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ENTRANCE EXAMINATIONS – 2020
Ph.D. Statistics

Hall Ticket No.	

Time : 2 hours
Max. Marks : 70

PART A: 35 Marks
PART B: 35 Marks

Instructions

1. Write your Hall Ticket Number on the OMR Answer Sheet given to you. Also write the Hall Ticket Number in the space provided above.
2. Answers are to be marked on the OMR sheet.
3. Hand over the OMR answer sheet at the end of the examination to the Invigilator.
4. No additional sheets will be provided. Rough work can be done in the question paper itself/space provided at the end of the booklet.
5. Calculators are not allowed.
6. There are a total of 70 questions in **PART-A** and **PART-B** together of **one mark** each.
7. The appropriate answer(s) should be coloured with either a blue or black ball point or a sketch pen. DO NOT USE A PENCIL.
8. This book contains **18 pages** including this page and excluding pages for the rough work. Please check that your paper has all the pages.
9. Given below are the meanings of some symbols that may have appeared in the question paper: $\mathbb{R}$-The set of all real numbers, $E(X)$-Expected value of the random variable X, $V(X)$-Variance of the random variable X, $Cov(X,Y)$-Covariance of the random variables X and Y, $\rho_{X,Y}$ denotes the correlation coefficient between X and Y, iid-independent and identically distributed, pdf-probability density function, $B(n,p)$ and $N(\mu,\sigma^2)$ denote respectively,the Binomial and the Normal distributions with the said parameters. $Rank(A)$ and $det(B)$ mean rank and determinant of the matrices A and B respectively.

Part-A

1. Let $X_1, X_2, ..., X_n$ be i.i. d. random variables with common distribution $U(0,1)$. Let $Y = 2log\left(\frac{1}{\prod_{i=1}^{n} X_i}\right)$. Then $E(Y)$ is

   A. 2n.
   B. 2.
   C. 4n.
   D. 4.

2. Let X be a random variable with $f(x) = 3x^2, 0 < x < 1$. Find the median of X

   A. 0.50.
   B. 0.79.
   C. 0.30.
   D. 0.62.

3. Let X be a random variable which take the values $0, 1, 2, 3, x$, where x is unknown. If each value of X is equally likely and mean of X is 6. Then

   A. $x = 24$.
   B. $x = 4$.
   C. $x = 30$.
   D. $x = 6$.

4. If A B are independent events, then which of the following statement is true

   A. A and B^c are dependent.
   B. A^c and B^c are independent.
   C. $P(A|B) = P(A)$.
   D. A^c and B^c are independent.

5. There are 3 true coins and 1 false coin with head on both sides. A coin is chosen at random and tossed 4 times. If head occurs all the 4 times, what is the probability that the false coin has been chosen and used.

   A. $\frac{16}{19}$.
   B. $\frac{1}{4}$.
   C. $\frac{1}{19}$.
   D. $\frac{15}{19}$.

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6. Two defective tubes get mixed up with 2 good ones. The tubes are tested one by one, until both defectives are found. Define

   I. $P(A_1)$=P(Last defective tube is obtained on second test)

   II. $P(A_2)$=P(Last defective tube is obtained on third test)

   III. $P(A_3)$=P(Last defective tube is obtained on fourth test).

   Arrange them in the order of increasing probability.

A. $P(A_3) > P(A_2) > P(A_1)$.

B. $P(A_3) > P(A_1) > P(A_2)$.

C. $P(A_2) > P(A_1) > P(A_3)$.

D. $P(A_2) > P(A_3) > P(A_1)$.

7. For any random variable X , if $g(X)$ is a convex function, then

A. $E(g(X)) \geq g(E(X))$.

B. $E(g(X)) \leq g(E(X))$.

C. $E(g(X)) = g(E(X))$.

D. None of the above.

8. Match the following inequalities

I. Cauchy Inequality	a. If $x_1, ..., x_n$ and $y_1, ..., y_n$ are two sets of numbers and $(1/p) + (1/q) = 1$ $p > 1$, then $(\sum x_i y_i) \leq (\sum x_i^p)^{1/p} (\sum y_i^p)^{1/q}$
II. Holder inequality	b. If $x_1, ..., x_n$ and $y_1, ..., y_n$ are two sets of numbers then $(\sum x_i y_i)^2 \leq (\sum x_i^2)(\sum y_i^2)$.
III. Minkowski inequality	c. If g is a convex function in (a, b) and X is a random variable whose support is contained in (a, b) and has the finite expectation then $g(E(X)) \leq E(g(X))$.
IV. Jensen's inequality	d. If $x_1, ..., x_n$ and $y_1, ..., y_n$ are two set of non-negative real numbers and $p \geq 1$, then $(\sum (x_i + y_i)^p)^{1/p} \leq (\sum x_i^p)^{1/p} + (\sum y_i^p)^{1/p}$.

A. I-b, II-a, III-d, IV-c.

B. I-d, II-c, III-a, IV-b.

C. I-c, II-a, III-d, IV-b.

D. I-b, II-a, III-c, IV-d.

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9. What percentage of the data do you expect to lie beyond the outside bars of the Boxplot if the data are $N(0, \sigma^2)$ with known variance σ^2

A. 0.7%.

B. 0.5%.

C. 0.3%.

D. 0.4%.

10. For a simple Null hypothesis against a simple alternate hypothesis based on a sample, Neymann Pearson lemma

A. gives a test of a given size and desired power.

B. gives a test with the highest $p-$ value.

C. gives a test of a given size with the highest power.

D. gives a test of least size.

11. The probability that in infinitely many throws of a coin whose probability of heads is 1/3, heads will show up in two consecutive throws infinitely often is

A. 0.

B. 1/9.

C. 1.

D. 1/2.

12. The random variables X_1 and X_2 are independent which are degenerate at -1 and 1 respectively, that is the distribution function of X_1 is $F_1(x) = \begin{cases} 0 & x < -1 \\ 1 & x \geq -1 \end{cases}$, the random variable $X_1 + X_2$

A. has Cauchy distribution.

B. is degenerate at 0.

C. is degenerate at 2.

D. has standard normal distribution.

13. $X \sim exp(\lambda)$, $Y = \lfloor X \rfloor$ where $\lfloor u \rfloor$ = the largest integer less than or equal to u, $\forall u \in \mathbb{R}$. Then,

A. Y is another continuous random variable.

B. $Y \sim P(\lambda)$.

C. Y has Geometric distribution.

D. $Y \sim B(1 - e^{-\lambda})$.

14. Bag 1 has 60 balls, 20 each of them numbered 0,1 and 2. Bag 2 also has 60 balls. But 25 each are numbered 0 and 1 and 10 are numbered 2. A ball each is drawn from Bags 1 and 2 and the numbers on the balls are X and $Y2$ respectively. Then

I. $P(X > 0) > P(Y > 0)$.
II. $P(X > 1) > P(Y > 1)$.
III. $E(X) > E(Y)$.
IV. $E\left(\frac{1}{X+1}\right) > E\left(\frac{1}{Y+1}\right)$.

A. Only I is not true.
B. Only II is not true.
C. Only III is not true.
D. Only IV is not true.

15. In order for the Poisson to give "good" approximate values for binomial probabilities we must have the condition(s) that:

A. the population size is large relative to the sample size.
B. the sample size is large.
C. the probability, p, is small and the sample size is large.
D. the probability, p, is close to .5 and the sample size is large.

16. A national consumer magazine reported that the correlation between car weight and car reliability is -0.30. They also reported that the correlation between car weight and annual maintenance cost is 0.20. Which of the following statements are likely to be true?

I. Heavier cars tend to be less reliable.
II. Heavier cars tend to cost more to maintain.
III. Car weight is related more strongly to reliability than to maintenance cost.

A. I and II.
B. I and III.
C. II and III.
D. I, II and III.

17. Which of the following statements are always true?

I. The P-value is greater than the significance level.
II. The P-value is computed from the significance level.
III. The P-value is a test statistic.
IV. The P-value is a probability.

A. Only I.
B. Only II.
C. Only III.
D. Only IV.

18. Which of the following statements are true?

I. Random sampling is a good way to reduce response bias.
II. To guard against bias from under-coverage, use a convenience sample.
III. To guard against non-response bias, use a mail-in survey.

A. Only I.
B. Only II.
C. Only III.
D. None of the above.

19. Which of the following statements are true?

I. A sample survey is an example of an experimental study.
II. An observational study requires fewer resources than an experiment.
III. The best method for investigating causal relationships is an observational study.

A. Only I.
B. Only II.
C. Only III.
D. None of the above.

20. Which of the following statements is true.

I. When the margin of error is small, the confidence level is high.
II. When the margin of error is small, the confidence level is low.
III. A confidence interval is a type of point estimate.
IV. A population mean is an example of a point estimate.

A. Only I and II.
B. Only III.
C. Only IV.
D. None of the above.

21. Which type of bias occurs because we do not obtain complete information about a population?

A. Non-response bias.
B. Sampling bias.
C. Response bias.
D. None of the above.

22. Consider the following statements.

I. We do not need to randomize if our sample size is sufficiently large.
II. A large sample size always ensures that our sample is representative of the population.
III. If all other things are equal, we need a larger sample size for a larger population.
IV. In a properly chosen sample, an estimate will be less variable with a large sample size and hence more precise.
V. Randomization ensures that we get precise and accurate estimates.

Which of the above statements are CORRECT?

A. Only II and III.
B. Only I and III.
C. Only IV and V.
D. Only IV.

23. Match the following:

I. Gamma Distribution	a. Mean: 4, Variance: 2.4.
II. Binomial distribution	b. Mean: 4, Variance: 4.
III. Poisson distribution	c. Mean: 4, Variance: 6.

A. I-a, II-b, III-c.
B. I-b, II-c, III-a.
C. I-c, II-a, III-b.
D. Incomplete options.

24. Match the following:

I. Markov Inequality	a. $P(X > t) \leq E(X)/t \; X > 0, \; E(\lvert X \rvert) < \infty$
II. Chebyshev's Inequality	b. $E(e^{tX}) \leq e^{t\mu} e^{t^2(b-a)^2/8}$ if $a < X < b$
III. Hoeffding's Inequality	c. $P(\lvert X-\mu \rvert \geq t) \leq \frac{\sigma^2}{t^2}$, $\mu = E(X)$, $\sigma^2 = Var(X)$

A. I-a, II-c, III-b.
B. I-b, II-c, III-a.
C. I-c, II-a, III-b.
D. Incomplete options.

25. Order the following landmark events in statistics in their chronological order

I. R. A. Fisher's Design of Experiments.
II. Student's t-distribution for the mean of small samples published.
III. Thomas Bayes proves Bayes' theorem.
IV. Gauss predicts the orbit of Ceres using a line of best fit.

A. I-II-III-IV.
B. III-IV-II-I.
C. IV-III-II-I.
D. II-I-III-IV.

26. Order the following infinite series in increasing order

I. $1-(1/2)+(1/3)-(1/4)+\dots$
II. $1+(1/4)+(1/9)+(1/16)+\dots$
III. $1-(1/3)+(1/5)-(1/7)+\dots$
IV. $1+1+(1/2!)+(1/3!)+(1/4!)+\dots$

A. I-IV-II-III.
B. II-I-IV-III.
C. III-IV-II-I.
D. I-III-II-IV.

27. The average of four consecutive even numbers is 27. Then the largest of these numbers is

A. 22.
B. 8.
C. 30.
D. 28.

28. The last two digits of $2^{12n}-6^{4n}$ when n is any positive integer are

A. 36.
B. 64.
C. 00.
D. 10.

29. Which of the following is an even function

A. $|x^2|-5x$.
B. x^4-x^5.
C. $e^{2x}+e^{-2x}$.
D. $e^{-x}-e^{x}$.

30. If $[x]$ denotes the greatest integer $\leq x$, then

$$\left[\frac{1}{3}\right]+\left[\frac{1}{3}+\frac{1}{99}\right]+\left[\frac{1}{3}+\frac{2}{99}\right]+\dots+\left[\frac{1}{3}+\frac{98}{99}\right]=$$

A. 32.
B. 33.
C. 42.
D. 62.

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31. The number of ways in which the letters of the word VALEDICTORY be arranged, so that all the vowels are adjacent to each other, is

A. 8!.

B. $\binom{8}{4}$.

C. $8! \times 4!$.

D. 8.

32. If X is a random variable, then

A. $2E(X^2) = E(X)$.

B. $E(X^2) \geq (E(X))^2$.

C. $E(X^2) \leq (E(X))^2$.

D. None of the above.

33. The greatest number of 5 digits, that will give us a remainder of 5, when divided by 8 and 9 respectively is

A. 99931.

B. 99941.

C. 99725.

D. 99936.

34. Let A be a 4×4 matrix with real entries such that sum of entries in each row of A is 1. Then sum of all entries in A^3 is

A. 3.

B. 4.

C. 16.

D. 5.

35. Let A be a 3×3 matrix with real entries such that $\det(A) = 6$ and trace of A is 0. If $\det(A + I) = 0$, where I denotes 3×3 identity matrix, then the eigen values of A are

A. -1,2,3.

B. -1,2,-3.

C. 1,2,-3.

D. -1,-2,3.

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Part-B

36. A parallel system is the one that functions as long as at least one component of it functions. A particular parallel system is composed of 4 independent components, each of which has a lifetime of exponential distribution with parameter λ. Let Y denote the lifetime of the system, which is the maximum of individual lifetime. Then $P(Y \leq y)$ is

A. $(1-e^{-\frac{y}{\lambda}})^4$.
B. $1-e^{-\frac{y}{\lambda}}$.
C. $(e^{-\frac{y}{\lambda}})^4$.
D. $(1-e^{-\frac{y}{\lambda}}))^2$.

37. Let $X_1, X_2, ..., X_{2n}$ be i.i.d. Normal (0,1) random variables. Define

$$U_n = \left(\frac{X_1}{X_2} + \frac{X_3}{X_4} + ... + \frac{X_{2n-1}}{X_{2n}}\right)$$

$$V_n = {X_1}^2 + {X_2}^2 + ... + {X_n}^2$$

. Then the limiting distribution of $Z_n = \frac{U_n}{V_n}$ is

A. Normal(0,1).
B. Cauchy(0,1).
C. $\chi^2(1)$.
D. Uniform(0,1).

38. Let $X_1, X_2, ..., X_n$ be i.i.d $b(1,p)$ random variables and T is a complete sufficient statistic for p. Then UMVUE of $p(1-p)$

A. $\frac{nT-T^2}{n(n-1)}$.
B. $\frac{nT}{n(n-1)}$.
C. $\frac{n-1}{n}T$.
D. T.

39. Let $P(s)$ be the probability generating function of the random variable X. Then probability generating function of $2X+1$

A. $P(2s+1)$.
B. $2P(s)+1$.
C. $sP(s^2)$.
D. $P(s^2+1)$.

40. A sample of size 1 is taken from an exponential distribution with parameter θ. If $X > 2$ is the critical region for testing $H_0 : \theta = 1$ against $H_1 : \theta > 1$. Then the probability of type 1 error is,

A. e^2.

B. e^{-2}.

C. $1 - e^2$.

D. $1 - e^{-2}$.

41. Systematic sampling is more efficient than SRSWOR when

A. $\rho < -\frac{1}{nk-1}$.

B. $\rho = \frac{1}{nk-1}$.

C. $\rho > \frac{1}{nk-1}$.

D. $\rho < \frac{1}{nk-1}$.

42. Let $X_1, X_2, \ldots$ be independent random variables with probability mass function is given by $P(X_n = \pm 1) = \frac{1}{2}, n = 1, 2, 3, \ldots$. Let $Z_n = \sum_{j=1}^{n} \frac{X_j}{2^j}$. Then $Z_n \xrightarrow{L} Z$ where Z is

A. U(0,1).

B. U(-1,1).

C. N(0,1).

D. $\chi^2(1)$.

43. Consider the following pairs

Concept	distribution
I. $\frac{1}{2} + \frac{1}{2}e^t$	a. Poisson(2)
II. $e^{2(e^t-1)}$	b. Exponential (1)
III. $\frac{1}{1-t}$	c. Bernoulli(1/2)

A. I-b, II-c, III-a.

B. I-a, II-b, III-a.

C. I-b, II-a, III-c.

D. I-c, II-a, III-b.

44. In a BIBD with t treatments in b blocks of k plots each and r replication. Then which of the following is not true

A. $rt = bk$.

B. $b \geq t$.

C. $r \geq k$.

D. $b \leq (r + t - k)$.

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45. Let $X_n, n > 0$ be a Markov chain with three states $\{0,1,2\}$ and transition probability matrix is
$\begin{bmatrix} 0.7 & 0.2 & 0.1 \\ 0 & 0.6 & 0.4 \\ 0.5 & 0 & 0.5 \end{bmatrix}$. The initial probabilities are $P[X_0 = i] = \frac{1}{3}, i = 1,2,3$. Then $P(X_2 = 1, X_1 = 2 | X_0 = 0) =$

A. 0.7.

B. 0.0.

C. 0.1.

D. 0.5.

46. Let $\mathcal{U}_g = \{T : E_\theta(T) = g(\theta), \forall\theta \in \Theta\}$ be the class of all unbiased estimator of $g(\theta)$. Then, the possible number of elements of the set $\mathcal{U}_g$

I. is 0. II. is 1. III. are infinitely many. IV. are finitely many.

A. Only I, III and IV are true

B. Only II, III and IV are true

C. Only I, II and IV are true

D. Only I, II and III are true

47. Let $X_1, ..., X_n$ be a sample from $U(\theta, \theta+1), -\infty < \theta < \infty$. Which are the following statements are correct.

I. Parameter is one dimensional and minimal sufficient statistics is two dimensional.

II. Ancillary statistic is function of minimal sufficient statistics.

III. The family is not complete.

IV. When can use Lehmann Scheffe theorem to obtain UMVUE of θ.

A. All of them.

B. Only I and III.

C. Only II and IV.

D. Only I, II and III.

48. Which of the following statements are true.

I. If X and Y independent standard normal then $\frac{X}{Y}$ follows Cauchy distribution.

II. If X and Y independent standard normal then $\frac{X}{|Y|}$ follows Cauchy distribution.

III. If X follows Cauchy distribution then $\frac{1}{X}$ follows Cauchy distribution.

IV. If $X_1, ..., X_n$ are random sample from Cauchy distribution then sample mean also follows Cauchy distribution with same parameter.

A. All of them.

B. Only I and III.

C. Only II and IV.

D. Only IV.

49. Match the following regression methods by minimizing the sum of squares of.

I. the vertical distance between the observations and line

II. the difference between observation and the line in the horizontal direction

III. the perpendicular distance between the observation and the line

IV. the area of rectangles defined between the observed data point and the nearest point on the l

A. I-b, II-d, III-a, IV-c.

B. I-a, II-d, III-c, IV-b.

C. I-c, II-a, III-d, IV-b.

D. I-b, II-a, III-d, IV-c.

50. If $F_n(x)$ is a sequence of probability distribution function converging completely to $F(x)$ and $\phi_n(t)$ is the corresponding sequence of characteristic function, then, $\phi_n t$ will converge to the characteristic function of F. The above is the statement of

A. Continuity Theorem.

B. Convergence in Distribution.

C. Almost sure convergence.

D. Convergence in probability.

51. Order the following convergence theorems according to stronger ones.

I. Almost sure convergence

II. Convergence in probability

III. Convergence in Moments

IV. Convergence in distribution

A. III-I-II-IV.

B. II-IV-III-I.

C. II-III-IV-I.

D. III-II-IV-I.

52. Let $Y_1, ..., Y_n$ be i.i.d. with pdf or pmf $f(y|\theta)$. The order of setups to required to find maximum likelihood estimator (MLE) of $g(\theta)$ are:

I. Find the derivative $\frac{d}{d\theta} \log L(\theta)$, set the derivative equal to zero and solve for θ.

II. Find likelihood function $L(\theta) = \prod_{i=1}^{n} f(y_i|\theta)$ and then find the log likelihood $\log L(\theta)$.

III. Apply invariance principle: If $\hat{\theta}$ is the MLE of θ, then $g(\hat{\theta})$ is the MLE of $g(\theta)$

IV. Show that $\hat{\theta}$ is the MLE by showing that $\hat{\theta}$ is the global maximimzer of $\log L(\theta)$. Often done by showing second derivative evaluated at $\hat{\theta}$ is negative.

A. II-I-IV-III.

B. II-IV-III-I.

C. II-III-IV-I.

D. III-II-IV-I.

53. Let $X_1, ..., X_n$ be a random sample from pmf/pdf $f(x|\theta)$. The order of steps required to find the UMVUE of θ are :

I. Find the minimum sufficient statistics T of θ.

II. Find the simple unbiased estimator conditional on T.

III. Apply Lehmann-Scheffe theorem.

IV. Show that the statistics T is complete.

A. I-II-IV-III.

B. II-IV-III-I.

C. II-III-IV-I.

D. III-II-IV-I.

54. Look at the columns below and match the entries:

I. Weak Law of Large Numbers	a. Convex functions
II. Strong law of Large Numbers	b. Convergence in Law
III. Central Limit Theorem	c. Convergence in Probability
IV. Jensen's Inequality	d. Almost sure Convergence

A. I-b, II-a, III-d, IV-c.

B. I-a, II-c, III-b, IV-d.

C. I-c, II-d, III-b, IV-a.

D. I-d, II-b, III-a, IV-c.

55. $\underset{\sim}{X} = \begin{pmatrix} X_1 \\ X_2 \\ X_3 \end{pmatrix} \sim N_3\left(\begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}, \begin{pmatrix} 4 & 2 & 2 \\ 2 & 6 & 3 \\ 2 & 3 & 4 \end{pmatrix}\right)$ $E(X_3|X_1 + X_2 = 5)$ and $V(X_3|X_1 + X_2 = 15)$ are equal to

A. $\frac{31}{7}$ and $\frac{171}{7}$ respectively.

B. $\frac{171}{7}$ and $\frac{31}{7}$ respectively.

C. $\frac{171}{7}$ and $\frac{171}{7}$ respectively.

D. $\frac{31}{7}$ and $\frac{31}{7}$ respectively.

56. Match the following columns regarding main and interaction effects in a 2^3 factorial design.

I.Main effect of factor A	a. $abc - bc - ab + b + ac - c - a + (1)$
II.Interaction effect of factors B and C	b. $abc - bc - ac + c - ab + b + a - (1)$
III.Interaction effect of all the 3 factors A, B and C.	c. $abc - bc + ac - c + ab - b + a - (1)$
IV.Interaction effect of A and C.	d. $abc - ac - ab + a + bc - b - c + (1)$

A. I-a, II-c, III-b, IV-d.

B. I-c, II-b, III-a, IV-d.

C. I-c, II-d, III-b, IV-a.

D. I-b, II-a, III-d, IV-c.

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57. Which of the following is a $BIBD$ for the 4 treatments T_1, T_2, T_3 and T_4?

I.

Block 1	Block 2	Block 3	Block 4
T_1, T_3, T_4	T_1, T_2, T_4	T_1, T_3, T_4	T_1, T_2, T_4

II.

Block 1	Block 2	Block 3	Block 4
T_1, T_2, T_3	T_2, T_3, T_4	T_1, T_3, T_4	T_2, T_3, T_4

III.

Block 1	Block 2	Block 3	Block 4
T_2, T_3, T_4	T_1, T_2, T_4	T_1, T_3, T_4	T_1, T_3, T_4

IV.

Block 1	Block 2	Block 3	Block 4
T_1, T_3, T_4	T_2, T_3, T_4	T_1, T_2, T_3	T_1, T_2, T_4

A. All of them.

B. Only I and III.

C. Only II and IV.

D. Only IV.

58. $X_1, X_2, \ldots$ are *iid* uniformly distributed random variables over $(0,1)$, let $S_n = X_1 + \ldots + X_n$, look at the following statements. and find out which all are correct.

I. $\frac{S_n}{n} \longrightarrow \frac{1}{2}$ almost surely.

II. $\{S_n < \infty\}$ almost surely

III. $\sqrt{12n}(\frac{S_n}{n} - \frac{1}{2}) \Rightarrow Z \sim N(0,1)$.

IV. The expected value of S_n exists.

A. Only I and III.

B. Only I, II and III.

C. Only II and III.

D. All of them.

59. $\log 10$ and $\log 5$ are two observations of the exponentially distributed random variable X with mean $\frac{1}{\lambda}$, an unbiased estimate for $\frac{\lambda}{\lambda+1}$ is

A. 0.3.

B. 0.15.

C. $\frac{\log 50}{2}$.

D. $\frac{\log 2}{2}$.

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60. The regression functions of Y on X and X on Y satisfy $E(Y|X) = 4 + 2X$ and $5E(X|Y) = -5+2Y$, so $(E(X), E(Y))$ and the correlation coefficient between X and Y are respectively are

A. $(0,0)$ and 1.

B. $(3,10)$ and $\sqrt{0.8}$.

C. $(5,10)$ and $\sqrt{0.5}$.

D. $(6,3)$ and 0.7.

61. $(X_1,\ldots,X_p)^T = \underset{\sim}{\mathbf{X}} \sim N_p(\underset{\sim}{\mu}, \Sigma)$, then the covariance vector of $X_p - E(X_p|X_1,\ldots,X_{p-1})$ and $(X_1,\ldots,X_{p-1})^T$ is

A. the $(p-1)\times 1$ vector of zeros $\underset{\sim}{0}$.

B. the $(p-1)\times 1$ vector of covariance of X_p with $(X_1,\ldots,X_p)^T$.

C. the $(p-1)\times 1$ vector with every term equal to the conditional variance of X_p given $(X_1,\ldots,X_{p-1})^T$.

D. the $(p-1)\times 1$ vector $(1,\ldots,1)^T$.

62. Consider the following Gauss Markov Model:

$$\begin{aligned} Y_1 &= 2\beta_1 - \beta_2 - \beta_3 + \beta_4 + \epsilon_1 \\ Y_2 &= \beta_1 + 2\beta_2 + \beta_3 + \epsilon_2 \\ Y_3 &= 3\beta_1 + \beta_2 + \beta_4 + \epsilon_3 \qquad ,(\epsilon_1,\ldots,\epsilon_5)^T \sim N_5(\underset{\sim}{0}, \sigma^2\mathbf{I}) \\ Y_4 &= \beta_1 + \beta_3 + \beta_4 + \epsilon_4 \\ Y_5 &= 2\beta_1 + \beta_2 - \beta_3 + \epsilon_5 \end{aligned}$$

Read the statements below and identify the correct ones.

I. β_4 is estimable.

II. $Y_1 + Y_2 - Y_3$ is a linear Zero function.

III. $\beta_1 + \beta_2$ is estimable.

IV. If R_0^2 =Residual sum of squares, $\frac{R_0^2}{\sigma^2} \sim \chi_2^2$ distribution.

A. Only I.

B. Only II, III and IV.

C. Only I, II and IV.

D. Only I and IV.

63. X_1, X_2 are a random sample from the variable X with probability mass function $P(X = x) = p^x(1-p)^{1-x}$, $x = 0, 1$ and $0 < p < 1$ which of the following statements is correct?

A. $X_1 - X_2$ is not a sufficient statistic for p.

B. $(X_1 + X_2)/2$ is not a sufficient statistic for p.

C. $(X_1 + X_2)/2$ is an unbiased estimator for p.

D. X_2 is a sufficient statistic for p.

64. The one step transition Probability matrix of homogeneous Markov Chain whose state space $S = \{1, 2, 3, 4, 5\}$ is

$$\begin{pmatrix} 1/4 & 1/3 & 1/5 & 1/6 & 1/20 \\ 1/5 & 1/4 & 1/4 & 3/10 & 0 \\ 1/6 & 1/6 & 1/3 & 1/6 & 1/6 \\ 2/5 & 1/5 & 0 & 1/5 & 1/5 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

A. This Markov Chain is irreducible.

B. All the states in S are recurrent.

C. The states 1 and 5 communicate with each other and are recurrent.

D. The states $1, 2, 3$ and 4 are transient and 5 is the only recurrent state.

65. Let X and Y be independent and identically distributed random variables with probability distributions given by $P(X = j) = \dfrac{1}{j(j+1)}$, $j = 1, 2, \ldots$ then the value of $P(X = Y)$ is in the interval

A. $(0, 1/4]$.

B. $(1/4, 1/2]$.

C. $(1/2, 3/4]$.

D. $(3/4, 1]$.

66. A coin is tossed 7 times and the outcomes are HTTHHTH, if the probability of head is p, then an unbiased estimate for p^2 is

A. 1/2.

B. 26/49.

C. 5/7.

D. 16/49.

67. Based on a sample of size n from the $N(\mu, 9)$ population to test $H_0 : \mu = \mu_0$ vs $H_1 : \mu > \mu_0$, the most powerful level α test is: reject H_0 if $\overline{X}_n > a$ where $\overline{X}_n$ is the sample mean. Now suppose variance is not 9 but 16. Let n' be the sample size required so that the test: reject H_0 if $\overline{X}_{n'} > a$ is the most powerful level α test. Then n'

A. is equal to n.

B. is equal to $2n$.

C. is more than n but less than $2n$.

D. is less than n.

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68. To study the factors that affect GPA of college students, fifty colleges are selected at random and the GPA scores of one male and one female student is collected from each college. We also classify each college as public or private. Among the following procedures, which would be the most appropriate here?

I. An independent 2 sampled t test on the male and female GPA.
II. A paired t test on the male and female GPA.
III. ANOVA on the four sets of GPAs: males at private college, females at private college, males at public college, females at public college.
IV. Two-way ANOVA using the college status as one factor and gender as the second factor.

A. Only I and III.
B. Only IV and III.
C. Only I, II and III.
D. Only IV.

69. Match the following, regarding modes of convergence for sequence of random variables,

I. Quadratic Mean	a. $F_n(t) \to F(t)$ at continuity points t
II. In probability	b. $E(X_n - X)^2 \to 0$
III. In distribution	c. $P(\lvert X_n - X\rvert > \epsilon) \to 0$ for all $\epsilon > 0$

A. I-a, II-b, III-c.
B. I-b, II-c, III-a.
C. I-c, II-a, III-b.
D. Incomplete options.

70. Suppose in stepwise regression with four regressors in the model we use p-value of two sided t-test is used to exclude variables from a model. Then what would be the desired order of removal of variables if the corresponding p-values for the regressors are

I. 0.0930
II. 0.0497
III. 0.6445
IV. 0.8447

A. I-II-III-IV.
B. IV-III-II-I.
C. II-I-III-IV.
D. IV-III-I-II.

University of Hyderabad

Entrance Examinations - 2020

School/Department/Centre : **Mathematics and Statistics**
Course/Subject : **PhD - Statistics - OR**

Q.No.	Answer	Q.No.	Answer	Q.No.	Answer	~~Q.No.~~	~~Answer~~
1	A	26	D	51	A	~~76~~	---
2	B	27	C	52	A	~~77~~	---
3	A	28	C	53	B	~~78~~	---
4	Benefit to all	29	C	54	C	~~79~~	---
5	A	30	B	55	D	~~80~~	---
6	A	31	C	56	C	~~81~~	---
7	A	32	B	57	D	~~82~~	---
8	A	33	B	58	A	~~83~~	---
9	A	34	B	59	B	~~84~~	---
10	C	35	D	60	B	~~85~~	---
11	C	36	A	61	A	~~86~~	---
12	B	37	B	62	B	~~87~~	---
13	C	38	A	63	C	~~88~~	---
14	D	39	C	64	D	~~89~~	---
15	C	40	B	65	B	~~90~~	---
16	D	41	A	66	B	~~91~~	---
17	D	42	B	67	C	~~92~~	---
18	D	43	D	68	D	~~93~~	---
19	D	44	D	69	B	~~94~~	---
20	D	45	B	70	D	~~95~~	---
21	B	46	D	~~71~~	---	~~96~~	---
22	D	47	D	~~72~~	---	~~97~~	---
23	C	48	A	~~73~~	---	~~98~~	---
24	A	49	Benefit to all	~~74~~	---	~~99~~	---
25	B	50	A	~~75~~	---	~~100~~	---

Signature
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