screen on which the interference pattern appears is 55 cms away. The angular position of the first minimum, in radians, is approximately

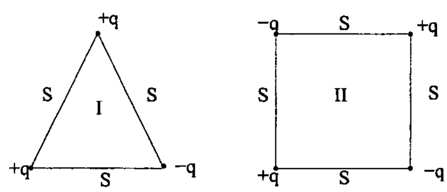
A. 0.023

B. 0.0034

C. 0.0023

D. 0.00023

75. To evaluate the dipole moments in the given configurations, one needs to have the knowledge of the origin



A. for both I and II

B. only for I and not for II

C. only for II and not for I

D. knowledge of origin is irrelevant in both the cases